Digital Signal Processing and its Applications
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Lecture – 24A
Review and Observations of Chebyshev Filter Design

So, warm welcome to the 24th lecture on the subject of Digital Signal Processing and its Applications. We have been discussing the Chebyshev lowpass filter in the previous lecture. And we have already talked about the Butterworth filter in a couple of lectures before that; where we observed some interesting engineering behavior of the Butterworth filter. Namely, if you wish to place more stringent demands, then you need to invest more. This is an observation that we made for the Butterworth filter. Now, we have in the previous lecture being able to come up with an expression for the order of the Chebyshev filter. Let us put down that expression.

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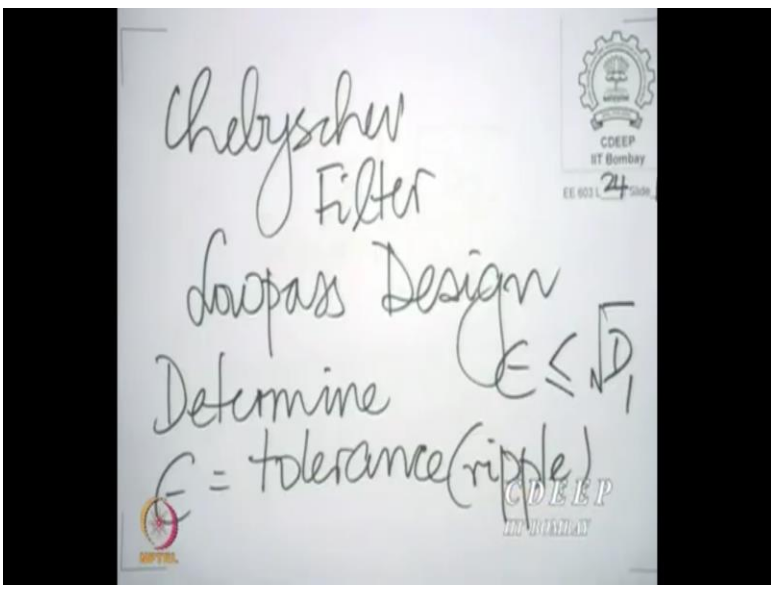

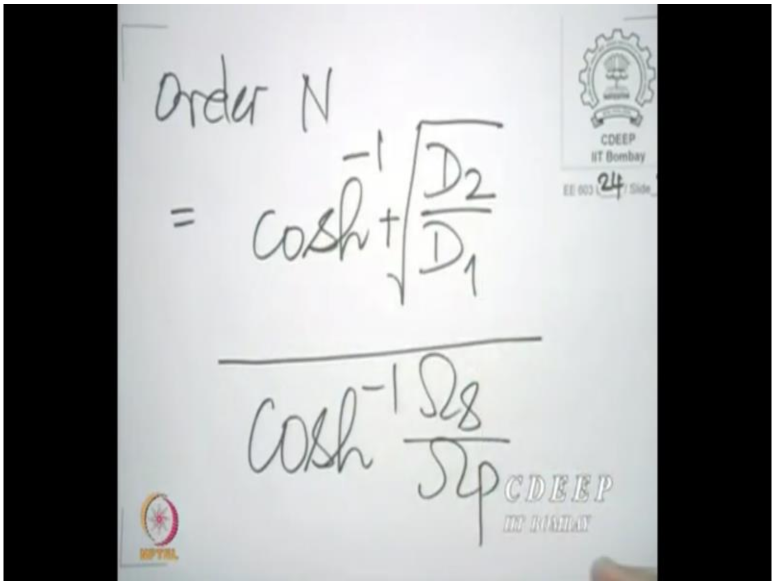

Now, essentially in the lowpass filter design we need to determine epsilon, which is what is called the tolerance parameter or ripple and the order. Which is essentially given by $\cosh^{-1}$, we see we have seen this last time have not we. $[\cosh^{-1} + (\sqrt{D_2 / D_1})]/ [\cosh^{-1} (\Omega_S/\Omega p)]$.

Now, in the tolerance or the ripple epsilon, we saw that epsilon is $\leq$; of course you always want to be small if possible, $\leq \sqrt{D_1}$. But, we also made an observation on why we should choose epsilon $=\sqrt{D_1}$ last time. In fact, we look at this expression. We now make some observations about the behavior of the order with respect to D_2, D_1, Ωs and Ωp; or rather the ratio Ωs and Ωp. Let us review a few ideas here.

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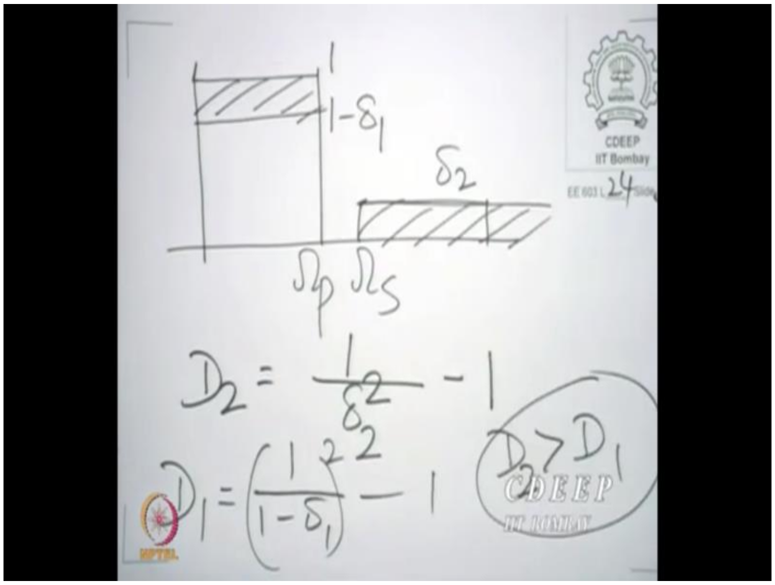

So, you recall that this is the specification with which we are working for the lowpass filter. Of course, this is an analog filter; so you, it goes all the way. And you have Ωp here, you have Ωs; you have $1-\delta_1$ here and 1 there and δ_2 here. And D_2 is $1/(\delta_2)^2 - 1$; and D1 is[$1/(1-\delta_1)]^2 - 1$,. Naturally, $D_2 > D_1$; otherwise it does not make sense. That is because δ_2 is definitely expected to be $<(1-\delta_1)$. And therefore $(1/\delta_2)^2 > [1/(1-\delta_1)]^2$.

That is to be expected; otherwise it does not make sense. You know if the passband tolerance is such that the passband amplitude goes below the stopband. It does not make sense to design the filter. And therefore it is meaningful to say $D_2 > D_1$; that is the least that you can ask. In fact, we expect that it should be reasonably greater; the better the filter, the greater it is. Better in the sense δ_2 goes lower and lower and $(1-\delta_1)$ goes higher and higher. So, let us make these observations. I mean this is not necessarily only for the Chebyshev filter; but for any lowpass filter. In fact, whenever there is a passband and stopband and adjoining; this is true.

So, yes there is a question. So, the question is in this slide should we put it from $(1+\delta_1)$ to $(1-\delta_1)$, or 1 to $(1+\delta_1)$. Now, you know in the Chebyshev filter we are in a position to design it with the upper limit not going beyond 1. So, we are going to put the specification between 1 and $(1-\delta_1)$ rather than $(1+\delta_1)$ and $(1-\delta_1)$; because we can afford to do it here. We know the Chebyshev magnitude can be constrained upwards by 1, and the same is true of the Butterworth magnitude. So, we do not need to go all the way to $(1+\delta_1)$ at all.

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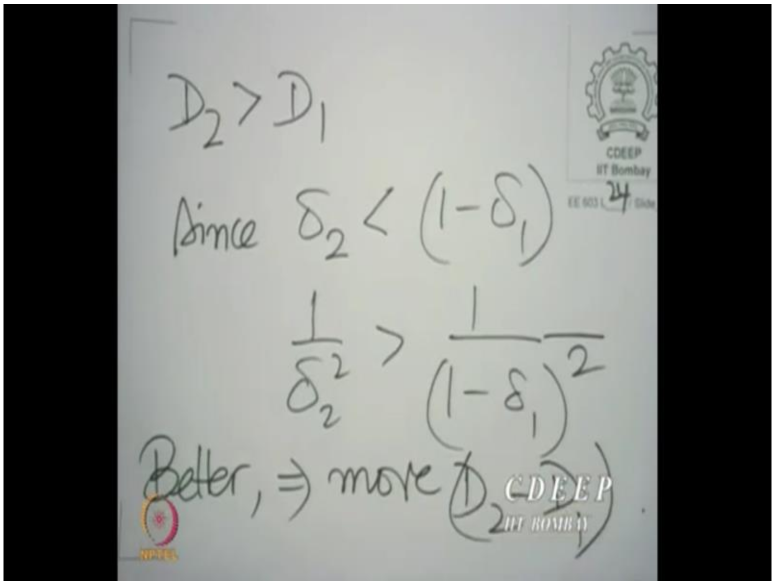

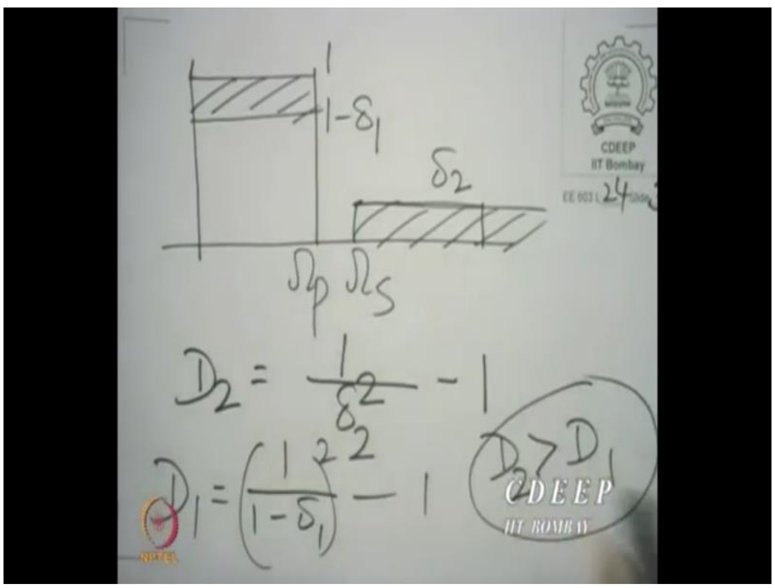

So, the first thing is D_2 is always expected to be > D_1, since $\delta_2 < (1 - \delta_1)$. So, therefore $1/(\delta 2)^2$ $>[1/(1 - \delta_1)]^2$. And better means more $D_2 - D_1$; the better the filter, the more the $D_2 - D_1$. Now, we are stepping into the territory of discrete time processing, where we need to appreciate engineering nuances. Before we talked about synthesis, we were looking at analysis; where we were looking at system properties and so on. So, there were fewer engineering nuances at that point in time.

But, now we are looking at synthesis, and synthesis as we can see is essentially a process of approximation by different approaches; approximation of the ideal. And that is an engineering problem, distinctly an engineering problem; and therefore, we begin to engineering nuances everywhere. Now, also $\Omega_S > \Omega_p$; and that is obvious the stopband edge must be after the passband edge.

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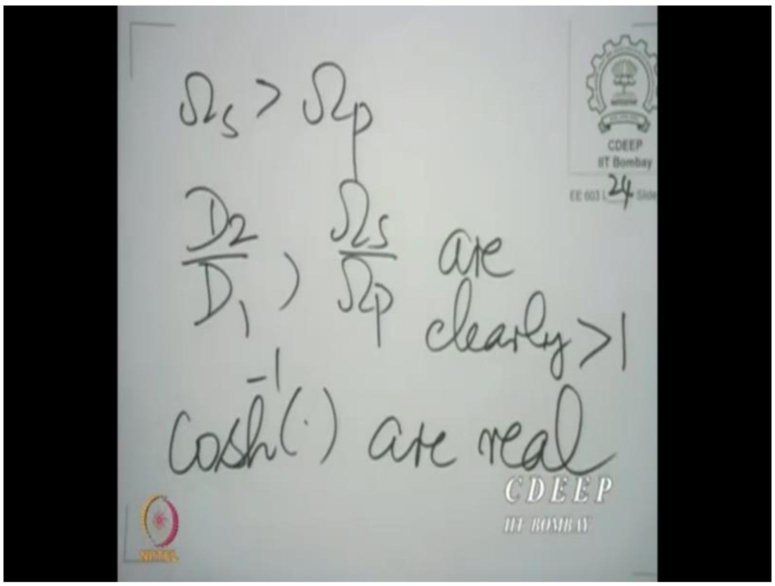

And therefore $\Omega_S > \Omega_p$; and therefore both D_2 / D_1 and $\Omega_S / \Omega_p > 1$. And therefore $\cosh^{-1}$ of both of these quantities are real; $\cosh^{-1}$ for each of these quantities are real. You see because, if you take the $\cosh^{-1}$ of an argument, which < 1; it would turn out to be complex. Right, there is no choice but for it to be complex. It is only for arguments > 1 that you can have the real $\cosh^{-1}$. In fact, let us recall a few properties of the cosh.

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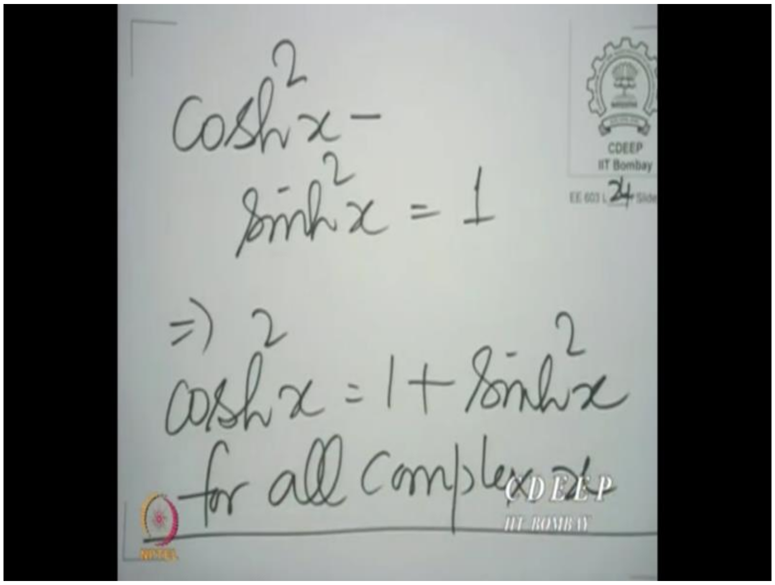

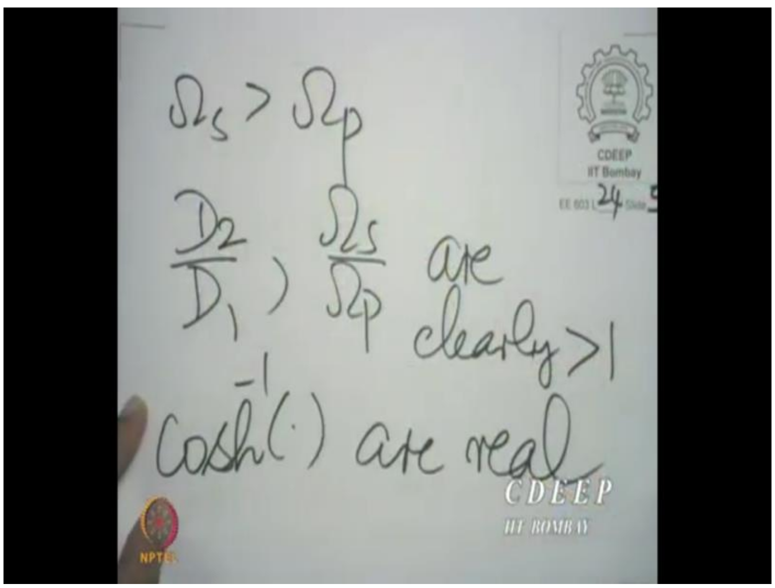

$Cosh^2$ x – $sinh^2$ x = 1; or $cosh^2$ x = 1 + $sinh^2$ x is true for all x, for all complex x in fact. And in particular if you want a real value for x here, $sinh^2$ x would be real for real x. And therefore if you want $cosh^{-1}$ and it is very clear that cos x has to >1. Because, $sinh^2$ x for real x is going to be a quantity, which is positive, non-negative at least. Therefore $cosh^{-1}$ must take an argument >1, if you want the output to be real. And that is what we verified when we had D_2 / D_1 and Ω_S /Ω_p >1 , is that correct.

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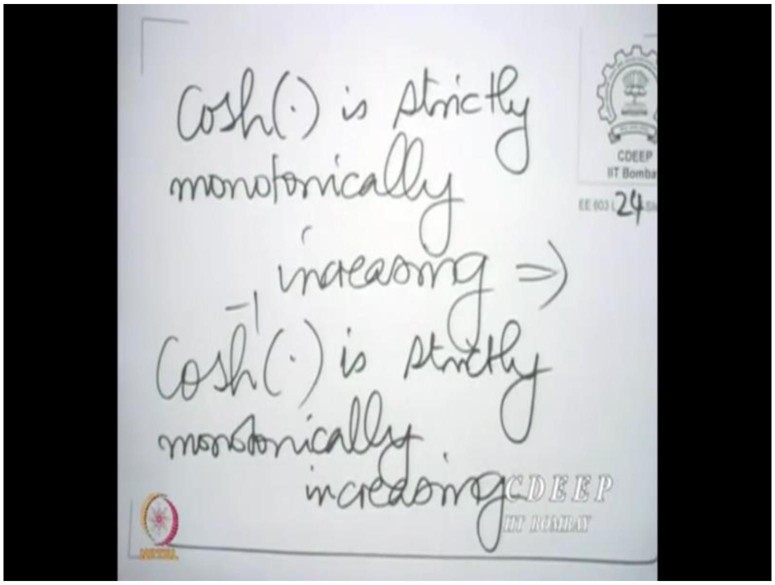

Now, also cosh just like cosh is a monotonically increasing, cosh is monotonically increasing, is strictly monotonically increasing in fact. It follows as a corollary that $\cosh^{-1}$ is also strictly monotonically increasing. This is easy to see because what strictly monotonically increasing, means between 0 and ∞ is that as you increase the argument from 0 to ∞; the cosine, the cosh also increases. So, it means if you are taking the $\cosh^{-1}$ of an argument that is increasing, the answer would also increase. You know the independent variable increasing, strictly increasing means independent variable and the dependent variable increase simultaneously.

And so both the inverse and the function itself must be increasing. And therefore, $\cosh^{-1}$, it is also a strictly increasing function. And now we have some interesting insights into how the order behaves.

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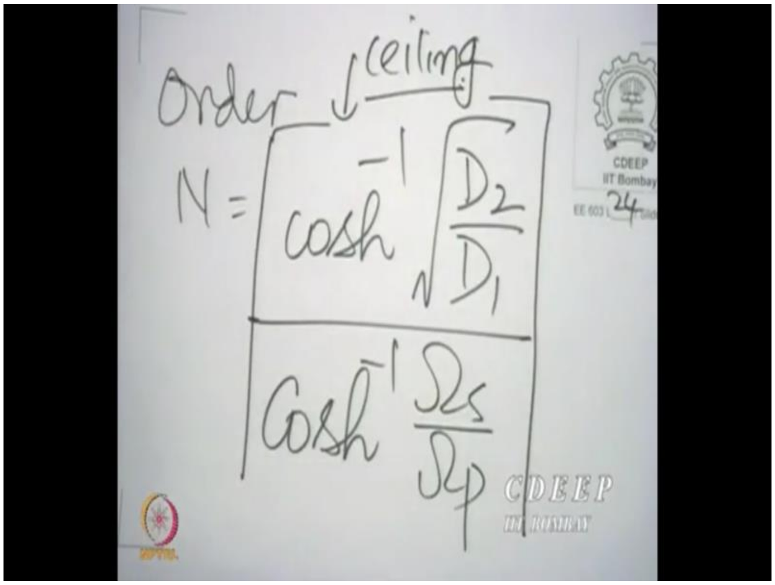

You see the order N, which is given by $\cosh^{-1} \sqrt{D_2/D_1} / \cosh^{-1} (\Omega_S / \Omega_p)$, P would clearly increase, if the numerator increases; and decrease if the denominator increases. Now, let us first look at the denominator fixing the numerator. When would the denominator increase? The denominator will increase if Ω_S / Ω_p is greater. Ω_S / Ω_p being greater means that the stopband is further away from the passband; which means you allowed a wider transition band. So, wider transition band translates to an increasing denominator, which means you are asking less.

And therefore you expect the order to decrease. Of course, one must remember that decrease is always in steps; because the order needs to be an integer. And the order N instead of saying it is equal to this; we should say it is equal to the ceiling of this here. So, ceiling function is a stepped function does not suddenly change; I am sorry does suddenly change. And it does not with a small within within between two integers; the ceiling does not change that is what I mean. So, where there is the possibility of transition from one integer to the other; there these factors do play a role.

So, if you ask for a larger tolerance or if you make the passband wider; I am sorry. If you make the transition band wider, you are in effect asking for less; and the order would tend to go down. Now, on the contrary let us look at the numerator; so numerator would increase if D_2 / D_1 is increasing. Now, D_2 / D_1 would increase as you can see if the filter becomes better, quote unquote better. Better means that either $(\delta_2)^2$ comes down, or $(1 - \delta_1)^2$ goes up; or both of them happen. That means you are asking for a more stringent tolerance in these passband and or the stopband.

In either case even if you fix the stopband, and ask for more from the passband; or if you fix the passband and ask for more from the stopband, you are increasing D_2 / D_1. That means, that the numerator is increasing, and consequently you are likely to be increasing the requirement of order. So, what we saw in the Butterworth filter holds good for the Chebyshev filter too. Ask for more and you have to invest more; so, we must write that down again.

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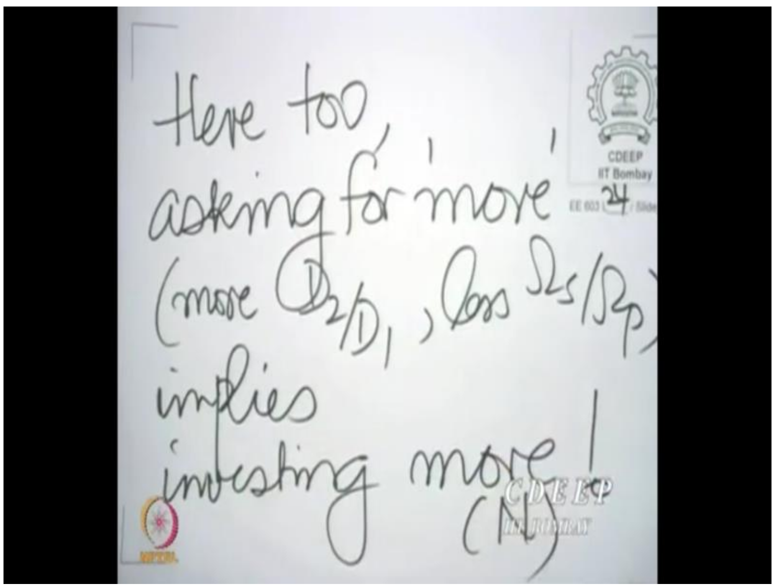

Here too, asking for more; that means more D_2 / D_1, or less Ω_S / Ω_p implies investing more, more N. N is a direct measure of investment; in fact, now we have seen why N is directly a measure of investment. N translates also into the order of the discrete time filter. How many delays you will need, how many multipliers; all that is implied by N. We have seen that in the Butterworth filter. So, it is very clear that N implies the resources required.