Digital Signal Processing & its Applications

Professor Vikram M. Gadre
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture 25 A

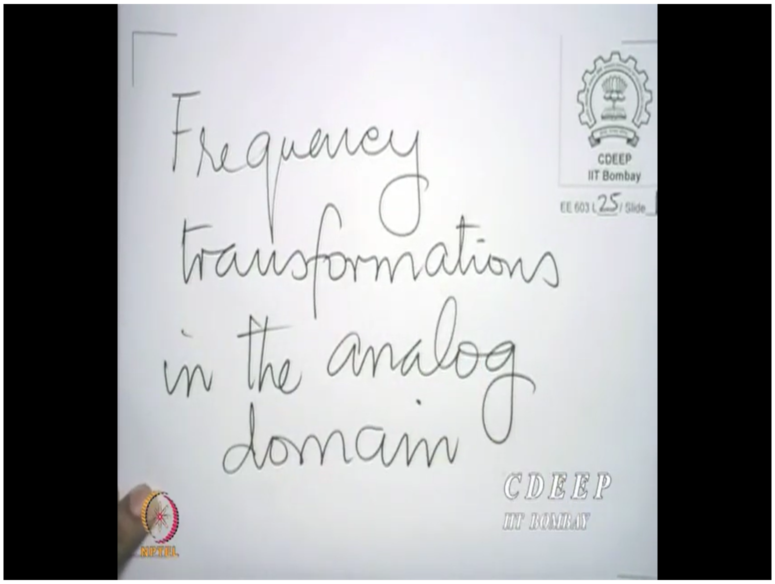
Introduction to Frequency Transformations in the Analog Domain

A warm welcome to the 25th lecture on the subject of Digital Signal Processing and its Applications. We begin with another aspect of infinite impulse response filter design today. So far, we have designed the Butterworth and Chebyshev low pass approximations for the analog domain. Now, we would like to design other kinds of piecewise constant filters of importance. They will be high pass, band pass and band stop.

We have of course assumed that we have brought the specifications into the analog domain already. So, using the bilinear transform, we have put down the specifications of the analog filter that we wish to design, whether it is high pass or band pass or band stop. But now, we wish to get down to actually designing the analog filter of given nature with appropriate nature for the pass band and stop band. By nature, I mean, equiripple or monotonic. So, we want the nature and the type of the filter, that is the kind of bands in the filters to be designed explicitly.

So, therefore, the process that we are going to follow is to again, as we have done before, take advantage of the designs that we have until date and then, use those designs to go further to build other kinds of filters. So, we have already designed Butterworth and Chebyshev pass filters, now we would like to use a transformation on these low pass filters to get filters of other kinds.

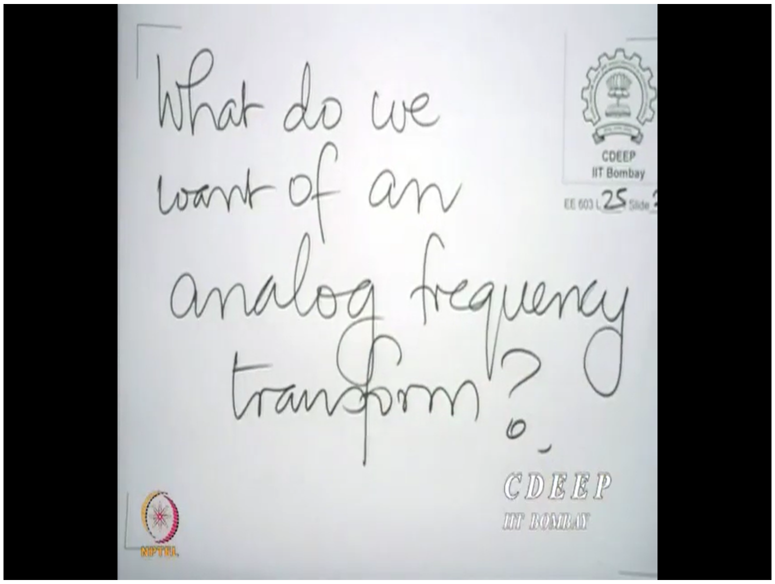
(Refer Slide Time: 02:08)



And therefore, what we are going to do today is to discuss frequency transformations in the analog domain. As the name suggests, frequency transformations essentially refers to a transformation on sinusoidal frequencies. And sinusoidal frequencies come from the imaginary axis in the s-plane.

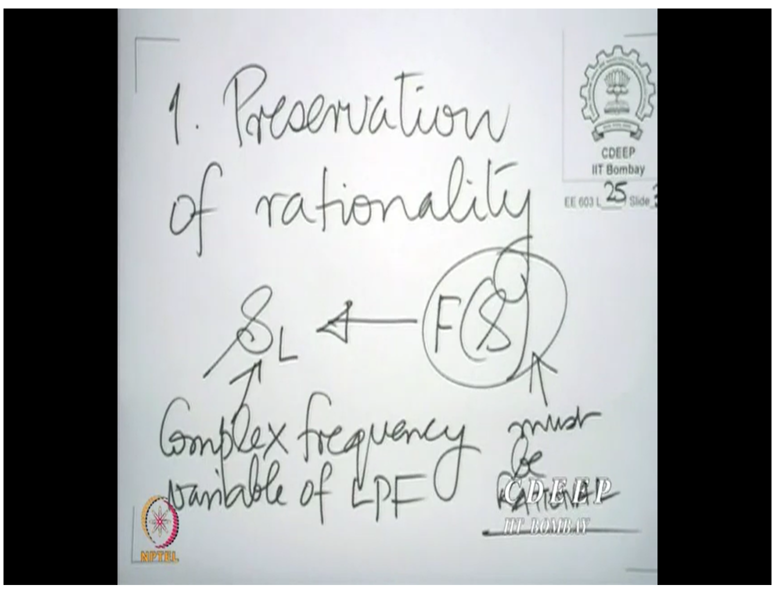
So, what we expect is a transformation that goes from the imaginary axis to the imaginary axis. And of course, the transformation must obey certain requirements. Let us first put down what we mean by a frequency transformation and what we insist that it should obey.

(Refer Slide Time: 03:05)



So, what do we want of an analog frequency transformation? We want three things, once again. As we did when we went from analog to digital or digital to analog.

(Refer Slide Time: 03:39)

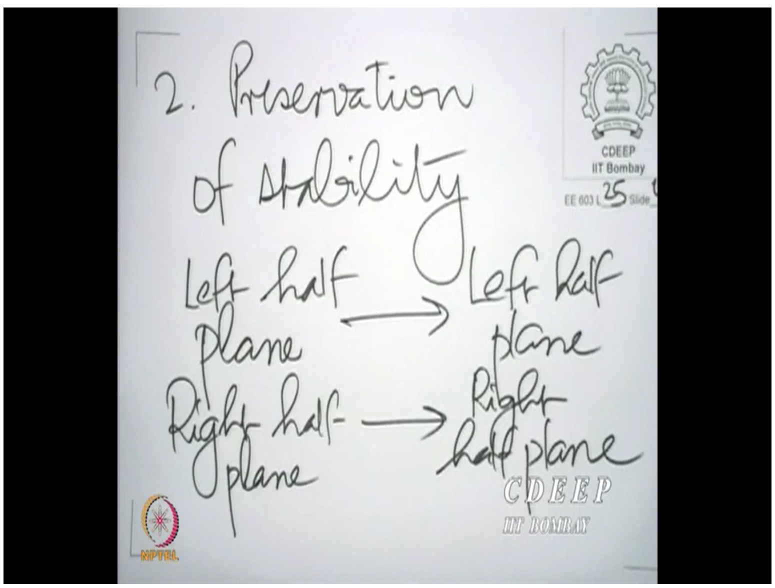


The first thing that we want is preservation of rationality. So, essentially what you are saying is that if you denote the complex frequency variable for the low pass filter by s_L, then you need a transformation on s_L. Now, please note, so far, we have always been just writing s for the complex frequency variable of the analog filter. But now, what we are saying is, we want indeed, to have the complex frequency variable s, the Laplace variable s.

But then, you want to go through and intermediate step. You want to design a low pass filter and you want to replace the complex frequency variable s in the low pass filter by another function of s. So, we need to distinguish between these two variables. The complex frequency variable in the low pass filter shall therefore, henceforth be referred to as s_L, s sub L. And s corresponds to the complex frequency variable in the filter of desired nature.

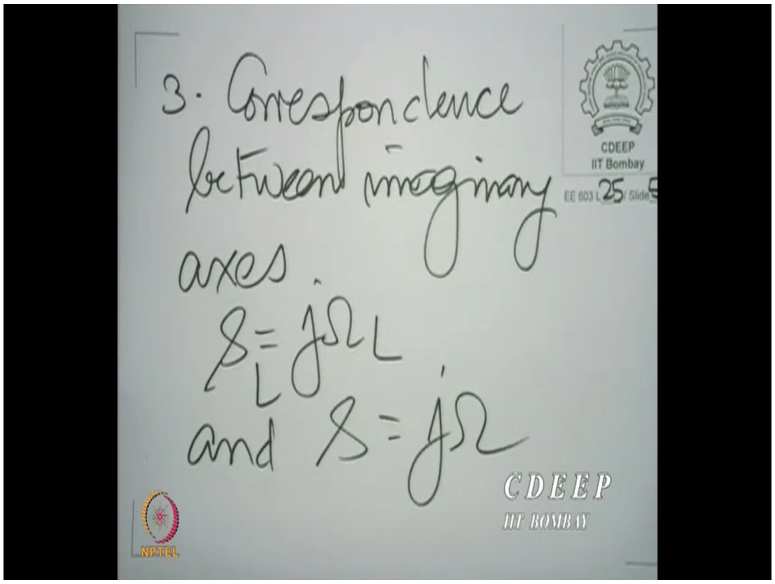
This is what we mean by an analog frequency transformation; replace the complex frequency variable in the low pass filter by an appropriate function of s. And obviously, if you want rationality to be preserved, so if you want the low pass filter which is of course, an analog rational function, to continue to be a rational function after the transformation, the only way is that F(s) itself must be rational. So, F must be rational, there is no choice. Of course, we do not need it to be bilinear or we do not need it to be first degree, it could be any degree, that does not matter. But it has to be rational.

(Refer Slide Time: 06:17)



Secondly, we want it to preserve stability. And that implies a correspondence between the left half plane and the right half plane. You see, it is the left half plane and the right half plane in s, in the complex, in the Laplace variable s. That determines stability or otherwise. So, we need left half plane to left half plane and right half plane to right half plane mapping. That would ensure that if the low pass filter is stable, as it is, then this transformation would not create an unstable analog system due to the rational change.

(Refer Slide Time: 07:29)



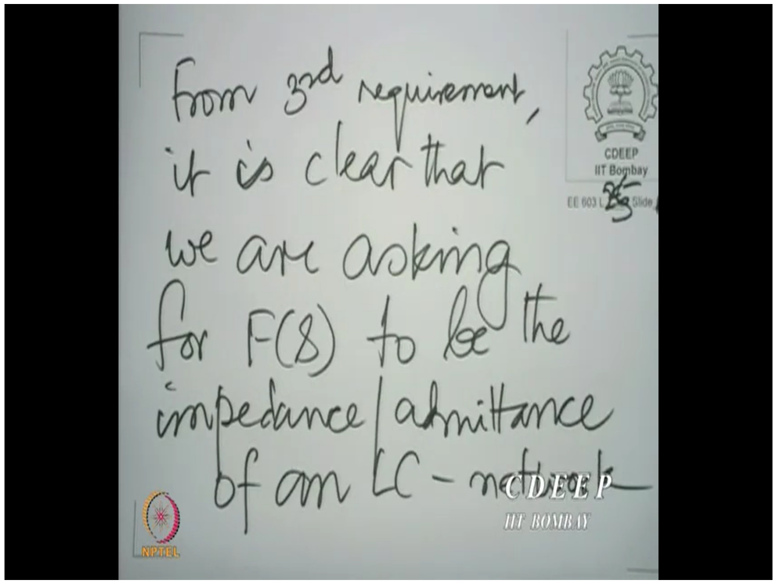
Finally, we need a correspondence between the imaginary axis. After all, we are talking about a sinusoidal frequency transformation. We want the imaginary axis to go to the imaginary axis, otherwise there is no meaning in this transformation.

After all, what we want is that a certain set of specifications on the kind of filter that you want to design should translate into corresponding frequency domain specifications for the low pass filter. So, the absolutely essential thing is that sinusoidal frequencies from the filter of desired nature must go into sinusoidal frequencies of the low pass filter. And that implies that the imaginary axis in the s plane must go on to the imaginary axis in the s_L plane.

Now, we might ask, what is a transformation or what is a set of transformations that meets all these requirements and how do we search for such transformations? Here again, our experience of analog functions should come to our aid. You see, after all, if we just reflect a little bit, what are those analog functions of s?

Let us begin with the third requirement. What are those analog functions of s, which when you substitute $s = j\Omega$, give you imaginary quantities in s_L? You know, it immediately rings a bell if we think of impedance. We might always visualize the function F(s) to be an impedance or an admittance. And if we do that, then it immediately rings a bell if we ask that $s = j\Omega$ should put us onto the imaginary axis. In other words, the impedance must be imaginary or the admittance must be imaginary. We know what kind of networks give us imaginary impedances. Essentially networks that comprise only of capacitors or inductors.

(Refer Slide Time: 10:05)

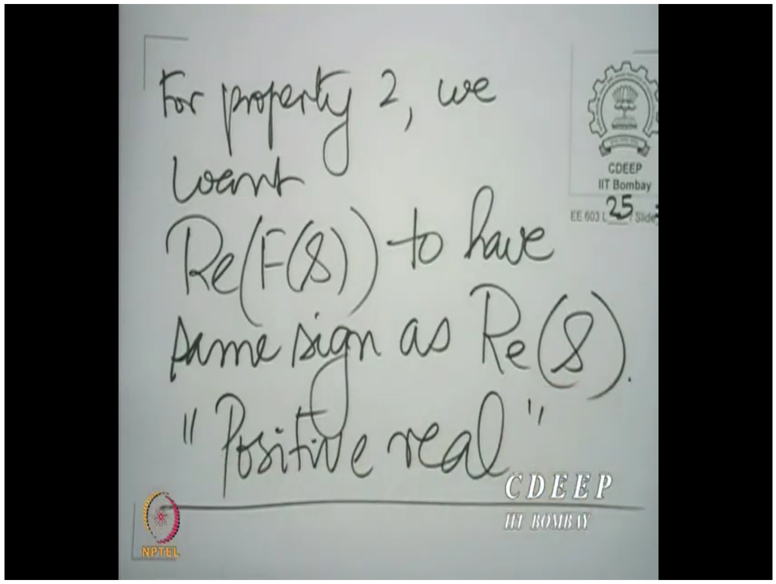


So, the first clue that we get from the third requirement, it is clear that we are asking for F(s) to be the impedance or admittance of an L C network; a network comprising only of inductances and capacitances.

And of course, we might ask whether, if we do indeed choose such L C impedances, they satisfy the other two properties as well. And we are fortunate, actually, that they do not. Now, the property of moving from the left half to the left half and the right half to the right half is actually a very important property of all impedance and admittance functions.

It is called the property of positive realness. Many of us might have encountered this property when we dealt with network theory, network synthesis in particular. But we probably have never looked at it from that perspective.

(Refer Slide Time: 11:38)



What we are saying in effect, for property 2, we want $Re(F(s))$ to have the same sign as $Re(s)$. And that is essentially a so called positive real property of network function. So, even without explicitly verifying this property, if we do indeed use L C networks, we are very likely to be satisfying this implicitly by design. But we shall, when we look at the frequency transformations, explicitly verify this property for each of them.

However, this tells us, this positive real property of impedance or admittance functions, assures us that we are likely to succeed in that endeavor. Finally, we go to the first of the requirements, rationality. Now, rationality is not a problem at all. The impedance or the admittance of L C networks is of course rational, in s, so we do not need to worry about that at all.

So, with that then, we have all the properties satisfied and we can look at the simplest of the three possibilities, we wish to go from low pass to high pass, low pass to band stop, low pass to band pass. And therefore, the simplest of them, as we expect, is low pass to high pass.