Digital Signal Processing & Its Applications

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Lecture No. 04 a

Answering questions from Previous Lectures

Good morning to all of you. And welcome to this fourth lecture on the subject of Digital Signal Processing and Its Applications. We had two lectures where we have not really answer any questions from the class. And therefore, let us begin this lecture by first asking whether you have any questions which need to be addressed.

So, I would request you to raise your hand and ask the question. I will repeat the question and I will answer the question in sequence. Yes. I have one question from here. So, the question is, there was a reference to analytic signals in the class. What exactly are analytic signals? Analytic signals are signals which have continuous derivatives of all orders.

The signal itself is continuous, its derivative is continuous and second derivative is continuous. That means, in some sense, an analytics signal is just about as smooth as you can get. It is the smoothest possible. It is a class of signals, analytic signals form a class of signals, which are as smooth as they can be. They have infinite number of continuous derivatives. Yes, any other questions?

Student: ...

Professor: Okay, that is a very good question. So, the question is, when you sample a signal, our reasoning was that the original spectrum is repeated at every multiple of the sampling frequency, is not it, you take the original Fourier transform of the signal, move it to every multiple of the sampling frequency, then you reflect it or mirror image it and put it also at every sampling frequency, and all these copies are added together to give you what constitutes the sampled signal.

And the question that is asked is, does this not mean that a sample signal has infinite energy? Because you have so many infinite copies, all of them contain some finite energy, so, therefore, the sum of all of them must diverge in terms of energy. Well, the answer is, yes. In fact, what happens when you sample let us see.

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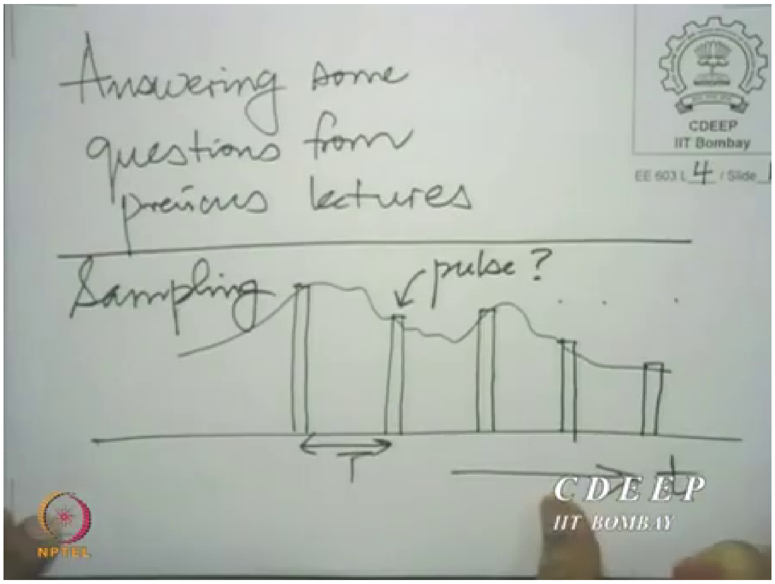

So, what we are doing here is first answering some questions. So, you see, take sampling, in sampling, what are you doing? You have this original continuous signal. Let us assume your sampling interval is *T*. That means, you take a sample here and what do you mean by taking the sample, essentially take a very narrow pulse and multiply it by the value of the signal at this point, then take another such narrow pulse or multiply by the value of the signal at a point spaced *T* from there and repeat this for every such spacing of *T* and so on.

Now, what really is this pulse? Where is this pulse going? Let us focus our attention on the point of sampling. What was happening when you added all those sinusoids, when we took a pure sine wave and we sampled it, and we looked at the point of sampling, considering that many sine waves have the same samples at those points, we said all of them when they come together, they form the, they add constructively at the points of sampling and destructively elsewhere.

What happens at the points of sampling? They are constructive, that means the amplitude starts diverging. And of course, at all other points, they are destructively. So, it is over an infinitesimally narrow region that you have a constructive interference, everywhere else it is destructive.

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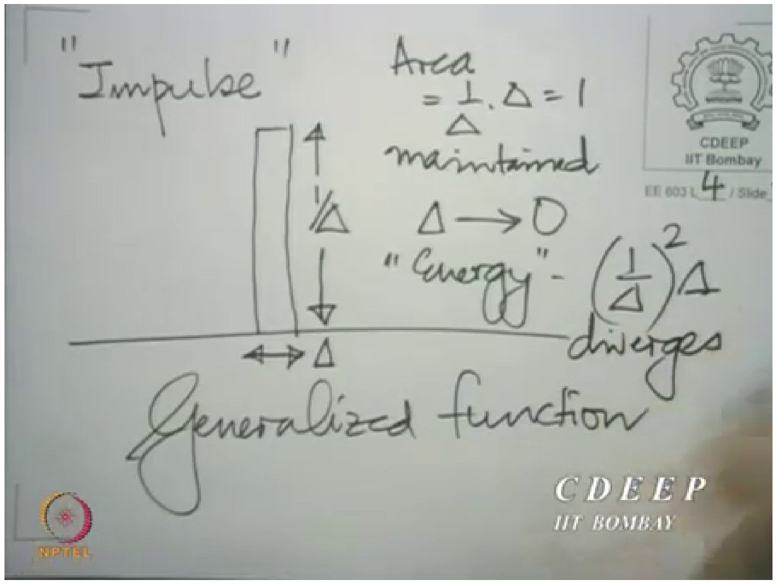

Now, how do we represent this process? We represent this process by a limiting concept called an impulse. What is an impulse? An impulse can be construed as an object. Note, I am not quite calling it a function, I am calling it an object. It is a generalized function, which has a width of Δ and a height of $1/\Delta$. And Δ goes toward *0*, what it means is, you maintain the area, the area of the impulse is unity, the area is maintained.

But if you look at the energy, or what can be called the energy here, maybe a little unfairly, the energy is essentially, how do you? Now, what do you mean by energy, you must think of this as a voltage being applied to a *1* Ω resistance to the current flowing is that resistance would be $1/\Delta$, over that interval of delta. So how much of power would be consumed, it would be $(1/\Delta)^2$ times *R*, that is *1*.

And what is the total energy for it is this happens for a time Δ. So, the total energy consumed is $(1/\Delta)^2 \times \Delta$, which is $1/\Delta$. So, it is $(1/\Delta)^2 \times \Delta$ and this diverges. So, the energy in the impulse diverges as Δ tends to, you see, for an impulse is the energy tends to infinity and that actually is what is happening in sampling.

So, in the limit, if you take idealized sampling, you are actually talking about an infinite energy process. But of course, sampling is never ideal. So, what happens in practice is you do not have impulses, you have pulses. So, they have finite energies for any small but not *0*, Δ, $1/\Delta^2$times Δ

is a finite quantity. So, in practice, you have finite sampling intervals. And therefore, you do not quite get idealized sampling.

And the manifestation in the frequency domain is that all the copies do not have the same height. Moreover, hopefully, if your pulses are narrow enough, the original spectrum, the first, the main, the real, the true person, the true spectrum is not affected seriously. But all other copies are distorted by the process of sampling. So, the answer is yes. In fact, sampling is indeed an infinite energy process and we have to treat it like that. Good. Very good. Any other questions before we proceed. Yes, there is a question there.

Student: ...

Professor: The question is, what is the difference between an analog signal and a digital signal? Well, the main difference is that the independent variable in an analog signal is continuous and a digital signal is discrete. But when you say digital signals, strictly speaking, it is also the dependent variable, which is discrete. But maybe the question should be reframed as, what are the differences between analog signals, discrete variable signals, and digital signals?

An analog signal has the independent and dependent variable continuous. A discrete variable signal has the independent variable discrete or sampled, but the dependent variable is continuous and that is the kind of situation we will deal with. A digital signal notionally has both the independent variable and the dependent variable as discrete. Yes. Any other questions? Yes, it is a question there.

Student: Is it necessary that the independent variable be discrete?

Professor: Okay, so the question is for a digital signal, is it necessary that the independent variable be discrete? Well, yes. Normally, we expect to deal with situations in a digital signal where the independent variable is discrete. It is almost, it is very rare that we would think so. Normally when we say digital signal, we are talking about discrete independent variable, discrete dependent variable. Yes. Any other questions? Yes, well, we will take question from here.

Student: Many times we are using a constructive interference and destructive interference, what is the meaning of constructive and destructive?

Professor: Constructive and destructive interference, destructive interference means all of them come together to annul one another. So, they go towards *0*, when they add, as you add more and more and more terms, they all fall to *0*, they all cancel one another out in the sum. That is what destructive interference means.

In fact, the word destructive interference can be thought of as in the context of waves, when you have many waves, which come together, they could either add constructively or destructively. If they add constructively their amplitudes go up, if they add destructively, their amplitudes go down, the sum has an amplitude, which keeps falling as you bring in more and more terms. That is what I meant by destructive interference. Yes.

Student: What is the difference between a Fourier series and a Fourier transform?

Professor: So, the question is what is the difference between a Fourier series and a Fourier transform? Well, a Fourier series applies only to a periodic signal. So, it is only when a signal is periodic, that you can talk about its Fourier series, that too, of course, it satisfies certain conditions called the Dirichlet conditions, in addition to being periodic, we will not go into that nicety here.

Anyway, for reasonable periodic signals, which are not too peculiar, you may allow me to say that, you can decompose them as a sum of sine waves all of whom have frequency which are multiples of the frequency of the original periodic signal. Now, the amplitudes and phases of those sine waves constitute what is called the Fourier series representation.

Now taking a periodic signal, and a periodic signal can be thought of as a periodic signal with its period tending to infinity. And therefore, the fundamental frequency tends towards *0*. And that means, instead of having a collection of discrete frequencies, those frequencies come infinitely close, infinitesimally distant from one another. And therefore, instead of a discrete set of points on the frequency axis, we now need to deal with the whole continuous frequency axis that becomes a Fourier transform. Okay. Yes, please. Yes.

Student: ...

Professors: Could you repeat the last part of your question? Why do you need two variables?

Student: ...

Professor: Alright, so I will repeat the question briefly, or I will try and put the question in context. You see, the question is when we were trying to understand the effect of sampling, we introduced, what we call a wraparound multiple. That is, we introduced the term $2 \pi N K$ and we said, if you add $2 \pi N K$ to the phase, it makes no difference to the sine wave at all. That is how we analyze that there are several sine waves which have the same sample.

The question is, why did we need to put N times K there? Well, we need to put N times K, because I need one time index and the other one, to give you the multiplicity of possible sine waves, which can come in so K is the index, which gives you the multiplicity of sine waves, which can play a role there and N is the index, which tells me the variation in time when you have time index and a multiplicity index together.

I need, I mean, $N K$ together makes an integer, but I need to recognize that there are two kinds of multiplicities which I can bring in, two kinds of ambiguities. Like one is, well, it is not really, time I would not call it an ambiguity. It is essentially just a function of time. The ambiguity comes from K. Yeah.

Student: If we are sampling a bandpass signal, which filter we have to use to reconstruct that bandpass?

Professor: So any other questions that yes. Well, the question is, if I am sampling a bandpass signal, which filter should I use? Well, he is asking me to answer the challenge, which I want. That is a part of your challenge. Yes. Any other questions? Okay, good. So, write down your answer now. Any other questions? Yes. Now let us take questions from other people also, yes, yes.

Student: If I want to analyse the periodic signal in frequency domain, so Fourier series, first I have to convert it into Fourier series and then I have to take Fourier transform or any other?

Professor: See, the question is, if I wish to analyze a periodic signal, do I first need to take a Fourier series and then construct the Fourier transform vice versa? Well, these are actually more matters of representation, what is more important is that you understand what you are doing. When you are taking a Fourier series, you are thinking of the periodic signal as a combination of sine waves.

Now, it does not matter whether you write them in one way or the other on the frequency, what is important is that you appreciate that when you go to the Fourier series, you are essentially thinking of the signal as a combination of discrete sine waves, sine wave with discrete frequencies, and then whatever you do should be done with our understanding.

It does not matter how you represent it. I mean, those are only cosmetic points, how you represent it, or those are not as important as understanding what you are inherently doing. I would always encourage the student to understand what he is doing in the transform domain rather than worry about the representation so much. Yes. Any other questions before we proceed to discuss further? Yes, we will take maybe a couple of questions.

Student: Sir if we are settling with the finite pulse, what will be the energy distribution?

Professor: Yes. That is a good question. So, the question is, you know, if you were to sample with a finite pulse, not an infinite pulse, so you know, here, let me go back to this drawing. Here, I am talking about a pulse width, width Δ, and height, $1/\Delta$, if I were to sample with $\Delta \neq 0$ here. So, I am saying.

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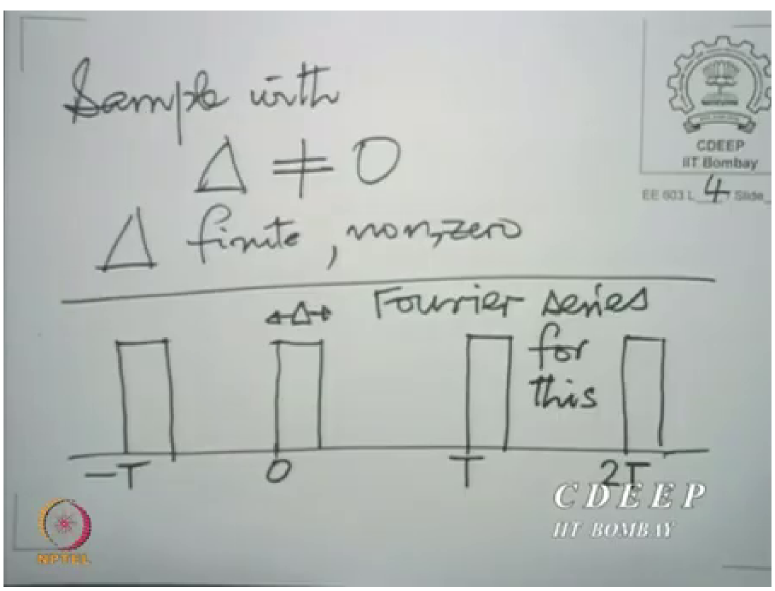

Sample with $\Delta \neq 0$ or Δ finite non-zero. How would I analyze this? Well, I give you the main steps, your analysis should proceed as follows. Think of the following periodic signal.

So, here we have a periodic signal, we assume that delta is much smaller than the sampling interval, we can write a Fourier series for this periodic signal.

Now the change that you need to make in your copies, is that if this $\Delta \rightarrow 0$, then all the copies are of the same amplitude. And the phase of the copies I have already explained, how to get? On the other hand, when you have a situation like this, where you have a finite pulse, each copy occurs at a multiple of the sampling frequency which is also Fourier series component, that Fourier series component has an amplitude and a phase.

So, the copy at that component, the copy of that multiple of the sampling frequency is modified in amplitude and phase by the Fourier series term at that multiple. So, the answer is write down the Fourier series for this, find out the Fourier series component at each multiple of the sampling frequency.

The component, the copy, which is brought to that multiple of the sampling frequency is modified in amplitude and phase by the Fourier series term falling on that multiple of the sampling. So, that means, typically what would happen is the Fourier series terms would decay as you go away from *0*. So, you would find the copies also decay in amplitude. And of course, would also undergo a change of phase.

And what is more is, it depends on. Now there are two ways of doing it. One way is you just gate the signal for some time. There is a subtle difference between these two things. You just allow the signal to pass for a small interval of time and then chop it away. That is one way to do this. The other way is not to gate it, but to capture the value of the signal at a point in time and hold it for a short while.

Now there is a very fundamental difference in the way you analyze these two. And as is my habit, I shall leave the difference to you as a challenge. How do you distinguish the two case? In the spectral domain? My answer is, I will give you a brief answer to the question, the brief answer is, when you gate it, that means when you allow it to pass for a short interval, and then stop it, then you do exactly what I said a couple of minutes ago, you just multiply the copies by the Fourier series component.

But when you hold, when you sample and hold; then you have trouble because not only are you multiplying the copy, you are also distorting that copy. You are also distorting that copy by, the spectrum of the pulse. So, I leave this to you as a challenge. Suppose I were to do these two different things. How would they differ in what happens on the frequency axis? So, it would need to be analyse based on basic exposure to signals and systems.

But anyway, I am not expecting that you are not thinking of it as a part of the syllabus. But since somebody posed the question, I put this to you as a challenge with this hint. Yes. By the way, I encourage students to write down their attempts to the challenging problems and submit them, oral discussions are all right. But no, I give credit when the solution is written down reasonably and submitted. Is that right? Yes.

Maybe we can take just one question if there is any other and then we will continue with our discussion. Yes. Any other question? You have a question?

Student: ...

Professor: Okay, so the question is, suppose I took a *0.7 kilohertz* sinusoid and sampled it at *1.4 kilohertz*, then the first mirror image would be at *(1.4 - 0.7)*. So, the original signal and the mirror image overlap and with the opposite phase, and therefore, we expect them to cancel, that is what very often happens, they cancel. In fact, they cancel depending on where you sample. And in fact, this is easier understood in the time domain.

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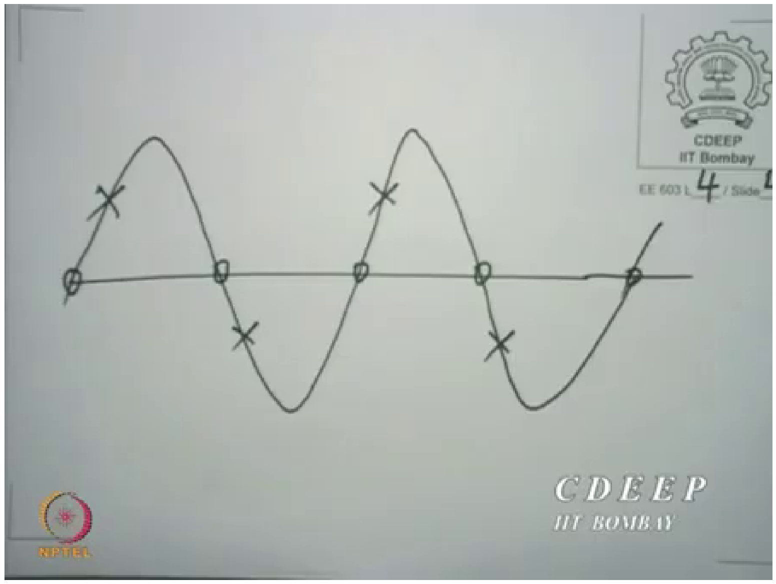

Suppose I want to take a sinusoid. Now, when you say that the sinusoid has a frequency of *0.7 kilohertz* and you are sampling at *1.4 kilohertz*. Another way to interpret this, is that you have a sampling rate twice of the frequency. In other words, in every period, you would have two samples. Now, this, the original spectral components, sinusoidal component, and the mirror image would cancel one another, if you sampled here, it would give you all *0* samples.

On the other hand, if you were to sample here, they would not cancel one another. So, it is not obvious that they would cancel one another, it would depend on the point of sampling, but there is a potential problem of they are cancelling one another if you happen to begin the sampling from a zero crossing. Okay. All right!

Now, that was several questions, we can, of course, encourage more questions in subsequent lectures, and not only in subsequent lectures in this lecture as well as we proceed. So, from now onwards, it should be made and I was very happy that we had several questions come up from the class, in future too we must make this course a dialogue, not a monologue, as I have told you in the first lecture.

In system, we think it is very important that you should participate wholeheartedly in the discussion. Otherwise, the classroom would not be as effective as it should. So, we should make this practice. So, whenever you have a question, you do not have to wait until I explicitly invite questions. Whenever there is a question in between, you must raise your hand. We shall attempt that question and decide how to deal with it at that point in time.