Digital Signal Processing and its Applications
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Lecture 25 B
High Pass Transformation

Because in a certain sense, when we go from low pass to high pass, what we are doing, is to reverse the roles of frequencies, in a way. I will explain what I mean.

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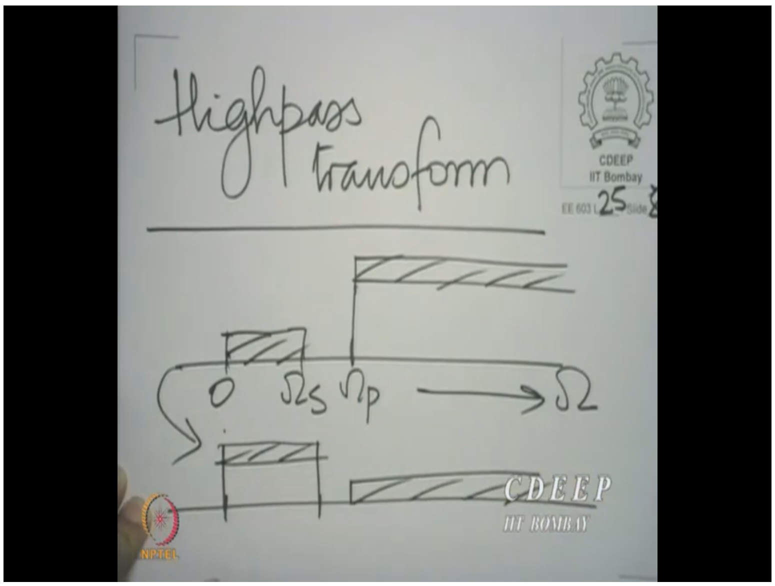

High pass transformation. Now, you see in a high pass transform, what we wish is that as the frequency increases, in a high pass filter, we have a specification something like this. So, you expect that you have a stop band and a pass band like this, so what we are saying is, as we go from 0 towards plus infinity, and of course the same thing is mirrored on the negative side, we do not need to keep drawing the negative side.

As we go from 0 to infinity on the Ω axis, we expect that we first encounter the stop band and then the pass band. So, it is very clear that if I want to go from here to a response like this, as I had for the low pass filter, then I should be traversing the frequencies in the opposite direction. So, when I move this way on the high pass frequency axis, I should be moving this way on the low pass frequency axis. That is what it is, it sort of suggests to us that we want an inversion of behavior. And then, it is very easy to think of an admittance which given me inversion. Either the admittance of an inductor or the impedance of a capacitor gives me inversion.

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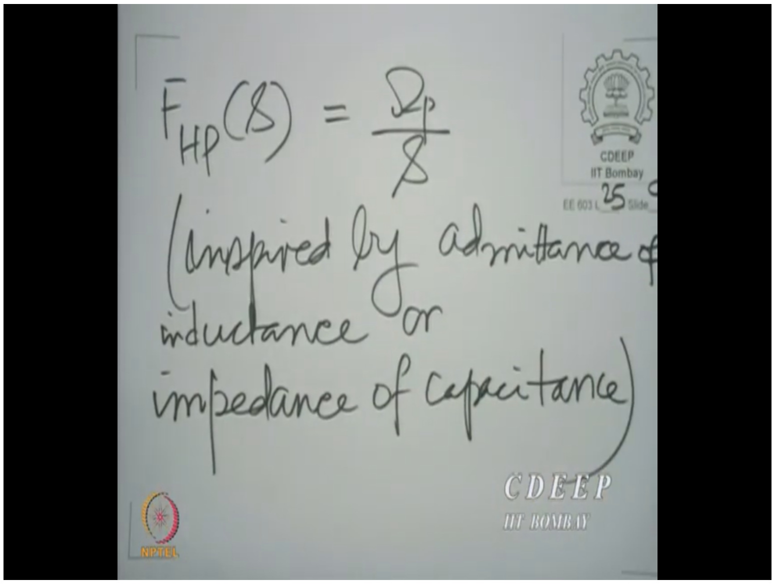

So, all that we need to do is to consider a transform like this, $F_{HP}(s) = \Omega_P/s$, simple. Inspired by inductance, admittance of an inductance or impedance of capacitance. Now, let us verify what this does to us. Let us start with the last property first, let us see what it does to the imaginary axis. Let us put a $s = j\Omega$.

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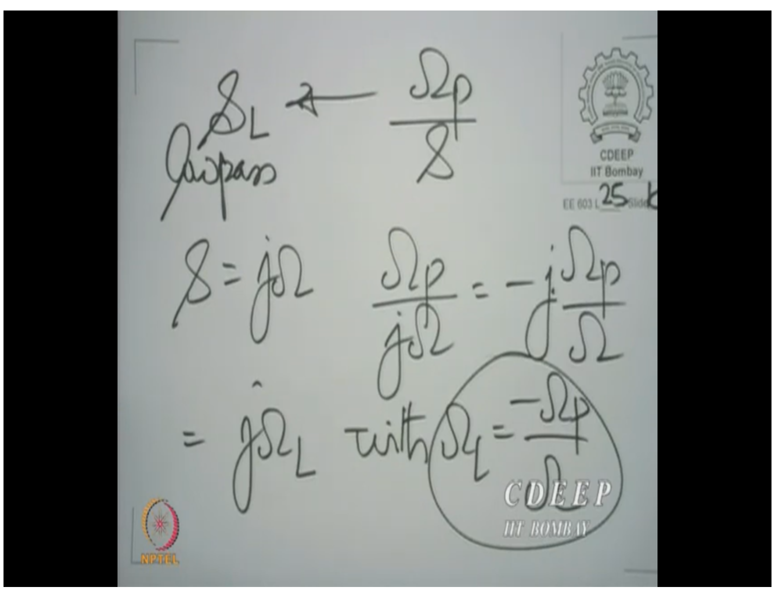

So, what we are doing essentially is, we are going to replace Ω, or rather we are going to replace $S_L \leftarrow \Omega_p/s$. So, what would be get? When we put S equal to $j\Omega$, we would get $\Omega_p/j\Omega$, which is $-\ j\Omega_p/\Omega$. And this is of the form $j\Omega_L$ with $\Omega_L =-\ \Omega_p/\Omega$.

So, indeed the sinusoidal frequency behaves as we expected to. And that was not surprising, you have taken an LC impedance anyway, it is bound to have an imaginary impedance. But the good thing is, if you now look at the way the frequency changes, so let us draw what we call the frequency plot, the frequency transform, the sinusoidal frequency transform or the frequency mapping. These are different names that are used for this.

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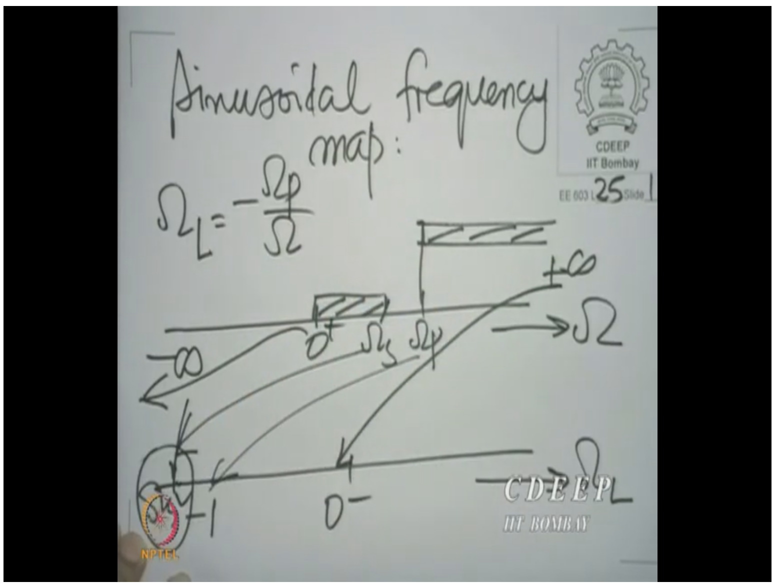

So, the sinusoidal frequency mapping. So, what we do in a sinusoidal frequency map, is we take Ω and map it to Ω_L. Of course, we know that $\Omega_L =-\ \Omega_p/\Omega$. So, what we are saying essentially is, as Ω goes, see you want, this is the kind of response that you want in the high pass filter.

Now, Ω_P would map to minus 1. Ω_S is less than Ω_P and therefore, $-\ \Omega_P/\Omega_S$ is a quantity less than minus 1. So, it is behind minus 1, so there we are. Ω_S would map there. And of course, 0, we must be careful about 0. We do not go to 0, we do not really go to 0, we say 0 plus. 0 plus means a very small positive value. And you can make that small as small as you desire to move to 0.

So, a very small positive value here would go to minus infinity. So, in fact 0 plus takes you to minus infinity. So, we can draw that, this goes to minus infinity. So, there we are. This pass band now moves from, let us call this Ω_{SL}, this point where the edge of the stop band maps. So, the stop band now maps between minus infinity and minus Ω_{SL}. And $-\ \Omega_{SL}$ is of course, behind minus 1.

Further, what happens to plus ∞? Now, $+\ \infty$ maps clearly to 0 minus, so $+\ \infty$ maps to 0 minus. 0 minus means, so when you say $+\ \infty$, you mean a positive number that grows without bounds. So, when you do $-\ \Omega_p/\Omega$, if Ω is a positive number that grows without bound, this would tend to a negative number that becomes as small as you desire in magnitude. That is what you mean by 0 minus. And therefore, the pass band is brought between minus 1 and 0. The stop band is taken between $-\ \infty$ and Ω_{SL}. And that indeed is what you expect for a low pass filter.

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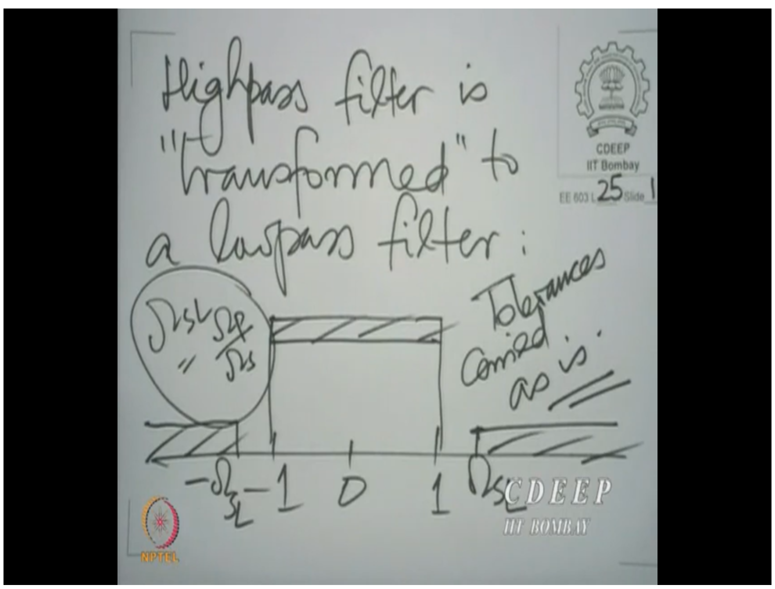

That means we have transformed, the high pass filter is thus transformed, is transformed to a low pass filter as follows. Plus 1 minus, of course, there is symmetry. So, the pass band is brought between minus 1 and 0 and therefore also between 0 and 1. The stop band is brought between Ω_{SL} and plus infinity and $-\ \Omega_{SL}$ and $-\ \infty$.

Remember, the pass band and stop band are carried as they are. The dependent variable is carried as it is, that is not affected. So, when you bring the frequencies from Ω to Ω_L, no effect is seen on the dependent variable that is carried. That means the tolerances are carried as they are, so that goes unaffected.

So, if you desire a certain set of tolerances in the high pass filter, the same set of tolerances are carried into the low pass filter. Tolerances carried as they are, carried as is. And of course $\Omega_{SL} = \Omega_P/\Omega_S$, for the high pass filter where Ω_P is more thanΩ_S.

Yes, there is a question. So, the questions is, would the maximum value of Ω be π? Now, please remember that we have already come to the analog domain. So, what was from 0 to pi has now been brought to 0 to ∞ by the bilinear transform. So, we are no longer dealing with the discreet angular normalized frequency, we are now dealing with an analog frequency which has emerged from the bilinear transform. Yes? So, so much so for the high pass. Now, you see therefore, the high pass filter design problem is easy to formulate now.

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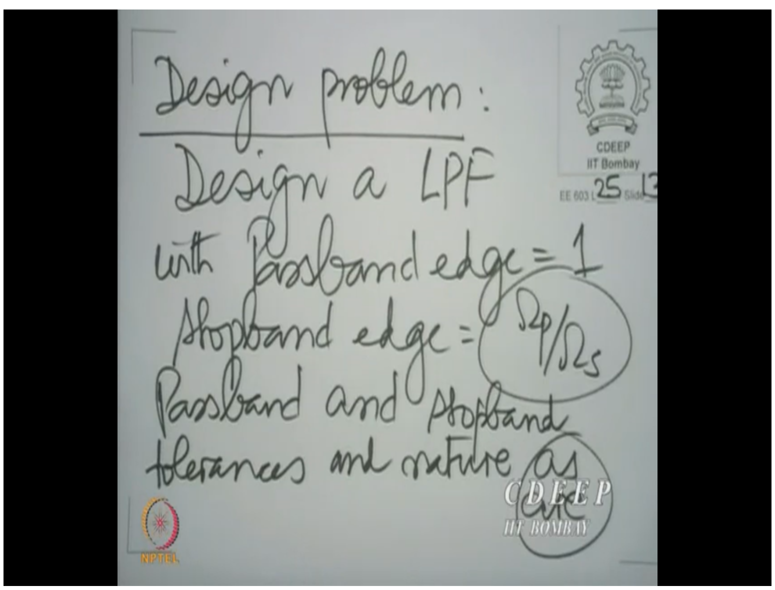

Design, a low pass, a corresponding low pass filter with pass band edge equal to 1, stop band edge equal to Ω_P by Ω_S. Pass band stop band tolerances in nature carried as they are, as are. What I mean by that is, if the pass band is monotonic, let it continue to be monotonic in the low pass filter. The pass band where equiripple, let it be equiripple here. So too for the stop band.

And that means, you now have the tools to do it, you can design it using the Butterworth approximation or the Chebyshev approximation, or if you are trying out any of the other two, inverse Chebyshev or elliptic, you could do that and then, all that you need to do is to. So, having designed this low pass filter, the next step would be to make a frequency transformation.

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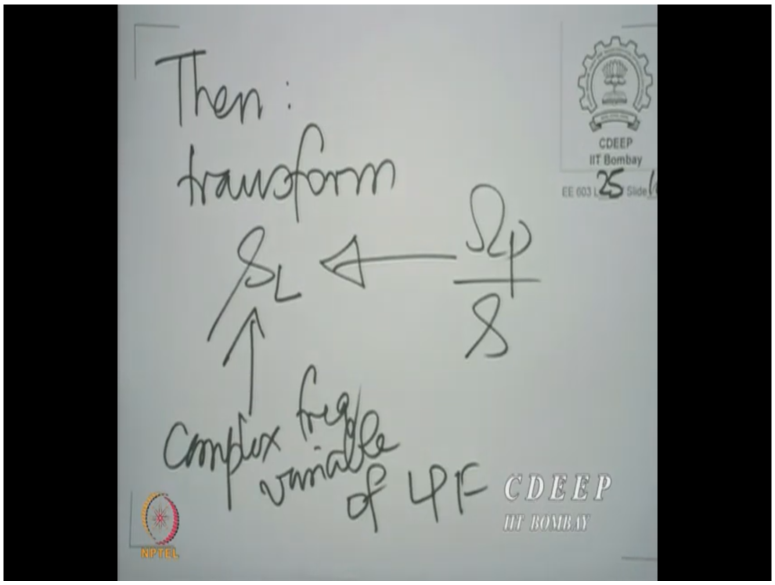

So, then, transform S_L, that is the complex frequency variable of low pass filter or Laplace variable of the low pass filter, using Ω_p/s. Replace $s_L \leftarrow \Omega_p/s$ and you get the high pass filter that you desire. Yes, there is a question. So, the question is - how do we prove the tolerance that are carried as they are? We do not need to prove anything there. You see, what we are moving is the independent variable.

Now, what value the function takes at that independent variable is unchanged. All that you are saying is I am evaluating the function at this value of the independent variable. And that is anyway specified in the original filter. So, I am saying the whole dependent variable is carried as it is. There is nothing to be proved there, you just have to note that you are making a transformation on the independent variable. And there is no change of the dependent variable, at every given point. That is carried as it is.

Now, having made this transformation, $\Omega_{SL} \leftarrow \Omega_p/s$, we now need to check whether we have obeyed the other two requirements of rationality and stability. Now, rationality is obvious. This is a rational transformation so if the original low pass filter is rational, as it is expected to be, then this transformation would retain the rational character. So, rationality is not a problem.

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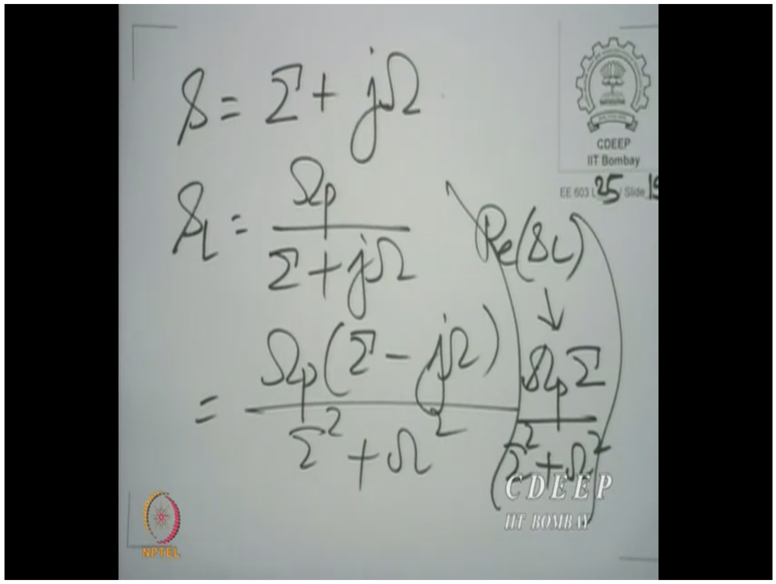

But stability, we need to check. So, we need to check what this does to the real part. So, let us consider a Σ to be, I mean $s = \Sigma + j\Omega$, whereupon $s_L = \Omega_P/(\Sigma + j\Omega)$, which can be rewritten as $\Omega_P(\Sigma - j\Omega)/(\Sigma^2 + \Omega^2)$. And therefore, the real part of $Re\{s_L\} = \Omega_P\Sigma/(\Sigma^2 + \Omega^2)$.

Now, it is very obvious that if Σ is positive, so is the real part of s_L, because Ω_P is a positive quantity and $\Sigma^2 + \Omega^2$ is bound to be positive. If Σ is positive, then $\Sigma^2 + \Omega^2$ has no choice but to be positive, it cannot be 0. And Ω_P is positive, so the sign of Σ is carried to the real part of s_L.

If sigma is positive, the real part of s_L is positive. If Σ is negative, the real part of s_L is negative. And therefore, the left half plane goes to left half plane and the right half plane goes to right half plane. What it means is, you cannot possibly have poles coming to the right half plane if you did not have them in the first place in the low pass filter. That is because a point of the right half plane in the filter of desired nature goes to a point in the right half plane of the low pass filter. And that could not possibly have been a pole. And therefore, the poles can only be in the left half plane.

In fact, we have also excluded the imaginary axis because the imaginary access goes to the imaginary access. So, the danger of poles going on to the imaginary axis also has been precluded in this process. Anyway, so that completes the design of high pass filters.