Digital Signal Processing & Its Applications
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Lecture No. 04 b
Desired Requirements from Discrete Systems

Now, we proceed then with a discussion of what we are just beginning on, what we just embarked on in the previous lecture, namely the question of discrete systems.

Now, you know, we will review once again what we mean by a discrete system. A discrete system is a system where you have a sampled input

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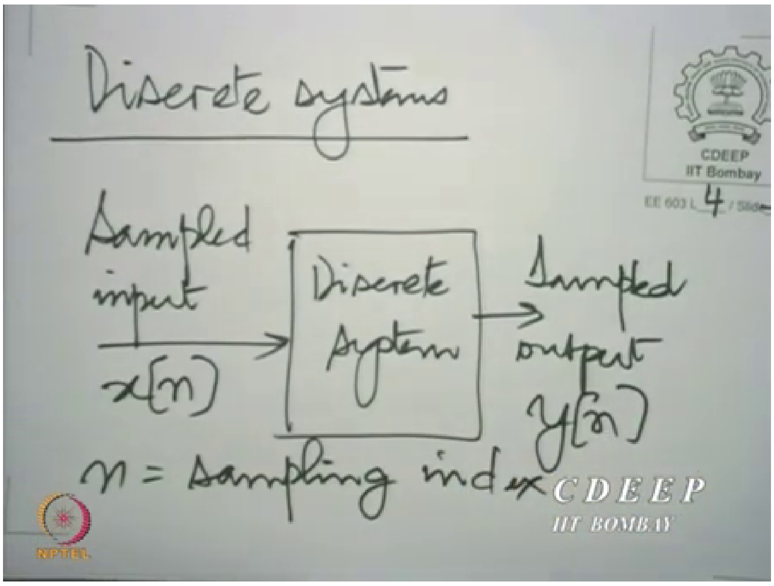

x

And a sample output is produced. We had agreed to call this input *x[n]* and output *y[n]*, and we associated *N* with the sampling index. In other words, when we say *N = 0*, we mean the *0th* sample, when we say *N = -1*, we mean the sample, the minus *1th* sample or the sample at *t = -T*, and so on. And we, of course, agree that the sampling instance are the same at the input and the output.

What we also agreed is our whole objective here is to see at least, in a major part of the course, if I can achieve from a configuration like this, what I might do with an analog system where you have a continuous input and a continuous output, hopefully the continuous input is band limited. So, when I sample it, I have sampled it sufficiently so that there is no aliasing.

And then, I am doing something to the sample that is what the discrete system is supposed to be specified. And doing something to a sample and producing a stream of output samples.

And the stream of output samples that I am producing, should be the same as if I were to sample the output signal from the analog system at the same point.

I am hoping to be able to do that. So, I have an analog system, where I would take a continuous input and produce a continuous output, I sample the analog input, I do something to the samples, and I produce the same samples as I would, if I were to sample the output of the analog system directly at the same points, I am hoping to be able to build such systems. And I will soon show, not so soon, it will take a while.

But we will see in due course that this is possible. And what is important is that for this to be possible, one of the first things that we need, as we saw is that when I put a sinusoid into the system, I should get a sinusoid as the output with the same frequency. In fact, we said that quite a number of sessions would be spent on building the properties of a system which ensures this.

Now, we will spend a minute in asking why we are so keen to deal with sinusoids. There are several reasons, sinusoids are natural in electrical engineering in a certain way and actually, because generation of electrical voltages often takes place with the ideal sinusoid as the pattern that is expected. So, if you think of a generator, and if it were an ideal generator, using the laws of electromagnetic induction, the voltage generated will be sinusoidal.

That is a reason pertaining to generation, there are several other reasons why sinusoids are natural in nature. But from a mathematical perspective, sinusoids are one class of waveforms which are analytic, they are probably the smoothest kind of waveforms that you can get and they are self-replicating under derivatives, integrals and addition.

Add two sine waves the same frequency, we have discussed this and you get a sine wave the same frequency, the amplitude, and the phase changes, take a derivative of a sine wave you get a sine wave of the same frequency, take the integral, the indefinite integral of a sine wave you get a sine wave of the same frequency.

Why are integrals, derivative and sum so important because if I have an electrical circuit, or if I have several different combinations of elements, I often need to do that. I need to add the quantities associated with each of the elements individually. Very often the relationships that govern elements involve derivatives or integrals. For example, an inductor relates the voltage and current through a derivative. Same is true for a capacitor.

So, the derivative of the integral is often ubiquitous in several systems, it occurs very frequently. And therefore, it is a good idea to see if you can have a waveform, which remains as it is in form, as it goes through different kinds of operations, the sine wave is a good answer. A more general answer is what is called the exponential. But we will come to that slowly later. At the moment, the sine wave is a good candidate, and therefore, we would like to deal with sine waves in a decoupled way. So, let us put that down. Yes, there is a question.

Student: ...

Professor: Yes. Well, the question is, sine waves are analytic. But then, we talk about negative. Now, let me postpone the answer to this question. There is a confusion in this question. The question was that negative frequencies should be zero. Here, we are not talking about positive or negative frequencies at all at the moment. So, let us not bring in that point in the discussion at all at the moment. It is a little premature.

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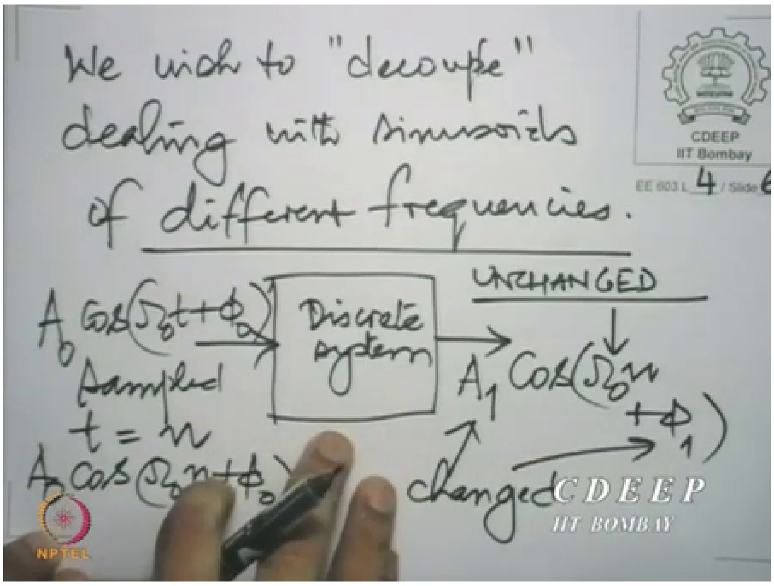

So, we wish to decouple dealing with sine waves of different frequencies. What I mean by that is, if I have a discrete system, I give it a sine wave $A_0 cos$. Now, it does not matter, we write cos or sine. Sampled at $T = N$. So, I get $A_0 cos(\omega_0 n + \phi_0)$. What appears is again, A_1 *cos* the same frequency ω_0 but changed in phase and amplitude. So, the phase and amplitude have changed, but the frequency is unchanged, that is important.

So, we are seeing two things in one here. We are saying that when I give a sinusoid as the input to the system that I am going to deal with, I need to get a sine wave. Point number one, I need to get a sine wave, that itself is a great thing. If I were to give a sine wave, and if I were to get some other kinds of wave, I am not happy. So, I need to get a sine wave, point number one.

Secondly, I need to get a sine wave of the same frequency. But I am willing to tolerate a change of amplitude and phase. This is what I want. This is the I mean, simple to say, not so easy to do. To be able to ensure this, now, what are the things, what are the obstacles to having this, the first obstacle is that when I put in a sine wave, I may not get a sine wave as the output.

Second obstacle is when I put in a sine wave when I get a sine wave of a but of a different frequency, or several obstacles, which I may need to deal with. So, you know, it is easier said than done. Ensuring that this happens for all sine waves is indeed a daunting task. And to achieve that, I need to put several properties on the system. Now, what I am going to require is something slightly more than this.

You know, and we will spend a minute on that. What I am going to require is not just that this whole for sine waves. I am going to require this whole for something called phasors. And I will now explain that idea. In some depth. You see, we would like to think of sinusoids as combinations of rotating complex numbers, and I will explain why. You see, take this very situation here.

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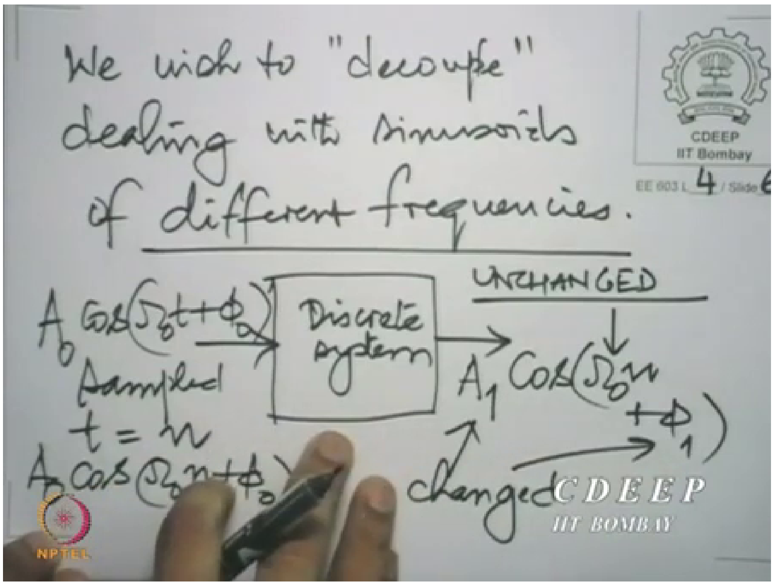

Let us assume that you are able to do this for whatever reason. I am able to get a sine wave of the same frequency and with a possible change of amplitude and phase. Even so, when I take a ratio of the output to the input, the ratio is not a constant. The ratio of course depends on N. In fact, you can write the ratio in a complicated way. But the ratio depends on it. What I want to do is to see if I can describe the system at particular frequency by just the ratio. And what I mean by that is, instead of dealing with sine waves, I need to bring in some other kinds of waveforms. Now, I introduced the word sequences.