Digital Signal Processing & Its Applications

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Lecture 32 a

Introduction to IIR Filters Realization and Cascade Structure

A warm welcome to the 32 lecture on the subject of Digital Signal Processing and its Applications. You will recall that in the previous lecture we had discussed the realization of rational causal system functions. We had looked at the direct form one and direct form 2 of realization. In both of which you are essentially taking the rational causal system function as it where and translating it into a hardware software structure.

We have talked about signal flow graphs and we had shown how to draw a signal flow graph corresponding to a realization. In fact, there was a very simple relationship between a signal flow graph and a translation into hardware, which was almost obvious and even into software, which require little more work but was not too difficult either.

What we do today is to look at other forms of realization, where we use decomposition in one way or the other. So, far we have not decomposed the rational system function at all. We have just taken it as it is and translate it into a signal flow graph and thereby, into a hardware structure if we so desire or a software program, if that is more convenient. Essentially list the 3 structures that we are going to study in greater depth in the lecture today.

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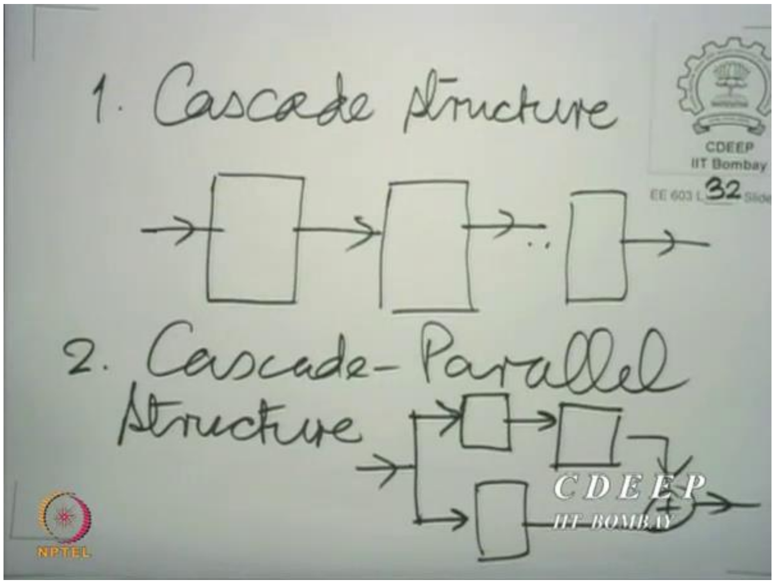

We are going to look at a cascade structure first. In a cascade structure, we are going to have a cascade as it were of system functions. Something like this and we did not give a hint to how you would do this in the previous lecture, we went into direct form 2 by using the idea of a cascade, but a very simple cascade.

The second structure that we would like to look at is a cascade parallel structure, so essentially structure something like this a combination and additive combination of cascades. We will see why we need to call it a cascade parallel structure, we could have been content with calling it a parallel structure. But there are situations where we must specify cascade parallel we will understand why.

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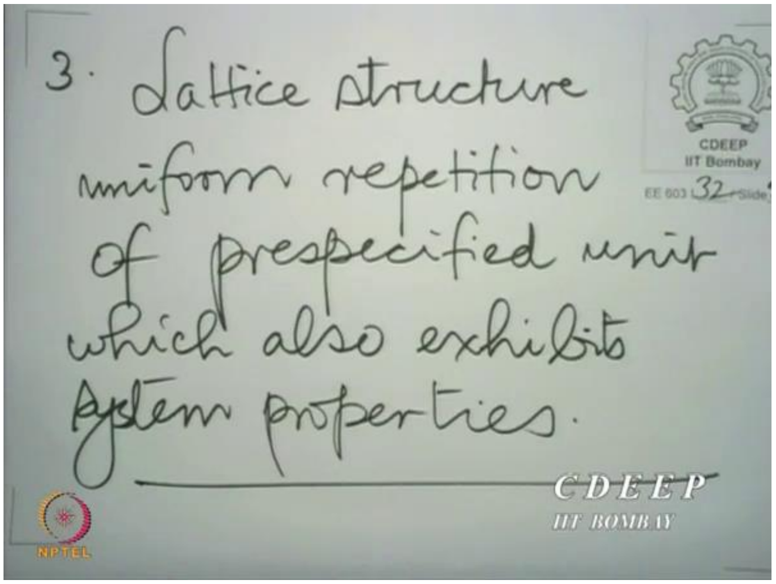

Finally, we look at what is called a lattice structure. In a way a lattice structure is a combination of the ideas of cascade in parallel. So, you could call it a cascade parallel structure in some generalized sense, but it is quite different really from both of them. In fact, the beauty of a lattice structure is that in addition to realization, it exhibits certain properties of the system.

And that is why we study the lattice structure its own right. So, it is essentially a uniform repetition of a pre specified unit which also exhibits system properties explicitly. We shall now look at the 1st of the cascade structure. Now, the idea on the cascade structure as we would expect is that a cascade implies the output of previous stage being given as the input to the next and this continues until we reach the final output.

Obviously, when you connect linear shift invariant systems in cascade, we have studied this right in the beginning of the course, when we connect linear system linear shift invariant systems in cascade, one of the things that we are sure is that they can be able to change without any change in the overall input output relationship.

So, it is very clear that if you have unequal elements in the cascade, what I mean by that is, if the different parts of the cascade are not identical, there are at least multiple realizations just based on this fact. That means a cascade realization is not going to be unique unless of course all the cascade elements are identical. Just an interchange just a permutation of them will give you a different structure.

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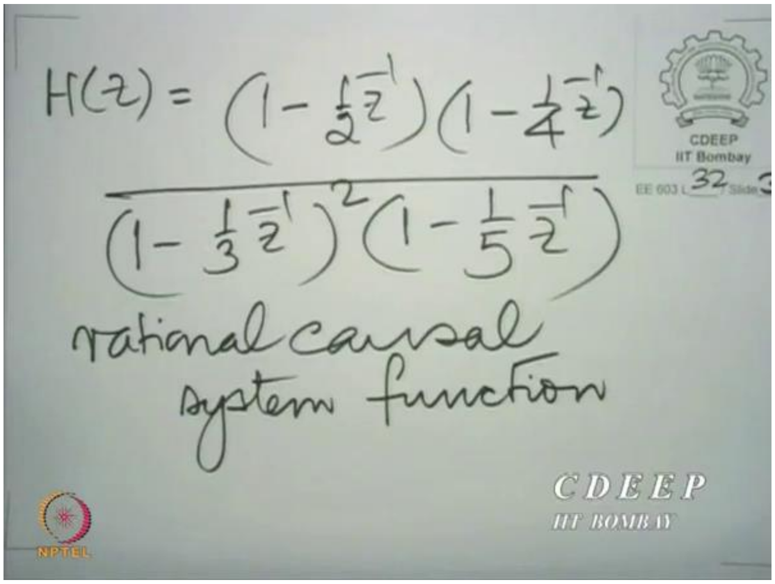

Let us, take an example. So, let us assume that we have a system rational system function that looks like this,[$(1-(1/2) z^{-1}) (1-(1/4) z^{-1}) / (1-(1/3) z^{-1})^2 (1-(1/5) z^{-1})$]let us assume this is H(z) this corresponds to a rational causal system function. Now there are several different ways of realizing this in cascade, I will take 1 as the first example.

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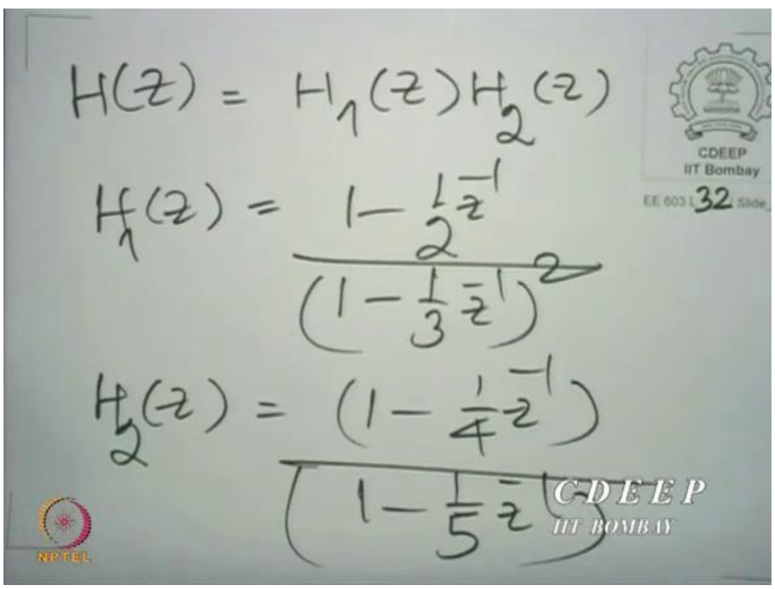

We could choose to divide the system function keeping the repeated poles together. So, H(z) could be written as [H_1(z). H_2(z)] where H_1(z) is [(1-(1/2) z $^{-1}$) / (1-(1/3) z $^{-1}$)^2] and H_2(z) is [(1-(1/4) z $^{-1}$) / (1-(1/5) z $^{-1}$)].

Obviously, that product e raise H(z). In fact, what I have done in this is to keep the rational system function in factored form. You could have, of course, you know how to express it in factor form that is not difficult all you need to do, though it is not always very easy is to find the poles to take complex conjugate pairs together.

So, here of course, we ensure all the poles and zeros are real. But that is not necessary. You could have complex conjugate poles and then or similarly complex conjugate zeros. And if you want to ensure real coefficients, these must be kept together. Anyway we could then realize each of these, in either direct from 1 or direct form, so let us just do one as an example.

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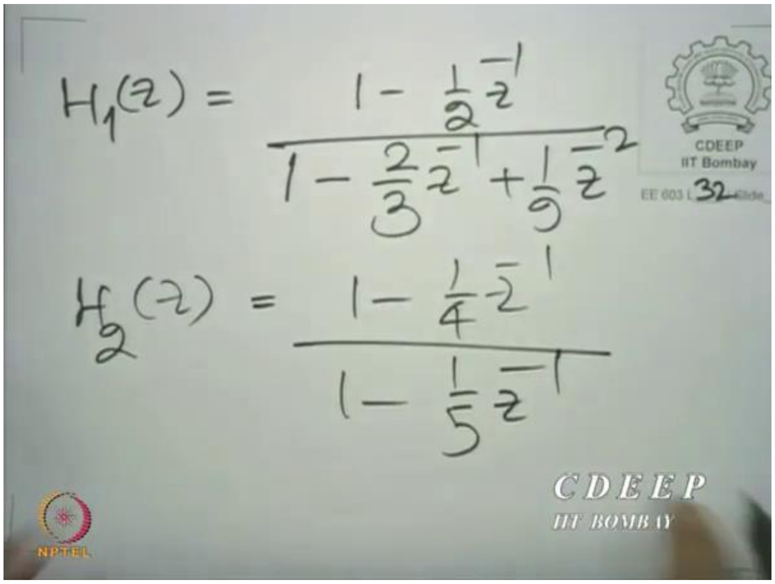

So, let us write H_1(z) in expanded form [(1-(1/2) z $^{-1}$) / (1-(1/3) z $^{-1}$)^2] would become (1-(2/3) z $^{-1}$ +(1/9) z $^{-2}$)) . And H_2(z) is (1-(1/4) z $^{-1}$) / (1-(1/5) z $^{-1}$) , whereupon, a cascade structure would look something like this. We realize them in direct form 2.

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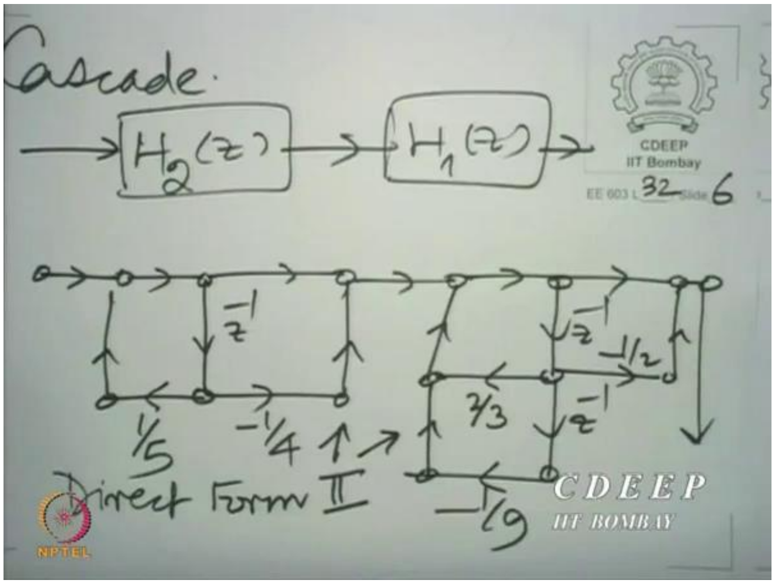

So, cascade would be essentially take $H_2(z)$ 1st and then $H_1(z)$ and that would essentially look like this draw signal flow graph corresponding to $H_2(z)$ you have a z^{-1} there you need only 1. A feedback of 1/5 and 1 and minus 1/4 that is H_2 for you and then we have H_1. In H_1 give a feedback of 2/3 and (-1/9) and then a feed forward of 1 and (-1/2). And there we go, this is the cascade structure corresponding to $H_2(z)$ followed by $H_1(z)$ now you see we have realized each of these in direct form 2, one and this one.

Now we could realize them in direct form 1 if we desire, though, that may not be economic. In fact, in a way that is splitting the cascade even further, if you think about direct from 1, even direct from 1 is a split of the numerator and denominator. So, we could realize direct from 1,2 there but that is not economical in terms of dealings. Now, this is not the only cascade, we could also of course, the simplest alternative is to exchange H_2 and H_1.

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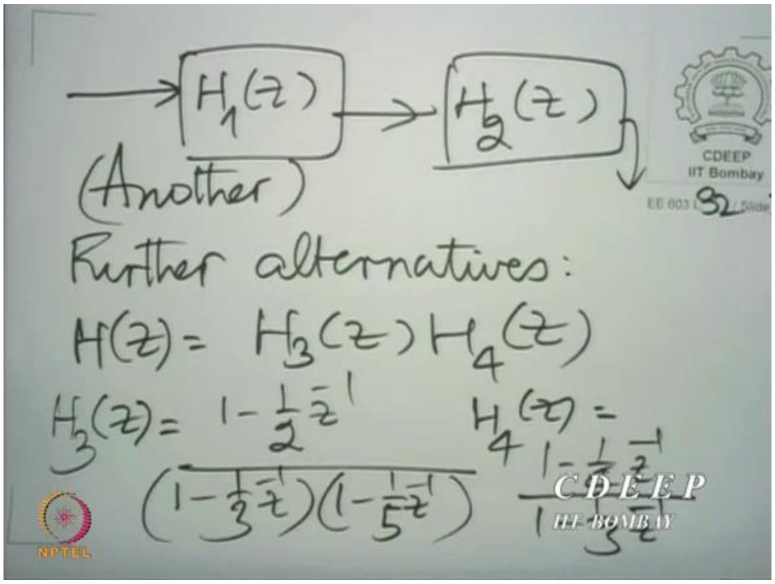

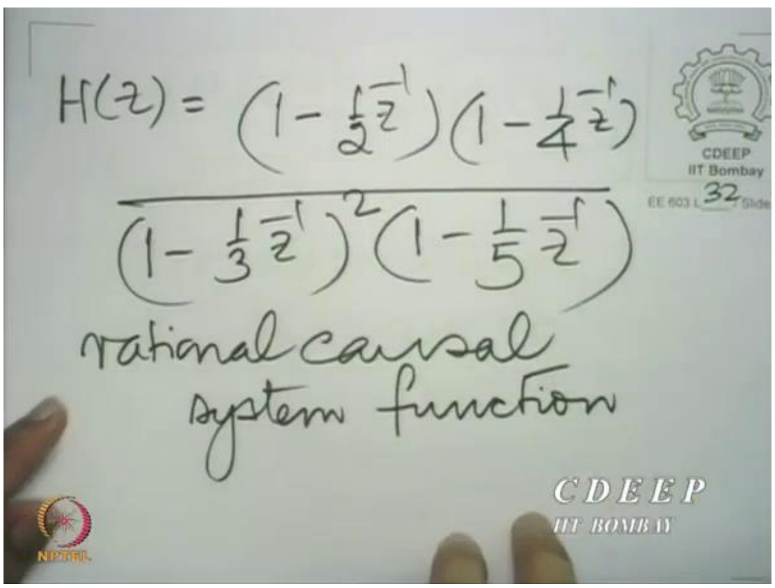

So, we could also have $H_1(z)$ followed by $H_2(z)$ and that gives me a different cascade structure. But there are even further alternatives. For example, express $H(z)$ as $[H_3(z). H_4(z)]$ and what we do is not to keep the poles up 1/3 together. So, we might go back to the original system function here. I have this, I do not need to keep the 1/3 poles together, I can take 1 (1/3) pole and 1 (1/5) pole and make that 1 system function.

And I can keep the other (1/3) pole aside so could for example have $H_3(z)$ is of the form $[(1 - (1/2)z^{-1}) /[(1 - (1/3)z^{-1}). (1 - (1/5)z^{-1})]$, and $H_4(z)$ in that case becomes $[(1 - (1/4)z^{-1}) / (1 -$

$(1/3)z^{-1})]$ that is another variation. Now of course, here again, you get two variations, because you could order it as $[H_3(z).\ H_4(z)]$ or $[H_4(z)\ .H_3(z)]$. Now, you could have still other variation.

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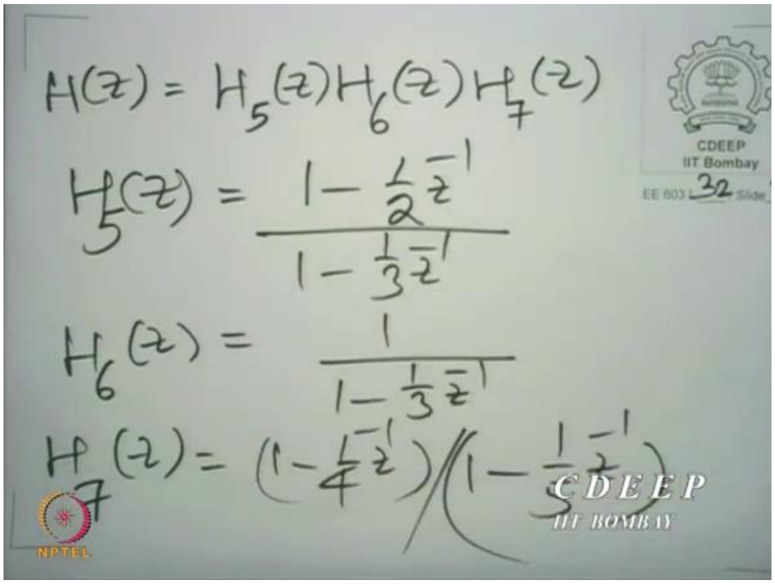

You can have the following possibility $H(z)$ could be written as $[H_5(z)\ .H_6(z).\ H_7(z)]$ where $H_5(z)$ can be simply $[(1 - (1/2)z^{-1}) / (1 - (1/3)z^{-1})]$ $H_6(z)$ can be simply $1 / (1 - (1/3)z^{-1})$ and $H_7(z)$ could be $[(1 - (1/4)z^{-1}) / (1 - (1/5)z^{-1})]$. And this gives me other possibility and now again, you can get rearrangements of these these are all distinct as system functions.

So, you could rearrange them again in any manner that you desire. So, you get several more combinations. Now, in fact you can see the trick that I am playing here. What I am doing is to decide on an association of zeros and poles of course, here as I said, I have all real poles and zeros. So, I have a you know the matter becomes simpler.

But if I have complex conjugates, let me emphasize and I want real coefficients I must keep complex conjugates together. So, that means in a cascade the smallest order smallest number of delays there I can use in once stage tends to be two at times because when you keep complex conjugate poles together, you would get a degree 2 term.

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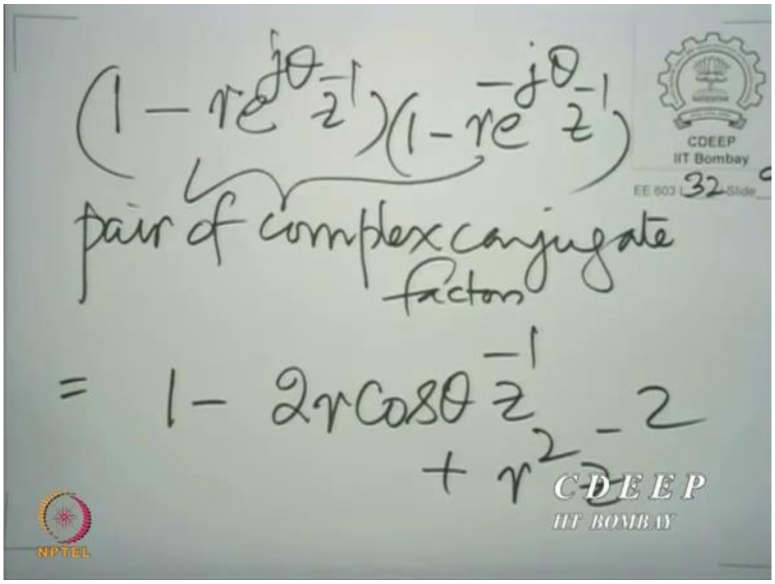

What would a term look like, if you get 2 complex conjugates you know, whenever you have a term of the form [(1- r cosθ) or $(1- re^{j\theta} z^{-1})$].[$(1- re^{-j\theta} z^{-1})$], this is a pair of complex conjugate factors. It becomes $1- 2r\cos\theta z^{-1} + r^2 z^{-2}$.

This is the typical term that you would get when you pair complex conjugate factors together and we do need to do it so you would sometimes not be able to go below degree 2 but you can definitely go to degree 2 that we are assured. So, in a cascade if you want to be very economical in the structure of one stage you must allow for a degree 2 stage degree means the number of delays that are involved there.

You must allow for a degree 2 stage but of course, if you have only real terms then you can even make do with a degree 1 stage. You see this is where we will later see that the lattice structures attractive. In the lattice structure in a much wider class of rational system functions, you can make do with a degree 1 stage and so although in many ways, lattice structures also like a cascade structure all somewhat like a parallel structure. It has this important advantage.