Digital Signal Processing & Its Applications
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Lecture 32 b
Cascade Parallel Structure

Anyway, now coming back to the next possibility, so we have understood what a cascade structure is, we have seen the various possibilities it is not one structure it is several different structures, we now move on to what is called that cascade parallel structure, I will explain to you why we need to call it cascade parallel and not just parallel.

Now, the idea is simple. In a cascade structure, you are trying to decompose the system function by addition. So, in the cascade structure your decomposed it by multiplication, you express the rational system function as a product of several rational system functions, each of which is casual in a parallel or in an you know in a structure which aim is to be parallel, you are trying to express the system function as a sum of different system functions.

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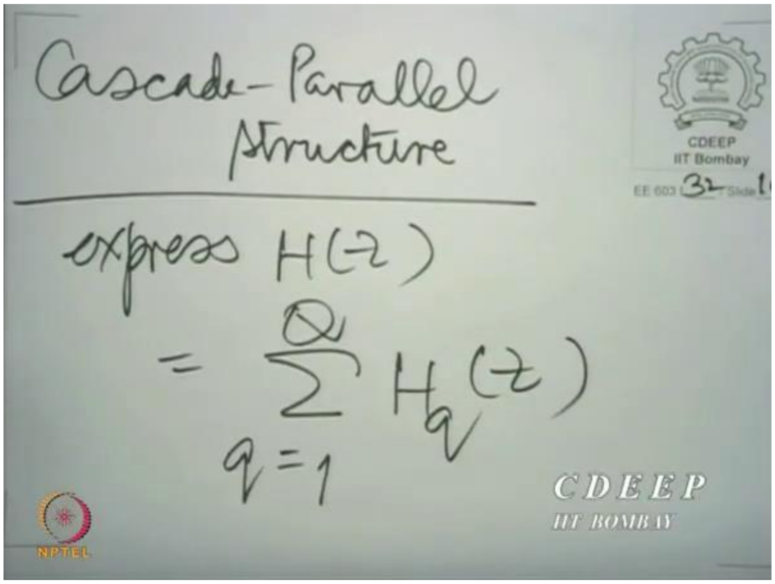

The idea is express $H(z) = \sum_{q=1}^{Q} H_q(z)$ Now, why we want to do this in this particular?

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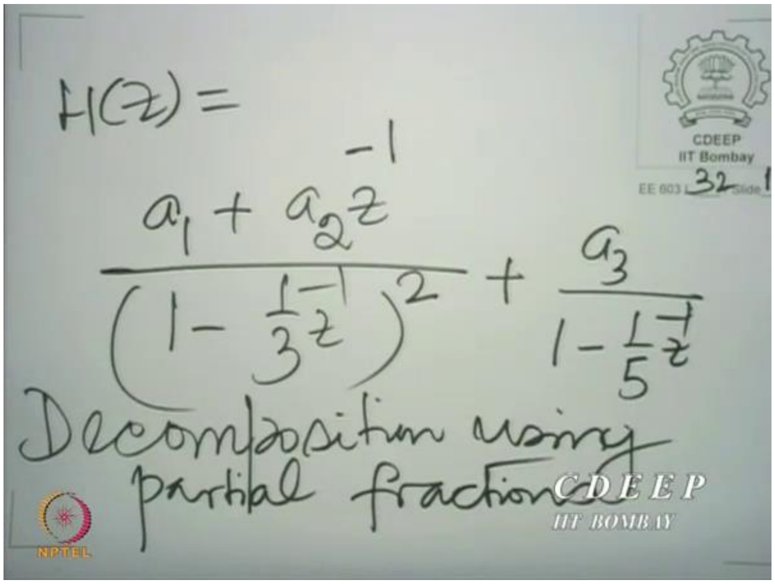

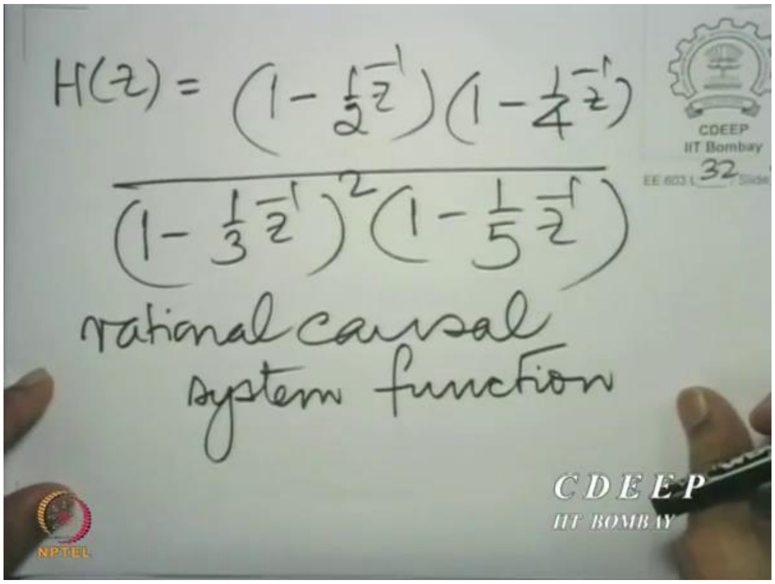

Let us, take the same system function again, here, so let us go back to a system function that we are using $H(z) = [(1-(1/2) z^{-1}) (1-(1/4) z^{-1})] / (1-(1/3) z^{-1})^2(1-(1/5) z^{-1})$. And let us try and see what ways we can decompose this as a sum. So, we could express this in 1 way as something divided by $1-(1/3) z^{-1}$.

Now here is where the problem comes,you see, you could also have this possibility,$(1-(1/3) z^{-1})^2$, sum $a_1 + a_2 z^{-1}$, if you please actually you will, let us see, we may need to allow this possibility $a_1 + a_2 z^{-1}$ you need to have the possibility of 1 degree less.

So, if this is of degree 2, you must allow a degree 1 term. And of course, $a_3 /1-(1/3) z^{-1}$ and this decomposition can be done by partial fractions. Incidentally, there are multiple forms of partial fraction decomposition. For example, the same system function could also be decomposed in the following way, by partial fraction expansion.

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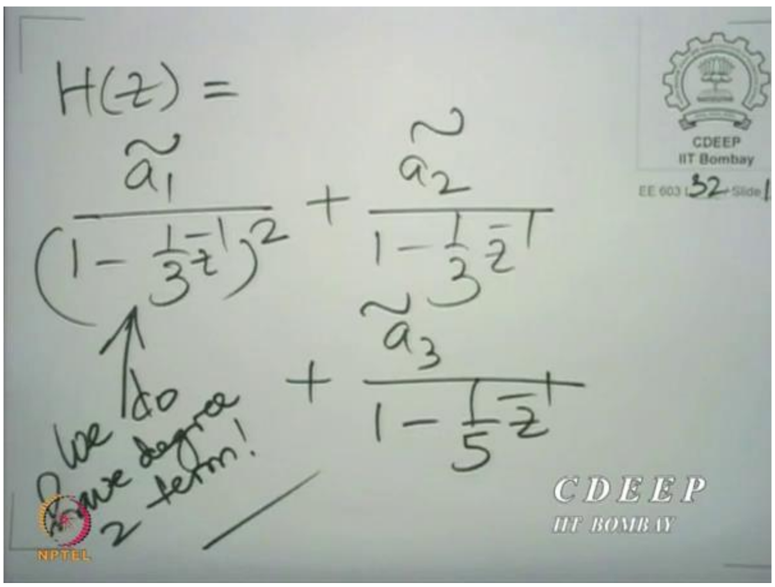

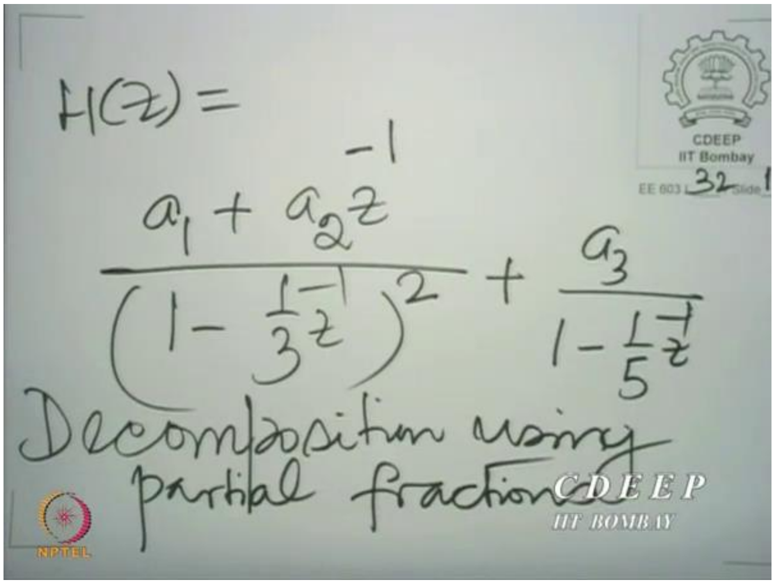

It is H(z) could have been written a sum a_1 / (1-(1/3) $z^{-1})^2$ + a_2 /1-(1/3) z^{-1} + a^3 divide or maybe we will call it a1 tilde they are not the same. You see, we should not write the same symbols a_1 tilde. So essentially only constants if you are insisting only on constants then this would be the form of the decomposition. But anyway what you notice is that we do have a degree 2 term here 2.

We cannot avoid that degree 2 term I am assuming that all of us are now familiar with how to decompose a rational system function into its partial fraction terms. I will not spend time on that here one could look up a standard text on complex analysis or series solution. But the point to be noted here is that whether we take this decomposition or we take this decomposition either of them this one or this one, we cannot avoid this degree two term here.

So, in a way we do not gain too much by this structure. We do have to realize the degree 2 term anyway. And that is why we call it cascade parallel because this degree 2 term here it is degree 2, because this power is 2, if the power where higher the root had a higher multiplicity, for example, suppose the root had a multiplicity of 4, then you would have a degree 4 term you could not go below degree 4. You see there is always this problem, when you have repeated roots, when you have repeated poles, you have no alternative.

But to have a degree term of that degree in the partial fraction decomposition, you cannot avoid it. And that particular term can either be realized in cascade form or direct form 2 or a combination of them. So, for example, suppose we took this form of the decomposition, $a_1 + a_2 z^{-1}$, we kept the entire polynomial on the degree two term as we have here and this,

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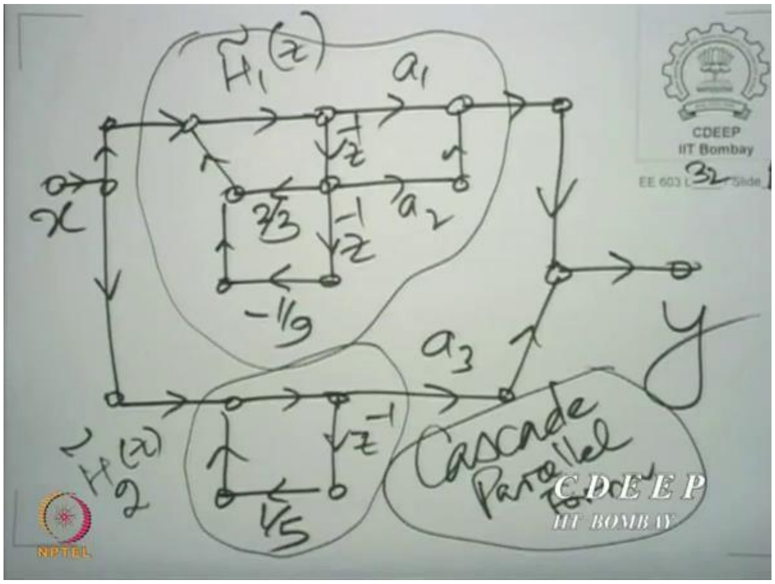

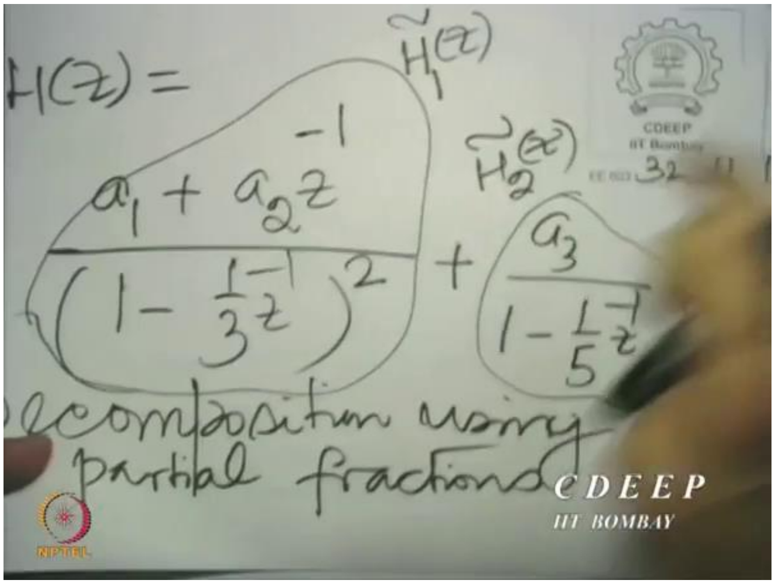

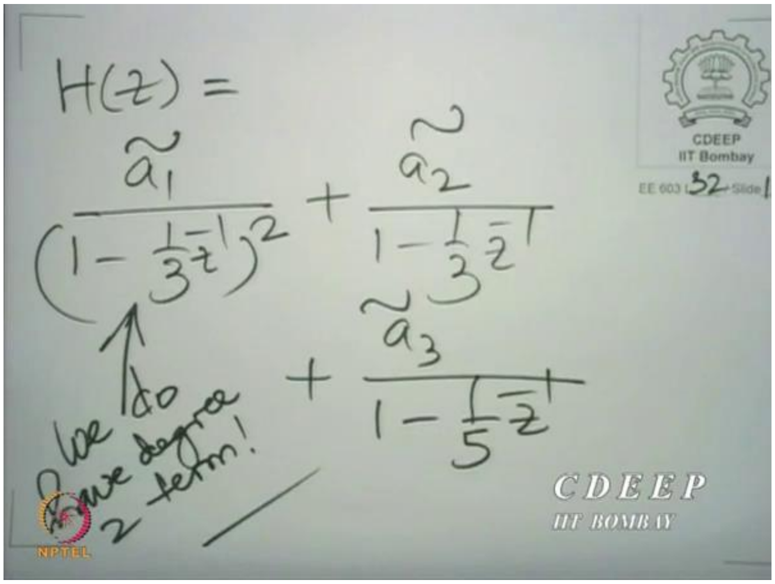

Then we could realize it as follow you see we could realize it is a sum of two functions. And on this branch, we could, of course, put the $(1-(1/3)\,z^{-1})^2$ here or you have 2/3 and (-1/9) and then the 3^{rd} branch, of course, you have $a_1 + a_2 z^{-1}$. So, you have 2, z^{-1} here a_1 and $a_2 z^{-1}$. And the 2^{nd} term of course is easy to realize. There is one fifth here and feedback nothing and will feed forward accept a_3.

So, these can be combined and there we to get y so you have x there and y here. So, this is a cascade parallel form corresponding to this expansion here. So, in $a_1 + a_2 z^{-1} / (1-(1/3)\,z^{-1})^2$ if we call this say H tilde 1 Z has been realized here. And if we happen to call this H tilde 2, then we realized H tilde 2 hear and they have been added. So, that simple.

Now, of course it is obvious if you want to realize this, you could of course, realize it in 2 ways. One is realize this as a cascade structure. So, here, you know, the, the name cascade parallel form is not quite clear. Why are we calling it cascade parallel? Because this really a parallel form. It is a parallel combination of 2 system functions, but it is in this case that the cascade parallel idea will become clear.

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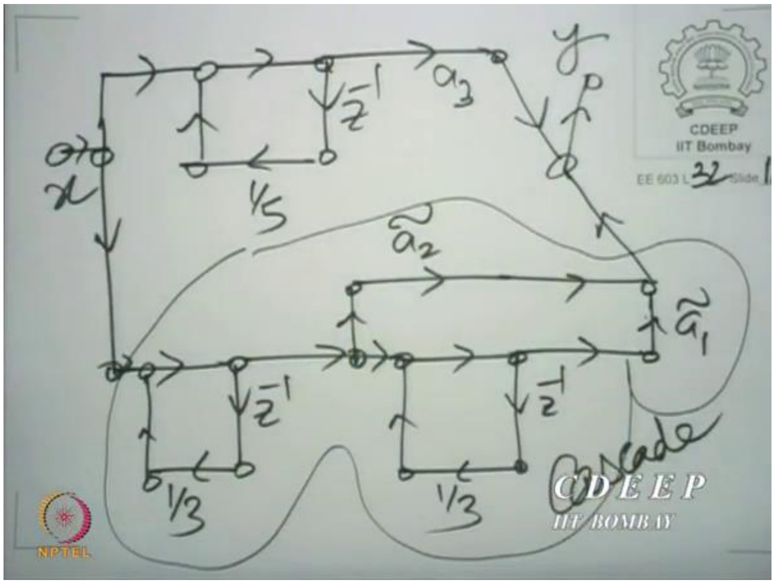

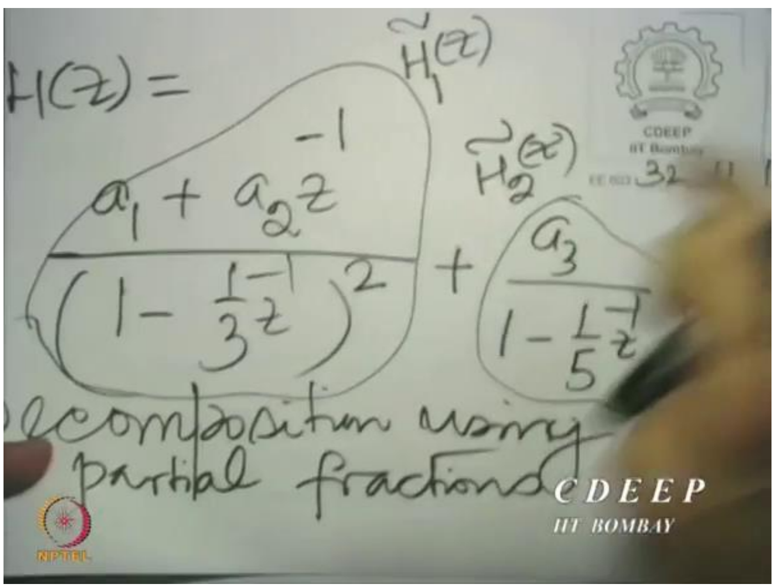

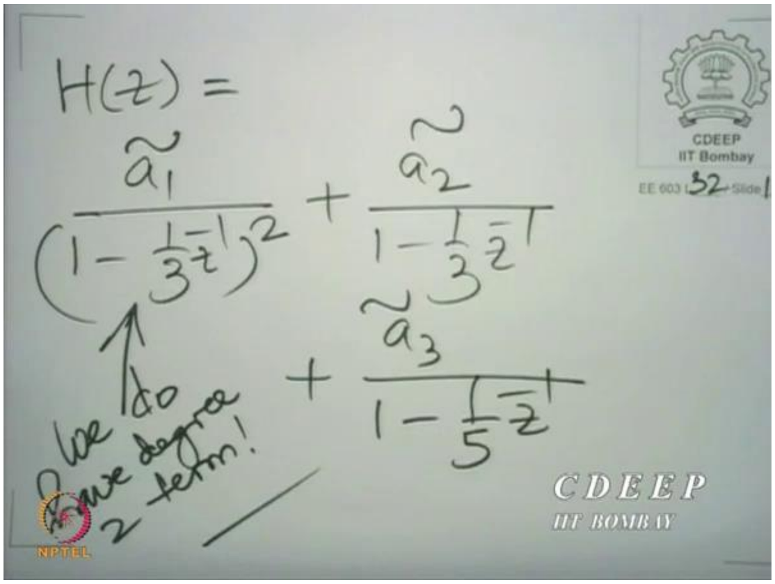

You see, in fact, what you can do is to realize this term as a cascade of 2 terms, which will help you them realize this term, so let us do that. So, we realize a cascade z^{-1} , 2 such terms of the form, $(1-(1/3)\, z^{-1})$. And now, this is multiply the output here is multiplied, maybe you can put a node in between that would make it easier to understand.

So, we cut this off and multiply this by a_2 tilde and cut this off and multiply this by a_1 tilde and sum them and then of course the upper realization is as it is. So, we have a_3 tilde as it is. In fact, a_3 tilde is the same as a_3 in the previous expression that is not difficult to see. So, a_3 in this expression is the same as the a_2 tilde in this expression.

So, there we are we have z^{-1}, 1/5 and this and we have a_3 here. So, now we could add these 2 and that completes y for you. Now, this is where you need a cascade that is why we are saying there is a cascade and a parallel combination, this is cascade this is parallel. So, we could also realize it using a cascade so now here this would really in the true sense be a cascade parallel form.

And even though in the previous case, we were able to make do with a sum of direct from 2 terms, here, we are actually using a cascade parallel. So, in general, we use the term cascade parallel because when you have higher degree terms, you can either realize them as a cascade or in direct form 2.

Anyway, those are some of the de-compositional realizations. These are what are called obvious de-compositional realizations. Now, we have seen the disadvantage of these obvious de-compositional realizations, one of the main disadvantages neither of them can guarantee a fixed unit degrees structure being repeated. In the cascade form or in the parallel form, the moment we have complex factors, you need to pair complex conjugates and therefore, degree 2 is what you need to be prepared for.

In the parallel form, if you have multiple poles and zeros, then you have no choice but to use a cascade parallel form or to use a direct form 2 term in the parallel realization in the partial fraction decomposition, which is of higher degree than maybe even 2, could be 3, degree 4 depending on the multiplicity.

And of course, where (())(12:28) if you have complex poles with multiplicity. Then for each complex pole, the complex conjugate makes a degree 2 factor and the multiplicity multiplies that number 2 by whatever multiplicity there is, so (())(12:46) is the say.