Digital Signal Processing & Its Applications
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Lecture 32c
Lattice Structure

Anyway it is obvious that we would like to have a uniform structure repeated if possible and let uniform structure could be degree one, if we can make it so. Now, what we are going to do is to conceive of the simplest possible degree one structure that we can envisage and then see whether that degree one structure can be repeated to get any rational causal system function.

We will find that we can do almost that, but not quite, there are a few pathological cases where we might not be able to do this, but otherwise in most cases, we shall indeed be able to decompose in the way that we have just described. And that leads us to what is called the lattice structure.

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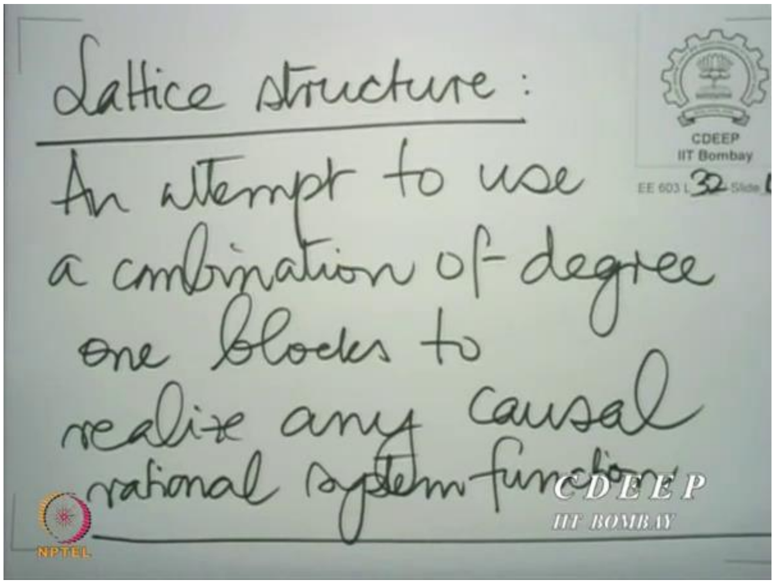

So, let me put down right in the beginning the motivation for the lattice structure. An attempt to use a combination of degree one blocks to realize any causal rational system function. Now, this is not as exciting if you look carefully at the direct form two structure the direct form two structure in a way is that you are using degree one terms after all. I mean, you are using one delay at a time, if you think of it that way.

And if you look at a degree, if you look at a direct form two, what you are doing is you are putting one loop for each additional term in the denominator or numerator. So, although this is exciting, it is not exciting enough. What I also said in the beginning of the lectures is that in addition to being able to realize each should also exhibit certain system properties that is where the lattice structure scores over any of the other structures and we will state that very clear.

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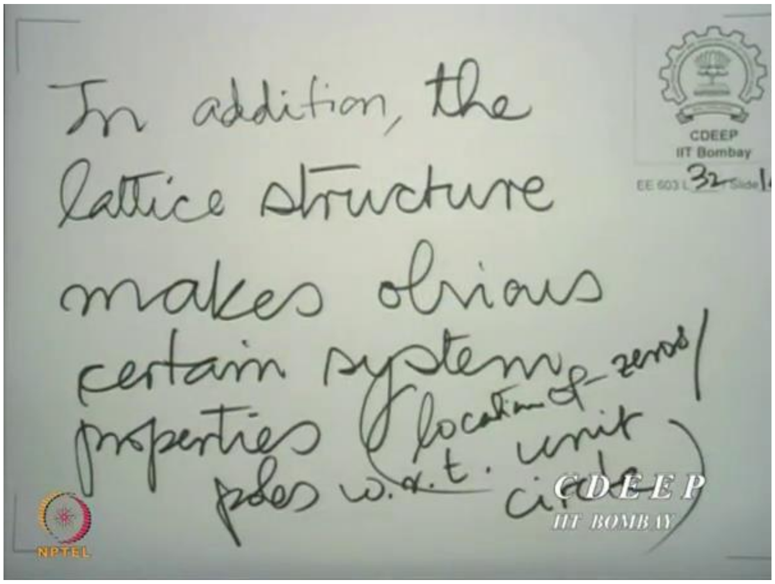

In addition the lattice structure make obvious certain system properties. In fact, location of poles with respect to unit circle. Why poles not only poles of location of zeros or poles, that is was attractive about the lattice structure, in addition to that degree, one repetition. So, in fact, before we you know, we cannot really at this point, understand all of this, we are just trying to put down a wish list and then see if the wish list is realized.

But what we will do right now is first to think of the simplest possible lattice structure that we can have. Now, you know, let us the smallest degree one structure that you can envisage should have at least one multiplier and one delay. Now, there again, you want to realize both the numerator and denominator or least you can do is to have two ranches so to speak, one which will give you a denominator kind of term, and the other which will give you a numerator.

Now, this is kind of intuitive. I mean, in a way what I am doing is I know what I finally have, and I am trying to arrive there by some kind of intuitive reasoning. So, do not worry too much if we do not understand all of this right now. After we have gone through the lattice structure in its entirety everything will fall into place.

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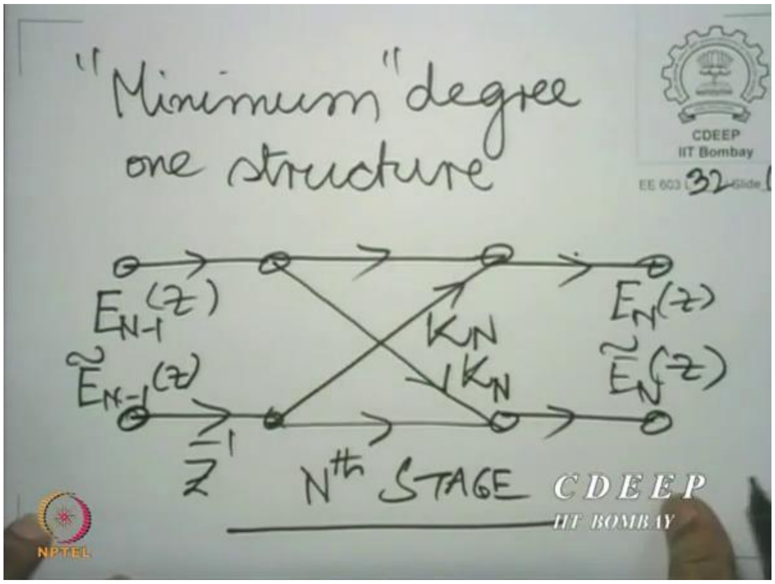

So, what I am saying is that minimum degree to degree one structure primary should look something like this. You need to have a delay somewhere. So, let us put a delay, let us put a delay here. I am showing two inputs and two outputs, I will explain why, you need to have a multiply, you need to have some summation and some multiplication. Let us, put that down. Let us, put, say K times or -K does not matter, K if you like. Now, you could have made done with this. But as I said later, we want to tweak the structure to give feedback as well.

If you have feedback in one direction, there must be some feed forward in some feedback. So, let us keep everything feed forward for the moment. But we will allow for one of them to get converted to feedback. So, let us keep this as K here, again as a multiplier and one there. So, this is what we call this is the minimal degree one structure that we are going to deal with.

And let us this is the, this is going to be repeated. At least write down the Z transforms of the sequences everywhere. So, this is let us assume that this is the Nth stage or the Nth such degree one structure. Therefore the sequence which is carved to here is the so called N- 1th sequence, let us call the sequence here, E_N -1 in the Z domain.

And we will call this E tilde (N-1) in the Z domain. And this is of course called E_N Z and E_N (tilde Z). And since this is the Nth stage, we should also number these coefficients here we will call this the Nth coefficient, K_N and K_N there as well. Let us, write down the relationships between E_N and E_N-1 from this drawing.

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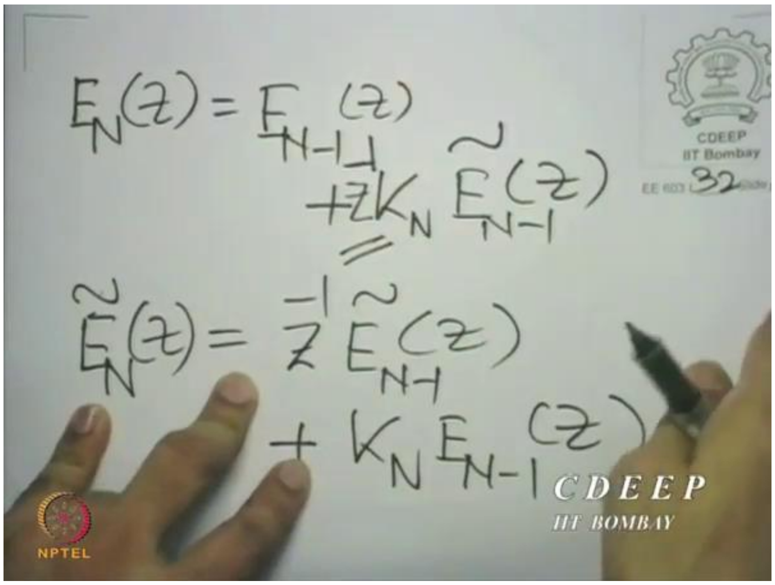

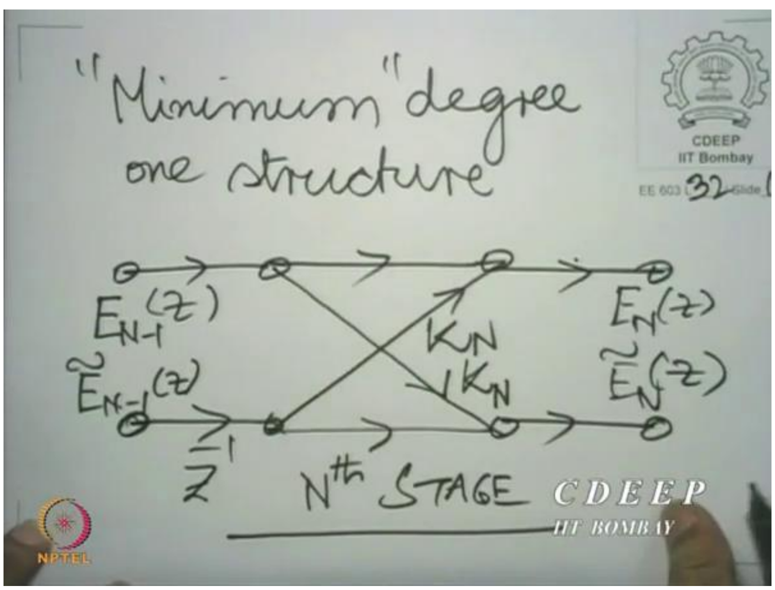

Clearly E_N Z is equal to E_{N-1} (Z) + K_N times E_N -1 tilde Z. I will just show that to you again, E_N (Z) is now see be careful this is E_{N-1} (Z) the truck brings it and transports it here and puts just E_N -1 here, this truck and this truck together carry E_N -1 and K_N times Z -1 E_N - 1 tilde Z times Z -1.

So, I need a Z^{-1} there two. Let us, not forget the Z^{-1} so Z^{-1} (E_{N-1} tilde Z) appears here multiplied by K_N.

What about E_N tilde? E_N tilde takes K_N times this so K_N E_{N-1} + Z^{-1} E_{N-1} tilde transported as it is. So, we have E_N (tilde Z) = Z^{-1} E_{N-1} tilde Z +(K_N times E_{N-1} Z). In the very beginning this is the Nth stage N can be 1, 2, 3 and so on. In the very beginning in stage number one you would have K1 here and you here you have E0 and E0 tilde here.

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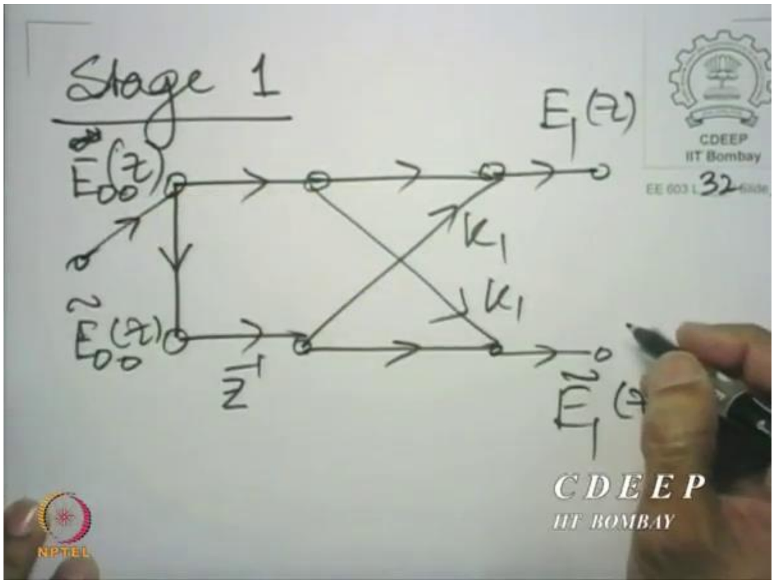

So, let us consider stage one. And let us derive some insights from stage one. You have E0 E0 tilde there or E0 Z and E0 tilde Z, Z^{-1} as you will K1 up there and K1 down here. E_1 Z here and E_1 (tilde Z) there let us explicitly write down the expressions and let us join them together and take this to be the input. Now, let us see what happens.

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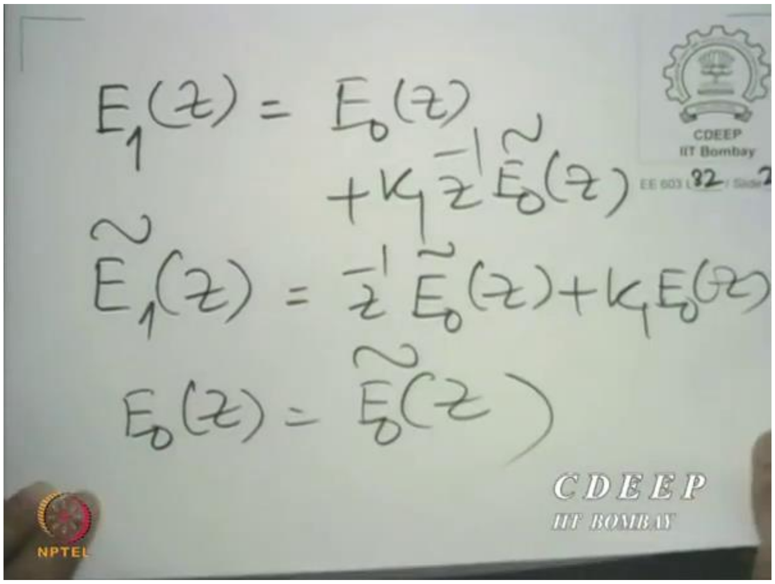

Clearly, E1 Z would be equal to you know it is going to be E0 (Z) + K_1 Z $^{-1}$ E0 (tilde Z) and E1 (tilde Z) is Z $^{-1}$ E_0 (tilde Z) + K1 E_0 (Z) but E_0 (Z) = E_0 (tilde Z) and therefore we could write down E_1 (Z) by E_0 (Z) and E_1 (tilde Z) by E_0(tilde Z).

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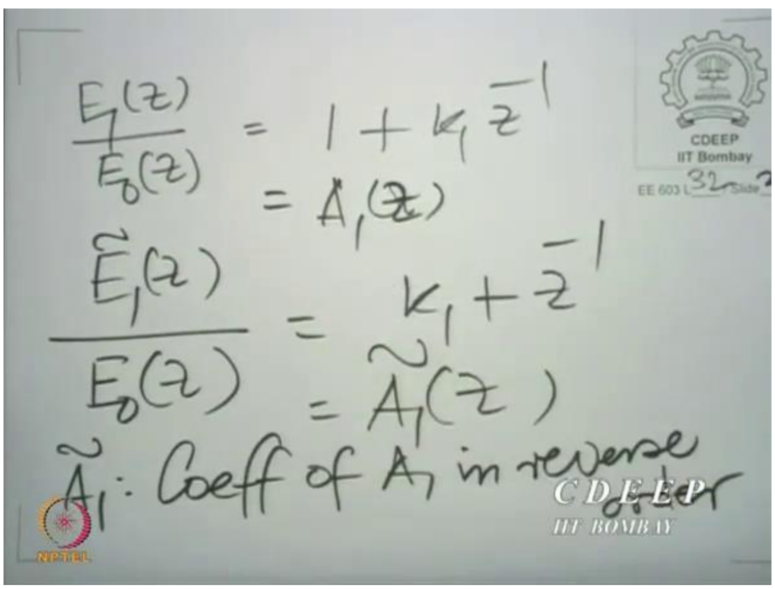

E_1 (Z) / E_0 (Z) or E0 tilde Z is does not matter is 1 + K_1 Z $^{-1}$ and (E1 tilde Z) /(E0 Z) =(K1 + Z $^{-1}$) and let us call this A_1 (Z) and let us call A_1 (tilde Z) and we note a something very interesting as a relation between A_1 and A_1 tilde is the coefficients of A1 reverse order so here the coefficients of the successive power of Z^{-1} are 1 and K1 here the coefficients of the successive powers of Z^{-1} are K1 and 1 we are in reverse order. And that can be expressed formally in the language of Z transforms as follows.

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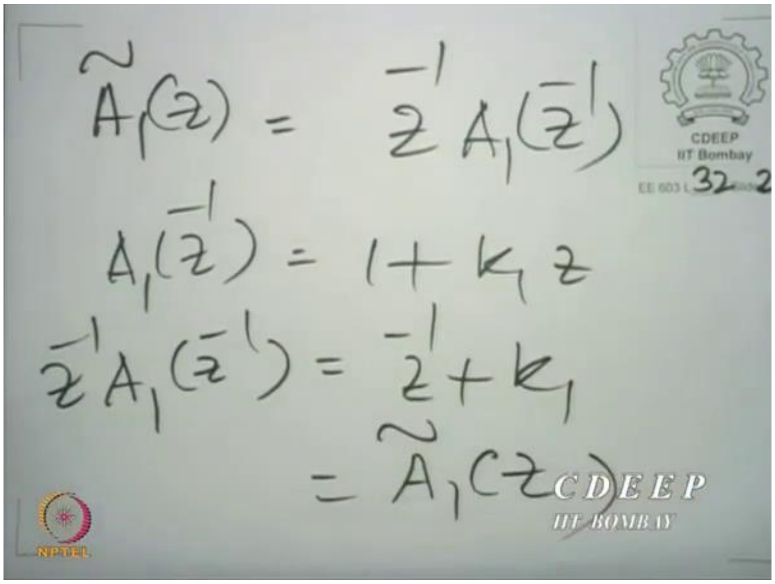

A1 (tilde Z) = (A1 Z^{-1}) * Z^{-1} that you can easily verify. So, A1 Z^{-1} could be (1 + K_1 Z) and therefore Z^{-1} A1 Z^{-1} would be (Z^{-1} + K1) which is indeed A1 tilde Z verified. Now, if we do indeed connect E0 and E0 tilde together as we have done here. We can now show the mathematical induction that this always be the case that means if we define A and tilde to be the ratio of EN/E0 mind you E0 and E0 E0 tilde are connected so, puts stages one after the other first stage second stage third stage but connect E0 and E0 tilde together.

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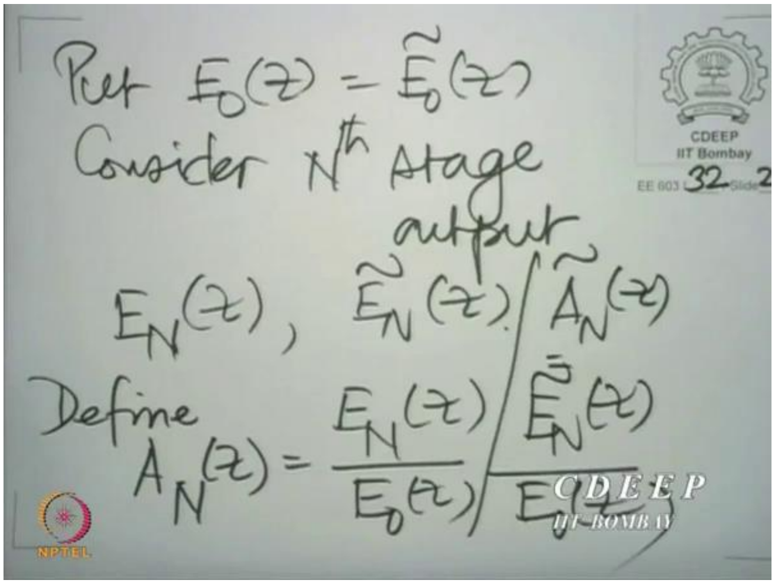

So, what I am saying is this in the lattice structure put E0 equal E0 tilde as we have done here and then consider the Nth stage output EN and EN tilde define A_N(Z) to be E_N (Z)/ E_0 (Z) and A_N (tilde Z) to be E (tilde Z) / E_0(Z tilde) for tilde and non-tilde for non-tilde.

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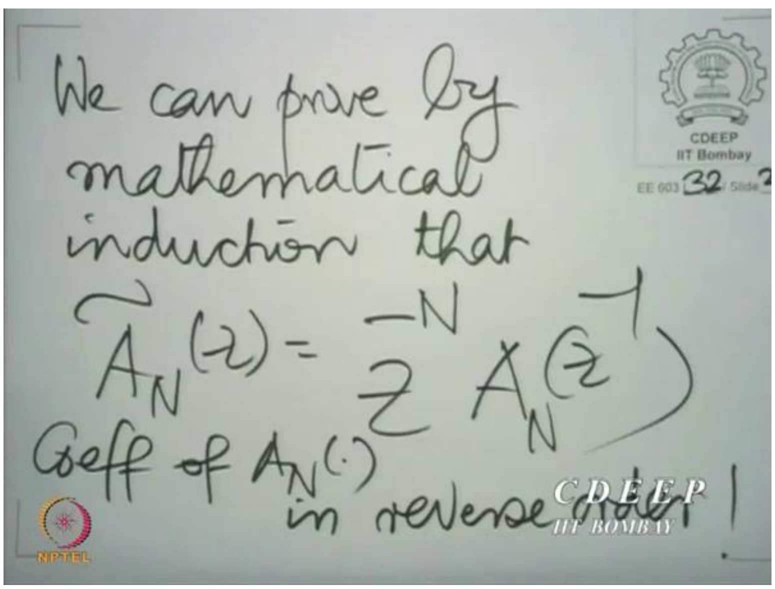

What we are saying here is that we can prove by mathematical induction that A_N (tilde Z) is Z $^{-N}$ An Z $^{-1}$ or the coefficients of A_N in reverse order this is the language for saying the coefficients of A_N (tilde Z) are coefficients of A_NZ in the reverse order we shall not prove this my mathematical induction. Now it is very simple we have already competed the base step because we proved it for N equal to 1.

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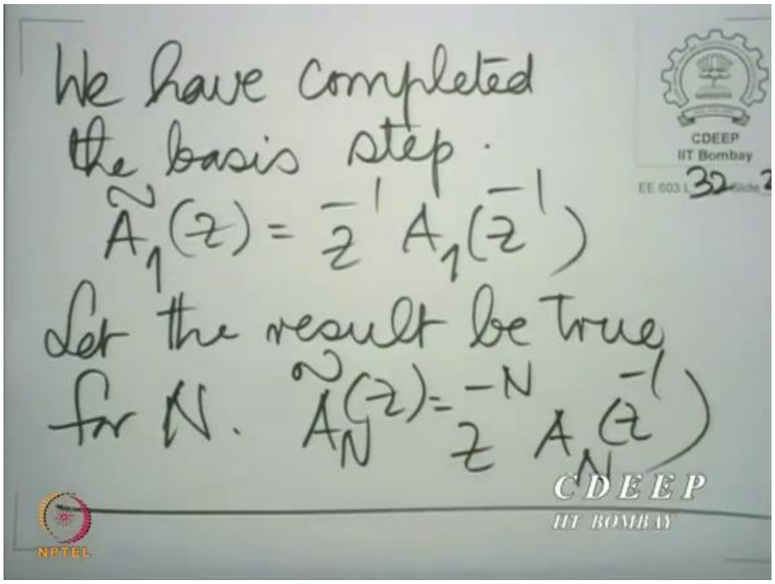

let us, now complete the inductive step so let us write that down, We have completed the base step, Indeed A1 (tilde Z) = Z $^{-1}$ A1(Z $^{-1}$). Let us this be true, let the result be true for N and N can begin with 1 and therefore An (Z) An (tilde Z) is indeed Z $^{-N}$ A_N (Z $^{-1}$) let this be true. We shall show that it is true for N replace by N+ 1.

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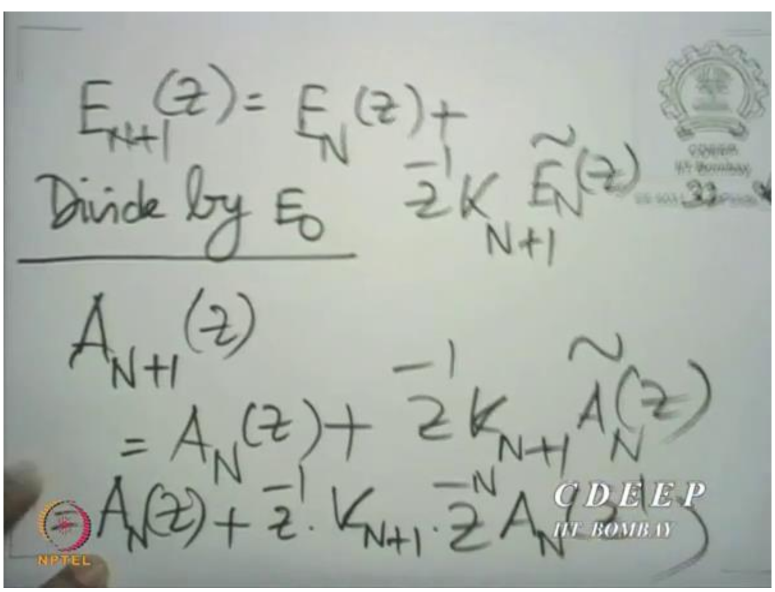

So, indeed let us consider the relationships between A_{N+1} and A_N you see E_{N+1} (Z) is of course as you know E_N (Z) +Z $^{-1}$ K_{N+1} E_N (tilde Z). Where upon the same relationship hold between the A's because you can divide by E0 on both sides.

Say that A_{N+1} (Z) is A_N (Z)+ Z $^{-1}$ K_N+_1 A_N (tilde Z) but we write them expression for AN (tilde Z) by mathematical by the assumption of mathematical induction, the inductive assumption. Then we write this as A_N Z here plus Z $^{-1}$ K_{N+1} (Z $^{-N}$)A_N Z $^{-1}$. And the beauty is we have got A_N+_1 entirely in term of AN there tilde anywhere. And assume it can be done to A_{N+1} tilde.

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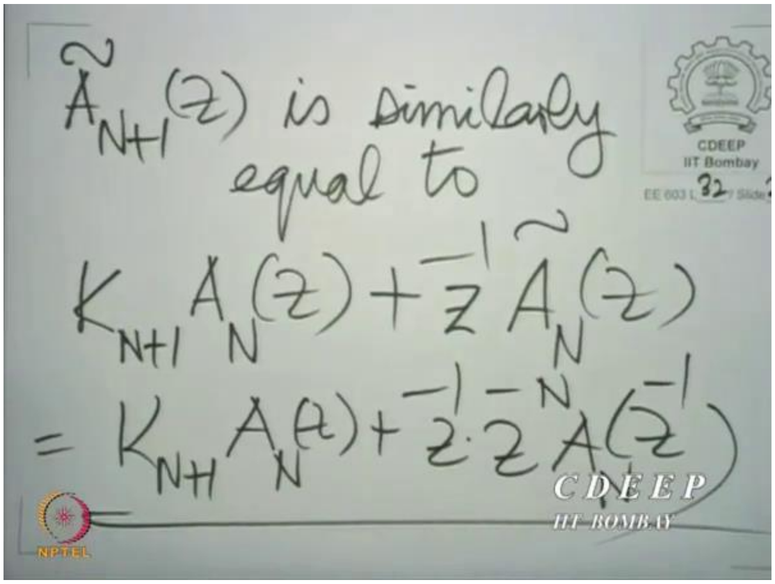

A_{N+1} (tilde Z) is similarly equal to K_N + A_N Z + Z $^{-1}$ A_N (tilde Z) which is of course, K_{N+1} A_N Z + Z $^{-1}$ times Z $^{-N}$ times A_N Z $^{-1}$. Now, it is a very simple matter. What we need to do is to compare the coefficient reversed form of A_N with A sorry, the coefficient reverse form of A_{N+1} with A_{N+1} tilde.

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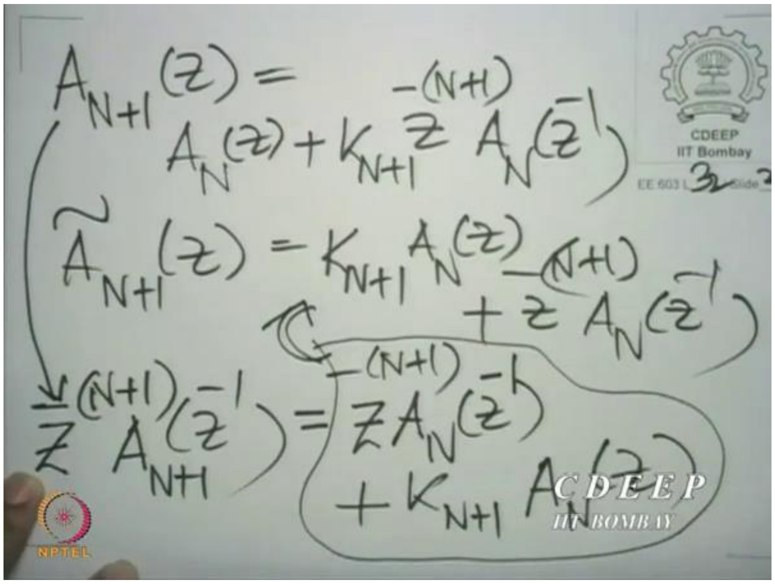

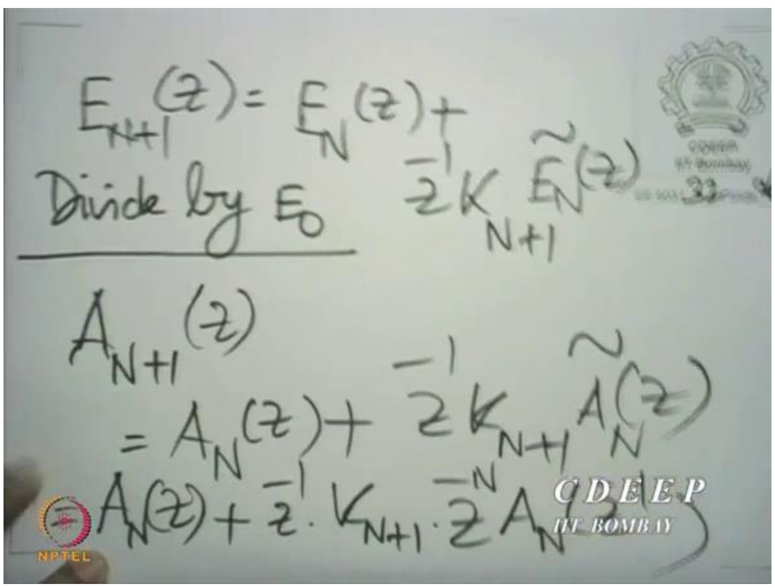

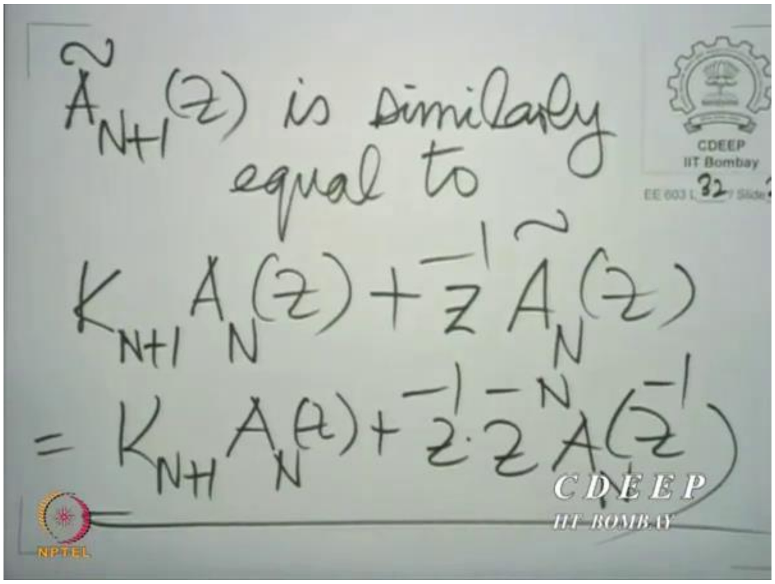

So, let us just write down you see let us write down the two equations together to make matter easy. So what we are saying is let us write down this equation A_{N+1} Z is from here this equation we will write it down explicitly. Essentially, AnZ + K_{N+1} Z^{-N+1} please note A_N Z^{-1} and A_{N+1} (tilde Z) from here is K_{N+1} A_NZ + Z^{-N+1} here.

So, K_{N+1} A_NZ +Z^{-N+1}, A_N Z^{-1}. And now, from here let us write down the coefficient reversed form. You see, it is very easy to show that Z^{-N+1} A_{N+1} Z^{-1} would involve two steps, it would involve replacing Z by Z^{-1} so you see this would become Z^{-1} here and this will become Z^{-1} so Z.

This would also get replaced by Z^{N+1} instead of Z^{-N+1} and then we will multiply by Z^{-N+1} there would be a factor of Z^{-N+1} coming in here and this Z^{N+1} would go away with the Z^{-N+1}.

So, it is very clear that this gives you Z^{-N+1} times An (Z) + K_{N+1} A_N Z^{-1} I am sorry so, it will be Z^{-N+1} A_N Z^{-1} you know Z Z gets replaced by Z^{-1} here plus K_{N+1} this term is got cancelled and this term has become A_N Z. Now, compare this expression and this expression here there exactly identical.

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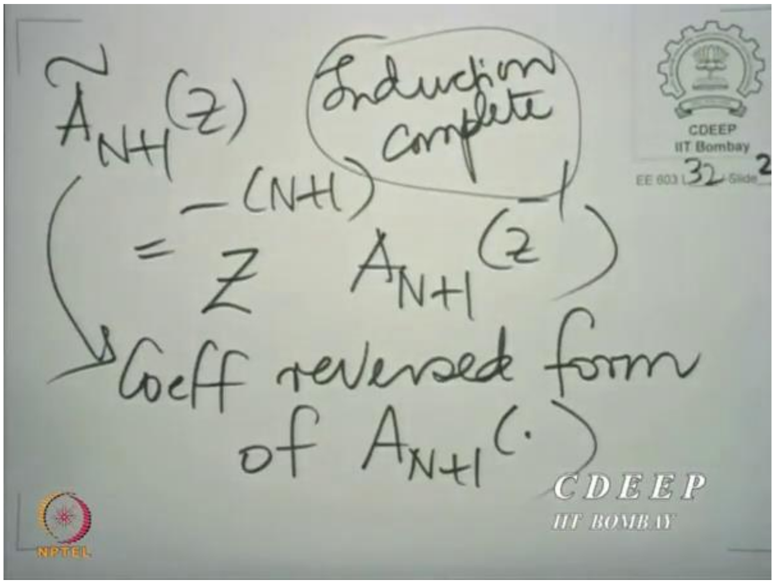

And therefore, it is very clear that A_{N+1} (tilde Z) is indeed Z^{-N+1} A_{N+1} Z^{-1} which means, essentially this is the coefficient reversed form of A_{N+1}. So, induction complete. What is true for N has also been shown to be true for N + 1. Now it is mystery why this property has excited us so much.

We will see that in the next lecture. Why is it that we are so excited about this coefficient reversal form. Anyway I mean, I would certainly urge you to verify that this operation of replacing Z by Z^{-1} first and then multiplying by Z raised to the highest negative power equal to the degree thus coefficient reversal. I urge you to verify it.

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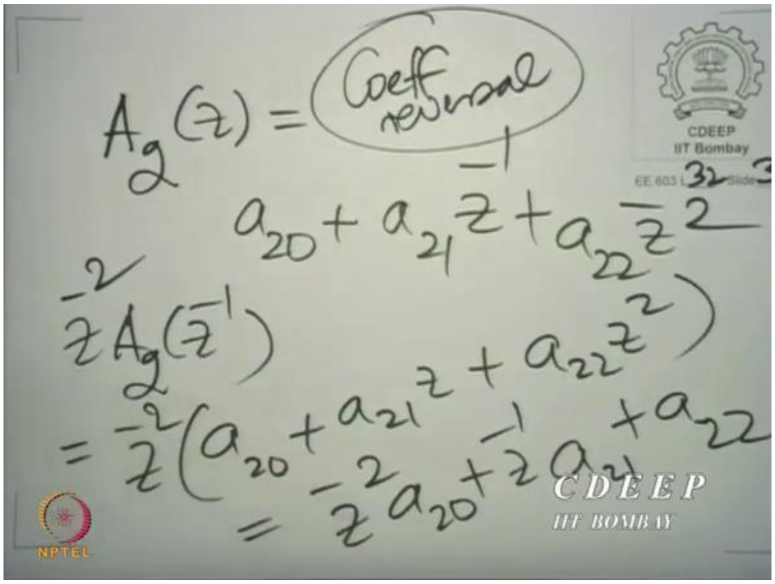

Anyway what I will do is illustrate by one example before we conclude. So, suppose we have second degree term so for example we had A_2 Z which is of the form some (a20 + a21) Z^{-1} + a22 Z^{-2}, then A2 Z^{-1} times Z^{-2} its degree 2 would become Z^{-2} times (a20 + a21) Z + a22 Z^2 which is very clearly Z^{-2} a20 + Z^{-1} a21 + a22.

So, it is the coefficient reversed form you see the coefficients in increasing powers of Z are a22, a21, a20. Whereas, the a20, a21 a22 here. And you can see from this the mechanism of coefficient reverse. How coefficient reversal occurs? The mechanism is clear from this example. This property of coefficient reversal will actually help us.

One way to look at it is it. So, it gives a very clear relationship between these two system functions AN and AN tilde. And it is a relationship that we will use very fruitfully data to derive another structure which has feedback, you see, the structure that we had in this lecture has no feedback. So, we could use it to realize only FIR system functions.

What we shall do in the next lecture is to investigate how we can use this structure to realize any of FIR system function. That means you see what we have right now is a forward recursion. But now we need a backward recursion. How do you go from AN + 1 to AN? We have gone from AN to AN + 1. But what we will have is the final structure and want to peel off stage by stage to get the individual case. That is what we shall do in the next lecture. Thank you.