Digital Signal Processing and Its Applications
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Lecture No. 34 b
Example realization of lattice structure for a rational system

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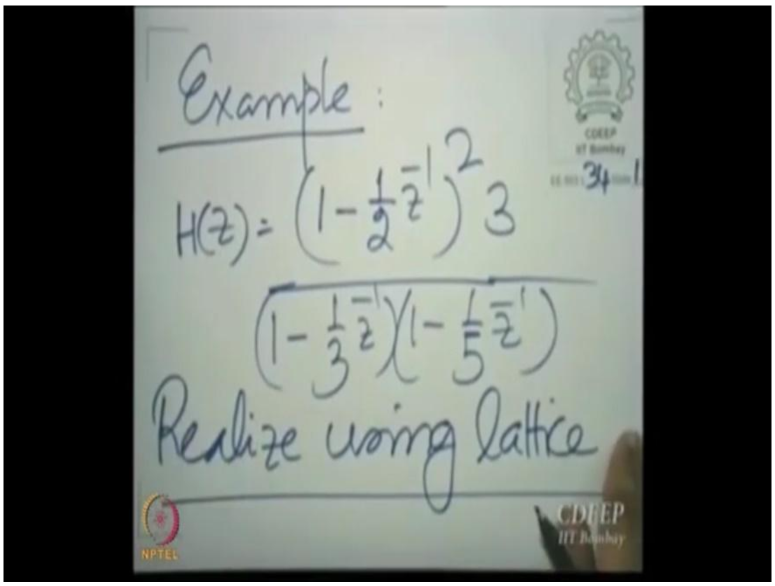

Let us construct $H(z) = \frac{3\left(1-\frac{1}{2}z^{-1}\right)^2}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{5}z^{-1}\right)}$ if you like using the lattice structure. Let us expand this. So we would need to expand this because it is not in the pole form or pole-zero form that the lattice structure can be realized; we need it explicitly as polynomials.

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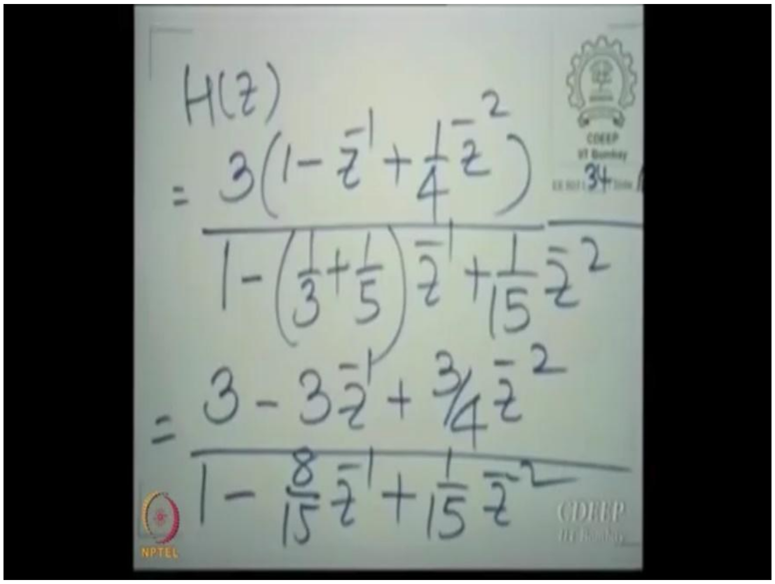

And therefore we need to expand this, let us do so. So $H(z) = \frac{3\left(1-z^{-1}+\frac{1}{4}z^{-2}\right)}{1-\left(\frac{1}{3}+\frac{1}{5}\right)z^{-1}+\frac{1}{15}z^{-2}}$. So that can be expanded as $\frac{3-3z^{-1}+\frac{3}{4}z^{-2}}{1-\frac{8}{15}z^{-1}+\frac{1}{15}z^{-2}}$.

Is that correct? Now let us start by realizing the denominator by $A_N(z)$. So of course very clearly now here I have conveniently chosen the numerator degree to be less than or equal to the denominator degree, if it were not I would first need to carry out long division of the numerator by the denominator.

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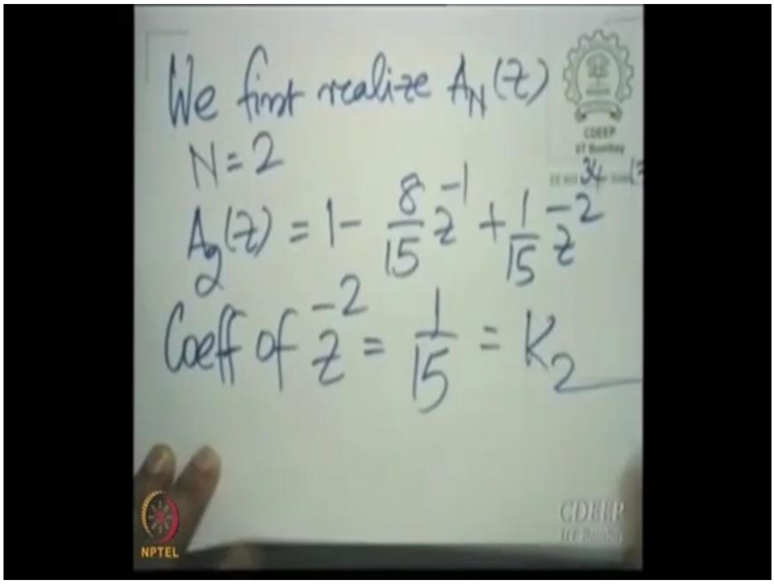

So we first realized $A_N(z)$. Clearly $N = 2$. So $A_2(z) = 1 - \frac{8}{15}z^{-1} + \frac{1}{15}z^{-2}$. And clearly the coefficient of z^{-2} in z^{-1} is $\frac{1}{15}$ and this should be equal K_2. Therefore $K_2 = \frac{1}{15}$. Now we know the relation between A_2 and A_1. Just for the sake of clarity though we have done it in general just for the sake of clarity let us write down that relationship explicitly.

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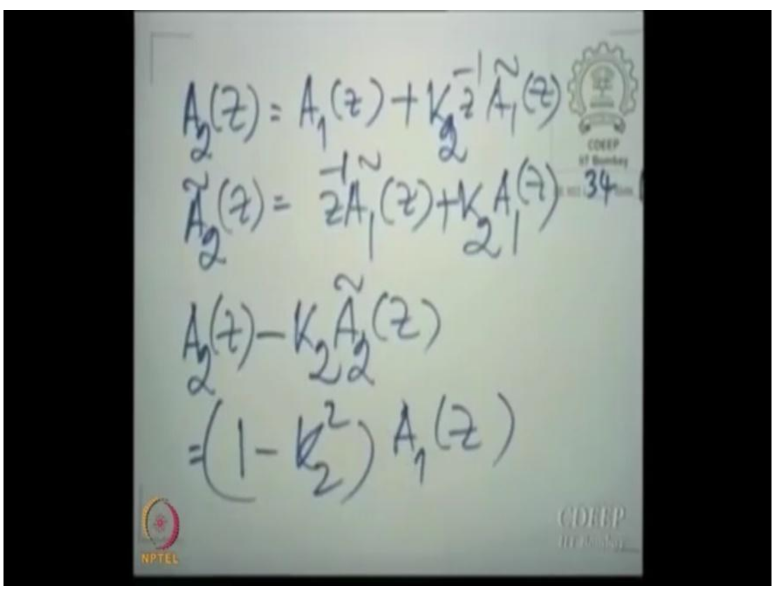

We know that $A_2(z) = A_1(z) + K_2 z^{-1}\widetilde{A}_1(z)$. And $\widetilde{A}_2(z) = z^{-1}\widetilde{A}_1(z) + K_2 A_1(z)$, is that correct? And therefore, if we take $A_2(z) - K_2\widetilde{A}_2(z)$, then this term would vanish and it would leave us with $\left(1 - K_2^2\right)A_1(z)$. I am just redoing that step to make it explicit though we have done it in general for $N + 1$. You do not need to do this every time.

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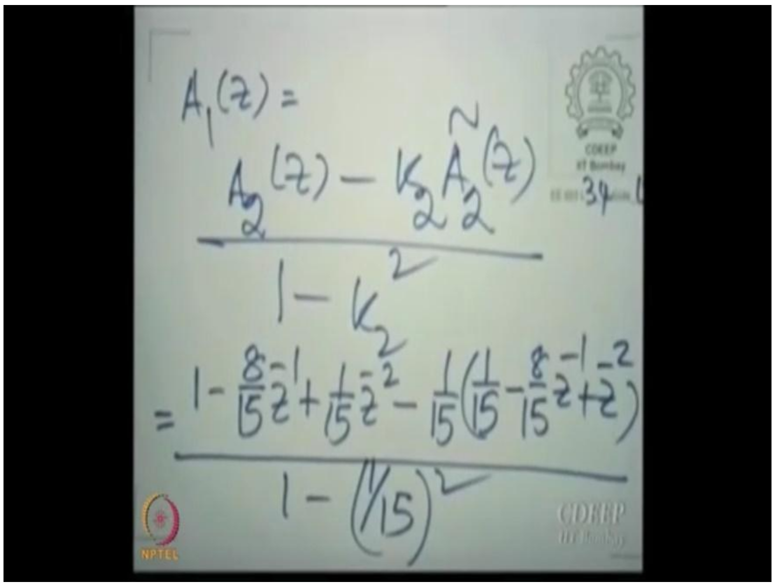

And therefore $A_1(z) = \frac{A_2(z) - K_2\widetilde{A}_2(z)}{1 - K_2^2}$ and this is very easy to write. Now $A_2(z) = 1 - \frac{8}{15}z^{-1} + \frac{1}{15}z^{-2}$ and $K_2 = \frac{1}{15}$. Now $\widetilde{A}_2(z)$ is essentially this written in reverse order. So we have $\frac{1 - \frac{8}{15}z^{-1} + \frac{1}{15}z^{-2} - \frac{1}{15}\left(\frac{1}{15} - \frac{8}{15}z^{-1} + z^{-2}\right)}{1 - \frac{1}{15^2}}$.

The question is what is the difference between a one person has asked what is the difference between $A_2(z)$ and $\widetilde{A}_2(z)$. They are very, very different. The coefficients are very different. You need to look very carefully to observe that they are very different. The coefficients are in reverse order. One needs to pay careful attention to understand the example. They are very different. The coefficients are in reverse order. It is 1, $-\frac{8}{15}$ and $\frac{1}{15}$ for $A_2(z)$.

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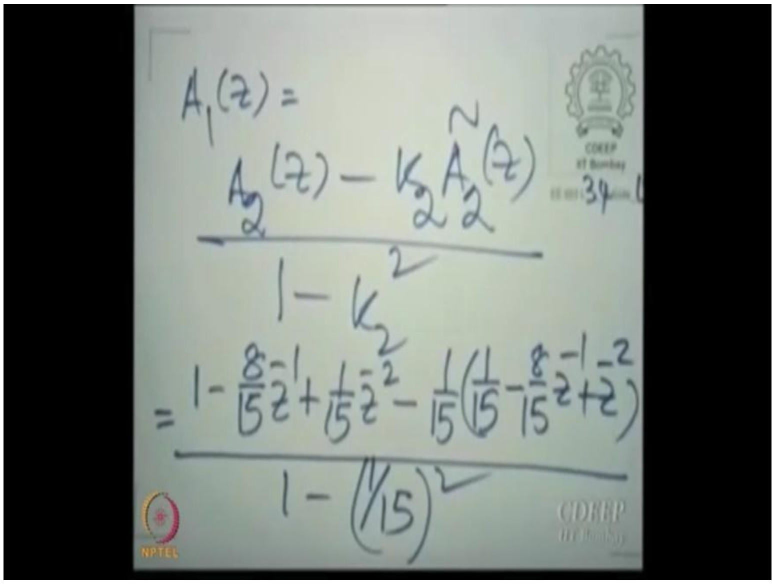

And here for $\widetilde{A}_2(z)$ it is $\frac{1}{15}$, $-\frac{8}{15}$ and 1, hence they are different. Now if you look at this difference $\frac{1}{15}z^{-2}$ is eliminated because you have $\frac{1}{15}z^{-2} - \frac{1}{15}\left(z^{-2}\right)$ and that is expected. That has to happen because the degree must go down by 1 right.

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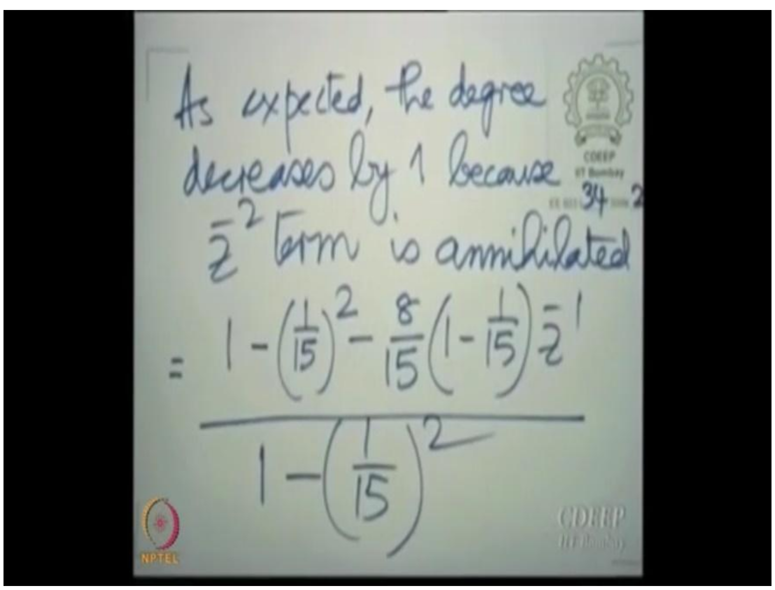

So as expected the degree decreases by 1 because the z^{-2} term is eliminated. And that leaves you with $\frac{1-\frac{1}{15^2}-\frac{8}{15}\left(1-\frac{1}{15}\right)z^{-1}}{1-\frac{1}{15^2}}$. So, in fact, we can even simplify this further, is that correct? We get this don't we?

Student: $1 - \frac{1}{2}z^{-1}$.

Professor: we can simplify this. So you see is this expression correct, this is correct? Yes. Now we can simplify because we can note that this is of the form $1 - x^2$ and this is $1 - x$. So you can write $1 + x$ here. You can cancel this.

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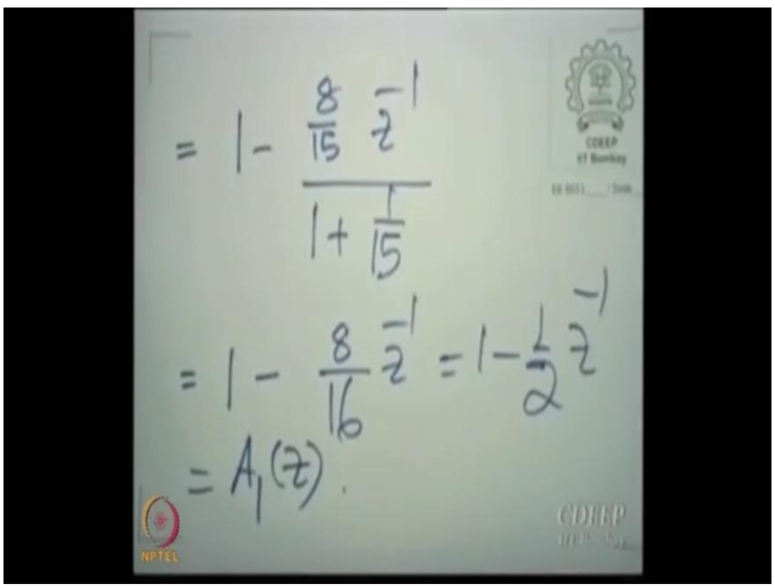

So you can take it separately in either way whatever you like. So you get $1 - \frac{\frac{8}{15}z^{-1}}{1+\frac{1}{15}}$ and that leaves you with $1 - \frac{8}{16}z^{-1} = 1 - \frac{1}{2}z^{-1}$. As expected this is again this is $A_1(z)$, is that correct? Now again it should be noted as expected that the coefficient of z^0 in $A_1(z)$ again comes out to be 1 as expected.

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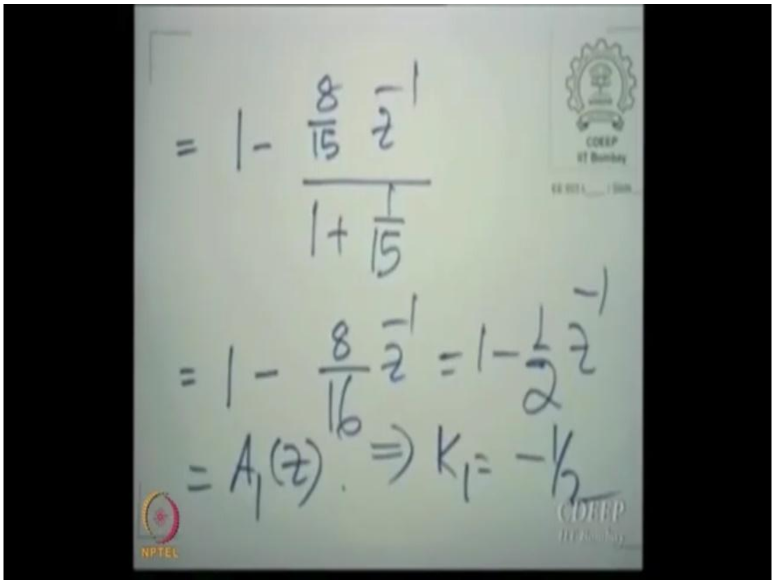

And therefore the coefficient of the highest power of z namely z^{-1} in $A_1(z)$ is essentially K_1 which means $K_1 = -\frac{1}{2}$. And once you have K_1 you have completed the lattice for the denominator. But now we need to write the lattice for the numerator. Essentially what we have is a lattice of the following form. We have an E_2 there.

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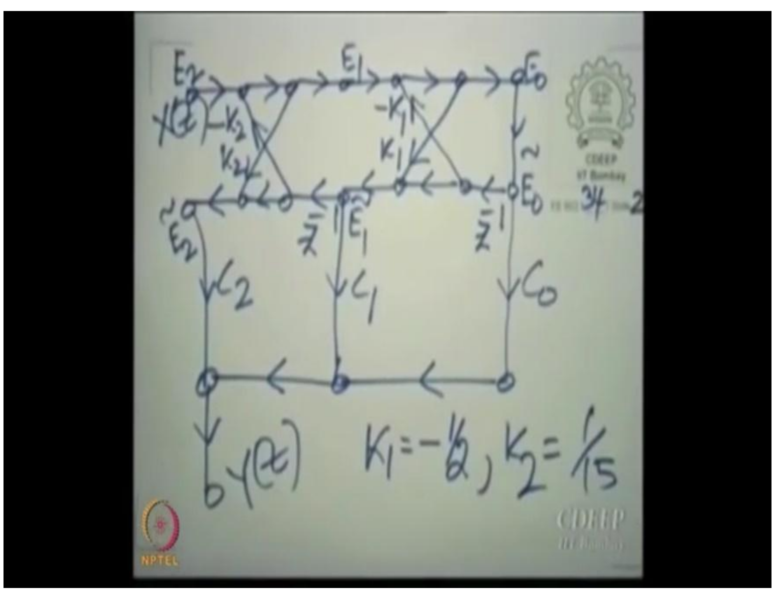

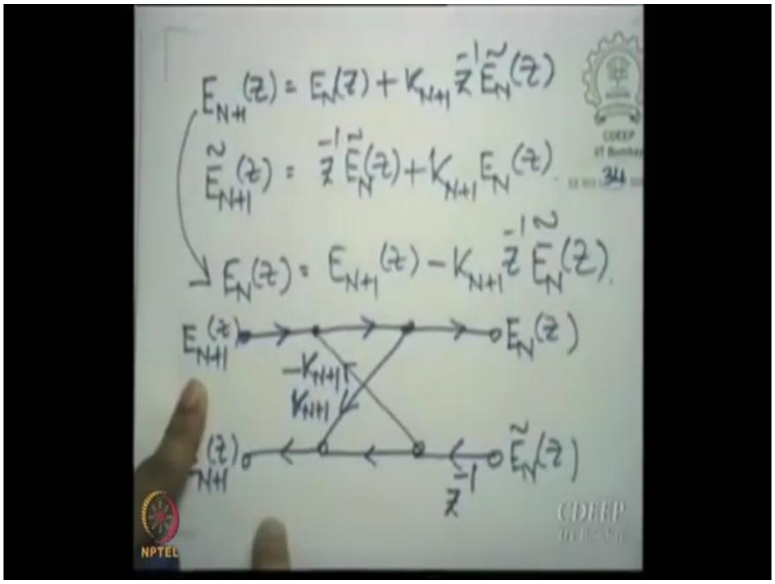

And let us go back to the generic structure and then draw it for this specific case. This is the generic structure. We need a $-K_{N+1}$ and a K_{N+1}. And when you have z^{-1} every time you take one step. So you need a z^{-1} here and you need a z^{-1} here and you need a minus you see remember it is E_2 here and $\tilde{E}_2$ here.

You do not need to keep writing the others but you have a $-K_2$ there and the K_2 here and the $-K_1$ here and the K_1 here. And this is E_0 and this is $\tilde{E}_0$. And of course this is E_1 and this is $\tilde{E}_1$. So we need to take c_0 times this plus c_1 times this plus c_2 times this and add them. So X is given here and Y is tucked off here. And we now need to obtain c_0, c_1 and c_2.

So we know the values we know $K_1 = -\frac{1}{2}$ and $K_2 = \frac{1}{15}$ and please note that we had assumed a stable causal system to begin with. In fact, we explicitly wrote the system in terms of its poles. You see that is why, that is the reason why I wrote the system in factor form in the numerator and denominator first. You see if you look back let me put that system function back here before you.

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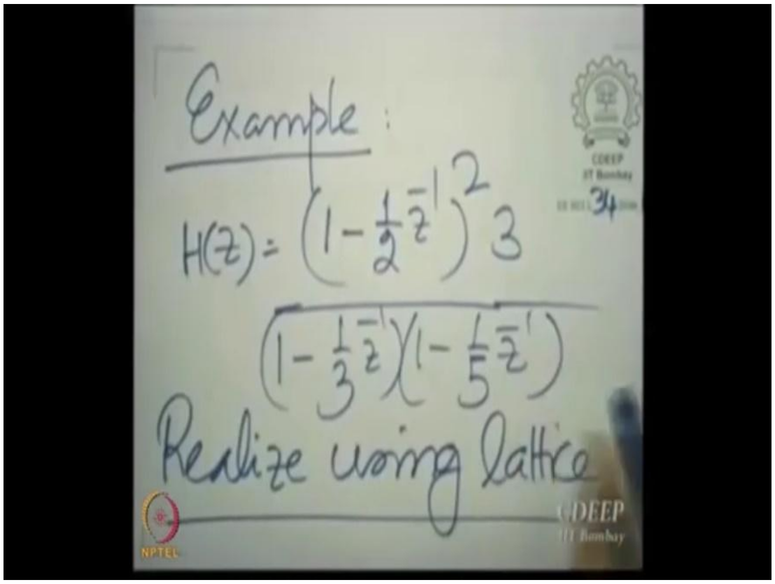

The system function has poles which are both inside the unit circle and therefore the system is stable.

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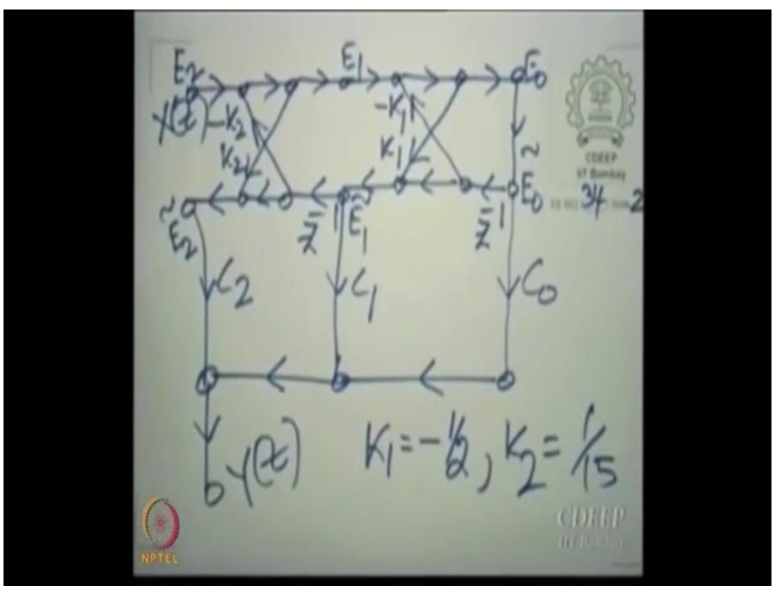

And as expected the denominator polynomial yields both lattice coefficients less than 1 in magnitude as was the necessary and sufficient condition. This is a verification; not a proof, but a verification. Now we need to determine the c_0, c_1, c_2. So let us begin. Now we know what these are, we know what the A's are.

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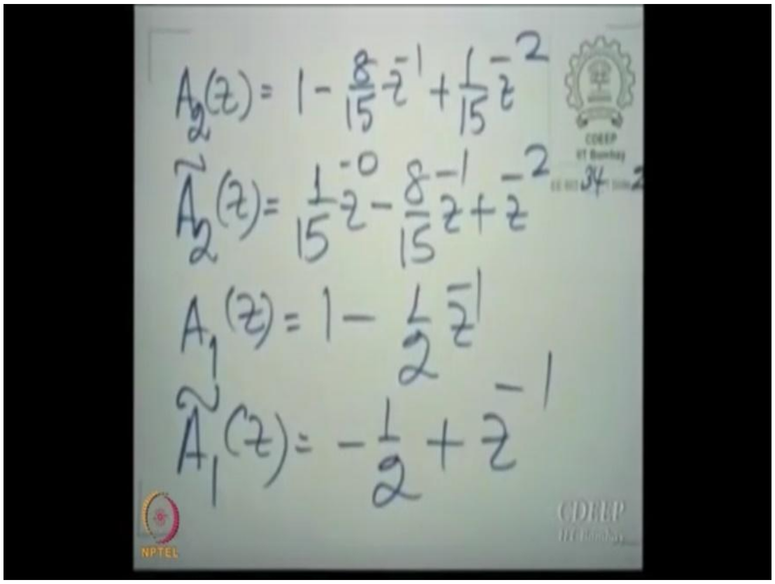

$A_2(z) = 1 - \frac{8}{15}z^{-1} + \frac{1}{15}z^{-2}$. $\tilde{A}_2(z) = \frac{1}{15} - \frac{8}{15}z^{-1} + z^{-2}$. $A_1(z) = 1 - \frac{1}{2}z^{-1}$. And therefore $\tilde{A}_1(z) = -\frac{1}{2} + z^{-1}$. And of course $A_0(z)$ and $\tilde{A}_0(z)$ are identically 1.

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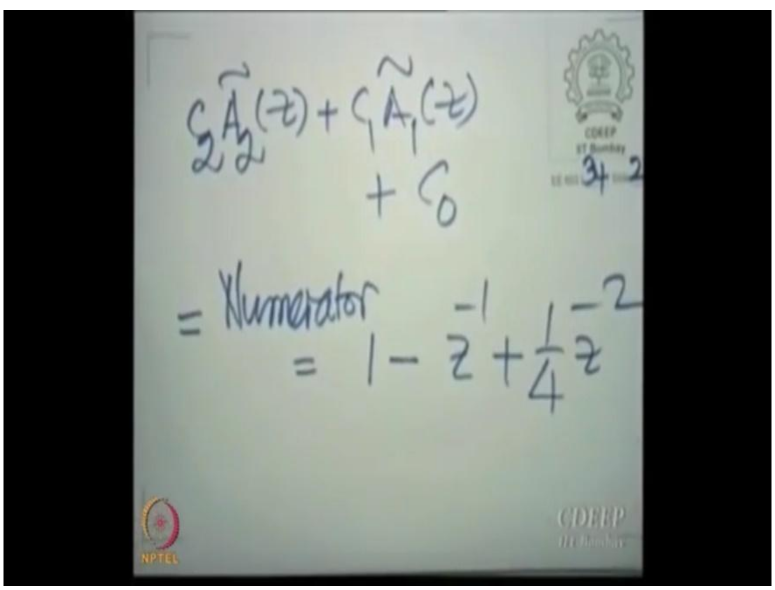

And therefore what we have you see $c_2\widetilde{A}_2(z) + c_1\widetilde{A}_1(z) + c_0\widetilde{A}_0(z)$. Now $\widetilde{A}_0(z) = 1$. This is equal to the numerator and the numerator is $1 - z^{-1} + \frac{1}{4}z^{-2}$. Now let us expand this relationship.

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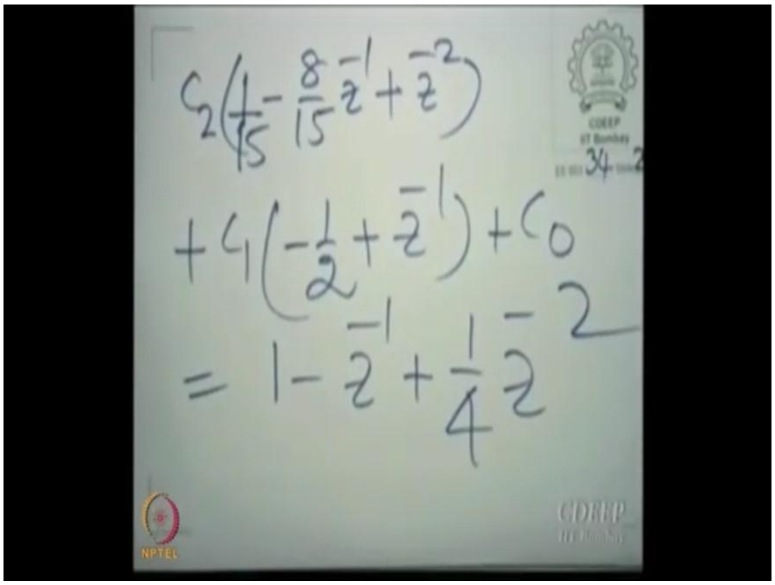

$c_2\widetilde{A}_2(z)$ which is $c_2\left(\frac{1}{15} - \frac{8}{15}z^{-1} + z^{-2}\right) + c_1\left(-\frac{1}{2} + z^{-1}\right) + c_0 = 1 - z^{-1} + \frac{1}{4}z^{-2}$. As expected the coefficient of z^{-2} can be compared on both sides; it comes only from here.

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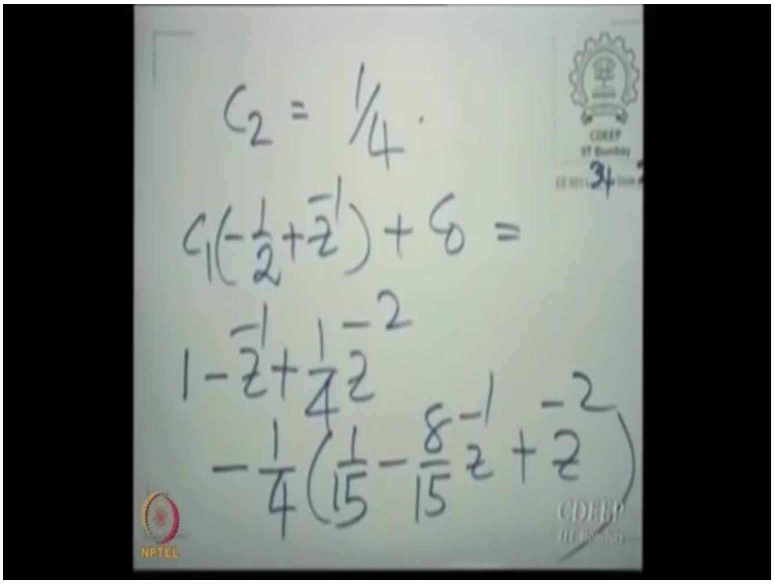

And therefore we have $c_2 = \frac{1}{4}$. And therefore we have $c_1\left(-\frac{1}{2} + z^{-1}\right) + c_0 = 1 - z^{-1} + \frac{1}{4}z^{-2} - \frac{1}{4}\left(\frac{1}{15} - \frac{8}{15}z^{-1} + z^{-2}\right)$, subtracting the term with c_2 from both sides. Now after you have subtracted as expected you see that $\frac{1}{4}z^{-2}$ is annihilated. So, $\frac{1}{4}z^{-2} - \frac{1}{4}\left(z^{-2}\right)$, so you get a degree 1 term only and then let us write that down.

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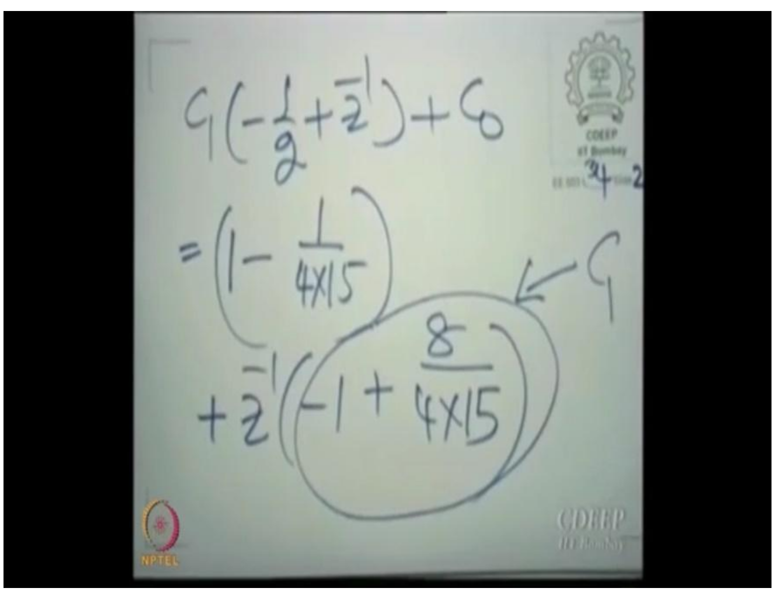

So, therefore $c_1\left(-\frac{1}{2}+z^{-1}\right)+c_0=\left(1-\frac{1}{4\times15}\right)+z^{-1}\left(-1+\frac{8}{4\times15}\right)$ and the z^{-2} term is annihilated. And therefore c_1 is now very clear as well. This is essentially c_1 now. The coefficient of z^{-1} is c_1. So once you have c_1 you can subtract this term from this side again and find c_0 finally. We can continue this.

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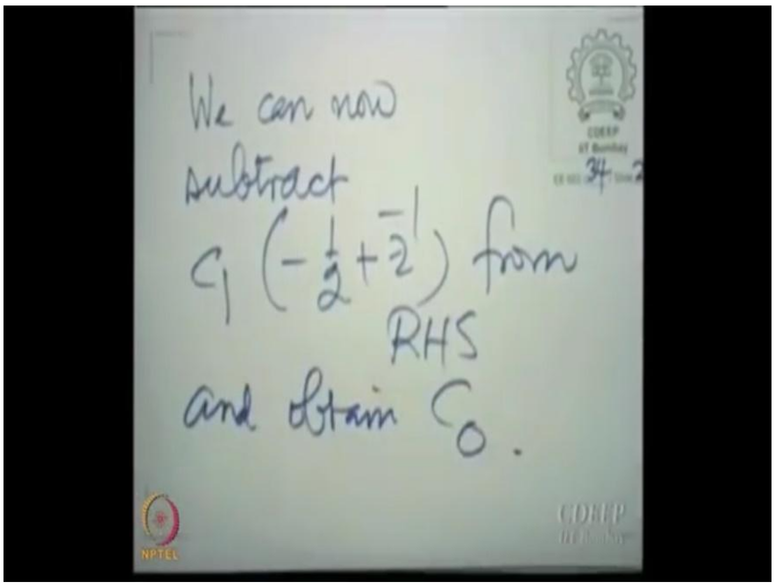

We can now subtract $c_1\left(-\frac{1}{2}+z^{-1}\right)$ from the right hand side and obtain c_0. And I will leave that to you as an exercise. So this is illustrated the process of obtaining a lattice realization of an $H(z)$.

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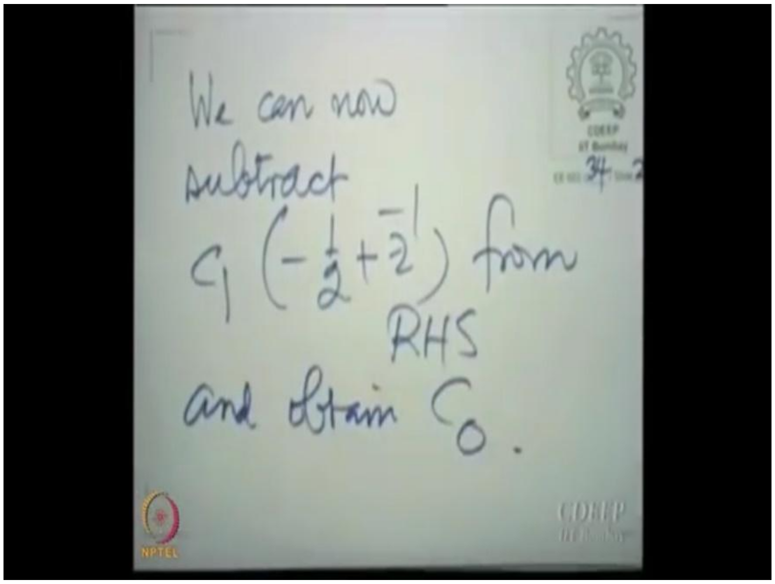

And this lattice realization can be used to realize either an FIR system function or an IIR system function. In the case of an IIR system function the lattice structure gives an insight into the stability. In the case of an FIR system function the lattice structure is just another structure which is regular. And in the assignment on filter design you are encouraged to realize both the FIR and the IIR structures using the lattice form.

Student: So we will have to add the final gain also.

Yes now you see what I have done now yeah that is a good point you see what I did here was to ignore the gain of 3 and one there are 2 ways in which you can do this. Either you could multiply all the C's by 3 or you could put one gain of 3 in the end. So I had taken the numerator without the gain of 3 here. But you could either incorporate that gain of 3 in all the C's, so you could multiply each of the C's by 3 or you could just put one gain of 3 finally, either way is fine.

That is a good point, somebody has pointed that out. So that completes the lattice structure and for the moment we are done with the subject of realization. In the next lecture we shall begin on a very important theme in discrete signal processing namely the discretization of the frequency axis. If you want to understand the frequency domain behavior we need to discretize the frequency axis. How do we do it and how do we do it efficiently? We shall discuss this beginning with the next lecture. Thank you.