Circuit Theory

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Lecture - 11
Amplitude and Phase of Network Functions

This is the eleventh lecture and today's lecture starts a discussion of amplitude and phase of network functions.

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In particular, we shall see the relationship between poles, zeroes amplitude phase, real part, imaginary part of network functions. In other words, we are going to discuss network functions in details, along with their anatomy and physiology. We are going break it up into different parts and look at different parts closely and so on. To do that, we first establish the fact that network functions of LLFPB networks are real and rational.

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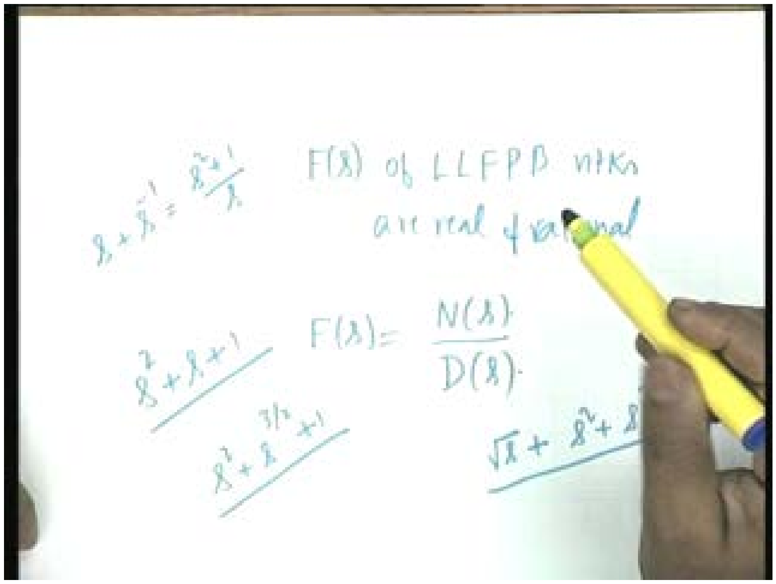

First let us understand what these terms real and rational mean? A function is rational, if it can be expressed as the ratio of polynomials. A function F of s is rational, if it can be expressed in the ratio of polynomials. What is a polynomial? Polynomial is a finite series with integral powers of the variable. For example, s squared plus s plus 1 is a polynomial in s, but s squared plus s to the 3 by 2 plus 1 is not a polynomial in s because it contains non integral powers. Root s plus s square plus s cubed is not a polynomial in s because it is it contains an irrational function square root of s, where s to the power half.

On the other hand, this can be, this is a polynomial in square root of s in s. You must specify the variable. A polynomial in s is a finite series. E to the s, for example, is not a polynomial because it is infinite. E to the s is 1 plus s plus s square by 2 factorial and up to infinity, it is not a polynomial. A polynomial is a finite series. It must terminate at a finite power of n and must not contain any fractional powers. It must contain only integral powers. Another restriction, the powers must be positive, powers must be positive integers. For example, s plus s to the minus 1 is not a polynomial in s. What it is here is, it is a rational function in s. That means the numerator is a polynomial in s, the denominator is a polynomial in s. So to repeat, a function is rational if it can be expressed in the ratio of the polynomials. A polynomial is a finite series containing positive integral powers of the variable.

Student: Sir, there must a polynomial for s, s power minus 1

Sir: Would it be a polynomial? No.

Student: Yes sir (..)

Sir: No. You cannot write s as the power of s to the minus 1. Again you will require a negative power. Now to show that all network functions are real and rational, oh, I have not said what is rational, I am sorry, I have said what is rational, I have not said what is real. A real rational function, a real function is one which is real when the variable is real. It is as simple as that.

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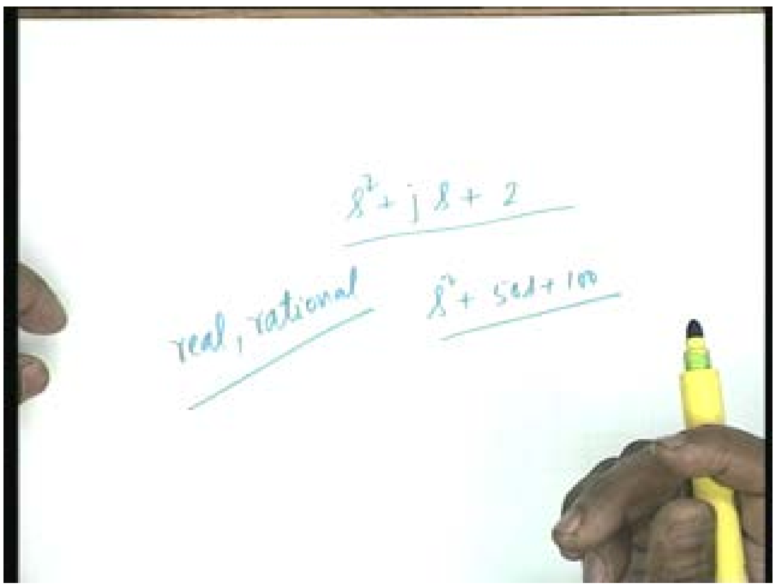

For example, s squared plus j s plus 2 is not a real function because, when s is real, it is complex. On the other hand, s squared plus 50 s plus 100 is a real polynomial because it is real when s is real, when the variable is real. So you understand the meaning of real and rational. To show that a network function is real and rational, we will not keep a strict proof, we will prove more by heuristics. We will take a simple example, but general enough to establish this fact.

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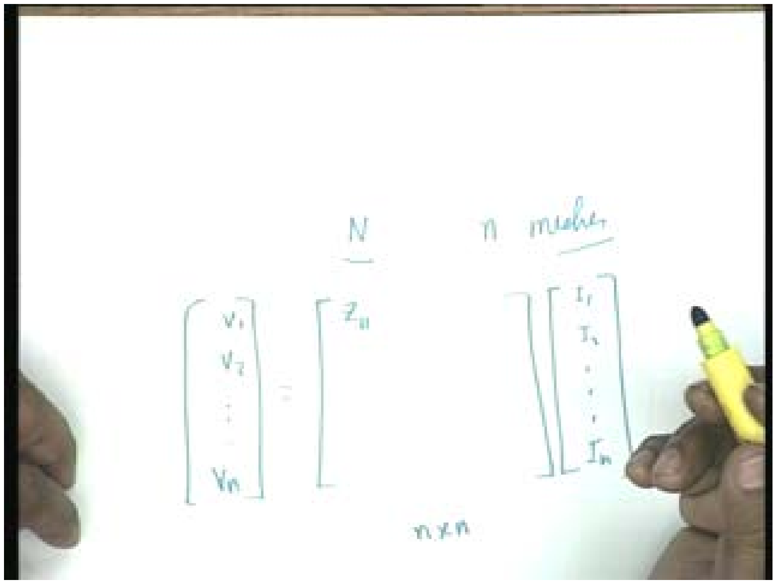

We take a network N which has n number of meshes. You identify, you take a network N containing LLFPB elements and sources, voltage sources, current sources and so on and so forth and you take a mesh basis analysis. To be specific, let us say that each mesh contains a voltage source. If it does not contain that, that voltage source shall be put equal to 0, replaced by a short circuit. If it contains a current source, take another element along with it and convert it to a voltage source.

So any network can be can be analyzed in terms of meshes, n meshes, then my equation in the transform domain would be V 1, V 2 to V n. These are the sources in the meshes would be equal to a coefficient matrix multiplied by I ,1 I 2, I n, where I 1, I 2, I n are the mesh currents and the elements here. Naturally, this matrix shall be an n by n matrix with elements, with entries which have the dimensions of impedance because voltage is being expressed as a multiplication of a current and a matrix a coefficient matrix which, obviously, must be impedance and if the impedance is in the position, let us say 1 1 position, first row, first column, when you know this is the self impedance of the mesh. You know these terms, do not you?

Students: No sir.

Sir: No? You do not?

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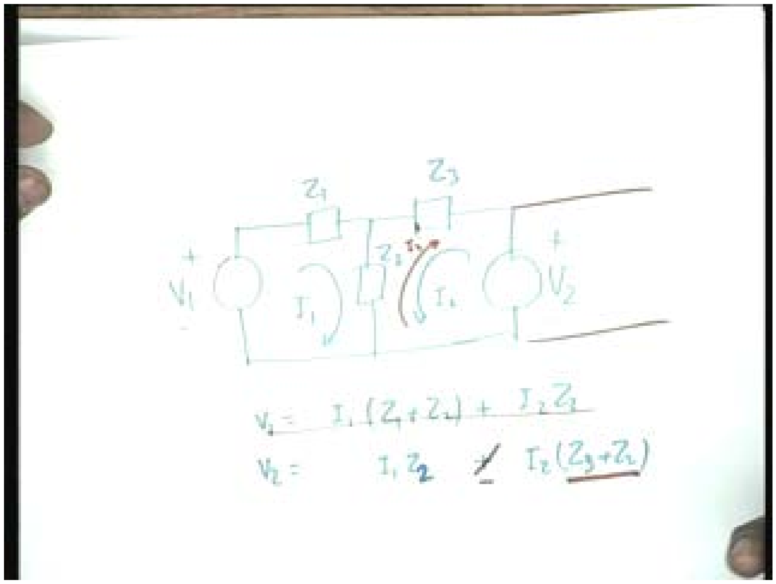

Let us consider a specific example, rather than an n term. Suppose, we have a network like this and suppose Z 1, Z 2, Z 3, then these 2 mesh network can be written as V 1 equal to I 1 Z 1 plus Z 2 plus I 2 Z 2, agreed? Is that clear? The first mesh equation, I 1 Z 1 plus I 1 plus I 2 Z 2, which I write as I 1 Z 2 plus I 2 Z 2. Similarly V 2 would be equal to I 1 Z 2 plus I 2 Z 3 plus Z 2. V 2 equal to I 2 Z 3 plus Z 2 and additionally there is a current I 1, so I 1 Z 2.

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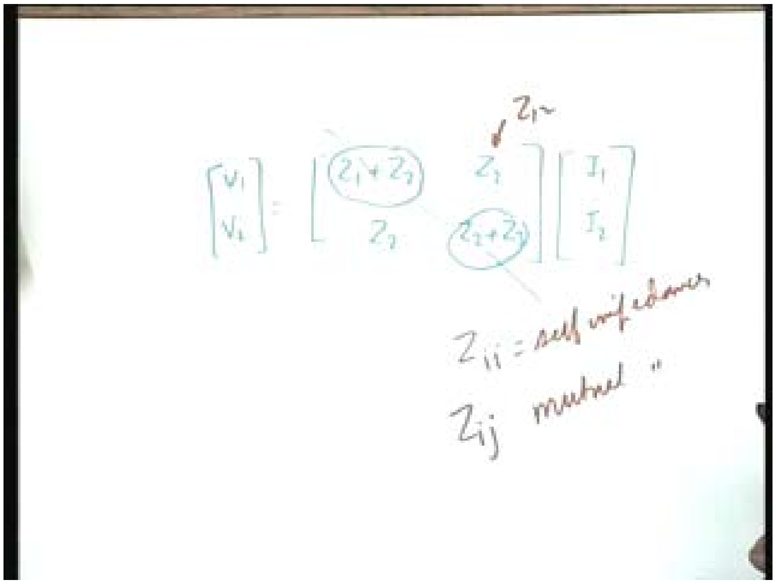

This, I can write in the matrix form as V 1 V 2 equal to Z 1 plus Z 2, Z 2, Z 2, Z 2 plus Z 3 multiplied by I 1, I 2. Now you see, what we had written earlier was an n mesh generalization. Instead of 2 meshes, we had V 1, V 2, V n impedance matrix multiplied by I 1, I 2, I n. Now let us see what these impedances means. Look at this 1 1 position element, Z 1 plus Z 2. Now if you look at mesh number 1, Z 1 plus Z 2 is the total impedance contained in this mesh and therefore, this impendence Z 1 plus Z 2 is called the self impedance of the mesh.

Similarly, you see the 2 2 entry? This is the 2 second row, second column, 2 2 entry, Z 2 plus Z 3 is the self impedance of mesh number 2, Z 2 plus Z 3, total impedance of this mesh. On the other hand, see these are the diagonal elements. The diagonal elements should be 1 1 2 2 3 3 4 4 and so on and so forth, ending in n n.

Student: Sir, how do we know that it is n 1 plus Z 2, if we that take the direction of current in the other direction that will be?

Sir: Even then, it will be Z 1 plus Z 2 self impedance.

Student: If I take I 1 in one direction and I 2 in other direction?

Sir: Oh! Does not matter if I 2 was in this direction.

Student: Yeah.

Sir: I 1, we dropped it to I 1, would have still been Z 1 plus Z 2

Student: Sir, but due to I 2.

Sir: The other one would have been minus I 2 Z 2?

Student: If (..) the reverse direction of I 1, then the self impedance will become minus Z 1 plus Z 2.

Sir: Let us do that.

Student: Sir, let us take the reverse action of I 2.

Sir: Pardon me.

Student: Sir, if we take the reverse direction of I 2 that will give a problem.

Sir: It will still be called self impedance. Self impedance has no sign. Self impedance is still the sum of these impedances. The current is minus I 1 in the other case. Self impedance has no sign. It is simply the sum of the impedances in (..) whereas, mutual impedance may have a sign like mutual inductance, you know. Mutual inductance has a sign if I 1 and I 2 do not both enter the transformer, then the mutual inductance is negative.

Student: Suppose, if we take the direction of I 2 as negative, this Z comes out to be Z 2 minus Z 1, that way, Z 1 as Z 3.

Sir: No, it shall not. This is the current in the mesh, the current in the mesh is like, if, whether you take it this direction or that direction, the impedance that multiplies the current in the mesh is the self impedance, whatever direction you take I 2.

Student: Sir, but by (()) (11:14) you will get that it will be I 2 Z 2 plus V 2 plus I 1 minus I 2 Z 1 is that 2.

Sir: I do not know what you are saying. Which mesh you want me to reverse this?

Student: Yeah.

Sir: I 2 prime. Then V 2 shall be equal to minus, this will not be plus.

Student: Yeah.

Sir: Minus I 2 Z 3 plus Z 2 minus I 1 Z 2. Minus or plus?

Student: Minus I 2 in the above equation. Plus sir.

Sir: No, first equation. Ignore this. This should be plus?

Student: Yes sir.

Sir: Now what are you saying is minus I 2 is the current now in the mesh and the sum of the impedances is still Z 2 plus it is not Z 2 minus Z 1. The mutual impedance can have a sign, self impedance does not have a sign. The choice of the current simply means it is the current is I 2 or minus I 2 and that sign is it reference to this source in this circuit. If the mesh current helps the sources or it source helps the mesh current, then the sign is positive. If it does not, then the sign of the current is negative. It does not the change the sign of the self impedance.

So the self impedance, Z i i, ith row and ith column, these are the self impedances and the off diagonal elements are called mutual impedances, that is, take for example, Z I j, Z 1 2, for example, here, first row and second column. This is the impedance common to meshes 1 and 2. This is called the mutual impedance. An impedance which is common between to meshes is called a mutual impedance. Suppose we had a third mesh here. Then mesh number 1 and mesh number 3 have nothing in common. So the mutual impedance would have been 0. Mutual inductance is an example of mutual impedance.

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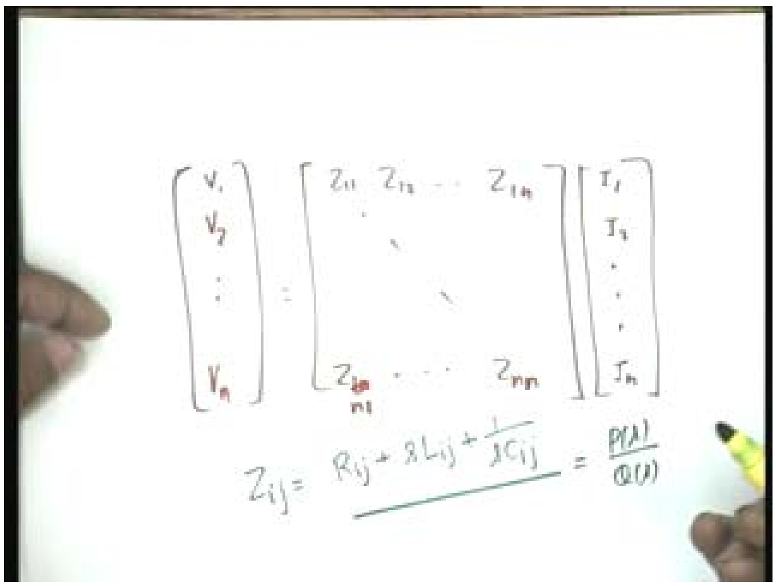

Now therefore, if I go back to my original n mesh network, I can write V 1, V 2, V n equals to Z 1 1, Z 1 2, so on. Z 1 n and Z n 1, Z 1 n, I am sorry, Z n n. This will be the form of the impedance matrix 1 1, 2 2, 3 3 up to n n.

Student: Sir, last time you said n 1 to the n n.

Sir: Z n 1 to n n, thank you, multiplied by I 1, I 2, I n. Now the question asked at this stage is, what is the form of the Z i j? In general, Z i j, what is the form? It can consist of an inductance, a resistance and a capacitance and also perhaps mutual inductance; mutual inductance is also inductance. So in general, it would be of the form R I j plus s L i j plus 1 over s c i j.

In general, this should be the form of every entry in this impedance matrix, whether it is a diagonal entry or off diagonal entry, it will be of this form and you see that this is rational. This is of the form of P of s by Q of s. Q of s you have simply is s c i j and P of s is s squared L i j C j etcetera. So this is rational. So any manipulation that you do in this matrix will give you rational function because each entry is rational. Multiplication of two rationales give a rational function; addition, subtraction, division, multiplication, any operation that you do and that is all you do in evaluating the determinant.

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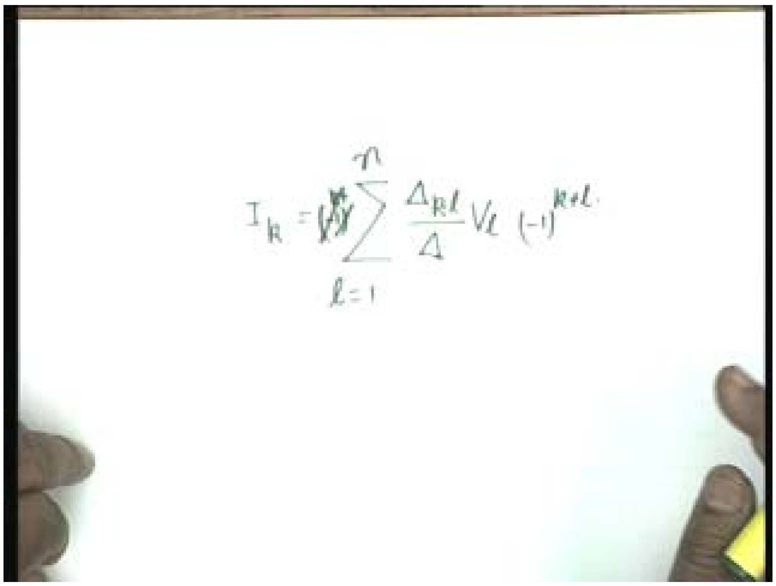

For example, to be specific, let us say I want to find out I k, the kth mesh current. Obviously, this would be del k l divided by del V l summation over, if you know the, whose formula is this, solution of matrix equation? No, I do not remember. Then this is l equal to 1 to n. That is, what you do is, you find out the determinant and find out its cofactor del k l. This, I hope you know.

Student: Yes sir.

Sir: del kl and then in addition, you have to multiply this by minus 1 to the power.

Student: l.

Sir: No.

Student: k plus l.

Sir: k plus l. This would naturally go inside minus 1 to the power k plus l. This is the determinant method of solving a set of linear equations. The exact form is not important. What is important is that its ratio. What is del k l ? del k l is, del with kth row and lth column eliminated. So the dimension would be n minus 1 by n minus 1. This, of course, you know from maths course and also perhaps, 1 1 0. The important point is that del k l, as well as del are rational functions and therefore, the ratio must also be a rational function and any network function now, for example, you could define i k by V j as a transfer admittance, that will also be rational. So we demonstrate that all network functions are real, not only rational. They are also real, why are they real?

Because the impedance, the resistance, inductance and capacitance are real quantities and therefore, the coefficients of s any power of s, they all will be real quantities and therefore, we established that all network functions are real and rational and I have already explained, what is real and what is rational. Do not forget this in your life.

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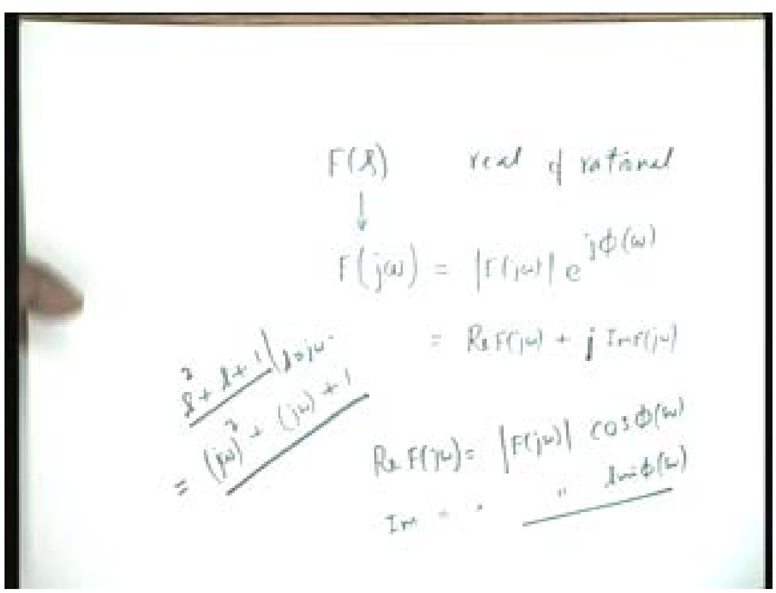

Now, therefore, a network function F of s which is real and rational, it is convenient to study networks in terms of their sinusoidal response and that is why you have studied this phaser concept in so much of details in terms of their sinusoidal response and as I said from F of s, if you want to find out the network function on the j omega axis, all you have to do is to find F of j omega and this network function now because it is a complex quantity, because it is a complex quantity, can be expressed in two ways, one is the Eulerian way, that is, the magnitude of F multiplied by e to the j phase of F, phi of omega; this is the Eulerian, that is, in terms of a radius factor and an angle or you could express this in terms of real part and imaginary part, which is the Cartesian way, j times imaginary part of F of j omega.

Student: Sir, this time, will the functional and real if we put (..), I mean sigma equal to 0 or s is equal to j omega?

Sir: The variable is j omega.

Student: Yes sir.

Sir: And if therefore, j omega is real, then the function is real.

Student: But sir, j omega is not real because omega is real.

Sir: Not necessarily, I can put any value of omega. As long as it is the variable, you identified the variable, you see, suppose my function is s squared my polynomial is s squared plus s plus 1. This is a real polynomial because the variable is s.

Student: Sir, but if s is equal to j omega.

Sir: If it is, if s equal to j omega, then I have j omega squared plus j omega plus 1

Sir: This is the value.

Student: Yes sir.

Sir: Under s equal to j omega, the nature of the polynomial has not changed. If j omega is considered as the variable, then the natural polynomial has not changed. The coefficients are all real and therefore, this is also a real polynomial in j omega but fortunately, you do not have to enter into that controversy. Our real rational shall always be referred to as a function of s and this is the reason why once you put s equal to j omega, the real and rational character, they are guaranteed because you obtained this from F of s, a function of s. Then we look at, we dissected, we take a dissection of this, you find the amplitude and phase, these are the two parts. Another way of expressing is, in terms of the real part and imaginary part and as you know, they are intimately related to each other.

For example, the real part of F of j omega is equal to F of j omega magnitude multiplied by cosine of phi omega and similarly, imaginary part of F of j omega is this magnitude multiplied by sin of phi omega. Imaginary part, as I have already explained, is a real quantity. Imaginary part is a real quantity. See, because we had multiplied by j and therefore, the imaginary part is this. On the other hand, if these are given, Eulerian parts, I am sorry, the Cartesian parts are given, you can find out the magnitude and phase as the magnitude is simply given by square root of real part square plus imaginary part squared and the angle phi omega is given by tan inverse, arc tan of imaginary part divided by real part.

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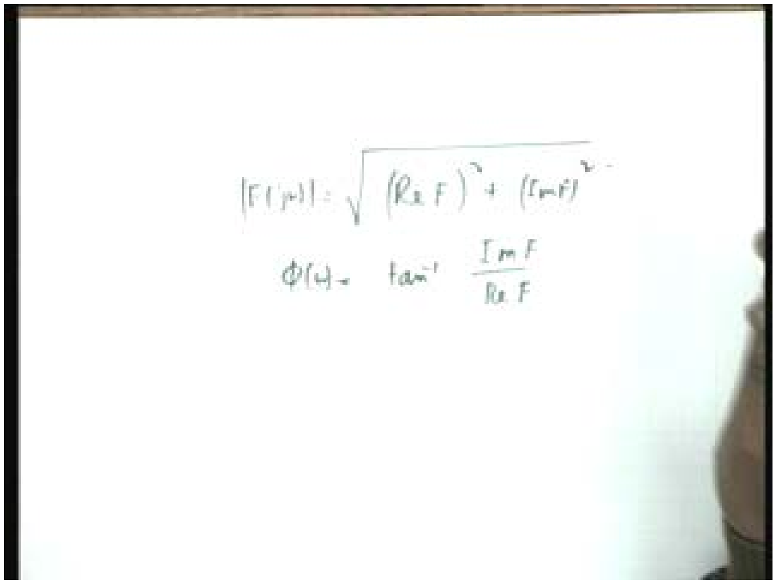

So the four parts of each arc of the network function, there are two ways of expressing and one is convertible to the other in a very easy manner but for later reference, particularly for network synthesis, let us be a bit more systematic and follow this derivation carefully because we are going to use these results again and again.

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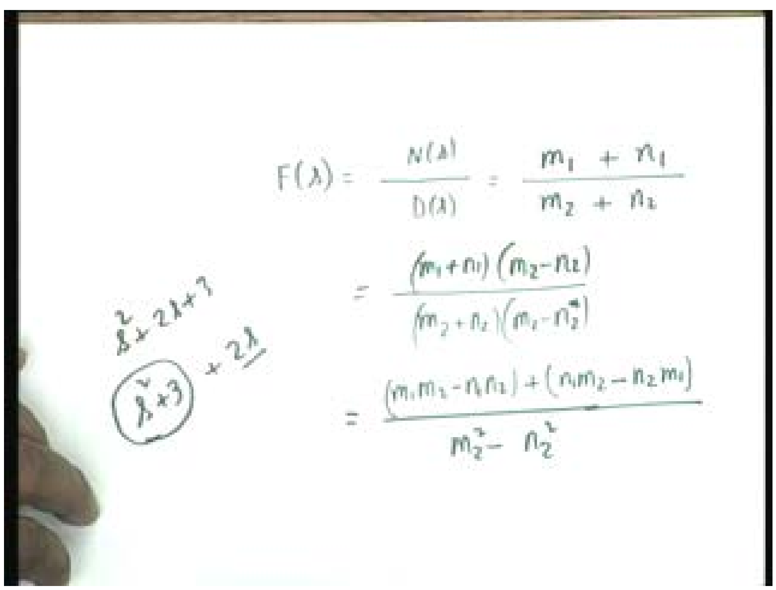

Suppose network function F of s, which you have said is real and rational, is equal to N of s by D of s. Now N of s in either polynomial N or D shall have some even powers of s and some odd powers of s. For example, if you had a polynomial s squared plus 2 s plus 3, s squared plus 3 contains even powers and twice s contains the odd power 1. The part of the polynomial, which contains even powers is called the even part and the part of the polynomial that contains odd powers are called the odd part and therefore, either of these two polynomials shall have an even part and an odd part.

Let us denote the even part of N of s by m 1. m 1 is a function of s. We are omitting this and the odd part as n 1, similarly for the denominator m 2 plus n 2. Now I can write this as, let us multiply the numerator and denominator by m 2 minus n 2. Then I get m 2 plus n 2 multiplied by m 2 minus n 2 and in the numerator m 1 plus n 1 m 2 minus n 2. You see, what I have done? Why do I multiply this? I make the denominator n even polynomial because the denominator becomes m 2 squared minus n 2 squared. Now even squared is even, odd squared is also even. Therefore, m 2 squared minus n 2 squared is an even polynomial of degree, twice the degree of m 2 or n 2, whichever is higher.

Student: Lower.

Sir: Whichever is higher not lower and in the numerator, I have a general polynomial, another general polynomial and I can write down the even and odd parts separately. For example, the even part would be m 1 m 2, this product shall give me even; even multiplied by even, then plus n 1 minus n 2, multiplication of odd polynomial by another odd polynomial shall again give me an even polynomial. So I shall have minus n 1 n 2. This is the even part of the numerator and the odd part would be, when n 1 multiplies m 2 and n 2 multiplies m 1 and if I put a negative sign n 2 m 1, this is the odd part. So I write now, F of s, there is no question of real part and imaginary part, yet F of s, I write in terms of even and odd part.

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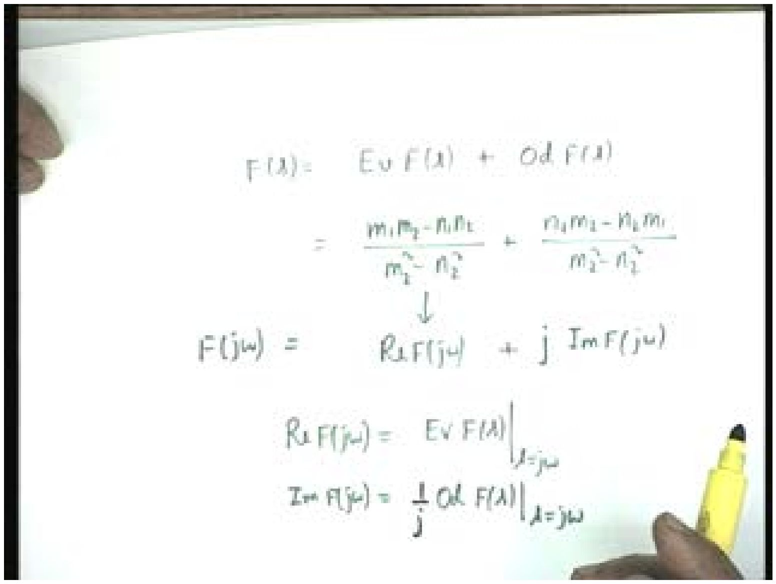

So I can write F of s as equal to even part of F of s. This is the symbol E v F of s plus odd part of F of s O d F of s. Where even part of F of s is m 1 m 2 minus n 1 n 2 divided by m 2 squared minus n 2 square and odd part of F of s is n 1 m 2 minus n 2 m 1 divided by m 2 squared minus n 2 squared. Now, if I put s equal to j omega, then this shall be purely real because it contains only even powers of s in the numerator, as well as the denominator. So this would become the real part of F of j omega and this would become, the denominator is even, the numerator is odd.

That means s is a factor and if you put s equal to j omega, then what you will get is j times a real quantity, that is, imaginary part of F of j omega. Is it clear? So we have this identification that real part of F of j omega is the even part of F of s with s equal to j omega and imaginary part of F of j omega would be equal to the odd part of F of s with s equal to j omega. Then, that is not complete. J do we divide by or?

Student: we divide by (..)

We divide by j. J times this equal to this with s equal to j omega and therefore, that is it and these expressions have to be remembered carefully. Now once you have found out the real and imaginary part, finally, the magnitude function and the phase function is not a problem, but you notice that real part will contain only even parts of omega. Is that right? Even this is derived from the even part. Imaginary part shall contain only odd powers of omega and therefore, the real part is an even function of omega and the imaginary part is an odd function of omega.

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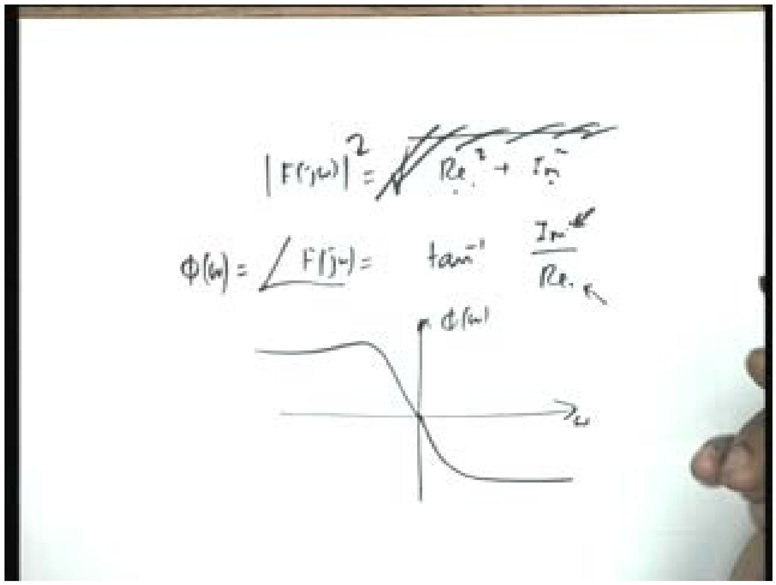

In consequence, the magnitude function F of j omega, which is equal to square root of real part plus imaginary part squared, will be an even function. The magnitude, by definition, cannot be real, cannot be negative and therefore, it is an even function and the angle of F of j omega which is the ratio of imaginary part divided by real part. Imaginary part is odd, real part is even, odd by even is odd and therefore, angle phi is an odd function of omega, which means that if I am given phi of omega versus omega for, let us say, this part of the diagram, then I can construct for the other part. It would simply be the negative of this.

Student: Sir, how mod of F j omega is even?

Sir: mod of F j omega, it cannot be.

Student: Square of odd sir.

Sir: Pardon me.

Student: Sir, it gives us sum of square of odd.

Sir: Sum of the real part squared and imaginary part squared. This is even this even so even plus even is even.

Student: (..) I mean, we can just say it is the square is

Sir: This is the even function.

Student: Sir and what about the phi omega?

Sir: phi omega is a ratio. There is no square rooting and this is an odd function and tangent is an odd function, tangent into minus theta equal to minus tan theta and therefore, then it is an odd function. Now this is the general derivation and you must remember this expression, that is, the even part and the odd part of F of s because we shall be using these expressions very often.

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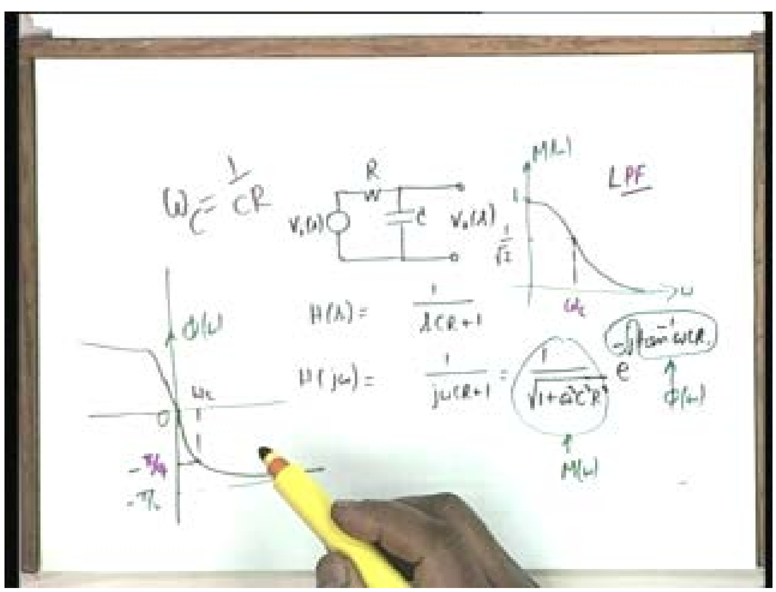

Let us take an example, let us take a simple R C network and let us say this is my V i s and this is V 0 s. The transfer function, which is now a dimensionless one, let us denote it by H of s is given by 1 by s c. This is simply 1 by s C R plus 1 and therefore, the function on the j omega axis is 1 by j omega C R plus 1, which will be broken up into its magnitude and phase as 1 plus omega squared C squared R squared and e to the power minus j tan inverse omega C R. Do you understand this? e to the minus j, this is the e to the minus e to the j phi. phi is minus because it is the angle of the numerator minus angle of the denominator. Angle of the denominator is imaginary part divided by the real part tan inverse.

Is there any problem here? No? Well, if we plot the magnitude function, let us call this as M omega and this as phi omega, the phase is minus tan inverse omega C R. Then the plot of M omega versus omega, it starts from 1 at omega equal to 0, it is 1 and at infinity it goes to 0, so it goes like this.

On the other hand, the phase function, if I plot it, phi, it is always negative. It starts at omega equal to 0, the phase is 0. It is always negative and omega equal to infinity, it goes to tan inverse infinity is

Student: phi by 2.

Sir: phi by 2 and therefore, the car goes like this, minus phi by 2. This is it. Now let us look at some critical points. This, obviously, a low pass filter, it prefers low frequency to high frequency. It is a low pass filter and the low pass filter is characterized by a cut off frequency which is defined as the frequency at which the amplitude falls by 3 decimals which is equivalent to 70 point 7 percent. So wherever the amplitude is 1 by root 2, which is exactly point 7 o 7, this omega C is called the cut off frequency. This is omega; omega c is the cut off frequency. Cut off frequency is an indication of the frequency band which the filter will pass with relatively little attenuation. Beyond the cut off frequency, the filter attenuates more and more.

At the cut off frequency, what is the cut off frequency in this case? When does it become 1 by root 2? Obviously, when omega squared C squared R squared is

Student: 1.

Sir: 1, the cut off frequency is 1 by C R, as simple as that, 1 by the time constant is the cut off frequency. At the cut off frequency, what is the angle?

Student: 45 degree.

Sir: 45 degree minus pi by 4, so negative minus pi by 4. The implication of this is, I had said that the phase function is, is this legible here? Can you read it? Here in the diagram? I told you that the phase function is odd function and therefore, the other part can be constructed very simply like this and if one actually plots this, then one would notice that the part of the phase curve between minus omega C and plus omega C is approximately linear.

Student: I do not know

Sir: Approximately linear, that means a straight line can be fitted to this curve between minus omega C and plus omega C. This is an observation. As we shall show later, the effectiveness of a filter or a network can be judged either from the magnitude response or from the phase response. Sometimes you require both but most of the times, it does not matter whether you take the amplitude response or the phase response.

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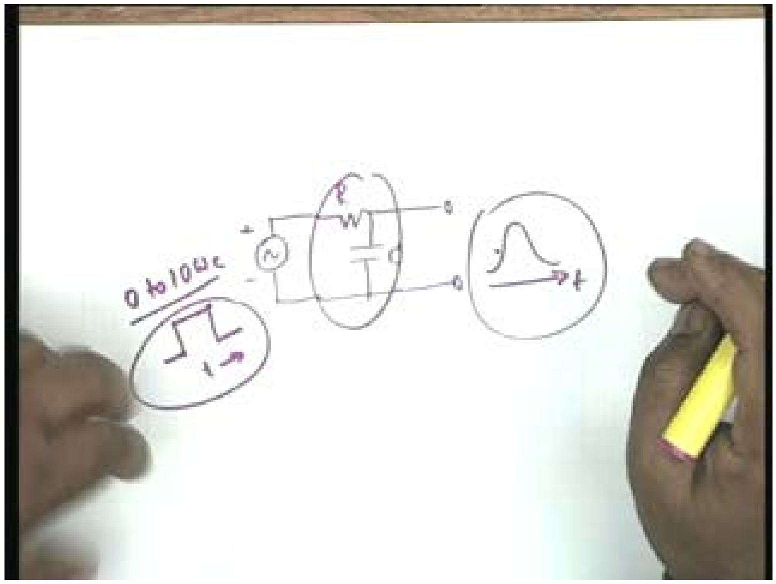

For example, to this network R and C, suppose I connect a signal source which contains frequencies, let us say 0 to 10 omega c. Suppose we have a signal source which contains whose spectrum extents from 0 frequency to 10 times the cut off frequency. The output, naturally, then shall contain only frequencies up to omega c with little attenuation. But beyond that, other frequencies are heavily attenuated and therefore, whatever shape of the waveform you feed here, the shape of waveform here would be quite different. Is not that clear? For example, if this contains all frequencies up to infinity, a square wave, a square pulse contains all frequencies up to infinity, it is a sine x by x form, then what would be form here? It would be

Student: Square in time.

Sir: Yeah, this is square in time. If you have a, forget about this.

Student: Same function.

Sir: Pardon me.

Student: Same problem frequency domain is a sine function

Sir: Frequency domain is a sine function which means that it contains frequencies up to infinity. Now at the output in the time domain, you will see a wave form like this. Why does it happen? Because all the high frequencies are cut off and this contains very high frequencies. High frequency is cut off, that means the rise shall be no longer be rapid.

Student: That is, frequencies contained in frequency domain.

Sir: This is also the time domain. We are considering, we are looking at the C R O wave forms. I am arguing that this square wave, this square pulse contains frequencies, all frequencies up to infinity. A low pass filter like this retains frequencies only up to omega C. Beyond that, it cuts off very heavily and therefore, all the high frequencies in this wave form shall go and the response, the time wave from on the oscilloscope, shall look like this. It is a gradual rise rises to a maximum and then a gradual fall because high frequencies have been cut off.

What I am saying now is that if you use a low pass filter like this and let us say, frequencies up to 10 omega C, then the output shall contain frequencies up to omega C only. All the rest will be discarded. So there will be pulse distortion, there will be distortion of the wave form but that is what may be you want. For example, if you have a biomedical signal as arises in ECG or EEG, which is very low frequency, 2 or 3 hertz is the kind of frequency that you get in biomedical signals, and the line frequency is 50 hertz, the line frequency, biomedical signals are very weak. The line frequency, if there is a line, power line, going here and I measuring here that, that distance is sufficient to induce a voltage here, magnetic induction and therefore, the biomedical signal that you get, weak signal at low frequency is mixed with a large amount of 50 hertz and you may want to get rid of 50 hertz.

So what you do is, you take a low pass filter and get rid of that. However, the other point that I am mentioning, do not take this very seriously at this point but it is worth mentioning, that instead of the amplitude response, if you looked at the phase response, as long as the phase response is linear, there can be no distortion. Linear phase is accompanied with no distortion.

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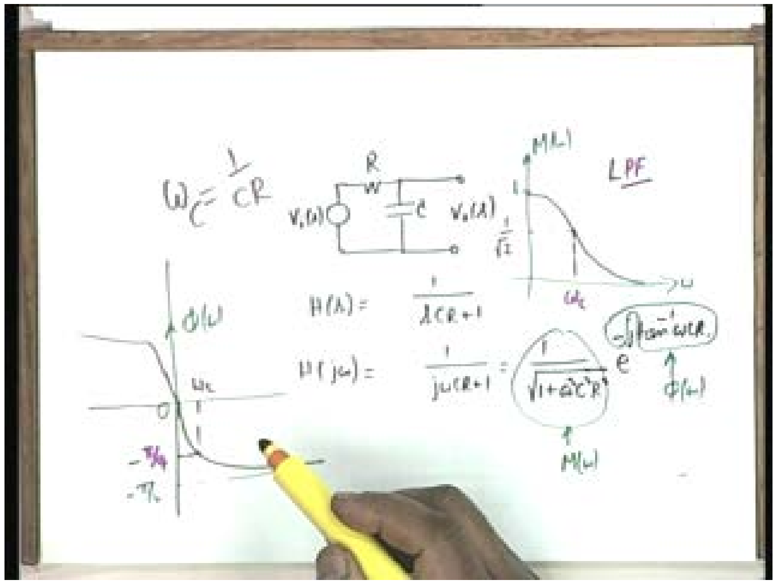

For example, here, as I said this part of the phase curve from minus omega C to plus omega C, they are approximately linear. That means, if the input contains frequencies from minus omega C to plus omega C, the shape of the waveform will be unaltered. On the other hand, if it goes beyond omega C, that is, beyond the linear region of the phase characteristics, then the pulse shape, then the shape of the waveform will be distorted. That means some frequencies would be rejected. Now this may be intentional, this may be unintentional but this was an example to illustrate the concepts of magnitude and phase.

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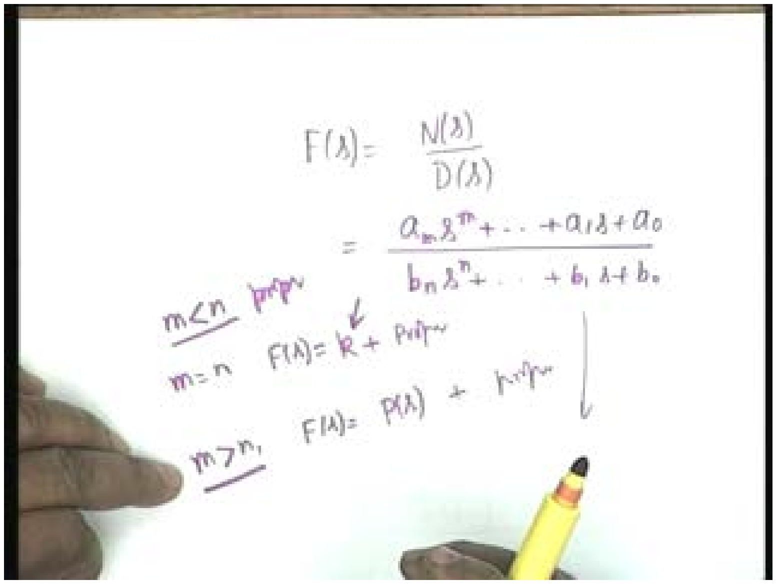

Now let us look at generalities again. Our network function F of s is a rational function, N of s by D of s and conventionally, well I can write this as if I expand the polynomial, let us say, the numerator is an mth degree polynomial plus a 1 s plus a 0 and the denominator is an nth polynomial plus b 1 s plus b 0. Conventionally, the first question is about the degrees. If m is less than n, then this network function is a called proper rational function. Proper, if m is less than n.

Student: The whole function of F?

Sir: Yeah, the whole function is called proper, if m is less than n. If m is equal to n, this is called proper. If m is equal to n then you can write F of s as some constant k. You can take out a constant k plus a proper rational function. On the other hand, if m is greater than n, then F of s can be written as a polynomial plus a proper rational function, agreed. Most of the network functions that we shall get, yeah, we will, examples of all the 3 types in practice, we will have examples of all the 3 types. Nevertheless, you should know these, these terms: proper and improper.

For example, if the function is proper, then in the impulse response there shall, the output, at the output, there shall be no impulse. Is the point clear? If this is a proper rational function, then in the impulse response of the system, there should be no impulse. On the other hand, if m is equal to n, then you shall have an impulse, in the impulse response, corresponding to k. The inverse of k is k delta t. Whereas, if m is greater than n, then in the impulse response, you will get not only impulse

Student: Derivatives.

Sir: But its derivatives also, agreed. These are the various links or threads between the time domain and the frequency domain and to an electrical engineer, they must be like 2 hands. Your right hand must know what the left hand is doing, left hand must know what the right hand is doing and you must be able interchange. You must be able to interchange the time domain into the frequency domain, whenever you want, whenever it is needed. You cannot, you should not be obsessed with any either of the 2 domain. As far as circuit theory is concerned, there is an obsession with the s domain, frequency domain, and the reason is very simple, because an engineer wishes to do a thing at the, by spending the least possible effort and the frequency domain affords that. With the frequency domain, you can do things with less effort than in the time domain.

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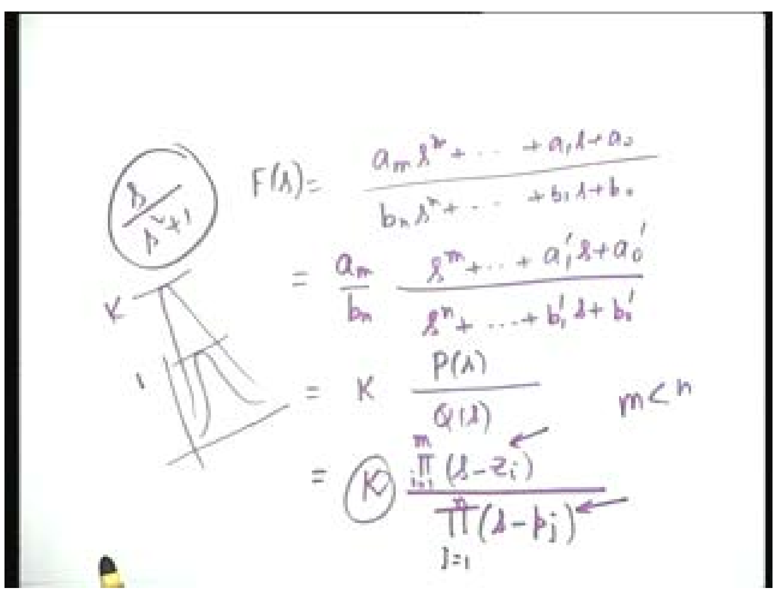

Now let us go back to this, F of s equal to a m s to the m plus etcetera plus a 1 s plus a 0 divided by b n s to the n plus etcetera plus b 1 s plus b 0. Now I can write this. It is conventional to write the numerator and denominator polynomials with the highest power term coefficient reduced to unity. In other words, I can take a m by b n common and then write s to the m plus etcetera plus a 1 prime s plus a 0 prime, where the prime is the un primed quantity divided by a m. Similarly, in the denominator, I have to, s to the n plus b 1 prime s plus b 0 prime and the reason for writing a function like this is the following.

You write K, let us change the nomenclature. Let us say P of s by Q of s, where P and Q are polynomials with leading coefficient. Leading coefficient means the coefficient of the highest power term equal to 1. Now, the reason for writing it in this form is that, P of s can now be written in terms of its factors in terms of its roots. That is, I shall have s minus Z I, continued product, i equal to 1 to, what will be the upper limit?

Student: m.

Sir: m. Similarly, in the denominator, I can write continued product s minus P j where, j equal to 1 to n. You understand why we wrote the coefficients, with coefficients equal to unity? Because the product of these terms shall give the highest power term coefficient is 1.

Student: oh, it is a term that is leading coefficients.

Sir: The leading coefficient is the highest power term. The coefficient of the highest power term is the leading coefficient. Now, obviously now, the poles and zeroes of the network function are obvious Z i's are the zeroes and P j's are the poles. Z i's are the zeroes and P j's are the poles and K is simply a scaling constant, a multiplying constant. The frequency response of the network either magnitude or phase or real part or the imaginary part, it is un-affected by K. The nature of frequency response, it is only a scaling constant. The magnitude of the scale up or down by the factor K, but the shape of the response shall remain the same.

It is the shape of the response that determines how it behaves towards different frequencies, for example, this shape of magnitude, obviously, is a band pass function. Now if it is multiplied by k, all that will happen is that instead of, let us say, instead of 1 here, this will become k. That is all. The nature of the function, that is, which discriminatory properties network functions are useful only when they are discriminated, unlike human beings. Unlike human behavior, this is, useful networks are useful only when they are discriminated.

So K only affect s, the scale Z i's and P j's, they are the ones which characterize the function. They determine the shape of the discriminatory property of the network and you can very easily see that the number of poles is the same as the number of the zeros. The number of finite zeros, here, if m is less than n, let us say, the number of finite zeroes is m, but there are n minus m zeroes at infinity. For example, s divided by s square plus 1, this function has one 0 at the origin and the other 0 at infinity. The 2 poles are a plus minus j 1, the number of zeros must be equal to number of poles, including the points at the origin and at infinity. If you take care of those points,

Student: Sir, how infinities and zeros are connected?

Sir: How infinity, put s equal to infinity, this function vanishes and that is a 0. Any value of s at which the function vanishes is a 0. Any value of s at which the function blows up is a pole.

Student: So that will not be defined, s is equal to?

Sir: This is the definition.

Student: No sir, s is equal to infinity, the function will not be defined.

Sir: Oh, does not matter. I know a capacitor will be a short circuited infinite frequency correct, that is good enough for me. Infinity is a concept, there are many things which are concepts here but you see, concepts are, this is the beauty of an engineer, he takes concepts, ideal concepts and apply that in practice, designs circuit which work and there is no success like success. We close at this point, we will meet after an hour.