

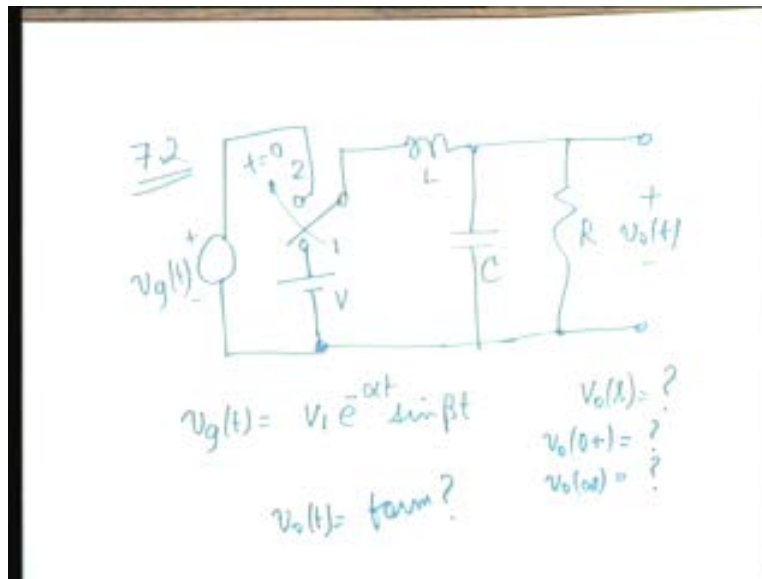
Circuit Theory
Prof. S. C. Dutta Roy
Department of Electronic & Engineering
Indian Institute of Technology, Delhi

Lecture - 12

Problem Session 3: Network Theorems Transform Methods

It is the twelfth lecture and we have our problem session 3. The first problem that we are going to discuss is 7 point 2.

(Refer Slide Time: 1:13)

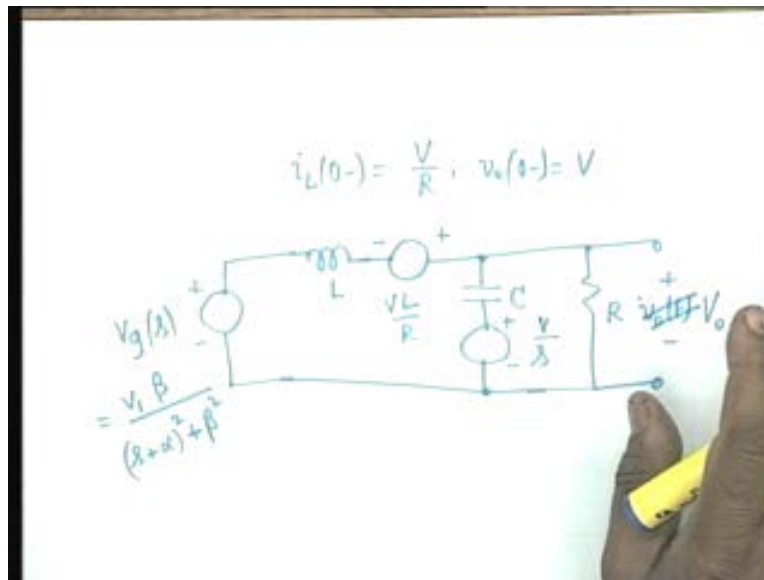


And the problem is this. The circuit consists of a switch which has two positions and it is thrown from position 1 to position 2 at t equal to 0. There is a battery of voltage V and the switch has an inductance L , a capacitance C and a resistance R . The response is the voltage across this, $V_0 t$ and 2, terminal 2 is connected another source $V \sin \omega t$ plus minus at t equal to 0, t equal to 0 minus, this switch is in position 1 and after having been at 1 for a long time, it is switched off to position 2.

The source voltage is $V \sin \omega t$ equal to $V_1 e^{-\alpha t} \sin \beta t$, source voltage is this. What you are required to find out is the transform of $V_0 t$, that is, $V_0 s$, then find the initial and final values, that is, $V_0 0$ plus and V_0 infinity. These are another two quantities to be found

out and then part C is, sketch one possible set of locations for the critical frequencies in the s plane and write the form of the response $V(t)$. Only the form is required and in the bracket it says, do not take the inverse transform, just the form of the response that is needed. Now looking at the circuit, you notice that, since this switch was in position 1 for a long time, if we indicate this current as i_L and then $i_L(0^-)$ is by inspection, it is V/R because capacitor is open, inductor is short.

(Refer Slide Time: 3:59)



And therefore, $i_L(0^-)$ is equal to V/R and the other quantity is the energy storage element. Its capacitance C and obviously, at $t=0^-$, C shall be charged to the voltage V and therefore $v_C(0^-) = V$. $v_C(0^-)$ is the voltage across capacitor. So $v_C(0^-)$ is equal to $C \cdot V$. These are the conditions at $t=0^-$. Now if I draw the transformed equivalent circuit for $t \geq 0^+$, then $V_g(s)$ and the transform of the given function $V_1 e^{-\alpha t} \sin(\beta t)$ would be $V_1 \beta$ divided by $s + \alpha$ whole squared plus β^2 .

We know this transform, $e^{-\alpha t} \sin(\beta t)$ and with this polarity, then we have the inductor L which has an initial current and therefore, it is the inductor L without any initial current plus a voltage source. We are not using a current source for obvious reasons.

Student: (..)

Sir: Yes, this will be minus plus and it would be $L \dot{i}$ minus, so it is V_L by R . This is the voltage source. Then the capacitor C can be looked upon as an initially uncharged capacitor in series with a voltage source whose value is

Student: (..)

Sir: But I am talking of the in transform domain. So it will be v by s . Is that okay? The transform domain and then I have the resistance R and V_0 . You must.. Yes?

Student: V_0/s

Sir: V_0/s , that is correct, because I am talking of the transform domain quantity. The thing that you should notice here is that I am not solving this problem by transfer function approach. I am not writing the transfer function at all, because there are initial conditions and if there are initial conditions in the network, the transfer function is not defined. You can define a transfer function now, you can define V_0 by V_g or any other network but I could not do it right from the circuit. I could not write in terms of impedances, admittances an inspection because there are initial conditions. You can now define a transfer function with those initial conditions. Now, if you look at this circuit, what approach should you follow to derive the value of V_0 ?

Student: (..)

Sir: Pardon me?

Student: Thevenin's theorem

Sir: Thevenin's theorem will require finding out an open circuit voltage here and the impedance, looking back and so on. There is a simpler way, just node equations, just one node equation. This

node whose value is V_0 and you see sum of the currents, this current plus this current plus current. Sum of them, it will be equal to 0.

(Refer Slide Time: 7:37)

The image shows a whiteboard with handwritten mathematical equations. The equations are:

$$V_0 - \frac{V_1 \beta}{(s + \alpha)^2 + \beta^2} - \frac{V_L}{R} + \frac{V_0 - \frac{V}{s}}{\frac{1}{sC}} + \frac{V_0}{R} = 0$$

$$V_0 \left(\frac{1}{sL} + sC + \frac{1}{R} \right) =$$

$$V_0 = \frac{1}{LC} \frac{V_1 \beta}{(s + \alpha)^2 + \beta^2}$$

And if I write systematically, it would be V_0 minus minus $V_1 \beta$ divided by s plus α whole squared plus β square minus V_L by R . There are 2 sources, so V_0 minus this source minus this source, that will be the drop in L . So you divide by sL , that will give the current in the inductor plus V_0 minus V by s divided by 1 over sC divided by the impedance. That will be the current to capacitor. Current to the capacitor will be V_0 minus this voltage, divided by the impedance plus V_0 by R would be equal to 0.

Now I have done the simplification, requires a bit of algebra. Keep the V_0 terms on the left hand side. You get 1 over sL plus sC plus 1 over R . This would be equal to, transfer everything else to the right hand side and simplify. This simplification algebra, I shall leave it to you. My final result after simplification, you must take all my results with a pinch of salt. You must verify because I am, I make many mistakes, I made many mistakes in life, I shall continue to do so and that is, I shall prove that I am a human being. My final result comes like this. No, let me give you the further simplified expression. I omit all the steps.

(Refer Slide Time: 9:36)

$$V_0(s) = \frac{1}{LC} \frac{V_1 \beta + \frac{VL}{R} (1 + sCR) [(s+\alpha)^2 + \beta^2]}{(s^2 + s \frac{1}{CR} + \frac{1}{LC}) [(s+\alpha)^2 + \beta^2]} = \frac{N(s)}{D(s)}$$

(b) $v_0(0+) = V$

Initial Value Theorem (IVT): $\lim_{s \rightarrow \infty} s V_0(s) = V$

$$\lim_{s \rightarrow \infty} s \left[\frac{1}{LC} \frac{V_1 \beta + \frac{VL}{R} (1 + sCR) [(s+\alpha)^2 + \beta^2]}{(s^2 + s \frac{1}{CR} + \frac{1}{LC}) [(s+\alpha)^2 + \beta^2]} \right] = V$$

$$\lim_{s \rightarrow \infty} \frac{VL \cdot sCR \cdot s^2}{s^4 LC} = \frac{VLC}{LC} = V$$

And I simply write V_0 equal to $\frac{1}{LC}$. This is what I get, $V_1 \beta$ plus v_1 by R plus SC R multiplied by s plus α whole squared, plus β squared divided by s squared plus s by C R plus 1 over L C . You see how I have put this, the living coefficient I made equal to unity multiplied by s plus α whole squared plus β square. I am not claiming accuracy of this. It may be right, it may not be right. But this is what I get.

If you get anything else than this, please do point out to me. Please do tell me. Now, this is obviously a rational function, I put it in the form of a rational function. So this is n of s by d of s and it is equal to the network function, not network function, it is V_0 of s . It is not a network function, it is a voltage function.

Student: How do we get VL by R ?

Sir: How do I get VL by R ? This is by simplification.

Student: (...) SC R ?

Sir: No, that S comes here $S C R$. Since you have raised the question, I have multiplied by $s l$. You see, what I get is this, VL by R . This VL by R is already here, divided by $s l$ then I multiplied by $s l$ both sides. You see, that is how I get s squared $l c$ plus $s l$ by R and so on and so forth. If it is wrong, let me know later. Dimensionally, it is correct, that I have checked. Now, therefore, take the next question is part b, says, find out V_0 of 0 plus. Now I could do this from the physical consideration. Physical considerations, there is no impulses in circuit, therefore, the voltage across the capacitor must be continuous. So V_0 of 0 plus should be equal to v .

But it is instructed to find out from this function, V_0 of s . If the transform is given by the initial value theorem, initial value theorem says that if the degree of the numerator is less than the degree of the denominator, initial value theorem says that V_0 of 0 plus should be equal to limit s tends to infinity s times V_0 of s . This is the initial value theorem and now if you multiply this by s

Student: (..)

Sir: That is correct. That means, in the response of the circuit, there should be no delta function. Now if you multiply this by s and take, leave s to go, allow s to go to infinity, obviously, the only the highest power terms shall be of any interest. Let us see, what are these highest power terms? You see s to this would be s squared. This gives an s . So it would be s cubed and you are multiplying by s , so it is s to the fourth. So what I shall have is $V L$ by R . This $V L$ by R multiplied by $S C R$. I ignored the 1 because I am only concerned on the highest power term multiplied by s squared from here divided by, what is the highest power term here? s to the 4 s squared and s squared.

Student: (..)

Sir: That is correct and this obviously is equal to,

Student: (..)

Sir: How come? We made a mistake. We left this $1/LC$. So this must come here and therefore, this is equal to V . It cannot be otherwise. No voltage can be the product of V and L/C . Dimensionally incorrect, so it should be immediately pointed out that we left out here. Now if I want to find out $V(0)$ of infinity, then the final value theorem says that final value theorem is valid under a certain constraint. What is the constraint? You do Laplace transform in signal plane system. What is the constraint on the final value theorem?

No poles in the right half plane. If there are poles in the right half plane, then what happens? The system is unstable, so it breaks into oscillation. Amplitude can go to infinity. Obviously, we cannot apply any logical procedure to find the value at infinity for an unstable system. So poles in right half plane is there guaranteed? Yes, it is guaranteed because $1/LC$ is a positive quantity, $1/CR$ is a positive quantity. So the roots of this shall lie in the left half plane minus $1/CR$ plus minus etcetera and by definition, the source that is given, α and β , we assume positive and therefore, these poles are also in the left half plane.

If poles are in the left half plane, then $V(0)$ of infinity would be $\lim_{s \rightarrow 0} s V(s)$ and you see, s tends to 0. Only the lowest power term dominates. So $1/LC$ and β^2 in the denominator and the numerator. It would be $V(1/\beta + VL/R)$ multiplied by $\alpha^2 + \beta^2$ but why am I doing all this? There is a multiplication by s and when s tends to 0, whatever the other quantities, unless it is infinite it is a big 0 and this also is, it tallies with the physical concepts. The source itself is an exponentially decaying source, $e^{-\alpha t} \sin \beta t$ and therefore, after infinite amount of time, the capacitor would be discharged completely. There will be no source for charging and therefore, the voltage shall be 0.

Student: When we solve for it, the power of s in the numerator and denominator are the same. So can we directly say this?

Sir: Power of s in numerator and denominator are not be same. This is 3 and this is 4.

Student: (..)

Sir: When you multiply by s, the power of s shall determine the behavior at infinity, not at 0. At 0, it is the lowest power term and then multiply this by s. The lowest power term, obviously, contains an s and therefore, when s goes to 0, it is 0. This should be possible to say this by inspection of this. This is a cubic polynomial. When we multiply by s, it becomes a fourth degree polynomial with the lowest power of term equal to some constant multiplied by s. In the denominator, it is also a fourth degree polynomial. The constant term is 1 by LC multiplied by alpha square plus beta square and therefore, as s goes to 0, the numerator goes to 0, the denominator remains a constant and the whole thing goes to 0.

It must tell you, whatever the complexity of the mathematical expression, the final result must tell you, it is common sense. If it does not then you better look into your mathematics. It is mathematics which can go wrong. Common sense often goes wrong but not for an expert engineer. Then last part of the question is, let me write this down once again.

(Refer Slide Time: 18:15)

The image shows a whiteboard with handwritten mathematical expressions. The top part shows the Laplace transform of the voltage response, $V_o(s)$, as a ratio of a numerator $N_1(s)$ to a denominator. The numerator is written as $\frac{A_1}{s + \alpha + j\beta} + \frac{A_1^*}{s + \alpha - j\beta}$. The denominator is $[(s + \alpha)^2 + \beta^2] [(s + \alpha)^2 + \beta^2]$. Below this, the time-domain response $v_o(t)$ is given as the sum of two terms: $K_1 e^{-\alpha t} \sin(\beta t + \theta)$ and $K_2 e^{-\alpha t} \sin(\beta t + \theta')$. Arrows point from the terms in the time-domain expression back to the corresponding terms in the Laplace transform expression.

V_o of s equal to 1 by LC times a cubic polynomial. A cubic polynomial divided by s squared plus s by CR plus 1 over LC times s plus alpha whole squared plus beta squared and in the numerator, you have a cubic polynomial. Let us write this as an n of s. I do not want to write this

expression again because part C says do not invert. You just keep it in form of the, choose a location possible, location of the poles and then give me the form of the voltage response in the time domain. Now possible location, these represent poles of the excitation and these are natural, these represent the natural frequencies of the network. Now depending on the values of R L and C, the poles can be on the negative real axis or can be coincident on the negative real axis or can be complex. Let us say they are complex, perfectly possible location

Student: (..)

Sir: These are minus alpha plus minus a beta. This come from the excitation function. That one part comes from h of s, the other part comes from e of s. That is network function multiplied by excitation transform. Now, obviously these are, let us say, if this is j beta and this point is minus alpha, then these two poles come from the excitation function. These generate two more poles which can be real, which can be complex. Suppose they are complex, a perfectly possible location would be like this, let us say, these poles are here. They must be complex conjugates and suppose, we use some other color.

Suppose this is j beta prime and this point is minus alpha prime. This is perfectly legitimate location of the poles. There are 4 poles or there are any more, no, because n of s is of degree 3. If the degree was 5, let us say then there would have been a pole at infinity. It is of degree 3, so these are only locations of poles and I can now write down n of V 0 of s. After choosing this, I can write down some, let us say, N 1 of s, some polynomial in the numerator which is of degree 3 which is simply 1 by L C multiplied by N of s divided by s plus alpha whole squared plus beta squared.

This is one set of poles and the other is s plus alpha prime whole squared plus beta prime squared. Now if I make a partial fraction expansion and take the inverse transform, does it stand to reason that V 0 t will be of the form k 1 e to the minus alpha t sine of beta t plus theta, due to these two poles and the other term would be k 2 e to the minus alpha prime t sin of beta prime t plus theta prime.

Student: Sir, but on looking at the expression of the denominators, we find that there are four poles.

Sir: That is right. There are 4 poles.

Student: Numerator is constant

Sir: Numerator can even be constant. It does not matter. Yes.

Student: Sir, if we can cancel the denominator by the numerator?

Sir: If there are cancellations, then you have to cancel them.

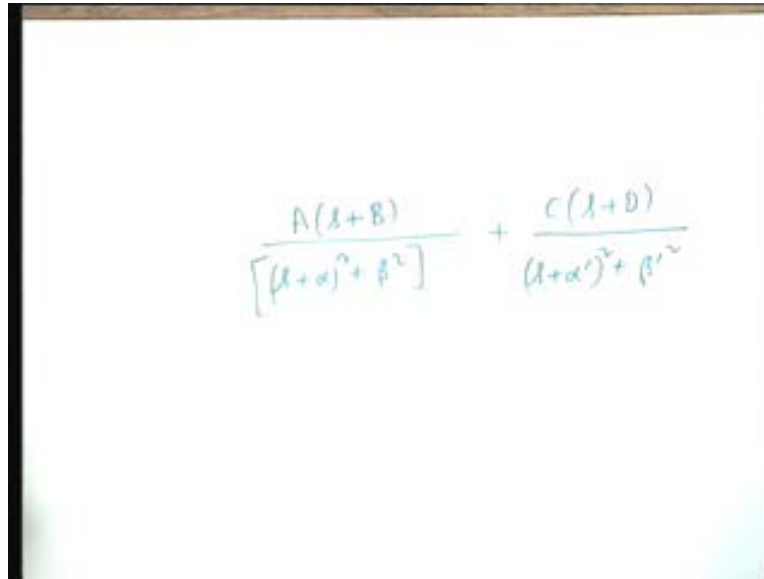
Student: (..)

We have to see, yes, in this case the possibility of cancellation does not arise because you are given the symbols on it. If the numerical values are given, possibly, they could. Yes, that is a good question. We assume here that values are such that there are no cancellations. If there are no cancellations, then this will be the form. There are 4 poles. Accordingly, there are 4 constants to be determined, K_1 , θ_1 , K_2 , θ_2 . You have to use 4 conditions to determine this and you know 2 of them; V_0 plus and the, no, you know V_0 plus V_0 plus V_0 infinity. You will have to possibly find out the primes also. But if you are working in the frequency domain, you do not do anything like that. You simply expand into partial fraction and get the results.

Now, in expanding into partial fraction, one trick of the trade is do not expand in terms of the complex poles. You can say, I will say A_1 divided by $s + \alpha + j\beta$ and the other is A_2 divided by $s + \alpha - j\beta$. You can do that $s + \alpha - j\beta$ you can do that and then find A_1 , A_2 , A_3 , A_4 . If you do that by mistake, even then, there is a saving grace that because the sum of the two has to give you real rational function, A_2 must be A_1^* . This simplifies things but do not do that. That is, this is a wrong route and you are trying to save time.

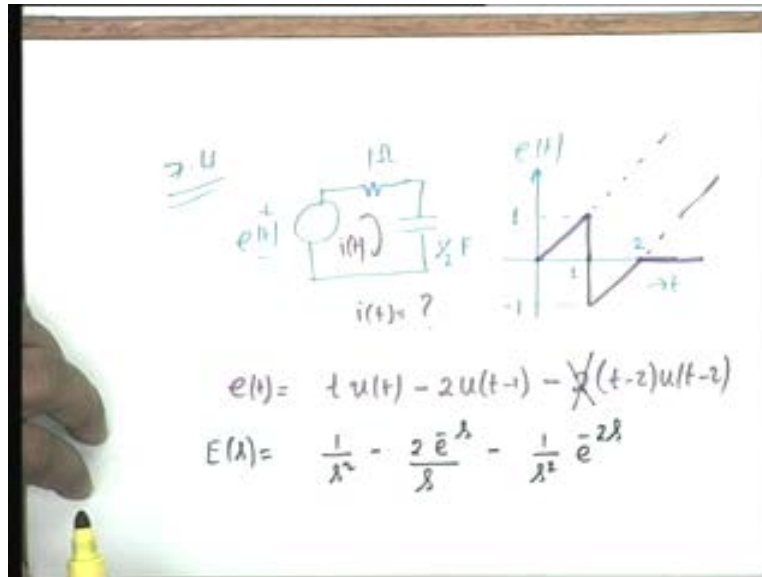
Saving time is possible if you look at this and the degree of the numerator is 1 less; it is less than the denominator so there are no poles at infinity. Then you simply write this in the following form.

(Refer Slide Time: 24:36)


$$\frac{A(s+B)}{[(s+\alpha)^2 + \beta^2]} + \frac{C(s+D)}{(s+\alpha')^2 + \beta'^2}$$

You write this as some A s plus B divided by s plus alpha whole squared plus beta squared plus C s plus D, some C s plus D s plus alpha prime squared plus beta prime squared. How do I know that this will take care of the numerator? The numerator is of degree 3. If you clear this out, obviously the numerator would be degree 3. There are 4 constants here A B C and D, you find out the constants and then invert term by term. You see, if your term like this, then obviously, it can be expressed, the inverse transform would be sum of a cosine and the sine which can be combined to a single sign. So this would be a shorter route to success. You can have a wrong route to failure. If you follow a wrong route, the most possible expectation is failure. Now let us go to the next problem. Next problem would be 7 point 4 and the problem is the following.

(Refer Slide Time: 25:52)



Student: In this problem, should we also find out the constant? (..)

Sir: You shall have to, otherwise, no. no. Since you are required to find out only the form, constants are not required. I chose this complex location to be able to write in very simple form. Now in, the question does not ask to find the constant. It is your choice now. If you want to waste time without any reward, you are most welcome to do it, but do not do it. My advice is do not do it.

7 point 4: There is a circuit of 1Ω and half Farad. This is the circuit. The voltage source $e(t)$ has the following wave form. This is to be seen carefully. It rises linearly to a value of 1 at t equal to 1, and then it falls at t equal to 1, it falls to minus 1 then it rises linearly again to 0 at t equal to 2 and then remains 0 throughout. This is the plot of $e(t)$ versus t .

Obviously, what you have to find out is the response $i(t)$ assuming 0 initial conditions, you have to find out $i(t)$. Obviously, all that you have to find out is the transform of this wave form. Then, the current transform would be the voltage transform divided by the impedance which is R

plus $\frac{1}{s}$. I can do this because there are no initial conditions. Otherwise I would have to write in terms of initial condition. Then apply whatever I wish to apply.

Now obviously, the first thing to do is express e of t in terms of the known functions step impulse and so on and my solution for e of t is that I take t of e of t . There is a systematic way of deriving such functions. You see, if I take t of u off to this unit ramp, what will you do? It goes like this. Now what I want to do is at t equal to 1, I want to bring down by 2 units. So I subtract $2u$ of t minus 1. It must occur only at t equal to 1. So I subtract $2u$ of t minus 1 then I shall get this. Then at t equal to 2, I want to kill the ramp and therefore I add I subtract minus.

Student: t Sir.

Sir: $2t$ minus $2u$ of t minus 2, agreed. You can do this by inspection.

Student: (..)

Sir: No t . Second term is only to bring this ramp down by 2 units to minus 1. Then it goes like this. Then I kill this. you must be very careful about writing this by observation

Student: (..)

Sir: t minus 2 is sufficient. Yes, I was wondering why it is taking so much time to point out the mistake. These are the types of mistakes one does by observation. So what is E of s ? I can write also this by observation $\frac{1}{s^2}$ minus $\frac{2}{s}$ to the minus s , minus what? $\frac{1}{s^2}$ minus $\frac{2}{s}$. So this is my excitation wave form.

Student: the Second term won't have s in the denominator?

Sir: Second term?

Student: s in the denominator?

Sir: Yes, of course, it shall, very good. Any other mistake? No.

(Refer Slide Time: 30:38)

$$\begin{aligned}
 I(s) &= \frac{E(s)}{1 + \frac{2}{s}} = \frac{sE(s)}{s+2} \\
 &= \frac{1 - e^{-2s}}{s(s+2)} - \frac{2}{s+2} e^{-s} \\
 &= \frac{1}{2} \left(\frac{1}{s} - \frac{1}{s+2} \right) (1 - e^{-2s}) - \frac{2}{s+2} e^{-s} \\
 i(t) &= \frac{1}{2} (1 - e^{-2t}) u(t) - \frac{1}{2} (1 - e^{-2(t-1)}) u(t-1) \\
 &\quad - 2 e^{-2(t-1)} u(t-1)
 \end{aligned}$$

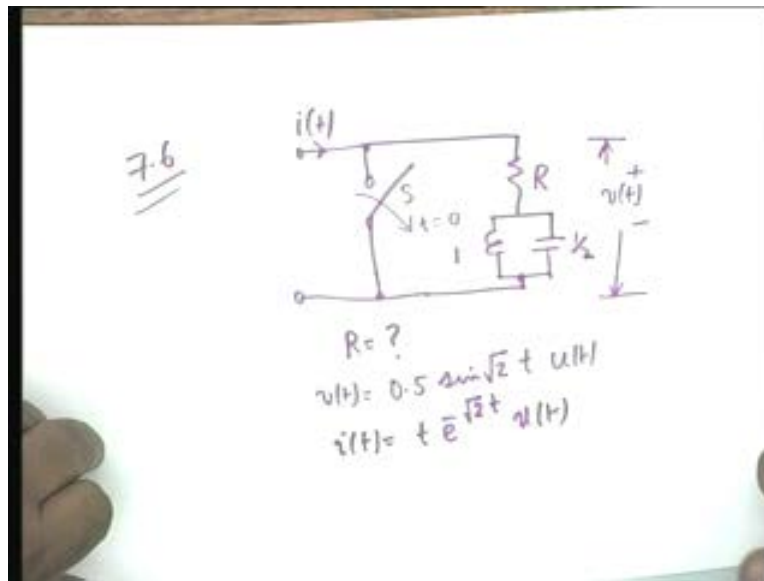
So my E of s either reproduce this here 1 by s squared 1 minus e to the minus 2 s I, combine the first and the third term minus 2 e to the minus s divided by s and therefore, the current transform would be E of s divided by 1 plus 1 by s C that would be 2 divided by s, agreed. This is impedance 1 ohm and half pored capacitor. So this is equal to s E of s divided by s plus 2 therefore, this equal to 1 minus e to the minus 2 s. If we substitute now, this is divided by s squared multiplied by s square.

So it would be s times s plus 2 minus, minus this s and s will cancel, so I will get 2 divided by s plus 2 e to the minus s, is that correct? Oh wonderful! So I can write this as 1 by s minus 1 over s plus 2 multiplied by half multiplied by 1 minus e to the minus 2 s. See everything by an inspection minus 2 divided by s plus 2 e to the minus s and the transform, inverse transform can now be written down term by term.

You see i of t would be equal to half 1 minus e to the minus 2 t u t. This takes care of this factor. Then you multiply by e to the minus 2 s so t will be replaced by t minus 2 that would be minus half 1 minus e to the minus 2 t minus 2 multiplied by u of t minus 2 then minus this term. This

term would be twice, yes, e to the minus 2, then no t minus 1 because of the e to the minus 2 and then you must not forget u of t minus 1. You see, the complete response has been obtained. There are many places, many ups and downs and one can make a mistake very easily. Doing a mistake, committing a mistake is absolutely no problem. Doing it correctly is a problem.

(Refer Slide Time: 33:49)



Our next problem would be 7 point 6. 7 point 6 concerns this circuit, a switch s which is opened at t equal to 0. Oh, I see, opened at t equal to 0 and there is a current source here, i of t . If it closed, the current flows like this. After opening the current flows in the main circuit and the main circuit is, well, for you can say personal bias, I do not like capital X to be used for resistance, capital X is reserved for reactance. So I will use capital R . Then you have a circuit 1 Henry and half Farad. The response required is v of t , not required, look at the condition of the problem, this circuit has 0 initial energy.

In other words, $i_L(0^-)$ is 0; $V_C(0^-)$ is 0. Initial energy is 0 at t equal to 0, this switch s is opened. Find the value of the register R . You are required to find out R such that the response is V t equal to point 5 sine of root 2 t $u(t)$. The response is this, if the excitation is $t e$ to the minus root 2 t $u(t)$. This is the condition. i t is this, v t is found to be this, find out R . So all you have to

do is to multiply the current transform by the impedance and equate that to the transform of the voltage.

(Refer Slide Time: 35:56)

Handwritten mathematical derivation on a whiteboard:

$$i(t) = t e^{-\sqrt{2}t} u(t)$$

$$I(s) = \frac{1}{(s + \sqrt{2})^2}$$

Transformation of $t e^{-\sqrt{2}t}$ is $-\frac{1}{(s + \sqrt{2})^2}$

$$V(s) = \frac{0.5 \times \sqrt{2}}{s^2 + 2}$$

$$\frac{0.5 \times \sqrt{2}}{s^2 + 2} = \frac{1}{(s + \sqrt{2})^2} \cdot \left[\frac{0.5 \times \sqrt{2}}{\frac{s^2}{2} + 1} \right]$$

Result: $R = \frac{1}{\sqrt{2}}$

Let us find out this a bit carefully. i of t equal to $t e$ to the minus root 2 t . So what is capital I of s ?

Student: 1 by s plus root 2 whole square

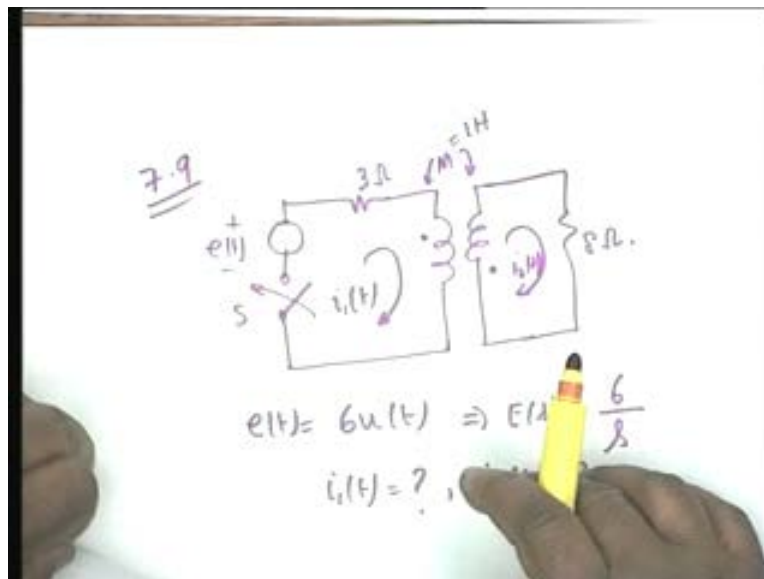
Sir: 1 by s plus root 2 whole square. That is absolutely correct. Does this require consulting a table? No, you should remember this all. To recall briefly, e to the minus root 2 t , the transform is 1 by s plus root 2 and if you multiply this by t and add a negative sign, then this would be differentiation of this which is equal to minus 1 over s plus root 2 whole square and therefore, the transform of $t e$ to the minus root 2 t , I have made a mistake, this must be multiplied by u t and the voltage transform v of s is point 5. The transform of sine omega t is simply omega divided by s squared plus omega squared, which is . So this is equal to current transform which is, no, 1 over s plus root 2 whole squared. This is the current transform multiplied by the impedance, which is R plus the impedance of a capacitor and an inductor in parallel is s L

divided by $s^2 LC + 1$. So L is 1 so s divided by $s^2 LC$, this would be s^2 divided by $2 + 1$.

So all that you have to do now, is to find out R such that these 2 sides are equal. Now the things have been so chosen that the capital R comes out as a constant independent of s . If you do it wrongly, then you can be assured that R will come out a function of s and then you know immediately that you made a mistake. My solution is R equal to $1/\sqrt{2}$ while obviously, it is not very difficult because s^2 by $2s^2 + 2$ will come. This will cancel and so on and so far that algebra you can do. My next problem would be 7 point 6.

Oh, 7 point 6 I have done, so 7 point 8. Have not I done 7 point 8? I have done that in the class. So I go to 7 point 9. 7 point 9 would turn out to be interesting.

(Refer Slide Time: 39:16)



For the transformer shown, the transformer is this, the switch s is turned on at t equal to 0. There is a 3 ohm register and then there is a transformer with dots in the reverse direction and capital M is the mutual inductance, is given as 1 Henry. Then you have 8 ohms and the mesh currents are indicator is $I_1 t$ and $I_2 t$. It is given that $e t$ is equal to $6 u t$ which means that capital E of s is

equal to 6 divided by s and that prior to the switching action, all initial energy was 0 and capital M equal to 1 Henry. You are required to find out I 1 t and I 2 t.

This is an exercise in which, the standard convention where capital M is consider positive has been done away with. The dots have been reversed, so M should have been negative. But you notice the current is also been reversed and therefore, you go back to our original equations 2 negatives make one positive. Is that clear? The current is also been reversed and therefore, my equations, I can write down now by inspection.

(Refer Slide Time: 40:53)

The image shows a hand pointing to a whiteboard with the following handwritten equations:

$$\frac{6}{s} = (3+s)I_1 + sI_2$$

$$0 = sI_1 + (8+2s)I_2$$

$$I_1 = \frac{\begin{vmatrix} \frac{6}{s} & s \\ 0 & 8+2s \end{vmatrix}}{\begin{vmatrix} 3+s & s \\ s & 8+2s \end{vmatrix}}$$

The equations for the 2 meshes 6 by s would be 3 plus, well, L 1 and L 2 are given. This is 1 Henry and this is 2 Henry. So it would be 3 plus s multiplied by I 1 plus, it would be a plus sign, a mutual inductance is 1, so its impedance is s M equal to s times I 2. The plus sign has been restored because dots have been reversed, current is also been reversed and the other equation would be 0 and the secondary, obviously, there is no source therefore 0 would be equal to s I 1 plus 8 plus 2 s multiplied by I 2 and now the problem is to find I 1 and I 2. Obviously, this is a simple problem 3 plus s 8 plus 2 s s s and the numerator, the first column replaced by the right hand side. On the left hand side, we have 6 by s 0 s 8 plus 2 s and now you can make simplifications.

(Refer Slide Time: 42:20)

$$I_1 = \frac{12}{s} \frac{s+4}{s^2 + 20s + 48}$$
$$\frac{12}{s} \frac{s+4}{(s+3)(s+16)}$$
$$\frac{12}{s} \frac{s+4}{(s+2)(s+12)}$$

My simplification gives me the result, I_1 equal to, once again a pinch of salt is to be added, s plus 4 divided by s squared plus 20 s plus 48. Unfortunately, I could not factorize this because I do not know. No factors of 48 add up to 28. Plus 6 is 14, 4 plus 12 is 16.

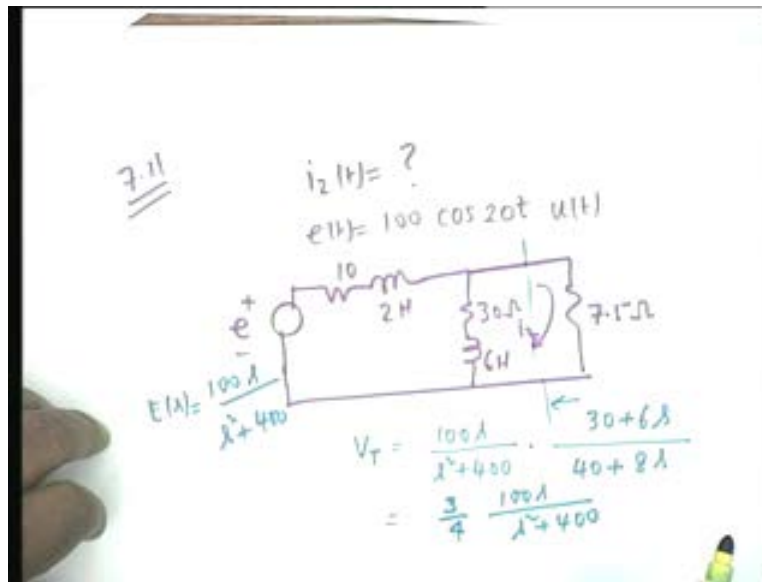
Student: Sir, s squared $(..)$ 14 plus 24, $(..)$

Oh I see, I have made a mistake there.

Student: And the solution is..

Sir: Just a minute, let me check. 3 plus s 8 plus 2 s minus s squared so twenty. Oh! Yes, I made a mess. This is twenty 4 this is 24 and this is 14. Now 12 and 2, oh wonderful! Therefore, I_1 equals to 12 by s , s plus 4. Is the numerator alright? So it becomes s plus 2 s plus 12 and obviously, this will be the sum of three terms. There will be a 1 by s term 1 by s plus 2, 1 by s plus 12 and you can find out $I_1 t$ by inversion. Similarly then, you find $I_2 t$. I have time for just 1 more problem and I will take 7 11.

(Refer Slide Time: 44:11)



7.11, at the bottom of this line, it is not come out well in my, find $i_2(t)$ using Thevenin's theorem. The excitation is $e(t)$ equal to $100 \cos 20t$, that t is missing, then multiplied by $u(t)$. Assume 0 initial energy. The circuit is this, e . Then you have 10 ohms and 2 Henry then 30 ohms and 6 Henry . Then you have 7.5 ohm and this is the current that you have to find out, i_2 and you have to apply Thevenin's theorem. Obviously, Thevenin's theorem means this is the load now.

So you will have to apply Thevenin's theorem to the left of this line and what is $E(s)$. It is 100 multiplied by s divided by $s^2 + 400$, wonderful. We know $E(s)$ therefore, V_T , the Thevenin voltage would be $100s$. I solved this by inspection. $s^2 + 400$, multiplied by $v(t)$ means this open and therefore, $30 + 6s$ divided by, what we get? $40 + 8s$. While do not you see that they will cancel out, $s + 5$ cancels out. So it becomes simply 6 by 8 which is 3 by 4 , $100s$ dot $s^2 + 400$.

This is a case of degeneracy. Normally, I should have a cubic polynomial in the denominator. It is a quadratic because there is a cancellation of terms.

(Refer Slide Time: 46:35)

Handwritten equations on a whiteboard:

$$V_T = \frac{75s}{s^2 + 400}$$

$$Z_T = \frac{(30 + 6s)(10 + 2s)}{40 + 8s}$$

$$= \frac{3 \times 2}{4} (s + 5)$$

$$= \frac{3}{2} (s + 5)$$

$$I_2 = \frac{V_T}{Z_T + 7.5} = \frac{3}{2} (s + 10)$$

A small circuit diagram is drawn to the right of the equations, showing a voltage source V_T in series with an impedance Z_T and a load impedance 7.5 .

So v_t equals to $75s$ divided by s squared plus 400 and what is Z_t ? Z_t is the parallel combination of 30 plus $6s$, 10 plus $2s$ divided by 40 plus $8s$ and once again, you see that there is a cancellation. I get 3 by 4 multiplied by $2s$ plus 5 . This is equal to 3 by $2s$ plus 5 Z_t . Why does this happen, because the time constant of this circuit is same as the time constant of this circuit. So they two combine to give you a simple RL circuit. Is that clear?

The ratio of R by L here is the same as the ratio R by L here. The time constants are the same and therefore, $i_{sub 2}$ is equal to V_T divided by Z_T plus, the current in this Z_T , plus 7 point 5 . Is this point clear? Our Thevenin's theorem says that we have V_T , then series Z_T and the 7 point 5 . The current therefore, is V_T by Z_T plus 7 point 5 and you can notice that here also, there is a simplification. 3 times 3 by 2 multiplied by 5 , 15 by 2 is 7 other 5 . So the denominator here shall be simply 3 by $2s$ plus 10 . That is correct and whatever the numerator is, you can now go ahead and solve it. You go ahead and do it. This is where we close.