Circuit Theory

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Lecture - 13

Poles, Zeros and Network Response

Circuit lecture and we are going to discuss poles, zeros and network response on the j omega axis, that is, for sinusoidal excitation.

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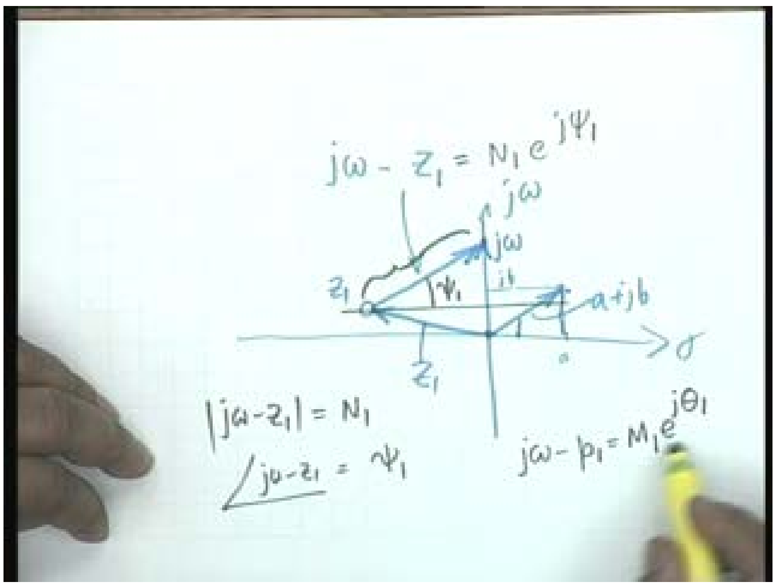

As we have seen last time, a network function f of s is real and rational, is of the form N of s by D of s and we express this in the form K times N of s by D of s, where the leading coefficients in both numerator and denominator are unity. We assume that the degree of N of s, this is the way it is written, is less than the degree D of s. That is, we assume that capital F of s is the proper rational function. We also showed that if we find the roots of N of s and D of s, then we can write this as s minus Z i continued product over, I divided by continued product s minus p j continued product over j where Z i at the zeros and p j at the poles.

What we wish to find out is capital F of j omega, that is, when s equal to j omega and then plot the magnitude and phase of capital F with frequency or the real part and imaginary part. Now in the process, you see that the terms now become of the form j omega minus Z i divided by continued product j omega minus p j and while analytically of course, we can find out the amplitude and phase, it is instructed to see how it can be obtained graphically and what further information can be obtained from the graphical procedure.

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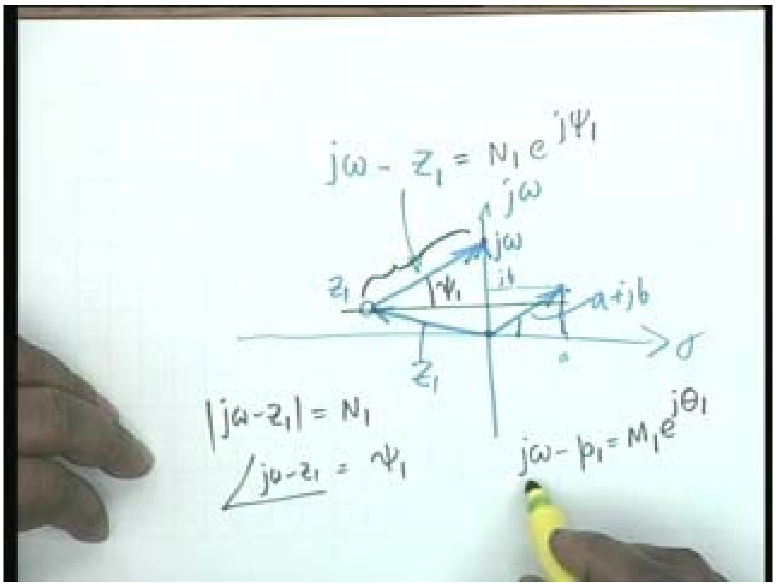

Let us represent the quantity j omega minus, let us say, Z 1, a typical factor in the numerator, a typical factor in the denominator, we shall also be found in the same fashion. Suppose you have the s plane sigma j omega and Z 1, let us say, is somewhere here. This 0 is a complex 0. Of course, Z 1 star shall also be but that is beside the point. Now if I find a point, if I take a point j omega on the j omega axis, then j omega in the complex plane is represented by a vector starting from the origin to the point, which is distant omega on the vertical axis.

j omega represents this vector just like a plus j b. If this is j b and this is a, a plus j b is a point and it is also a vector. This is a plus j b. Its magnitude is equal to magnitude of this and the angle is equal to this angle. a plus j b is in the complex plane. It is a vector whose, the length, the distance from the origin is its magnitude, square root of s squared plus b squared and this angle which is 10 inverse b divided by a is the angle.

So j omega is a vector which lies on the vertical axis, j omega axis and similarly Z 1 is a complex quantity; it is a point. It is also a vector like this and by the triangle of vectors, this is the vector Z 1. By the triangular vectors j omega minus Z 1 would be this vector, agreed? j omega minus Z 1 is this vector because Z 1 plus j omega minus Z 1 is j omega by the triangle of vectors and therefore, j omega minus Z 1 in the s plane is a vector whose length, this length is the magnitude of j omega minus Z 1, let us call that as, let us call this as N 1 and the angle that it makes with the sigma axis, let us call this angle as psi 1.

This gives the angle of j omega minus Z 1 s equal to psi 1. Is that clear? The graphical representation and therefore, j omega minus Z 1 can be written in terms of the polar form as N 1 e to the power j psi 1 and this can be done for all the 0 factors, for all the factors of the denominator and all the factors of the numerator and therefore, if I represent j omega minus p 1 by its magnitude is m 1 and angle is, let us say theta 1, exactly like the 0 factor, we can represent the pole factor like this.

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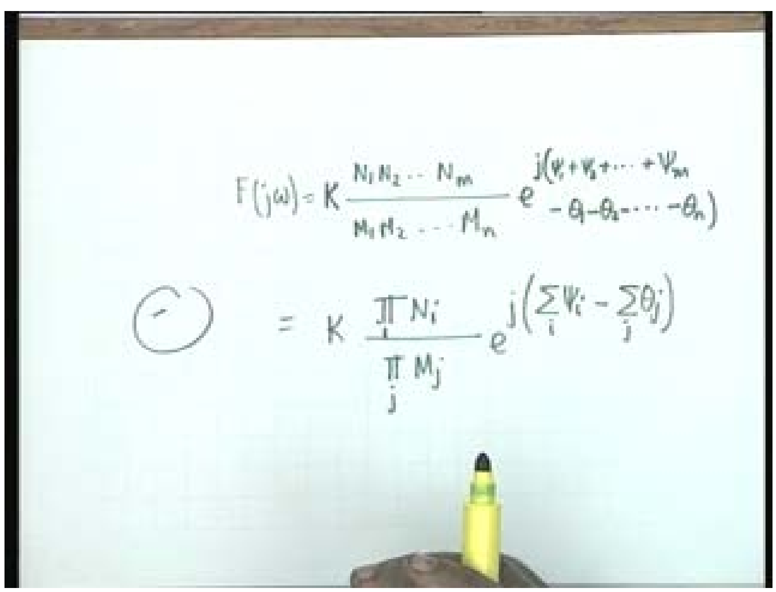

If the magnitude and angles are M n theta, then obviously, a network function F of j omega can be represented as N 1 N 2 up to N m with the degree of N of s is M divided by M 1 M 2 up to M n, if the degree of denominator is N multiplied by k, the constant and e to the power j, the angles of the numerator factors which s psi 1 plus psi 2 plus etcetera plus psi m, then minus because this comes with M 1 e to the power j theta 1. When it goes up it becomes minus theta 1, minus theta 2, minus etcetera, minus theta n. This is the representation of the complete network function. I can abbreviate this as continued product of N I, continued product of M j, continued product over I, over j e to the power j summation psi i over i minus summation theta j over j.

This helps us to pictorially see the shape of the network response either its amplitude response or the phase response or both and this is a graphical procedure for evaluating a network function. Although it is hardly used to actually compute network function, it helps us to visualize and to solve, wherever there is a question whether the phase is plus 180 or minus 180. If a quantity is negative, the angle can be plus 180 or minus 180. So same thing but if we slightly differ from the frequency at which the phase is 180, it will matter whether it is positive angle or negative angle because we go either in the second quadrant or in the third quadrant and therefore, in solving such riddles or such places where there is a chance of confusion, this picture, this pictorial diagram helps us tremendously and we shall illustrate this with an example. Is this graphical procedure clear? We take an example of a network function, a simple enough network function.
Let us say f of s is equal to 4 s divided by s squared plus 2 s plus 2.

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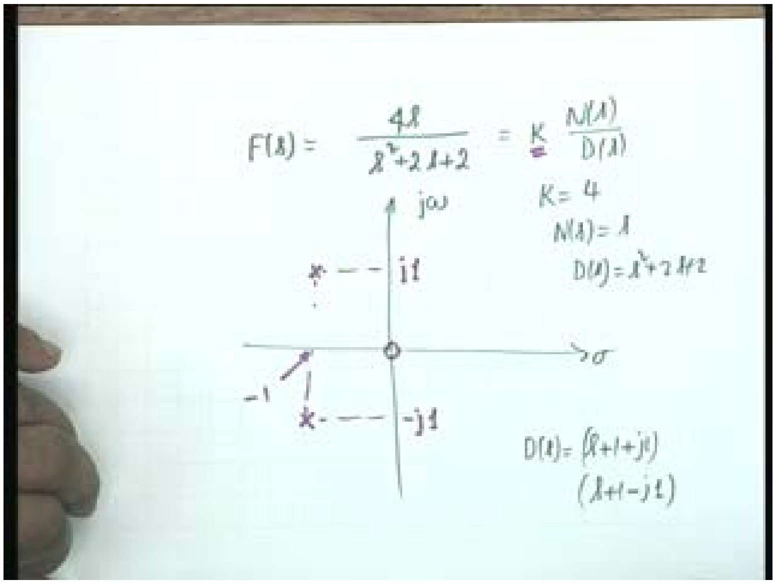

Obviously, we have a 0 at the origin. This is 4 times so, if you write this as k N of s by D of s then k is equal to 4. N of s is simply equal to s and D of s equal to s square plus 2 s plus 2 and it is not difficult to see that D of s the poles are the factors are s plus 1 plus j 1 multiplied by s plus 1 minus j 1. So the pole 0 diagram would be like this. We have a 0 at the origin and a pole here and a complex conjugate pole here, where this is minus 1 and this point is j 1, this point is minus j 1, agreed?

This is the pole 0 diagram and all that is required to know to know the network function is the multiplying constant k. Once you know the poles and zeros, you can construct the network function, provided you know the multiplying constant or two, within an arbitrary multiplying constant, you can find the network function. Now suppose we want to find out, let us say I will draw the diagram again.

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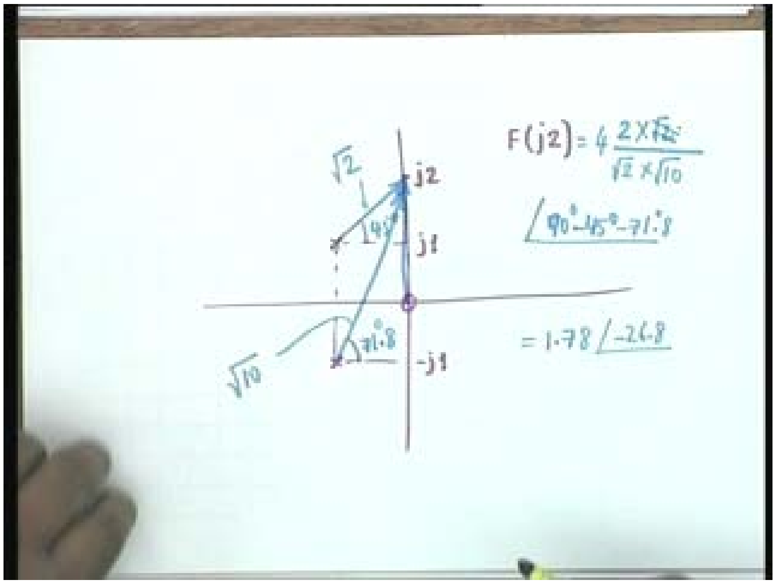

Suppose you have this and this and this is the 0. This is j 1, minus j 1. Suppose you want to find out F of j 2, at the frequency 2 omega equal to 2, you want to find out what the network function is. So mark the point j 2 here and then draw the vectors from the 0 and also from the two poles. From the 0 obviously, the magnitude of the vector is 2, well, F of j 2 will be 4 multiplied by, this length is 2, and the angle is 90. Then from this, the vector is this and obviously, this is 1. So the length of this vector is root 2 and this angle is 45. So this will be 2 multiplied by root 2 and it would be plus or minus?

This is the angle of pole and therefore, minus 45 and then you have to draw another vector from this to the point j 2 and one can find out this length is 3 and this length is 1 and therefore, you can see that this is equal to square root of 10 and the angle is tan inverse 3 by 1. tan inverse 3 is 71 degrees, 71 point 8 degrees and therefore,

Student: Sir, should not we divide by?

Sir: Yes. This root 2 should come in the denominator and root 10 should also come in the denominator because this is a pole factor and the angle should, again, subtract 71 degrees point 8 and this comes out as 1 point 7 8 angle minus 26 point 8. In a similar manner, we can find out at a few representative values and then plot capital F of j omega magnitude and its angle, that is, its phase. Let us look at some more interesting features of this network function.

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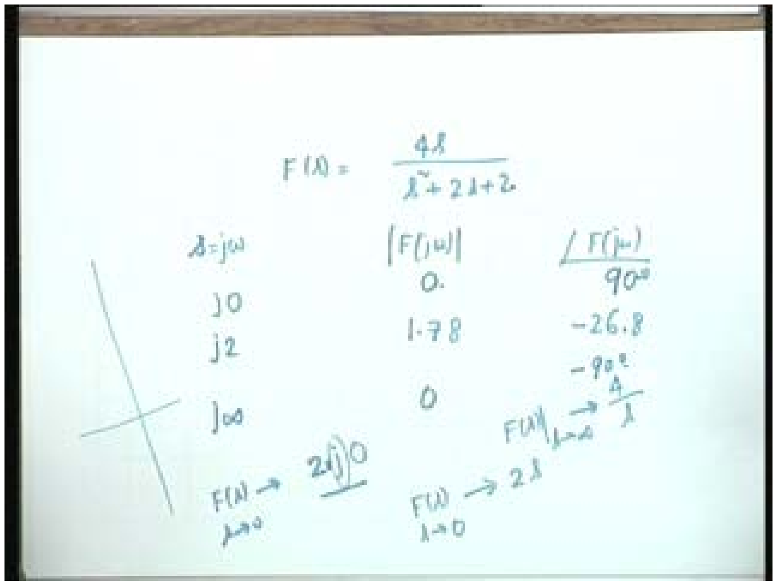

Our function is F of s equal to 4 s divided by s squared plus twice s plus 2 and we have just found out that s equal to j omega. The value of F of j omega, magnitude and the angle of F of j omega, we have just found out one representative point, that is, at j 2. At j 2 the magnitude is 1 point 7 8 and the angle is minus 26 point 8. We can find out at some other frequencies but the frequencies that are most easily calculated, the network function is calculated are the extreme frequency that is 0 and infinity. Suppose s equal to j 0, now if s tends to 0 then obviously F of s tends to the lowest power here and the lowest power here. So you get 4 by 2, that is, 2 times j 0. The magnitude obviously is 0 but the angle is plus or minus? Plus 90. The angle is plus 90.

Student: Sir, how does it?

Sir: Because s equal to j 0. This behaves as 4 s by 2 s tends to 0 which is equal to twice s and I have substituted s equal to j 0. The magnitude is 0 but the angle is 90.

Student: Sir, Won't it of the form 0 plus 0?

Sir: 0 plus, no, I have put s equal to j 0.

Student: Sir, but the total form is of the form 0 plus 0. (..)

Sir: So?

Student: The angle is not actually defined (..)

Sir: Then 0 by 0 is not defined. But you see the value, as s tends to 0, 0 plus j 0, 0 minus j 0, minus 0 plus j 0, minus 0 minus j 0, they are all the same point. As s tends to 0, either of this, any of these 4 combinations, this tends to twice s. There is no doubt about that? Now you put s equal to 0, s equal to j 0 and the angle we are traversing, the j omega axis not the real axis, if we have traversed the real axis we now put s equal to 0, now or minus 0, whatever it is. So the angle is phi by 2 because of the factor j.

In a similar manner if you look s tends to infinity, then F of s, s tends to infinity tends to the highest power factors. That is, 4 s by s squared. So it will be 4 by s 4 by s, not 2 highest power factors. These are the ones that dominate when s tends to infinity and therefore, the magnitude at s equal to infinity, j infinity would be 0 but the angle would be minus 90 and therefore, we know how the magnitude varies, say magnitude starts from 0 goes to 0. In between it must go through a maximum and the angle starts from plus 90 and goes to minus 90. So in between it must pass through 0. The angle must pass through 0. Indeed, if you take a few more frequencies on the j omega axis and find out the values, then the plot, you can show that the plot is like this.

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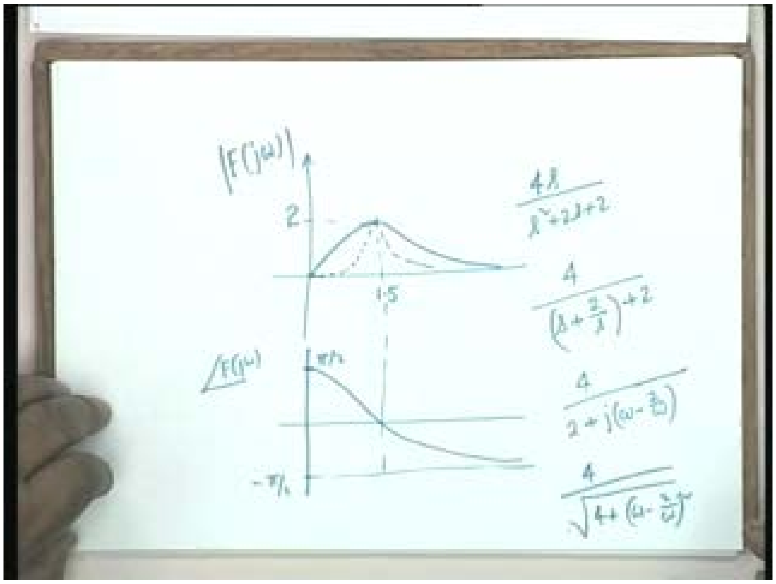

The magnitude that is capital F of j omega magnitude is like this. It starts from 0, goes to a maximum and then infinity and this point happens to be approximately 1 point 5 and the maximum approximately is 2. On the other hand, the phase, angle of F of j omega starts from plus 90 plus phi by 2 and goes to minus phi by 2. Can you tell me where a phase will be 0? Guess it is this frequency, 1 point 5. You can see by putting 1 point 5 that the phase is indeed 0. So it goes like this. The plot is like this. I am tempted to tell you another, one more feature of this transfer function that it is, it looks like a band pass filter. Is not it? This criminates against low frequencies, it discriminates against high frequencies and it accepts a band of frequencies around 1 point 5.

So it is a band pass filter but it say bad band pass filter. It is a lousy band pass filter because discrimination is not strong enough. We would have called it strong enough if we had a curve like this. So there you see, there is a weak resonance here, weak resonance here at this frequency, 1 point 5 and if you look at the function 4 s by s square plus 2 s plus 2, it is not very difficult to find out to show that it is indeed, a band pass filter. You see, you can write this as 4 divided by s plus 2 by s plus 2, which I can write as 4 divided by 2 plus j s equal to j omega.

We can write this omega minus 2 by omega and now you can see the magnitude will be 4 divided by square root of 4 plus omega minus 2 by omega whole squared and if you look at this, obviously, the maximum will be reached because this is a whole squared whole squared term cannot be negative. The maximum will be reached when this is 0. That means omega squared equal to 2 which means omega is 1 point 4 1. So this maximum is exactly at root 2; not 1 point 5. It is at root 2 and the maximum value there is 4 divided by square root of 4. It is equal to 2.

These are matters of common sense. I am not done any analysis or anything I have just written down the expression and I am trying to interpret. Is that clear, that the magnitude at omega equal to root 2 shall be equal to 2 and look at the phase, when omega is equal to root 2, this term vanishes. So the angle becomes exactly 0 degree. Is not that right? This term vanishes, the imaginary part of the denominator and therefore, it becomes purely real quantity, positive quantity, the angle must be 0. You can also see that when omega tends to 0, the angle is 90, plus 90.

How, if omega tends to 0, this term becomes negligible compared to this term and so the denominator goes to minus j infinity and minus j in the denominator is equal to plus j in the numerator and plus j corresponds to an angle of 90. In a similar manner, when small omega goes to infinity, the second term becomes negligible. The angle of the denominator becomes plus 90 and therefore, the angle of the network function becomes equal to minus 90.

These are the things that an engineer looks at, when he looks at an expression and he should immediately, as an electrical engineer and as an as a student of SCDR in your future life, whenever you look at such a function, you should be able to say that this is a band pass function, maximum will occur here, this will be the maximum value, the phase will start from here, phase will end somewhere else at another angle and it passes through 0 phase at such and such frequency. This should be obvious. No analysis should be needed.

Now let us look at some other interesting cases. Suppose I have a network function, now I am going to talk of only common sense nothing else.

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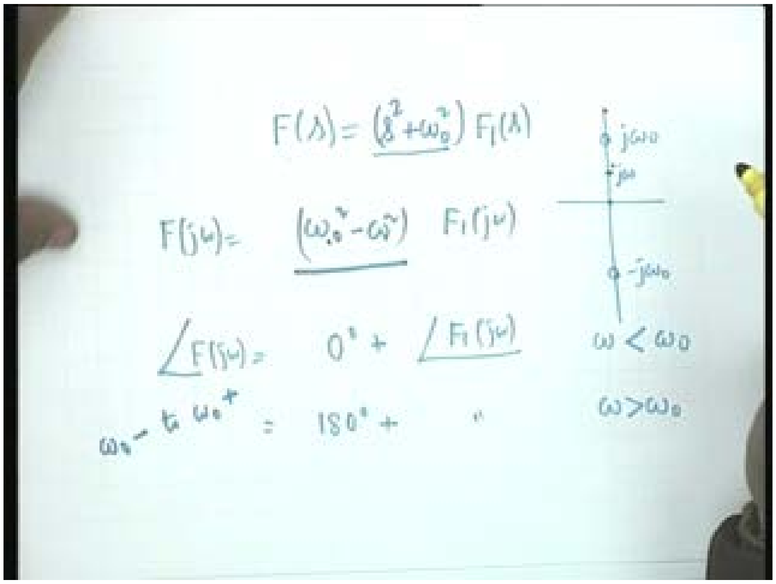

Suppose you have a network function F of s which has a pair of zeros at plus minus j omega naught pair of zeros on the j omega axis j omega naught and minus j omega naught. Why do they occur in conjugate pairs? Why do these complex poles and zeros occur in conjugate pairs?

Students: Coefficients are real.

Coefficients are real. There is another way of saying this, what is it? If the coefficients are real, the function is called a real function. It is a rational function and I also said it is a real function. That is, the function is real when the variable is real, then that is called a real function. So complex conjugate poles, therefore, corresponding to these two complex conjugate zeros, corresponding to these two, we shall have a term s squared plus omega naught squared in the network function multiplied by some other network function, let us say F 1 of s. So you are considering a network function F of s which has a pair of zeros on the j omega axis. Now let us see what happens.

Suppose I have a frequency j omega which is less than j omega naught, that is, I consider a frequency omega below omega naught, then what would be the angle? Do not look at the expression. What would be the angle due to these two zeros? The angle would be from 0 to j omega, no, what we have is from j omega naught to j omega, this vector has an angle of minus 90. From j minus j omega naught to j omega, the angle is plus 90. So the angle should be 0. Now let us look at the expression, F of j omega, obviously, is equal to omega naught squared minus omega squared F 1 of j omega.

If omega is less than omega naught squared, then this factor which is contributed to by the two imaginary zeros contributes a phase of 0 degree; plus 90 and minus 90. So the angle of F of j omega would be equal to 0 degree plus the angle of F 1 of j omega, agreed? Provided omega is less than omega naught. What happens when omega exceeds omega naught?

Student: (..)

Sir: The angle becomes 180 degrees. Plus or minus?

Student: Plus

Sir: Plus because you take it here. This angle is 90. This angle is also 90. So it is plus 180 degree. It can also be seen from the expression, if omega exceeds omega naught then this becomes negative. So it contributes a phase of 180 degrees. But it is not quite clear from the mathematical expression whether it is plus 180 or minus 180. That becomes clear if we take help of the diagram. So the angle is equal to 180 plus angle of this, if omega is greater than omega naught.

In other words, at omega equal to omega naught there is a jump of phase from 0 degree to 180 degrees. Is that clear? At omega equal to omega naught, the phase, whatever the phase is from omega equal to omega naught minus to omega naught plus, there is a jump of phase by 180 degrees contributed to by the factor s squared plus omega naught squared now. Yes?

Student: Sir, we do not have any information about the phase of F 1 of j omega.

Sir: We do not need because angle of F 1 of j omega, that may also have other j omega axis poles or zeros but what I am saying is, whatever is contributed by angle of F 1 of j omega. That is different. Due to these two there shall be a jump of phase from 0 to 180.

Student: So F 1 of j omega may also have a pair of zeros

Correct. So if it also has a pair of zeros, at some other at plus minus j omega naught, then the jump would be 360.

Student: Sir, you are saying that phase would be zero degree plus F 1 j omega to omega less than omega naught. Sir, but when omega is less than minus omega naught, it will be minus (..)

Sir: No, I am considering a positive frequency. See, conventionally, that is a good question. You see, in this plot, I plotted only for positive frequency because I know that for negative frequencies, the angle is an odd function and therefore, I can easily construct it. Similarly for magnitude, I do not have to construct again. It is an even function, so it simply repeats. I consider only for positive frequency. That is a good pointer, good point to question.

So due to a pair of j omega axis zeros, I repeat there shall be a jump of 180 degrees. I am considering only this factor. I am not considering the jump that may occur due to F 1. Well, if it does then we will have to add it. On the other hand, if instead of zeros these were poles, if these were poles, that is, if we had a situation like this:

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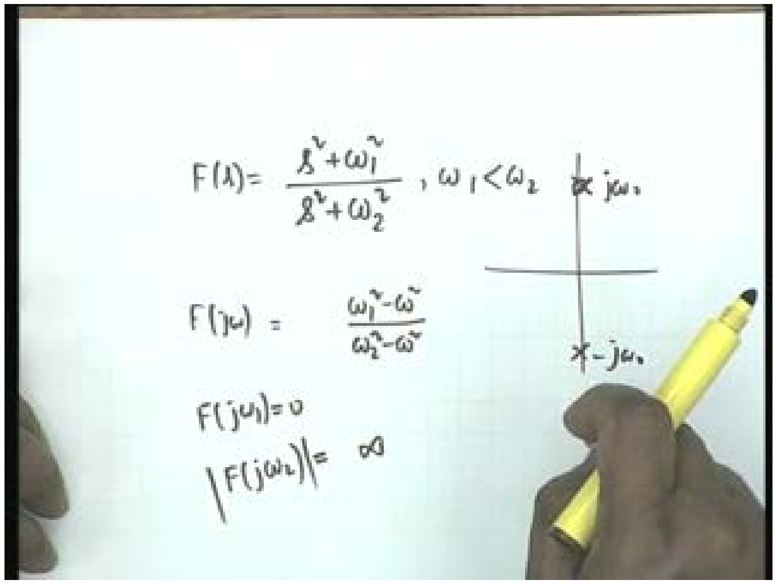

Let us say, pole plus j omega naught and minus j omega naught, then the jump would have been from 0 to minus 180. Is that clear?

Student: Sir, can you explain?

Sir: Can I explain? For the zeros, if there is a pair of zeros, the jump is from 0 to plus 180. Now if there are a pair of poles plus 180 jump in the denominator corresponds to minus 180 for the total function. So due to a pair of poles on the j omega axis, the jump in phase from omega 0 minus to omega 0 plus would have been 0 to minus 180 and if I consider a function like this, let us say F of s equal to s squared plus omega 1 square divided by s square plus omega 2 squared.

Suppose you have a function, total function is this, where omega 1 is less than omega 2 and if I plot its magnitude and phase. The magnitude, obviously, at omega equal to omega 1 shall be 0. Is not that right? Because it will omega 1 squared minus omega square and if omega equal to omega 1, the magnitude shall be 0. On the other hand.

Student: Sir, what about the other contribution of minus j omega 1?

Sir: I have already said that this is omega 1 is less than omega 2. Let us write down F of j omega equal to omega 1 square minus omega square divided by omega 2 square minus omega square. The contribution due to this, whatever it is, if it is not 0, then we do not bother. 0 divided by any quantity is 0 and therefore, the magnitude at j at omega equal to omega 1 shall be equal to 0. F of j omega 1 shall be equal to 0 and there shall be a jump of phase at omega 1 from 0 to 180. If omega exceeds omega 1, if omega is omega 1 plus, obviously, the quantity shall be negative and the phase shall be 100 plus 180.

On the other hand, at omega equal to omega 2, what is the value? The magnitude is infinity. At omega equal to omega 2, the magnitude is infinity and the angle shall jump from whatever value it was by minus 180 degrees, whatever value it was. Now it is very

Student: Will it be minus infinity?

Sir: Pardon me.

Student: Will it be minus infinity since omega 1 is less than omega 2?

Sir: Magnitude. Now if you reflect on this a little bit, then you can show, you can see that the magnitude of F of j omega, I have said omega 1 is less than omega 2.

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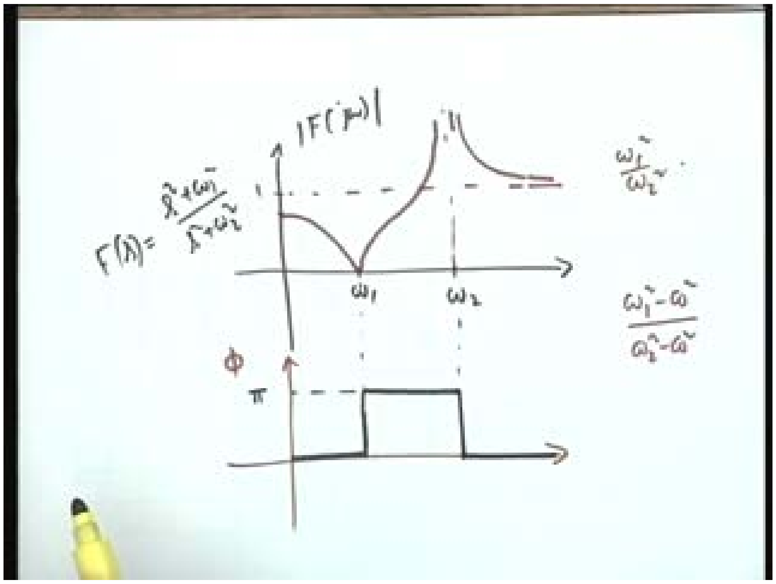

You can see that the magnitude is always positive. The magnitude shall pass through 0. Magnitude, what shall it starts from? At s equal to 0, what is the value magnitude omega 1 square divided by omega 2 squared. This is less than 1, right, because omega 2 is assumed to be greater than omega 1. So if 1 is here, it starts from somewhere here. Then it must pass through a 0 at omega equal to omega 1. At omega equal to omega 2, it goes to infinity. So it must go like this. Infinity and then when omega goes to infinity, what is the value?

My function is omega 1 squared minus omega squared omega 2 squared minus omega squared. When omega goes to infinity, the value is unity and therefore, it comes back to this. This is the plot of the magnitude. I cannot show infinity on a finite plane and therefore I simply break it. This is the magnitude. What about the phase? Let us look at the phase, phi. The phase you see, if omega is less than omega 1, obviously, phase is what? 0. If omega is less than omega 1 this quantity is positive this is also positive and so the phase is 0. At omega equal to omega 1, there is a jump from 0 to 180. So this phase will be pi plus pi.

Between omega 1 and omega 2, the phase remains at 180 and at omega 2, the jumps from 180 to 0. That is, jump is from 0 to minus hundred and 80 and therefore, this is the phase plot. Is this okay? My F of s is s squared plus omega 1 squared divided by s square plus omega 2 squared. If you have understood this diagram, then the next discussion is going to be very easy to follow. In this discussion, we have assumed poles and zeros on the j omega axis. Suppose we shift the poles on the j omega axis slightly to the left, poles cannot be shifted to the right, cannot be shift to the right half plane because then the system would be unstable. We cannot have a LLF PB network which is unstable.

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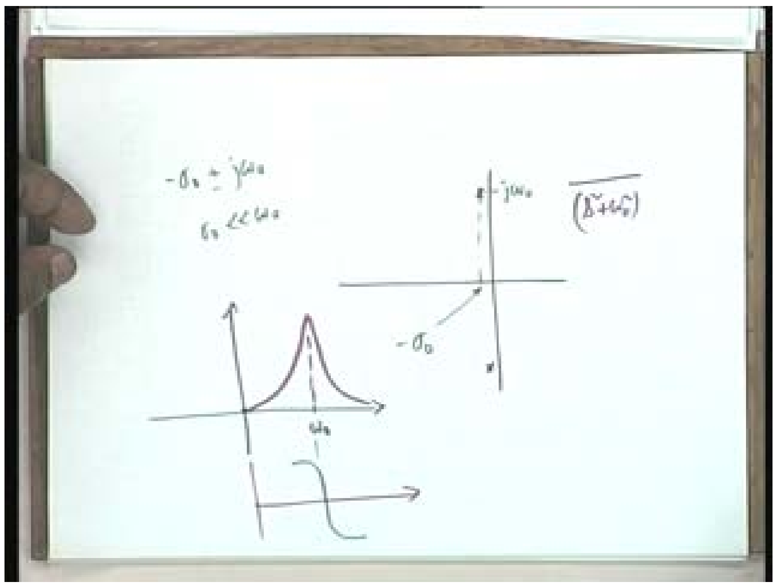

So if I shift the poles slightly to the left, suppose they are here and the coordinates are, let us say, this is sigma naught minus sigma naught and this is j omega naught. So we have 2 poles at minus sigma naught plus minus j omega naught where obviously sigma not is much less than omega naught. If this is the story then what kind of magnitude response do we expect? You see, if this was exactly on the j omega axis then the magnitude at that point should have been infinity, agreed? If I shift slightly to left, obviously, it cannot be infinity.

Therefore, what will happen is, it will show a peak like this but it shall not be quite infinity. This will be a band pass type of response. Due to this, it would be a band pass type of response and what about the phase?

If these poles were on the j omega axis, exactly on the j omega axis, then the magnitude at omega equal to omega naught would have been infinity, right? If we have a factor s square plus omega naught square in the denominator then at omega equal to omega naught, this factor becomes 0. So 1 by 0 becomes infinity but suppose the pole is not at plus minus j omega naught, but at minus sigma naught plus minus j omega naught, it is slightly shifted to the left. Then infinity cannot be reached, but it would be very high at omega equal to omega naught and what about the phase if I plot the phase, whatever be the previous case, there shall be a jump through approximately 180 degree. Jump down or jump up?

Student: Down.

Sir: Down. So at around omega naught, the phase will very rapidly shift from whatever value it was by minus 180 degrees approximately, because it is not exactly on the j omega axis. On the other hand, if we have a pair of zeros, is this point clear, if we have a pair of zeros on the j omega axis plus, I beg your pardon, slightly shifted from the j omega axis.

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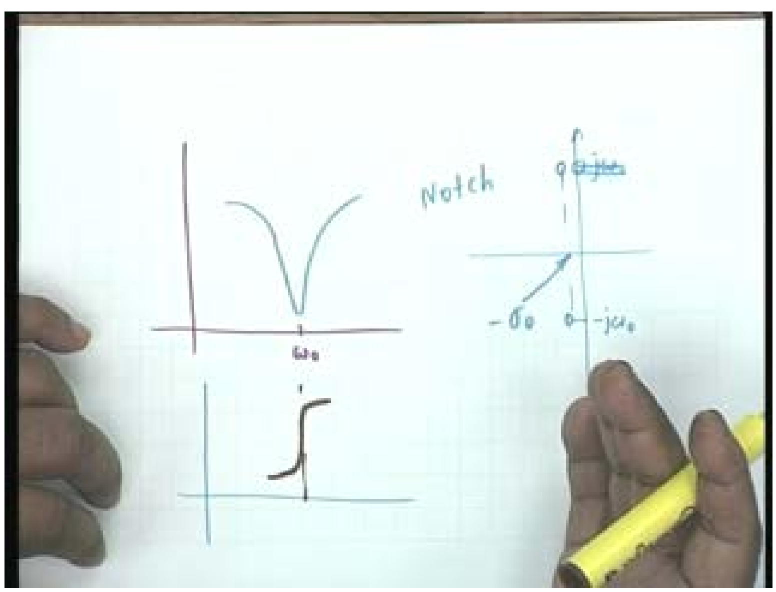

Let us say this is minus j omega naught and this is minus sigma naught. Then the magnitude function at omega naught will not show a 0, but will show a dip. It will not reach exactly 0, but it must show a dip like this, agreed? If these are shifted to exactly on the j omega axis, it will go through a 0. Such a filter, such a response is a band stop response or sometimes called a notch, response notch. There is a notch here and what about the phase? Whatever be the phase earlier, the phase at omega equal to omega naught must increase rapidly through approximately 180 degrees.

These are the strongest tools of circuit theory, an electrical engineer who knows circuit theory. By looking at a network function, he should be able to say what are the most critical frequencies and what happens at those critical frequencies, what happens to the phase, what happens to the magnitude and so on and so forth. Now we said that you cannot transfer poles to the right half plane. There cannot be poles in the right half plane, can there be zeros?

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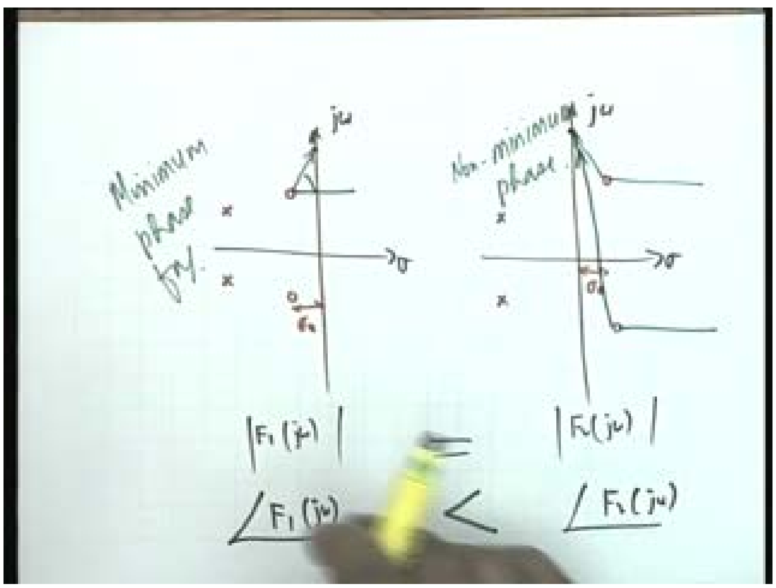

Let us consider two situations. There cannot be any poles in the right half plane, the question is can there be zeros? Yes, of course zeros. I am not restricted. There can be zeros. Suppose, I have a situation in which there are two zeros like this and there 2 poles like this and a situation in which this zeros are shifted, let us say to the right half plane by the same amount. If this distance is sigma 0 then we take sigma 0 in the right half plane and shift these zeros here and here. That is, we consider j omega axis as the mirror and shift zeros to their mirror images, agreed?

Now the poles remain intact; poles cannot be shifted. Suppose we take a point j omega on the j omega axis where we want to find out the magnitude and the phase. As far as magnitude is concerned, the distance from here to here and the distance from here to here is the same, agreed? So the magnitude of the network function, the two network functions, let us say F 1 and F 2, will there be a difference? At any point on the j omega axis, the magnitude of this network function and this network function shall be the same. What about the phase? These two are equal.

What about the phase, angle of F 1 j omega and angle of F 2 j omega? Let us consider one of these situations. For this, the angle is this much, whereas for this vector, the angle is obtuse, is not that right? It is more than 90. This is less than 90 and similarly for here, once again the angle is more than 90 and therefore, the angle of F 2 shall be greater than the angle of F 1. Is that clear? Is it okay?

Student: Excuse me, Sir.

Sir: Yeah.

Student: Sir, we are considering the reciprocal systems

Sir: It is not a reciprocal system. No, no, systems are reciprocal. You mean reciprocity in the other sense?

Student: Yes

Sir: Reciprocity? Yes, this is reciprocity.

Student: Then again if we invert the response in the excitation

Sir: Oh you cannot invert the response. You see, if you invert this and these zeros will become poles and they will go to the right half plane, no. That is not been the meaning of reciprocity.

Student: We had that the same excitation gives a response and if the response is applied at the excitation state we will have the output as the

Sir: The transfer function does not become a reciprocal. Then it cannot be because when it becomes unstable, reciprocal of these will have zeros here and poles in the right half plane.

Student: No sir

Sir: Wait a second, what he says is reciprocal of this will not exist, will be unstable. Reciprocal of this will stable but not for this. This is a perfectly legitimate network function so you cannot simply take a reciprocal of a network function and realize it. Reciprocal of an impedance is admittance. That is perfectly all right. If this is an impedance, its reciprocal can be realized. Can this be an impedance?

Student: No

Sir: No, because its reciprocal will be unstable, will be admittance but the point that I am mentioning is, let us go back to, you have any other question? A point that was mentioning is, if there are zeros in the right half plane and the magnitudes are the same, then this zeros contribute to a larger phase as compared to the zeros in the left half plane, although their magnitudes are same. Therefore, well, there is a terminology now that I shall introduce, network functions whose zeros are restricted to the left half plane are called minimum phase functions.

If this is not so, if the zeros are not restricted to the left half plane, then we shall call them simply non minimum phase function, not maximum phase. We do not know what is maximum. In fact, this terminology is also unfortunate. We do not know if this is minimum but between the two, the magnitudes are the same. The phase of this is smaller than the phase of this. Now what is important is a minimum phase network function is one in which the poles, of course, have to be in the left half plane, there is no question, in which the zeros are not allowed to be to lie in the right half plane. A minimum phase network function is onw in which the zeros are restricted or constrained to be confined within the left half plane. Is the point clear?

Let us consider minimum phase and non minimum phase; it will play dominant role throughout your life, not only here, in electronic circuits and control theory in communication, everywhere. So you remember, simple, zeros are in the left half plane. That is minimum phase and I have told you why it is called minimum phase. Although the term minimum is a point in a continuum where is they are only two discrete situations, it should not be called minimum but unfortunately, this is the terminology that has gone through.

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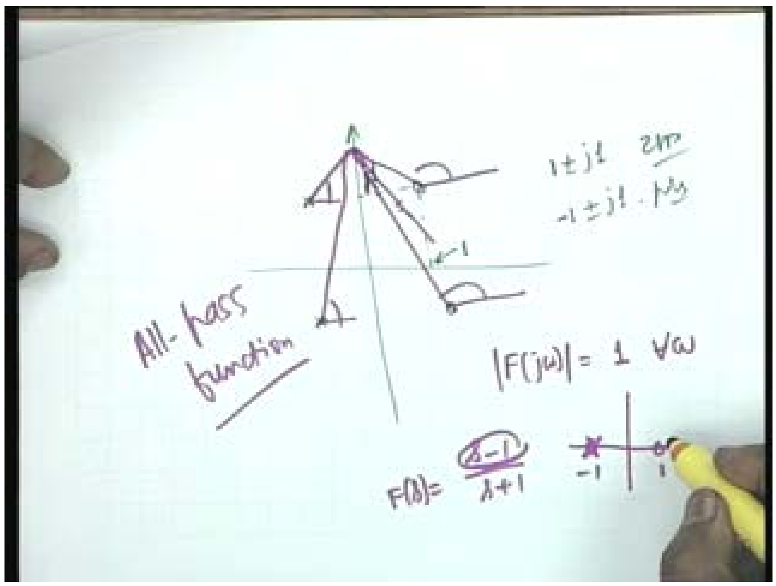

Let us consider another interesting situation, a non minimum phase function in which the poles are the mirror images of zeros. If the j omega axis is considered as a mirror and this is the object, then this is the image or if this is the object, then this is the image and let us say these values are 1 and j 1. That is, the zeros are a 1 plus minus j 1 and the poles are at minus 1 plus minus j 1. This is a perfectly valid network function. It is stable but non-minimum phase. Now if you take the magnitude for any omega, any point on the j omega axis, if you draw the vectors from the poles and from the zeros, they would be equal. The vector from here to here is the same as the vector from here to here.

Similarly the vector from here to here is the same as the vector from here to here, magnitude wise and therefore, the magnitude of this function would be equal to 1. For arbitrary omega, for all frequencies, for all omega, it shall be equal to 1 but the angle, of course, shall be different. No, the angle is not 0. What is the angle? Angle is from the zeros, I beg your pardon, so it is this angle plus this angle minus this angle minus this angle. So the angle is not 0 and the angle varies with value of omega. Such a function, in which the poles are mirror images of the zeros, which have been necessary property that the magnitude is a constant for all frequencies, is called naturally, an all-pass function.

That is, as far as the magnitude is concerned, it does not discriminate against any frequency. All frequencies are passed. The simplest example is s minus 1 divided by s plus 1. You can see, by putting s equal to j omega that the magnitude is 1. Simplest example is a first order all pass filter in which there is a 0 at plus 1 and a pole at minus 1. Such a function is called an all pass function. Now, if it does not discriminate against any frequency, what is its use then? The phase varies with frequency. So it can be used as a phase compensator. As a phase compensation network, it can be used and phase and delay which we have not yet come to are very intimately related. So such networks,

Student: Will not it be the other way round?

Sir: Pardon me.

Student: Will not the s minus 1, s plus 1 be the other way round?

Sir: Other way round? How can that be? The 0 is a plus 1, if it is other way round, then this 0, pole would be a plus 1. Pole at plus 1 is not permitted. What do you mean by other way round? Reciprocal?

Student: (..)

No. You see, where does this vanish at s equal to 1? 1 is the real quantity, so it is on the real axis.

Student: But j omega is equal to

Poles and zeros are defined in the s plane not with j omega. I am finding out the response at j omega. Now therefore, what I am saying is, all pass function is used to compensate for phase or delays as you shall see later and therefore, they are called, all pass network functions are called phase equalizers.

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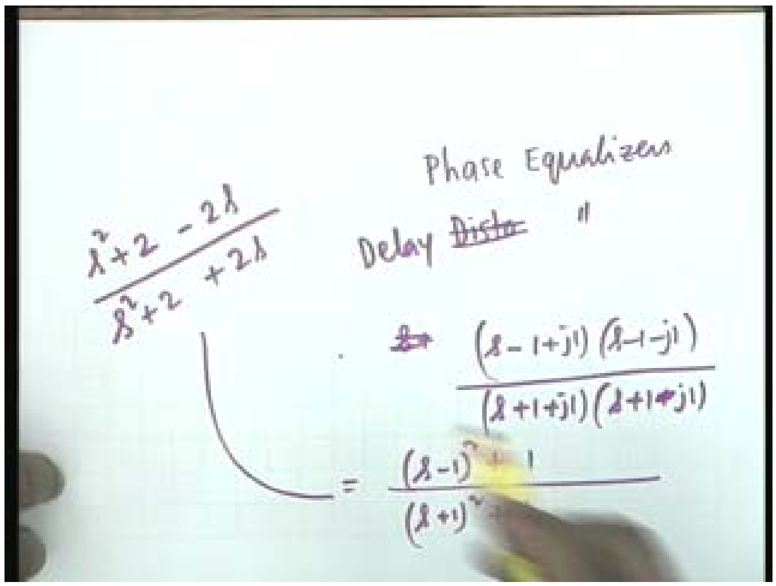

You do not need them in stereos, very sophisticated stereos or also called delay equalizer. Now this function, that is, the function that we are taken is s plus, I beg your pardon, s minus 1 plus j 1 s minus 1 minus j 1. These are the zeros and the poles were s plus 1 plus j 1 s plus 1 minus j 1 and the function is s minus 1 whole squared plus 1 s plus 1 whole squared plus 1 and you can easily see by putting s equal to j omega that the magnitude is unity because this is s squared plus 2 minus 2 s and in the denominator, we have s squared plus 2 s, is not it right?

This is s squared minus 2 s plus 1 and 1 makes 2. If you put s equal to j omega now this becomes real. 2 minus omega square minus 2 j omega, that becomes imaginary part and the magnitude is real part square plus imaginary part square, which would be the same for the numerator as well as denominator.

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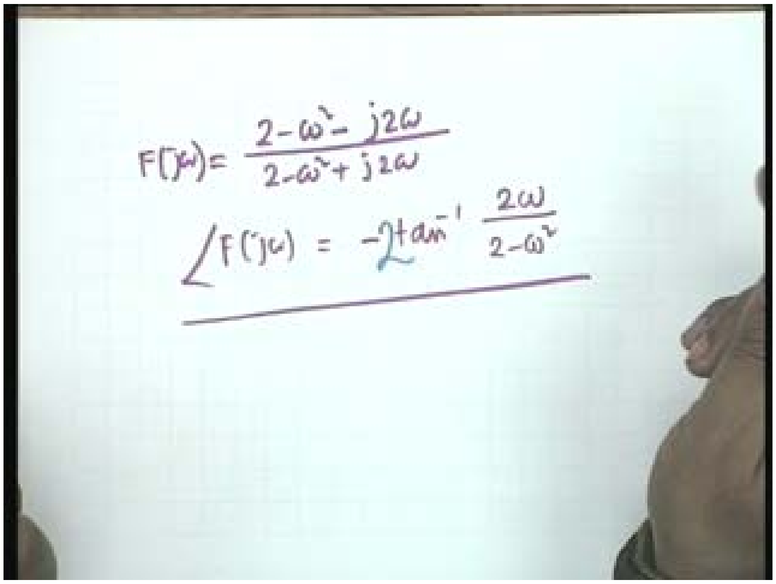

But the angle of F of j omega, obviously, would be minus tan inverse 2 divided by 2 minus omega squared 2 omega divided by 2 minus omega squared, where I made a mistake. Can anybody tell me what the mistake is? What I said was 2 minus omega square minus j 2 omega. This is the network function, This is F of j omega and I wrote the phase as this. Is this correct? Where is the inaccuracy? This is the angle of numerator.

Student: 2 times

Sir: 2 times, that is the correct expression because the angle of the denominator is exactly identical, except for a sign and when it comes in the numerator, it becomes minus and therefore minus 2 tan inverse 2 omega by 2 minus omega squared and this is where we shall start from, the next time.