Circuit Theory
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Lecture - 14
Single Tuned Circuits

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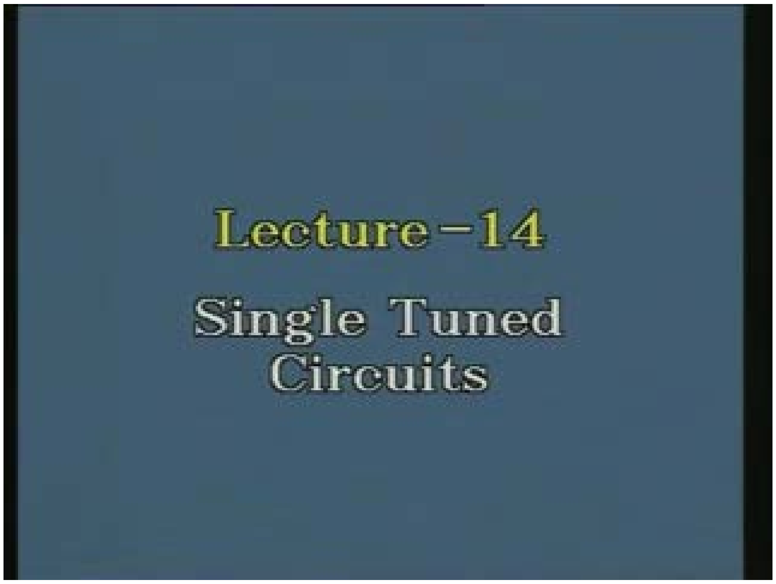

This is the fourteenth lecture on single tuned circuits. Now, before I do single tuned circuits, the couple of things which are left from the previous lecture, let me recall, what I did in a hurry at the last moment in the previous lecture. That was about an all pass network function.

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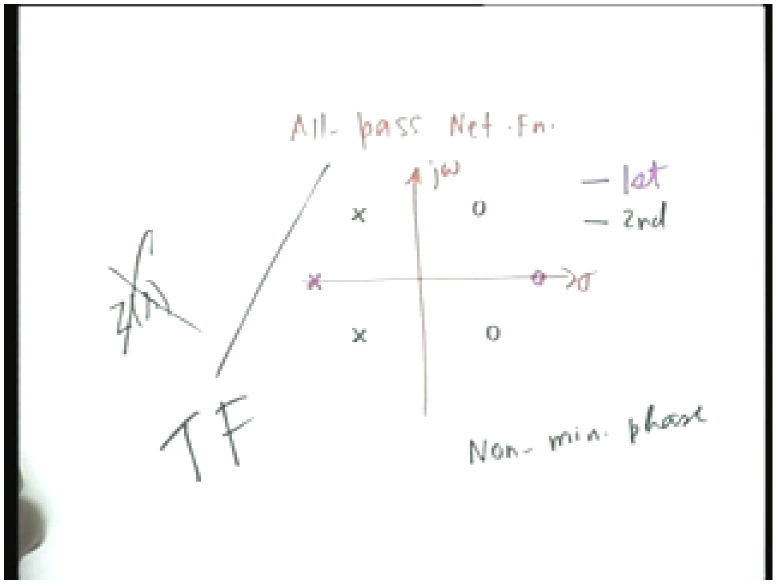

And I said that a network function will be all pass if the poles at the mirror images, at the zeros and since poles have to be in the left half plane, if we have poles like this, then zeros which are mirror images of these poles with the j omega axis considered as a mirror. This acts as an all pass function and this is a second order, 2 poles and 2 zeros. A first order one would be with, first order one can have a pole only on the real axis. So a first order one would be like this; a pole here and the pole 0 and the 0 here. This is first order, whereas the black one is the second order transfer function.

Similarly you could have third order. If you have all the 3 poles and all the 3 zeros will be a third order. Similarly you could go on increasing the order. The characteristic is that the poles must be mirror images of the zeros. Now and therefore, it is of necessity, all pass functions are of necessity, non minimum phase functions. A minimum phase function is one whose zeros are restricted to the left half of the s plane. The borderline, that is, between the borderline, is j omega axis. If you have zeros on the j omega axis, this is also called a non minimum phase function.

A minimum phase function should have zeros in the open left half plane. Is this sentence clear? Open left half plane, that is, j omega axis is excluded. All zeros for minimum phase function should be in the open left half plane, whereas a non minimum phase function can have zeros anywhere including the j omega axis. So an all pass network function is necessarily a non minimum phase function. Let us look at the second order all pass function in a little more detail. Let us say we have another point that needs to be clarified.

An all pass function cannot be a driving point function or the other way round, a driving point function cannot be all pass. Why? If you have an impedance which is like this, which has zeros in the right half plane, then obviously the admittance shall have zeros, shall have poles in the right half plane which means that the admittance will be unstable. Well, this cannot be. Driving point impedances of necessity, have to have their poles and zeros all in the left half plane. However a driving point function can have zeros on the j omega axis. It can have poles also in the j omega axis but not in the right half plane. So an all pass function cannot be a driving point function. All pass function network function must of necessity be a transfer function.

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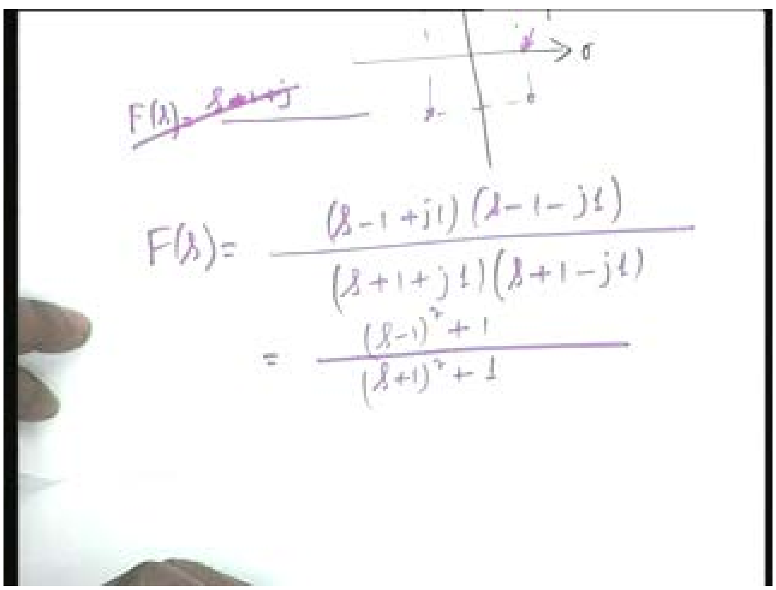

Now let us look at the second order case in a little more details and to be specific, what we need was we took an example, a simple example like this, we took zeros here where this distance is 1 and this distance is also 1. This is j 1. So this is minus j 1 and then the poles are somewhere here; a very simple example. The transfer function, the all pass function F of s will now be equal to s plus 1, no I beg your pardon, s minus 1 plus j 1. Let me write it again here. F of s would be the pole factors, that is, s plus 1 plus j 1. You understand why plus 1 comes in the denominator, because the location of the pole is minus 1 plus minus j 1 and the other factor would be s plus 1 minus j 1.

In the numerator it will be s minus 1 plus j 1 s minus 1 minus j 1. In addition, we can have, you can have a constant multiplying factor which can be chosen at will. So we do not pay attention to this and if you simplify this, you notice that this is s minus 1 whole squared plus 1 divided by s plus 1 whole squared plus 1. It is a real quantity. It is a real rational function and if you put s equal to j omega, but before that let us simplify this. You see that this is s squared plus 1 minus plus 2 minus 2 s and s squared plus 2 plus 2 s.

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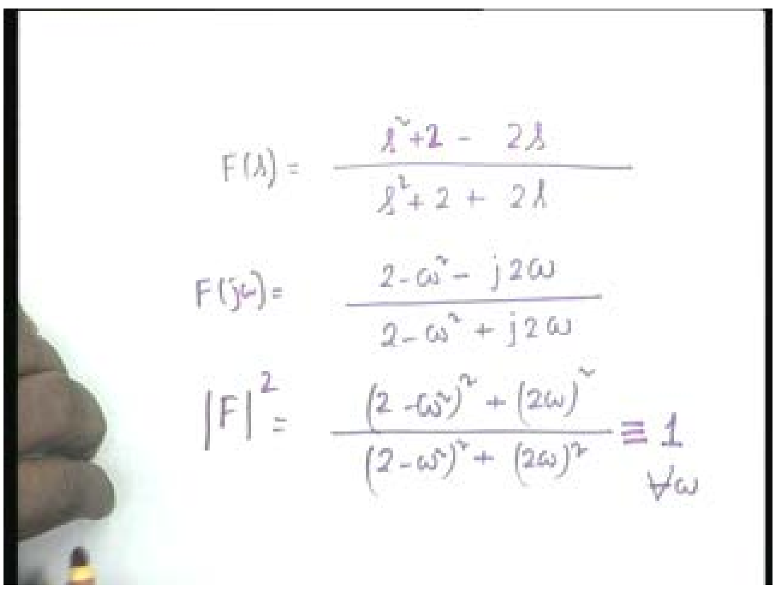

You understand why this 2 comes, from s plus 1 whole squared and 1 and another one which is addition and therefore, F of j omega is equal to 2 minus omega squared minus j 2 omega divided by 2 minus omega squared plus j 2 omega and the magnitude, you can see, the magnitude squared is equal to 2 minus omega squared whole squared plus 2 omega whole squared divided by 2 minus omega squared whole squared plus 2 omega whole squared and this is identically equal to 1, for all omega, irrespective of the value of the omega, which proves that it is all pass.

It does not discriminate. As far as magnitude is concerned it does not discriminate against any frequency.

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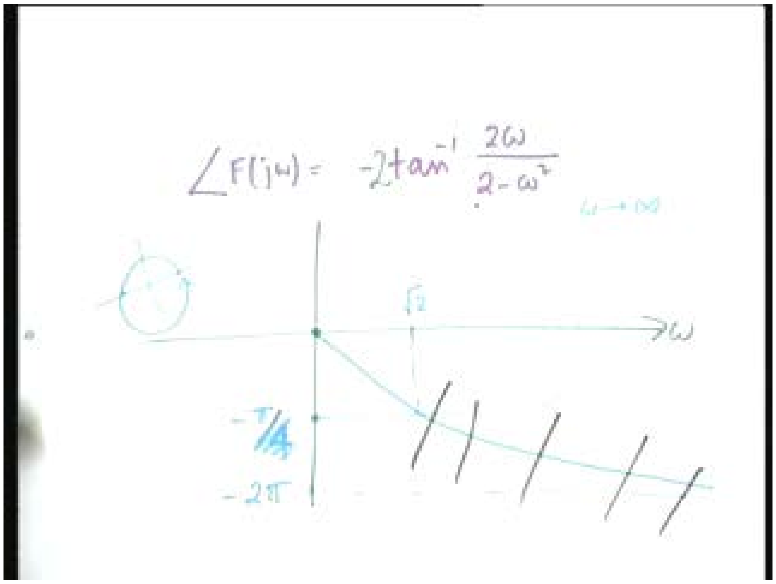

On the other hand if you find the angle, the angle of F of j omega is given by the angle of the numerator which is minus tan inverse 2 omega divided by 2 minus omega squared. Have I done this correctly? 2 minus omega squared and the angle of the denominator is the same and therefore, we simply, all that we have to do is to take a factor of 2 here and if I now plot the angle verses omega, then you see at omega equal to 0, the angle is 0. At omega equal to omega squared equal to 2, that is, omega equal to root 2 the angle is

Student: minus pi by 4

Sir: Minus pi by 4. The negative sign is important. Minus pi by 4.

Student: Minus pi.

Sir: Oh, minus pi, I beg your pardon. tan inverse infinity pi by 2 multiplied by 2. This is minus pi and then when omega goes to infinity, the angle is again 0. It is minus pi and therefore, the curve must be continuous. So at infinity, it goes to not 0 but minus 2 pi. This is, this must be understood. The angle is always negative, so it starts from here, minus from 0 to minus pi and then it goes to minus 2 pi not plus 2 pi. The angle of the direction of rotation is important. So this is the second order function.

Student: Excuse me Sir.

Sir: Yes.

Student: Did you say that the phase function always lies between minus pi and pi?

Sir: No, I never said that. That is not correct. The phase could be as large as 6 pi, 8 pi. Why not? Phase depends on the order of the function.

Student: Sir, what if it goes to 0? It can also go to 0.

Sir: Sure it can also go to 0. Phase can have can have non-monotonousity. It can go like this for I can device a function in which the phase goes like this, (..) the minimum and then comes to 0. Sure, a phase behavior is not restricted.

Student: At infinity also, it goes to 0.

Sir: Where?

Student: Sir, at infinity

Sir: It goes to minus 2 pi. Minus 2 pi is the same as 0, as far its sine, cosine, tangent, cotangent or any function is concerned. But one must realize if the phase is always negative and therefore, this 0 here means minus 2 pi. Similarly, plus 2 pi also is 0. Plus 4 pi is also equivalent to 0 because you take any trigonometric function, the value is the same.

Student: Sir, this negative has come because of the minus sign?

Sir: That is correct.

Student: Sir but s omega is tending towards infinity

Sir: Correct.

Student: (..) that the phase angle inside will be tending towards negative 0. That means, from (..) minus 2 pi. And this minus will make it plus 2.

Sir: Absolutely wonderful. You see, look at this, I expected, I was expecting this question much earlier. You see, when omega squared is less than 2, then this quantity is positive and therefore, the phase is negative. But an omega squared goes beyond to what happens?

Student: It will become negative sir, we have..

Sir: So the phase will become positive.

Student: Yes sir

Sir: And therefore, at omega squared equal to 2 there would be a transition in phase and all this is wrong.

Student: Yes sir

Sir: Agreed. All this is wrong. At omega squared is 2 minus, the phase is negative. Omega squared 2 plus, the phase is positive and therefore, there is an abrupt change through minus pi to plus pi. So this is wrong. You corrected it. Incidentally it is also wrongly given in the text book, check and correct this. We next consider the main topic of the day, that is, the single tune circuit.

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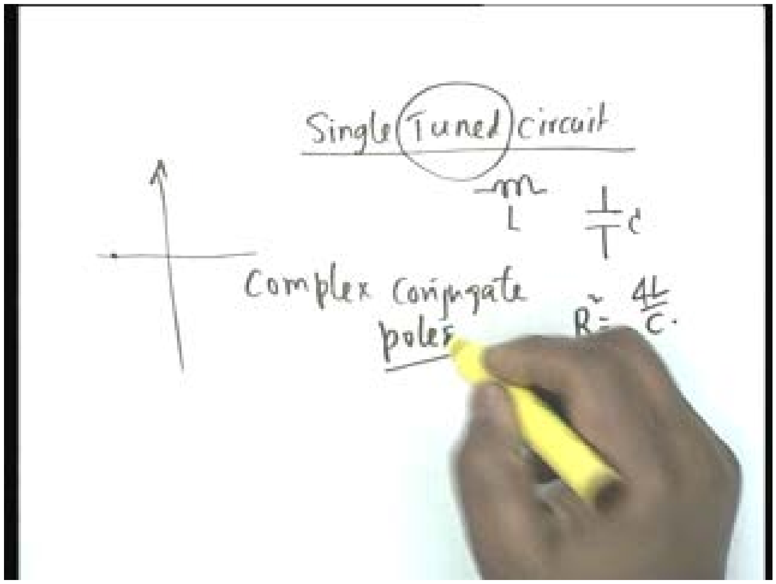

Single tune circuit is a circuit in which there is one inductor and one capacitor connected in a certain fashion so that they resonate. That is, the reactance of the inductor is positive; the reactance of the capacitor is negative and the two can conspire to show no reactance outside at a particular frequency. This as you know, is the phenomenal of resonance. Resonance also goes by the name tuning and single tuned circuit means we are considering a resonance circuit which has resonance at only one frequency, single tune.

We can have double tune circuits also. We can have 2 resonances as we shall see later but that will require more reactive elements. A single tuned circuit is formed by one inductor and one capacitor and usually a single tuned circuit is interesting only if it produces a pair of complex conjugate poles. Otherwise it is not very interesting. If a single tune circuit, you know the resistance, inductance and capacitance can be such that the circuit is over damped. If it is over damped, the poles are on the negative real axis and that case is not is not of much interest. You understand what I mean?

I have already taken the differential equation of a series R L C circuit and shown that the natural frequencies can be either complex or can be real. If they are real, they should lie on the negative real axis and if in the case of an over damped circuit or the critically damped, critically dampled means both the natural frequencies are on the negative real axis at a particular point coincident of a particular point, which happens when R squared equal to 4 L by C. I think that is the, is that correct?

Student: Yes sir.

Sir: Now if the resistance, we will find out in a moment, if the resistance goes below this, then we have complex conjugate poles and this is complex conjugate natural frequencies and this is the case, that is, have interest as far as single tuned circuit is concerned or any circuit for that matter, any electrical engineering circuit where you want to produce a band pass type of response, that is, where you want to select a particular band of frequencies around the center frequency, a single tuned circuit if complex conjugate poles is of interest. So let us look at complex conjugate poles and introduce some nomenclature.

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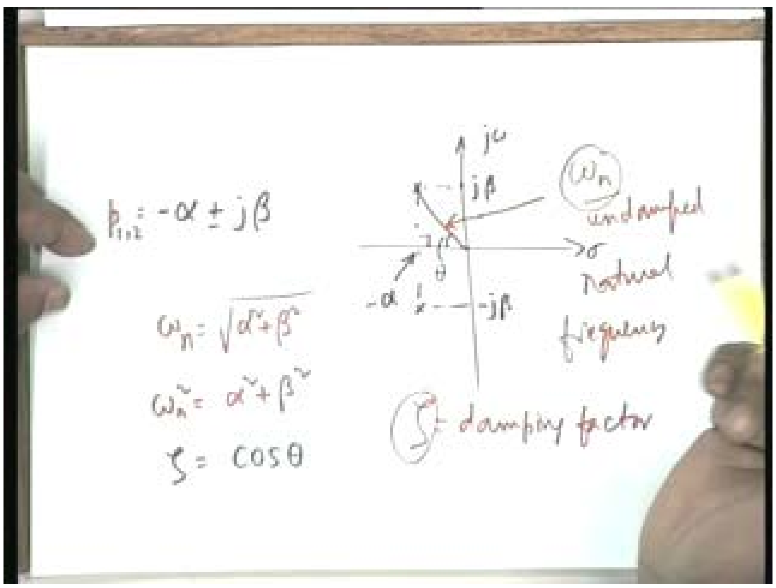

Here is a pair of complex conjugate poles. Let us say, this is j beta. This is minus j beta and let us say, this is minus alpha. So the poles are at minus alpha plus minus j beta. It is conventional to characterize these poles instead of writing p 1 2 equal to minus alpha plus j beta. It is conventional to characterize these poles by means of two parameters called omega n, the undamped natural frequency and the other is zeta, the damping factor and there are defined like this. These definitions would be pretty standard and you shall use this in control, in communications and anywhere that resonance circuits, are used anywhere there is a question of selectivity omega n and zeta these terms shall be used. They are defined like this. Draw the radius vector from the origin to one of the poles, any pole, let us say, the upper pole. Then the length of this line is called omega n, the length of this line is called the undamped natural frequency omega n and obviously, omega n is equal to square root of alpha squared plus beta squared. Is not that right?

This, in this triangle, this angle is 90 and therefore, the hypotenuse squared is equal to alpha squared plus beta squared or omega n squared equal to alpha squared plus beta squared and this angle that it makes with the negative real axis is called, is given the name zeta and zeta, no sign, is just the magnitude, the value. Forget about the sign. This angle theta zeta is defined as cosine of theta. Zeta is defined as cosine of this angle, the damping factor zeta is defined as cosine of this angle and therefore, we can express alpha and beta in terms of omega n and zeta.

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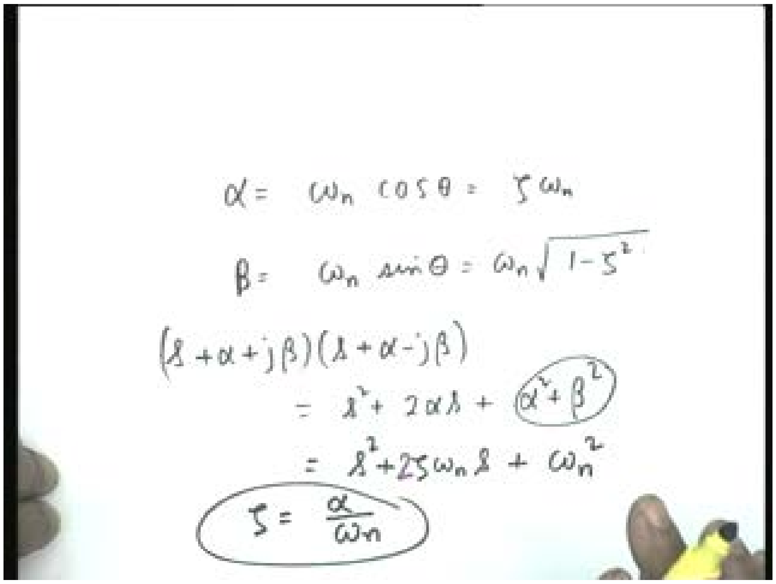

For example, if you look at the figure, do not you see that alpha, just the value alpha is simply omega n, the hypotenuse multiplied by cosine theta, that is, simply equal to 0 omega n and beta is equal to omega n times sine theta. That is, equal to omega n under root of 1 minus zeta squared. So the location of the roots, alpha and beta are expressed in terms of zeta and omega n in this fashion. Otherwise also, you see the polynomial, the denominator, it is the poles. Then the denominator polynomial of the transfer function would be of the form s plus alpha plus j beta s plus alpha minus j beta and this is, obviously, equal to s squared plus twice alpha s plus alpha squared plus beta squared and this is written as alpha s squared plus zeta omega n s plus omega n squared.

Alpha squared plus beta squared, we have already found out, to be equal to omega n squared and therefore, there is a factor of 2 here. Twice zeta omega ns plus omega n squared. So zeta is equal to how much? zeta omega n is alpha. Therefore, it is alpha divided by omega n. This is the relationship between zeta alpha and omega n. 2 alpha is equal to 2 zeta omega n and therefore, zeta is equal to alpha divided by omega n. Is that point clear?

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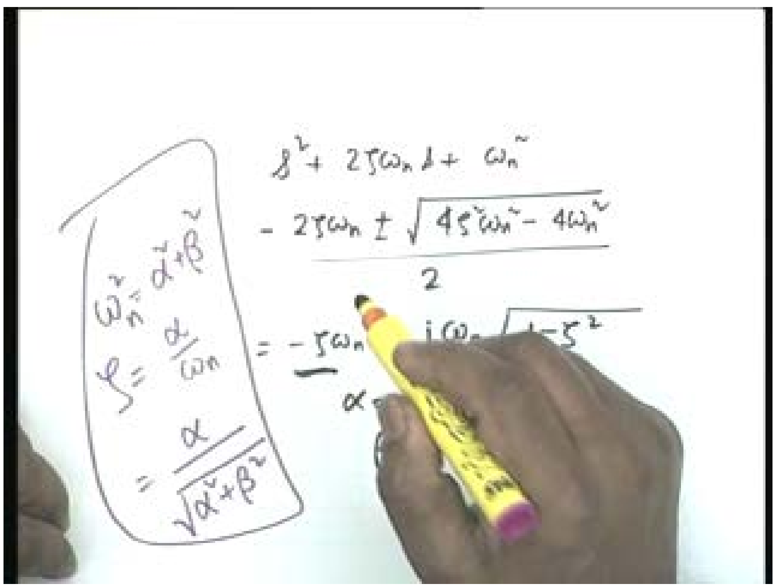

If you find the roots of this polynomial, what do you find? If you find the roots of this polynomial s squared plus twice zeta omega n s plus omega n squared, where are the roots? Minus 2 zeta omega n plus minus square root of 4 zeta squared omega n squared minus 4 omega n squared divided by 2 and therefore, this is minus zeta omega n plus minus j omega n under root of 1 minus zeta squared. Why did I take j out, because as I said, otherwise, the case is of no interest. If j is not taken out, if this is greater than this, then we lose interest in the pair of poles. It is only when the complex conjugate that interests lasts and you see that this is equal to alpha therefore, alpha is obviously equal to zeta omega n and beta equal to omega n square root 1 minus zeta squared.

The same as we derived previously in terms of the angle theta and the radius factor which was omega n. The thing to remember, to summarize, the thing to remember is that under the natural frequency is given by alpha squared plus beta squared and zeta is equal to, in terms of alpha and omega n, it is alpha divided by omega n which is equal to alpha divided by square root of alpha squared plus beta squared. We shall require these relations again and again, not only in this course but also in control and communications. Any question at this point?

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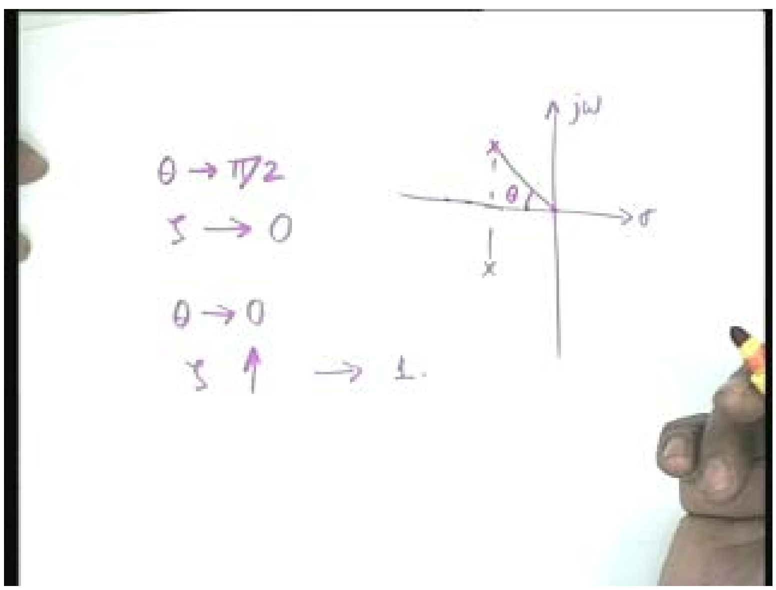

If there are no questions, let us look at this figure again. Sigma j omega. We have a pair of poles and as I said, this angle is theta. When theta tends to pi by 2, the poles move close to the j omega axis and zeta then tends to 0. If theta tends to pi by 2, zeta, which is cosine of pi 2 tends to 0 and therefore, the damping decreases as theta increases. On the other hand, when theta tends to 0, the damping increases and damping tends to the value 1. As I said, this is not of interest. That is, if the poles now move towards the negative real axis, well they could be coincident or they could separate out, but this is not of not of much interest. So there is a relationship between the damping factor theta and the angle zeta. By looking at the function, you should be able to say whether it is critically damped, where critical damping, what is the value of zeta.

Student: 1

Sir: Pardon me.

Student: 1

Sir: 1 okay. If zeta exceeds 1, then of course, the poles separate on the negative real axis and that case is of no interest. So zeta less than 1 is under damped response, zeta greater than 1 over damped, zeta equal to 1 is critical response.

Student: What does it mean if zeta is greater than 1?

Sir: Zeta greater than 1 will mean that the poles separate on the negative real axis. Now let us look at, before we pass to the single tuned circuit, let us look at the network function, F of s; it will be some constant K divided by s squared plus twice zeta omega n s plus omega n squared.

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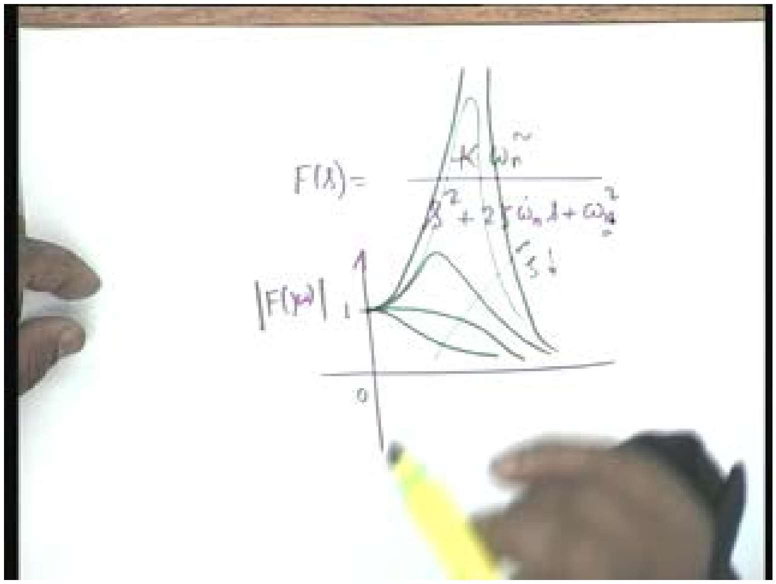

Complex conjugate poles are always represented in terms of zeta and omega n. Now if you look at the magnitude response, capital F of j omega, if you look at the magnitude response, let us make K equal to omega n squared for simplicity. If you look at the magnitude response, obviously, at omega equal to 0, it starts from 1 and then it can go like this or it can go like this, depending on the value of zeta and the case is of interest only when zeta is such that, there occurs a peaking, there occurs a peaking. As zeta decreases, the peaking goes on increasing. When zeta is equal to 0, then the amplitude goes to infinity. Amplitude goes to infinity at what frequency?

Student: Omega n.

Omega n, is not that right? When zeta goes to 0, the denominator becomes omega n squared minus omega squared. So at omega equal to omega n, the amplitude, the magnitude goes to infinity and this is why omega n is called the undamped natural frequency. That is, if there no damping, then this is the frequency at which the circuit will show infinite magnitude, infinite amount of selectivity. Is it, point clear?

If you look at the phase, we will find out for what value of zeta it is critically damped. I mean the magnitude response. Our critical damping, if you recall these terms under damped, over damped and critical damping, was with respect to what? With respect to?

Student: Response

Sir: Yes, but response in which domain?

Student: Frequency domain

Sir: No, time domain. It was in a time domain, agreed? We have shown that in the time domain, critical damping corresponds to the border line region between oscillations and no oscillations. So it is in the time domain. In the frequency domain, when does a peaking occur? We shall investigate this point in details. That is, there will exist a value of zeta below which there will be peaking, above which there will be no peaking and we are talking of frequency response manner. It is not time domain response. So things will be slightly different, but we shall be able to find out. Again before we go to the actual circuit, let us look at the phase response.

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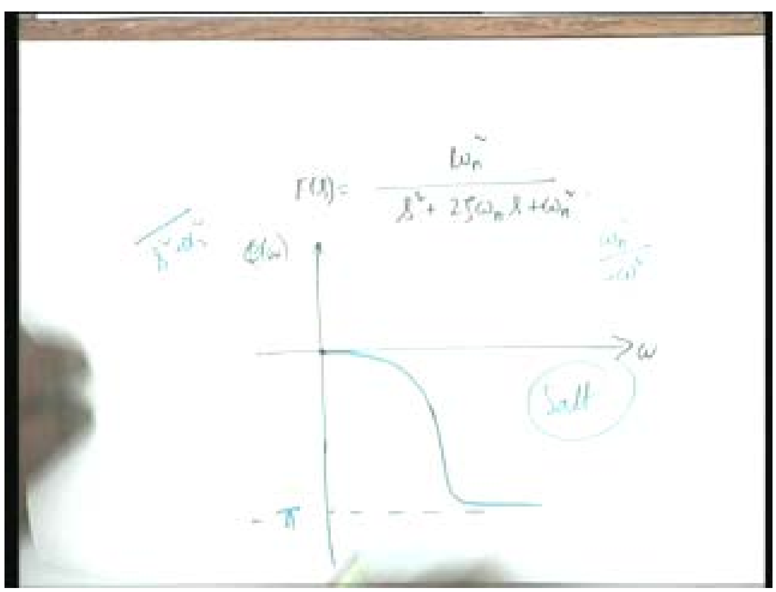

We have omega n squared divided by s squared plus twice zeta omega n s plus omega n squared, the phase response. As you see, when s is 0, when the frequency is 0, phi omega. When the frequency is 0, what is the phase? Phase is 0. So it starts from 0 and then at omega equal to omega n, the phase, if zeta is small, if zeta was 0 at omega equal to omega n, the phase goes through a jump of how many degrees? We have already explained this that if we have s squared plus alpha 2 squared in the denominator, then the jump is through.

Student: Minus 180

Minus 180 and therefore, at omega near omega equal to omega n, zeta is small zeta is not exactly 0, it goes through a jump like this. What happens at omega equal to infinity? At omega equal to infinity, the function behaves like omega n squared by minus omega squared and therefore, it must go to minus pi. Take it as a pinch of salt and correct it, I do not want to explain everything in this course. There are some things which you shall have to answer.

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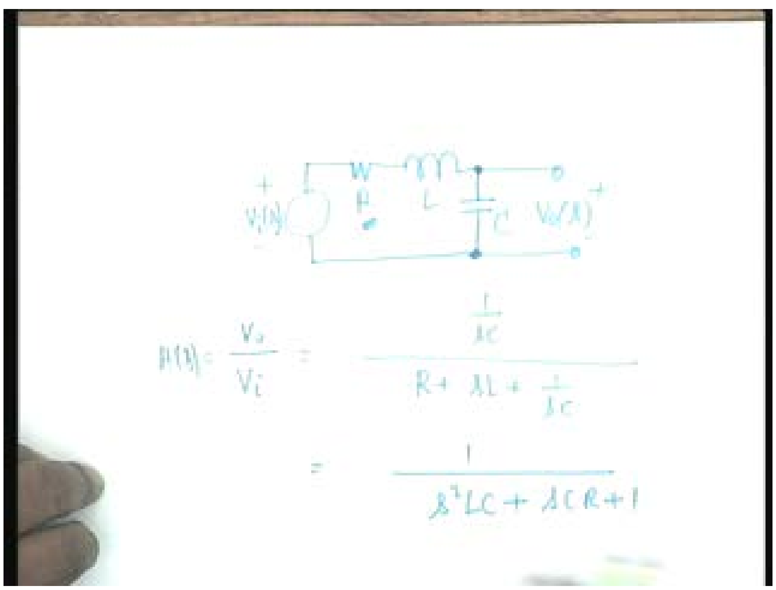

Now let us look at this circuit. We have a single tuned circuit, is a resistance, inductance in series, there are various forms. We take a typical form and a capacitor C. This is my transform of the input voltage and this is my output voltage V 0 s. This is a typical single tuned circuit. The voltage output being taken across the capacitor of value C. There are various other forms: you could take the output across L, you could connect L and C in parallel and so on and so forth. Typically this resistance is that of the inductance because as you perhaps know by now, capacitances can be made relatively loss free. That is, the resistance associated with the capacitance can be made as small, the series resistance, as small as possible or the parallel resistance can be made as large as one wants to but whenever you wind a coil to make an inductance, the wire itself has a certain specific resistance and therefore, you cannot make an inductance without resistance unless you go to super conducting temperatures.

The ordinary temperatures, there must be a resistance. So this resistance may be the resistance of the inductor or may be added externally but if you take the transfer function V 0 by V i which we shall now call H of s, obviously, this is equal to 1 over s C divided by R plus s L plus 1 over s C which I can write as 1 by s squared L C plus s C R plus 1.

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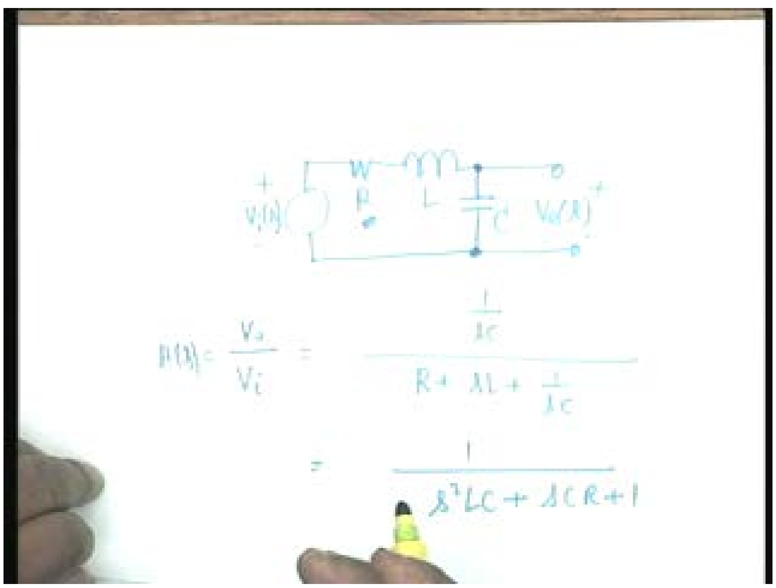

This can be simplified as 1 over L C 1 by s squared plus s R by L plus 1 over L C. Obviously, the poles p 1 2 of this transfer function shall be minus R by 2 L plus minus square root of yes, 1 by L C minus R squared divided by 4 L squared, is that okay? Times j. Otherwise, it is not of interest. If it cannot produce complex conjugate poles, then it is not of interest. So this is equal to minus alpha plus minus j beta. The point that we are looking for is, now where is the amplitude a maximum? But before that we can calculate the magnitude of this function and the phase by graphical methods.

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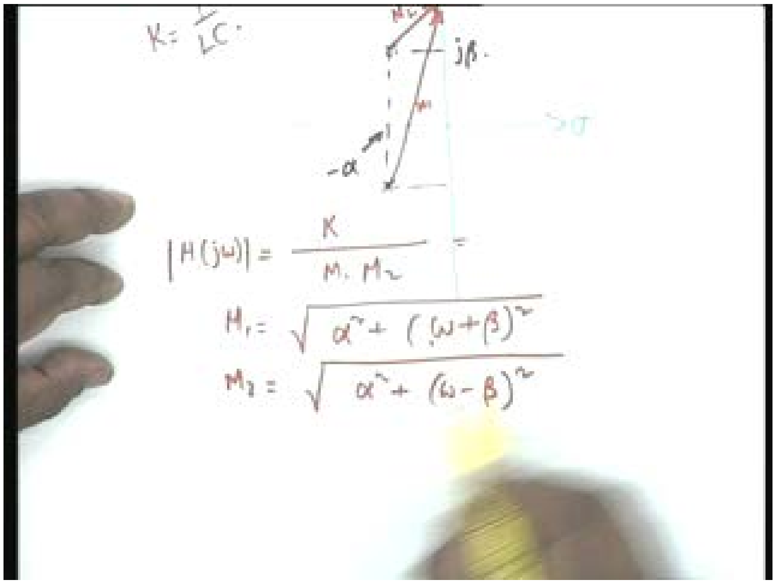

That is, what we do is, we have a pair of poles like this. This, is in a sense, repetition but it is worth doing. This is my, this is j beta and this is minus alpha and you take a certain frequency, let us say, somewhere here, let us say. Let us say this is j omega and we want to find out a magnitude at this frequency. So what we do is, we draw a vector from this pole to this point and we draw another vector from this pole to this point. We call this as M 1 and this as M 2, so my transfer function magnitude, H of j omega would be simply the constant K which was 1 by L C K equal to 1 by L C. How is it related to alphas, alpha and beta? The constant K is equal to 1 by L C. How is related to alpha and beta? It is simply alpha squared plus beta squared which means that it is equal to omega n squared.

So let us forget about that. K is 1 by L C. So it is K divided by M 1 M 2. That is it. The magnitude and if you notice the magnitude well, M 1 you can find out geometrically, M 1 is, how much is this distance? It is omega plus beta not j beta. The distance, I am trying to find is this, the hypotenuse of this right angle triangle. This distance, the perpendicular is omega plus beta. This much is beta and this much is omega and this is alpha and therefore, this is K, well M 1 is equal to square root of alpha squared plus omega plus beta whole squared and in a similar manner you can find M 2 to be equal to square root of alpha squared plus omega minus beta whole squared.

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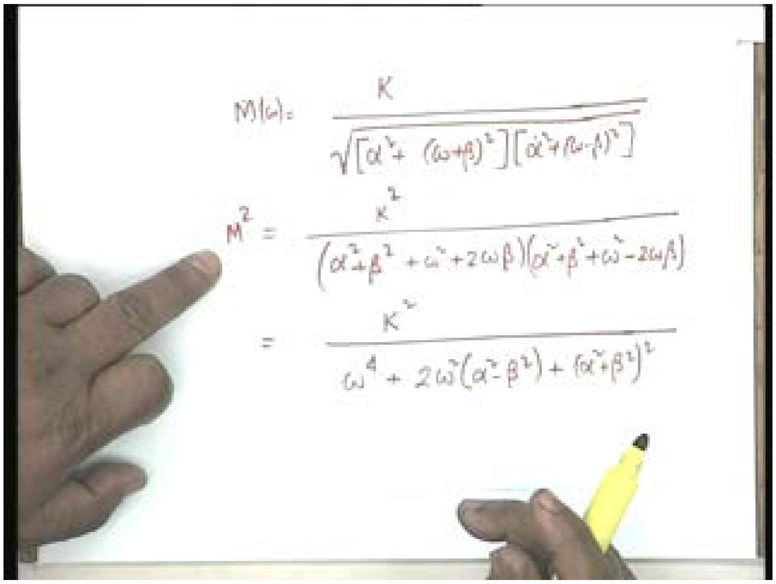

Therefore, the magnitude, we will call that M, M of omega would be equal to K divided by square root of alpha squared plus omega plus beta whole squared multiplied by alpha squared plus omega minus beta whole squared. We want to find out where would the peak occur, where would this function be a maximum. Now to that end, we square the function. Capital M of omega, obviously, has to be a positive quantity because it is a magnitude and therefore, the maximum of capital M shall correspond to the maximum of M squared. Is the point clear? Therefore, what we do is we find out M squared and that I can write as, notice this, alpha squared plus beta squared plus omega squared plus twice omega beta.

Student: K squared.

Sir: K squared, multiplied by alpha squared plus beta squared plus omega squared minus twice omega beta. Now you can simplify this to the following. You can write this is as K squared divided by omega to the fourth, you can see where it comes from, plus omega squared 2 omega squared alpha squared minus beta squared plus alpha squared plus beta squared whole squared. This is as simple as that and we are interested in the maximum of capital M. You can go ahead and differentiate with respect to omega squared, because this can be treated as a function of omega squared and find out the frequency of maximum. A simpler way rather than differentiating would be to look at the denominator.

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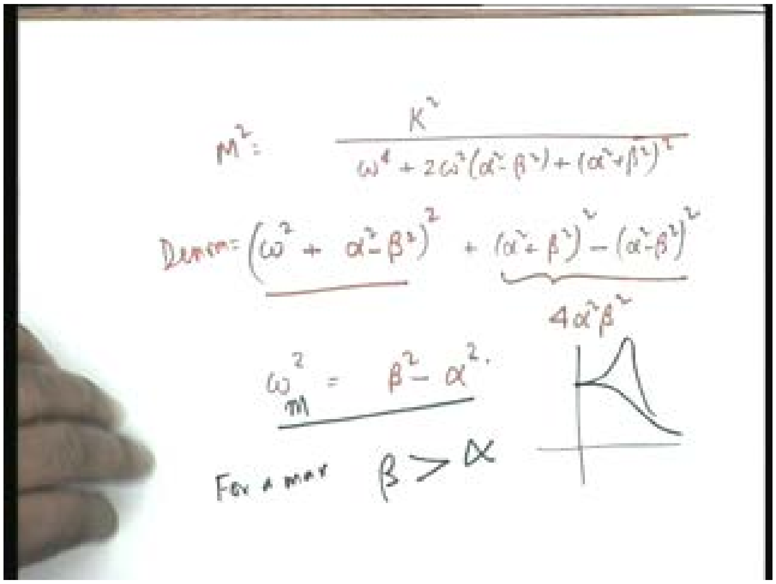

Let me repeat, these are some tricks of the trade which we should have learnt in high school, plus 10 or earlier. Of course, no differentiation is taught there, but one is often encountered, one often encounters the problem of maximizing or minimizing. Now this, the denominator of this can be written as omega squared plus alpha squared minus beta squared whole squared. Then plus alpha squared, this is the denominator, alpha squared plus beta squared whole squared minus alpha squared minus beta squared whole squared.

Make it a whole squared so that omega squared terms in, comes inside and this quantity, as you can see, this 4 alpha squared beta squared a constant. It does not vary with frequency, this is the term that varies with frequency and because it is a whole squared, it cannot be negative and therefore, the minimum value of this will occur when this is equal to 0 and that occurs at omega squared is equal to, this is equal to 0. So it should be beta squared minus alpha squared and since this is the frequency of maximum, we put a subscript of m here. Omega m squared equal to beta squared minus alpha squared.

Now we have done quite a bit of mathematics, simple algebra, but you must not lose sight of what you are doing. This is the frequency at which the denominator is a minimum and therefore, M squared will be a maximum and therefore, this will be the frequency of peak, if at all there is a peak, if at all. You see, you notice that if beta is greater than alpha which means that the imaginary part of the root is greater than the real part. Then obviously, omega m shall have a value. On the other hand, if alpha is equal to beta, that is, if the root is such, if the pole is such, that is, real part and the imaginary part are equal then obviously, this maximum occurs at the origin at omega equal to 0.

What does that mean? Maximum occurs here and then it falls. So it is a case of a low pass filter. If beta is equal to alpha, it gives you a low pass filter. On the other hand, if beta is greater than alpha, I am sorry, if beta is less than alpha, then omega m squared is negative, omega m becomes imaginary, which means that there is no maximum. No maximum exists. Is that okay? So for a maximum, beta must be greater than alpha. Beta cannot be equal to alpha, because then, the maximum occurs at omega equal to 0, which is low pass filter. Only when beta is greater than alpha, we shall get a peaking like this, agreed? Now let us look at this expression in terms of zeta.

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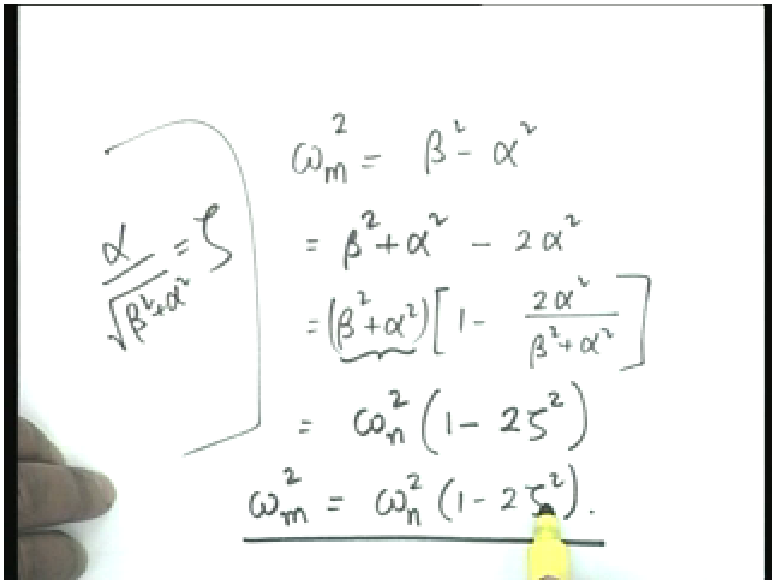

We have shown that the maximum occurs at a frequency omega m squared equal to beta squared minus alpha squared. We want to put this in terms of zeta, that is, the damping coefficient. Well I can write this as, there are many ways of doing this, but I do it like this. I write beta squared plus alpha squared minus 2 alpha squared and I can write this as beta squared plus alpha squared, take common, 1 minus 2 alpha squared divided by beta squared plus alpha squared.

I write this as beta squared plus alpha squared obviously is omega n squared if you recall and inside bracket minus 2 alpha squared plus beta squared plus alpha squared, if you recall alpha by square root of beta squared plus alpha squared is zeta, the damping coefficient, and therefore,1 minus 2 zeta squared, is that okay? So my omega m squared, the frequency of maximum is related to the natural frequency, undamped natural frequency by this relation 1 minus 2 zeta squared and now we can interpret the concepts. We can interpret the results that we got in terms of alpha and beta, in terms of zeta now a single variable. You notice that there will be a peak, provided, 2 zeta squared is less than 1.

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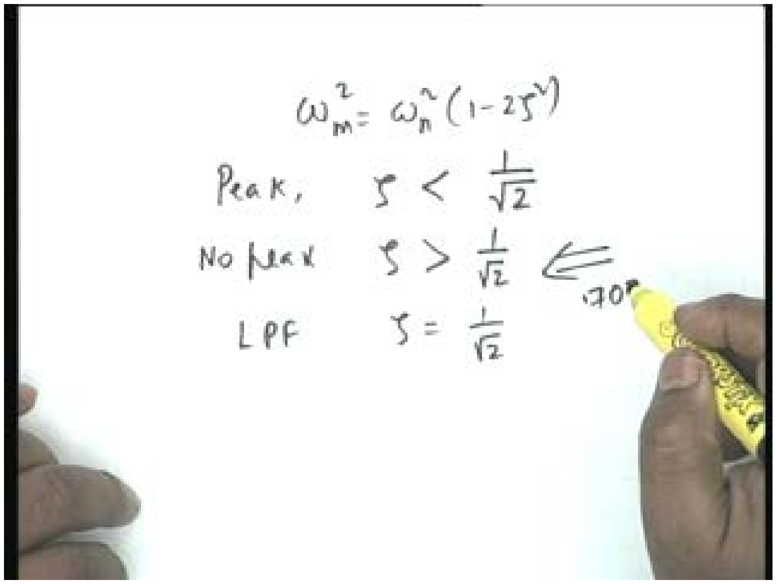

So if I recall omega m squared is omega n squared 1 minus 2 zeta squared, then peak occurs if zeta is less than 1 by root 2. Is that clear? What happens if zeta is equal to 1 by root 2? It becomes a low pass filter. That is maximum occurs, but maximum is 0. No peak if zeta is 1 by zeta greater than 1 by root 2 and low pass filter, if zeta equal to 1 by root 2. So the damping coefficient now characterizes the frequency response also, but the critical values are not the same as in the time domain. In the time domain, zeta equal to 1. Is it okay?

In the time domain, again I gave you a pause because there is a distance between time domain and frequency domain, there should be some time gap between the time domain and frequency domain. Zeta equal to 1 was the critical damping value in the time domain. What did that mean? It meant that if zeta is less than 1, the step response will show oscillations. If zeta is greater than 1, step response will not show oscillation. On the other hand, the critical value here is 1 by root 2 which is .707. If zeta is less than .707, there will be a peaking. If zeta is less than .707, I am sorry, if zeta is less than .707 there will be peaking, if zeta is greater than .707, there shall be no peaking.

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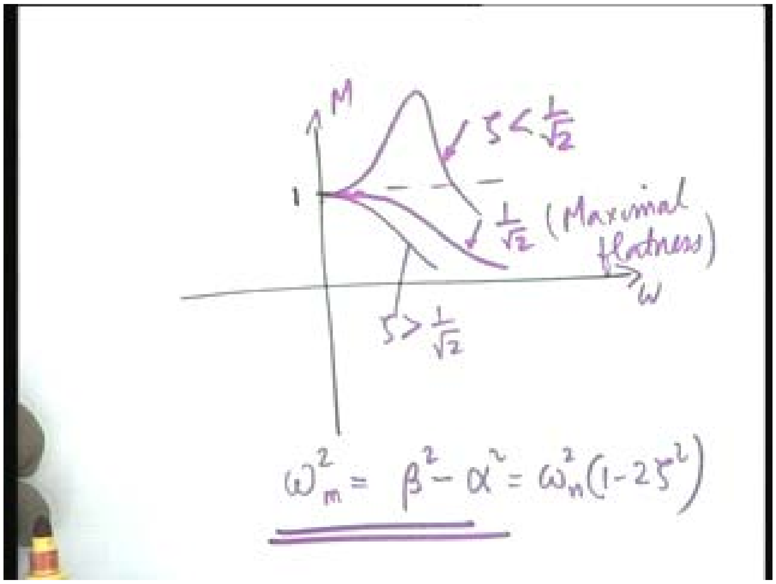

And therefore, this figure should be imprinted in your mind. Where do we start from? We start from 1 and this is omega, this is magnitude function. Zeta equal to 1 by root 2 is something like this; zeta greater than 1 by root 2, you will have a shape like this, and zeta less than 1 by root 2, you will have a shape like this. So zeta equal to 1 by root 2, this is a case of interest. It is a case of interest not only because it defines a critical damping in the frequency domain, but also because of another characteristic. You see, zeta equal to 1 by root 2, what is the slope at omega equal to 0?

Student: 0.

Sir: 0. zeta greater than 1 by root 2, this slope is positive. This condition zeta equal to 1 by root 2 is known as the condition of maximal flatness or zeta equal to .707, is the condition for maximally flat response. Beyond this, the slope is negative and omega m is equal to, let us recall the value, beta squared minus alpha squared and in terms of the natural frequency oscillation, this omega n squared 1 minus 2 zeta squared. This result, that is, the maximum can also be visualized graphically.

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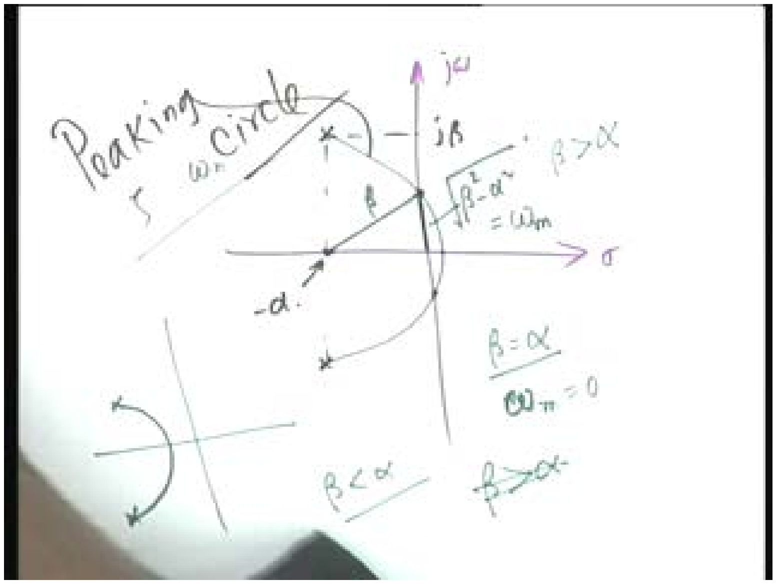

That is, you have the s plane sigma j omega. Let us say, we have the poles at j beta minus alpha. Now, look at this geometric construction. Suppose with this point as the center and this as the radius, what is the radius, radius equal to beta, you draw a circle. Well, will this circle, if beta is greater than alpha, then obviously, this circle will intersect the j omega axis at 2 points. My circle is not too good, nevertheless it shall do at 2 points. One is this and the other is this and because of symmetry, the intersection at this point shall be at the same distance from the origin as the intersection at this point.

Now if I take, what is this distance? This is equal to beta squared minus alpha squared square root, why, because this is beta and this is alpha. So this distance must be square root of beta squared minus alpha squared which is precisely equal to,

Student: under root of

Sir: Omega n, not under root. Precisely equal to omega n and therefore, we can find out without any calculation given the pole, given the 2 complex conjugate poles, we can find out the frequency of maximum simply by drawing a circle. If this is the center and this is the radius, intersection on the j omega axis gives you frequency of maximum response. You also notice that if beta equals to alpha and this circle shall intersect the j omega axis only at a single frequency single value. That is at the?

Student: Origin.

Sir: Origin, which means that the j omega axis shall now be a tangent to the circle. So beta equal to alpha maximum, omega m becomes equal to 0 and if beta is greater than alpha then this circle shall not intersect the j omega axis.

Student: beta is less than alpha

Sir: I beg your pardon, beta is less than alpha, this circle shall not intersect the j omega axis and therefore, there shall be no maximum. This is the way of visualizing the peaking of the response of a single tuned circuit or the response of a complex conjugate pair of poles. Now it is not necessary that you have physically L C and R as you shall see later, many communication channels can be modeled by a second order system, many control systems, process control, there are no actual inductances or capacitances, but their behavior is that of a second order system.

And whenever there is a second order system, zeta and omega n shall come into play and these concepts that you are learning not only are true for physical circuits which you can build in the laboratory and observe the performance in a oscilloscope, they are also useful for characterizing control systems, communication system and even mechanical systems, systems from transmission, systems for other purposes. Even mechanical systems or acoustic systems, vibration, loud speaker characterization or microphone, all of them can be characterized in terms of usually, second order system and there the values of zeta and omega n are of extreme importance.

The point that I was mentioning now is that, this circle therefore is worth a thousand words. If you draw this circle, then you know whether a maximum occurs or not. If a maximum occurs, at what frequency does the maximum occurs and therefore, this circle is given a very interesting name. It is called the peaking circle. If you draw a peaking circle, then you shall not only know whether peaking occurs or not, you shall also know where the peaking occurs.

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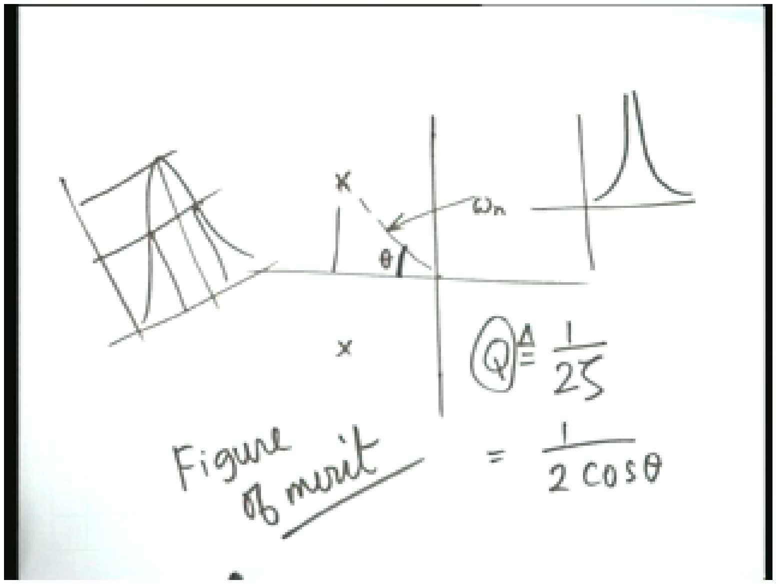

Now with reference to the location of the poles, you recall that this distance is omega n and this is theta. The parameter, a figure of merit is defined like this. You have defined Q in 110, w 110 in a different manner. Now we shall define a Q in terms of a general complex conjugate pair of poles. We are no longer associating Q with an inductance or a capacitance. You know there is an energy definition of Q for an inductance and a capacitance and so on.

There is also a definition for a band pass circuit or for a selective circuit in terms of this selectivity, that is, the maximum, frequency of maximum, divided by the bandwidth. That is a frequency difference between two frequencies at which the response is 70.7 percent, that is, 1 by root 2 or in terms of decimals 3 decimals down. We are giving yet another definition of Q and in usual circumstances this definition shall give you the same result as any other definition under usual circumstances.

It is defined as 1 over 2 zeta, as simple as that, 1 over 2 zeta which is equal to 1 over 2 cosine theta and Q is a figure of merit. Before we go into the details of Q, let us ask some very elementary questions. What happens when the poles go towards the imaginary axis? Q increases. If the poles lie exactly on the imaginary axis, then what is the value of Q?

Student: Infinity.

Sir: Infinity. Now, if the poles lie exactly on the j omega axis, do not you recall that the magnitude goes to infinity? If the magnitude goes to infinity where are the (..) points? Infinitesimally close to that and therefore, Q magnitude goes to infinity and the bandwidth becomes approximately

Student: 0.

Sir: 0 and therefore, Q goes to infinity. On the other hand, if zeta is less than, I am sorry, if zeta is greater than 1 by root 2, what does it mean? This angle is less than?

Student: 45.

Sir: 45. Then there will be no peaking and therefore, theta equal to 45 is the critical angle. If theta exceeds 45, there shall be peaking. If theta is lower than 45, there shall be no peaking. So theta is another way of characterizing the figure of merit Q; peaking and no peaking. If theta tends to 0, there is a definition of Q. Q becomes equal to half. If theta tends to 0, zeta tends to 1, Q becomes equal to half but it cannot be interpreted in terms of a selective response. Why not? Because there is no peaking. The definition of Q still stands, but it loses its physical meaning. We shall continue this on Friday.