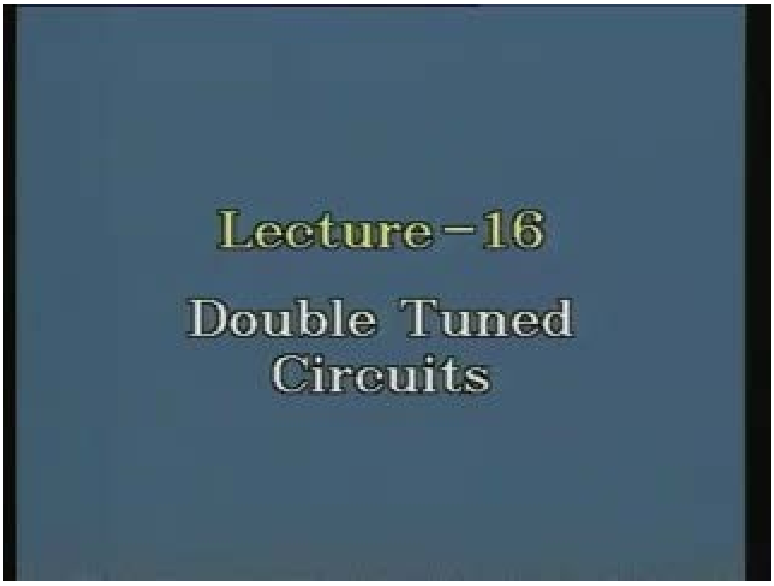
Circuit Theory
Prof. S.C. Dutta Roy
Department of Electrical Engineering
Indian Institute of Technology, Delhi

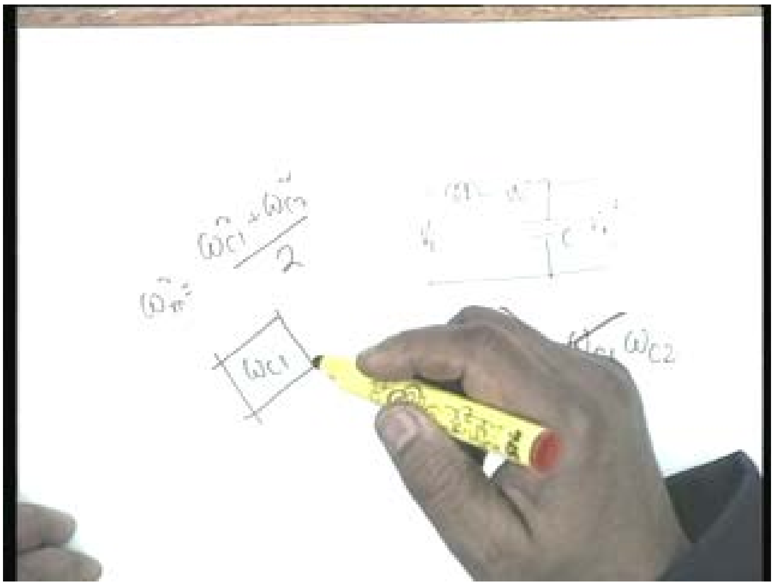
Lecture - 16
Double Tuned Circuits

(Refer Slide Time: 00:52)



This is the sixteenth lecture and our topic for discussion today is double tuned circuits. Before I taken double tuned circuits, I like to mention that in response to the challenges that I have said last time and I had said, not all statements that I make are true. I am not under oath to make all true statements. I do make intentional mistakes and one of the things that mean is pointed out, is this relationship.

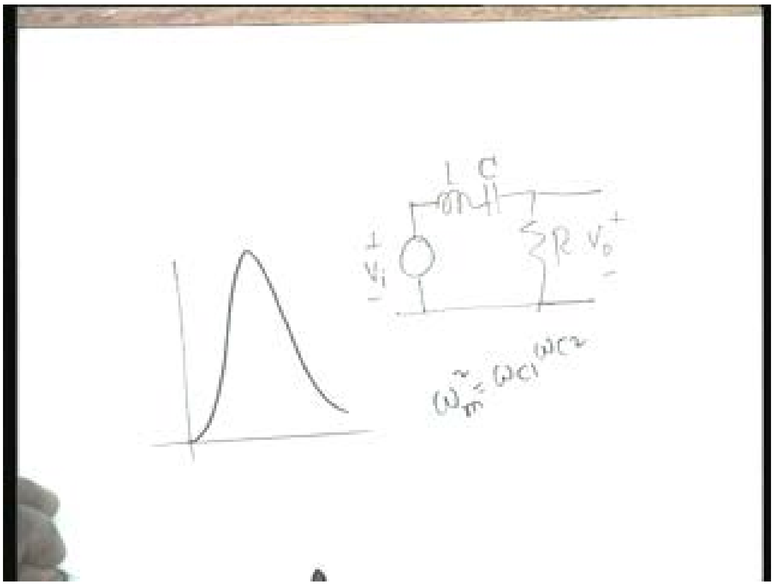
(Refer Slide Time: 1:32)



In a single tuned circuit, if we take the output across a capacitor c and this is V I, this is V 0, then the central frequency or the frequency of maximum, I had claimed that this will be the geometric mean of the 2 cut off frequencies, omega c 1 and omega c 2. This happens to be a wrong statement. In fact, what is true is that omega m squared is equal to omega c 1 squared plus omega c 2 squared divided by 2. That is, it is the arithmetic mean of, not of the half power frequencies, but of the square of the frequency of maximum response is the arithmetic mean of these squares of the half power frequencies.

That is number , and number 2 I had set another challenge which has been very successfully met is how to get omega c 1. Omega c 2, we had found out, both the reports, Mayank and Manish, both the reports, I got only 2, have shown how to construct this omega c, geometrically, and I am very happy about it. May I also mention, in passing, that the statements that I have made is needs true. Once again, with a pinch of salt, is indeed true if you take the output across the resistor.

(Refer Slide Time: 3:09)



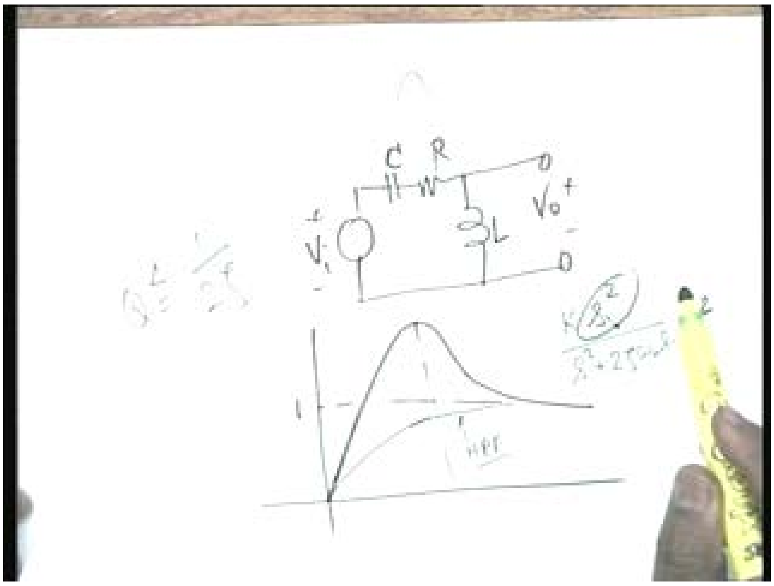
That is, if the output voltage is proportional to the current in this circuit. Suppose, I have a circuit like this, then you can see from physical reasoning that at DC, at a DC, the output shall be 0, because the capacitor will charge to the DC voltage. At infinite frequency, the output shall again be 0. Why, because the inductor offers an open circuit. j omega L, omega goes to infinity therefore, the impedance goes to infinity and therefore, the response of this circuit shall truly be a band pass response. It would be like this. It would start from 0, it would end up in 0. Truly a band pass response and one can easily so that here omega m squared is indeed equal to omega c 1 times omega c 2. It is the geometric mean.

Now you notice that same circuit, same tuned circuit, can give you a low pass response and also a band pass response. A low pass response flows if you take the output across the capacitor and zeta is less than, zeta is a greater than or less than

Student: Greater than.

Sir: Greater than 1 by, greater than the critical balance. Similarly, the same tune circuit, if you take the output across the inductor,

(Refer Slide Time: 4:48)



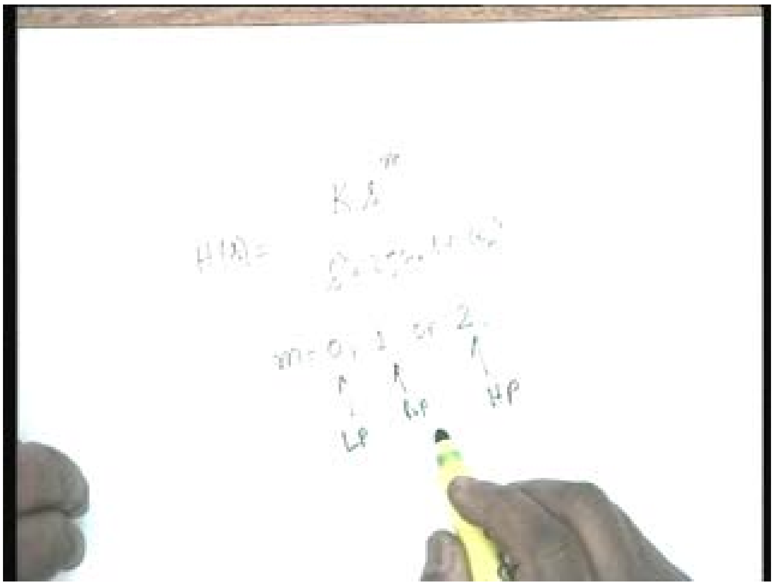
if you take the output across the inductor, then you see that, again from physical arguments, at DC the output should be 0, because the inductor axis is short circuit. At infinite frequency, the output should be the same as the input, that is, the transfer function magnitude will be equal to 1 and therefore, in between, in between there are maybe there may be an maximum, there may not be a maximum. For example, if zeta is less than a certain value, I will get a response like this. Were the finite, the final, the infinite frequency value is unity, starts from 0 at DC, then it may so a peaking, depending on the value of zeta and then it goes (…) to unity at infinite frequency.

On the other hand, if zeta is greater than a certain value, that if zeta exits the critical damping, you might get a response like this, which is truly a high pass response, a high pass filter. So the single tuned circuit it an extremely interesting circuit from which you can get high pass, low pass, as well as band pass responses. You see, even in the high pass mode, it will act as a band pass if this goes high enough, if this is selective enough. Even in the low pass mode it can act as a band pass filter if the Q, once again I go back to definition of Q, Q is 1 by 2 zeta. If Q is sufficiently high or zeta is sufficiently low.

So in a any of the 3 modes, the circuit can act as an band pass filter, provided, the resistance in the circuit is not excessively large. It is this resistance which gives the dumping affect and therefore, the resistance, if the resistance is not excessively large, it may act as a band pass filter. But it may be of interest to find out, in the high pass case, in the high pass mode, what is the frequency of maximum, what are the conditions for a peaking, what is the condition for critical damping, that is, what is the condition for a transition between high pass and band pass, that is, peaking or no peaking, and how to find out the 3 db cut off?

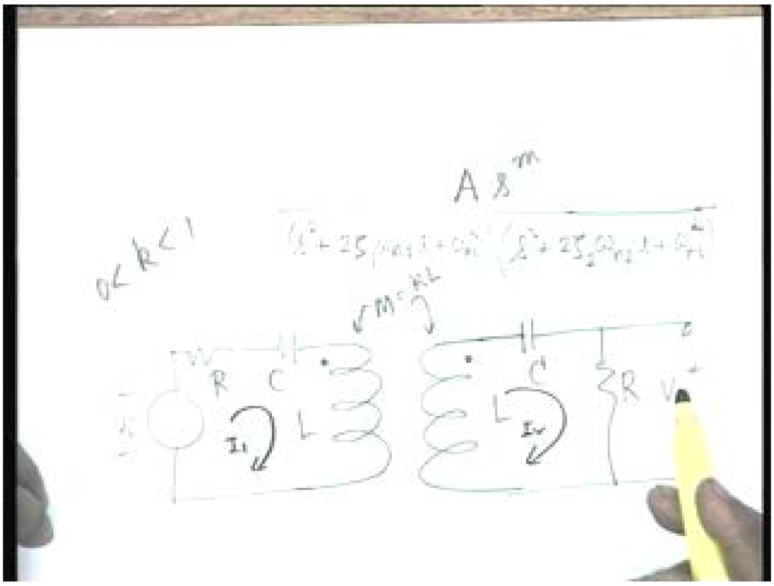
It is worth investigating. May I also mention that in the high pass mode, it is very easy to see that the transfer function would be of the form s squared, some constant K divided by s squared plus s squared plus twice zeta omega and s plus omega n squared. Do you see this? That it would be s squared, because this is s L divided by R plus s L plus 1 over s C so it will be s square. In the low pass mode, the numerator was a constant, s square term was not there. In the band pass mode, the numerator has a power of s. So the single tuned circuit is indeed of versatile circuit, in which, you can get K s to the power m divided by s square plus twice zeta omega n s plus omega n squared.

(Refer Slide Time: 08:26)



A transfer function of this type, where m can either 0 1 or 2, agreed? m is equal to 0 is the low pass mode, m equal to 2 is the high pass mode and m equal to 1 is the band pass mode. It is worth investigating all these 3. Now we next consider the case of a pair, 2 pairs of complex conjugate roots, that is, we consider transfer functions in which there are 2 pairs of complex conjugate roots. For example, twice zeta 1 omega n 1 s plus omega n 1 squared and s squared plus twice zeta 2 omega n 2 s plus omega n 2 squared. We consider 2 pairs of complex conjugate roots.

(Refer Slide Time: 9:15)

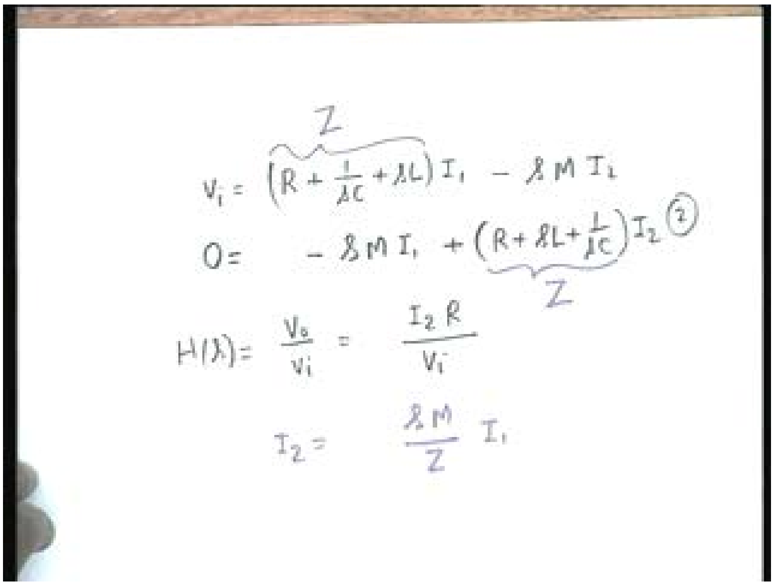


The numerator may be a constant multiplied by some power of s. Obviously, the power n, can it be negative? No, it cannot be negative. So it is either 0, 1, 2, 3 or it can also be 4, depending on how you take the output. Let us see and a practical circuit which gives this kind of a response is the, so called, double tuned circuit or a circuit which is magnetically coupled each other, the primary, as well as the secondary, both are tuned circuits. That is, 2 tuned circuits coupled to each other by means of their inductances by means of a mutual inductance.

We consider this circuit, this is my V I, Laplace transform voltage and then I have, let us say, an inductor and the resistor and this is my V 0. This is a typical mode in which it is operated, and to keep life simple, let us suppose that there are identical tuned circuits. That is, the resistance capacitance inductance are all identical and that there is a mutual inductance between them, let us say these are the dots. There is a mutual inductance which is equal to, as you know, square root of L 1 L 2, k times square root of L 1 L 2, so this would be k times L and k is the coefficient of coupling, k lies between 0 and 1.

We shall investigate this circuit, it is 1 example of producing, 1 example of a circuit which produces 2 pairs of complex conjugate poles and we shall see how this simple circuit can behave in a wide variety of manners. To be able to analyze this, let us suppose, let us consider to loop currents I 1 and I 2. You notice that m is the dots are in the favorable direction, but I have reverse the direction of current and therefore, the mutual inductance term shall come with the negative sign. Is this clear?

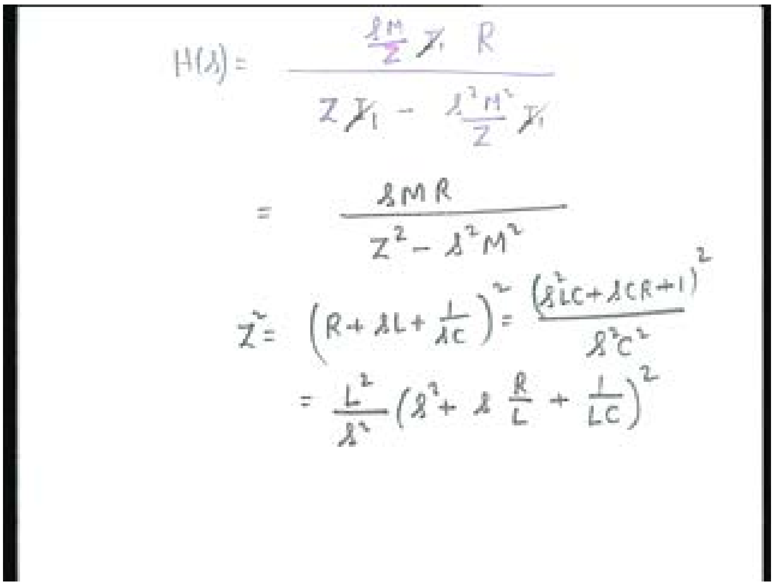
(Refer Slide Time: 12:44)



So my equation shall be, V i would be equal to R plus 1 over s C plus s L multiplied by I 1, this is the self impedance term. My primary question becomes R plus 1 over s C plus s L times I 1 and then the mutual inductance terms, as I said, it should come with a negative sign s M I 2 and for the second loop, that is, for the secondary, 0 would be equal to minus s M I 1 plus R plus s L plus 1 over s C times I 2. These are the 2 loop equations and my transfer function H of s is equal to V 0 by V i V 0 is I 2 R.

The current I 2 flowing through the resistance R, divided by V i and from the second equation, from equation number 2, you can see, I would like to put some abbreviations let us call this Z and this one as Z. Then obviously, you would notice from the second equation that I 2 equal to s M by Z time I 1. From the second equation I 2 equal to s M by Z times I 1 and if I substitute this in the expression for transfer function, I get H of s equals to s M by Z times I 1 times R I 2 R divided by V i which is Z I 1 minus s M times I 2.

(Refer Slide Time: 14:23)



So it would be s squared M squared by Z times I 1 and you can now cancel I 1 and get this as s M R divided by Z squared multiplied by Z also, minus s squared M squared. Now let us simplify Z squared. Z squared is R plus s L plus 1 over s C, whole squared, which I can write as s squared C squared and then s squared L C plus s C R plus 1, whole squared, and this I can write as if I take L C out. Then I shall get L squared by s squared, is that okay? This s squared, I shall cancel it, if I take L C out it will come L squared C squared, C squared C squared cancel, multiplied by s squared plus s R by L plus 1 over L C whole squared.

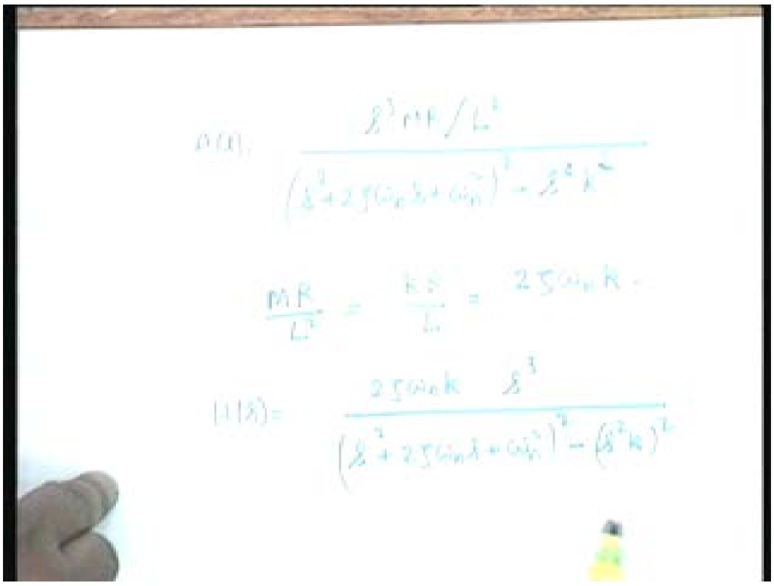
Now I introduce the notations, let me write these equations again, H of s equal to s M R divided by Z squared minus s squared M squared and I have found out this Z squared as equal to L square by s squared, s squared plus. Now R by L, if you recall, I represent this as twice zeta omega n. In terms of zeta, the dumping in factor and omega n the natural, undamped natural frequency, so I get twice zeta omega n s and I also make 1 by L C as omega n squared. Therefore, I get omega n squared. Therefore, my H of s the transfer function becomes s M R divided by L squared.

Student: It would be a whole squared sir.

Sir: It would be a whole squared, thank you. L squared, the s square that occurs here, I can take it in the numerator as s cubed. Then, in the subtraction, minus s squared M squared, I must have s to the fourth M squared. This s squared, I am taking, I am multiplying both numerator denominator by s squared. So this becomes s to the fourth M squared and here I shall have s squared plus twice zeta omega n s plus omega n squared. A bit of algebra, not too difficult though.

I can simplify this further by putting M equal to K L. Have I missed a term? No. what I will do is, if I write this again, H of s is equal to s cubed M R, I divided by L squared, both the numerator and denominator. Then in the denominator, this becomes s squared plus twice zeta omega n s plus omega n squared. Whole squared minus, I divided by L squared so I get the s to the fourth time, K squared

(Refer Slide Time: 18:45)

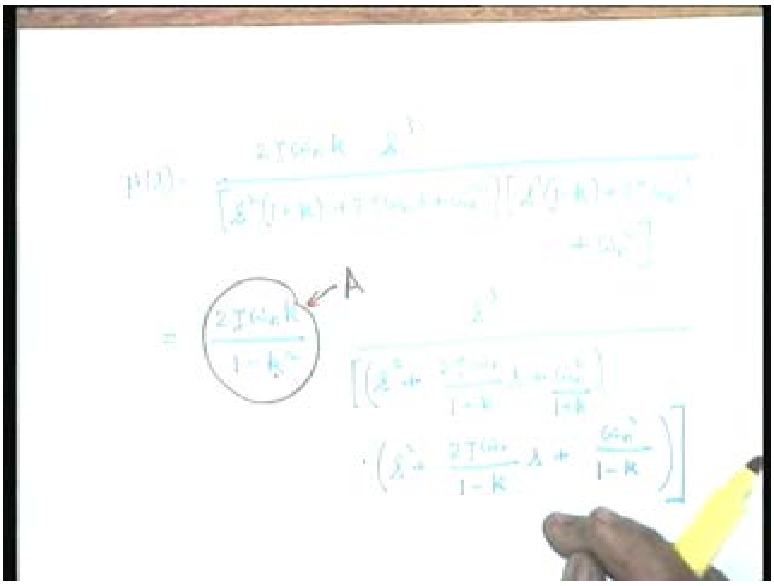


Simply because, M is equal to K times L, so what is M R divided by L squared? The M is K L and therefore, it is K R divided by L and what is R by L?

Student: (…)

Sir: Omega n and therefore, I get twice zeta omega n times K. This quantity becomes twice zeta omega n K therefore, my write each H of s is equal to twice zeta omega n K multiplied by s cubed divided by s squared plus twice zeta omega n s plus omega n squared whole squared minus s squared K, whole squared. No simplification, no approximation so far. It is exact. Now the denominator, you can see, it is of the form s squared minus B squared and therefore, you should be able to take, the factorize it to A plus B and A minus B and the result is the following.

(Refer Slide Time: 20:13)



I get H of S is equal to twice zeta omega n K s cubed divided by s squared, if I add the 2 terms A plus B, then I get 1 plus K plus twice zeta omega n s plus omega n squared and the second factor would be same, except for a change of sign of K. That is, I shall get s squared 1 minus k plus twice zeta omega n s plus omega n squared. Let us do a little more simplification, let us take 1 plus K out from here and 1 minus K out from here. Then I get twice zeta omega n K divided by 1 minus K squared, multiplied by s cubed; do you understand what I am doing? 1 plus K I have taken out, and 1 minus K, I have taken out.

So the product is 1 minus K squared, that is what happens, that is what appears here and the factors that will be left would be s squared plus twice zeta omega n divided by 1 plus K s plus omega n squared. This is the 1 of these factors, multiplied by is

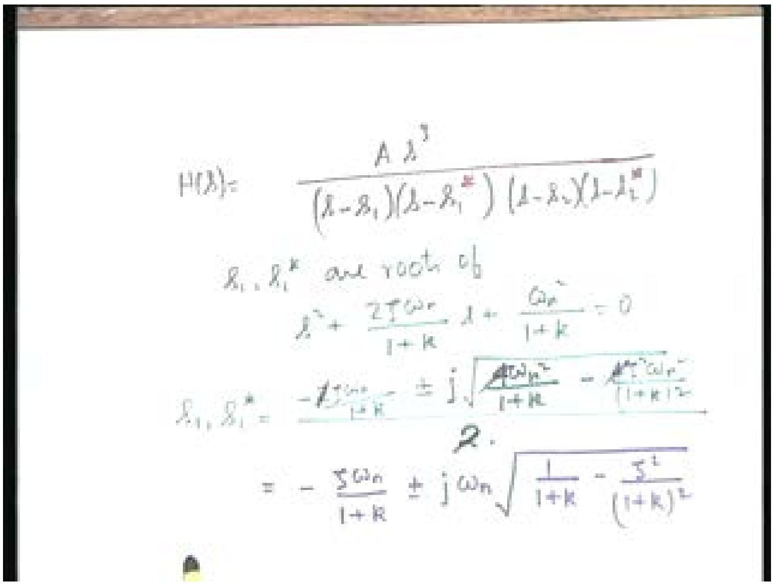
Students: (…)

1 plus K, yes, I must not miss that, multiplied by s squared plus twice zeta omega n divided by 1 minus K s plus omega n squared divided 1 minus K. Now we have the expression in the

form of a constant A. A constant A multiplied by s cubed therefore; there are 3 zeros at the origin. 3 zeros at the origin because of s cube. In the denominator, I have 2 quadratic factors and each quadratic factors shall have 2 roots and therefore, I have 4 poles and, as you know, if the poles are on the real axis, the case is of very little interest. The real axis poles do not offer selectivity.

If the case would be of interest and are our trouble of introducing 4 energy storage elements; 2 inductors, 2 capacitors. In fact, 5 energy, the mutual inductance also stores energy, 5 storage elements would be successful provided the poles are complex conjugate. This is a real polynomial, real polynomial, so poles shall be complex conjugate. Now assuming that the poles are complex conjugate, I can write my H of s as equal to A s cubed, divided by s minus s 1 multiplied by s minus s 1 star. s 1 and s 1 star are the roots of the first polynomial, then s minus s 2 s minus s 2 star.

(Refer Slide Time: 23:14)



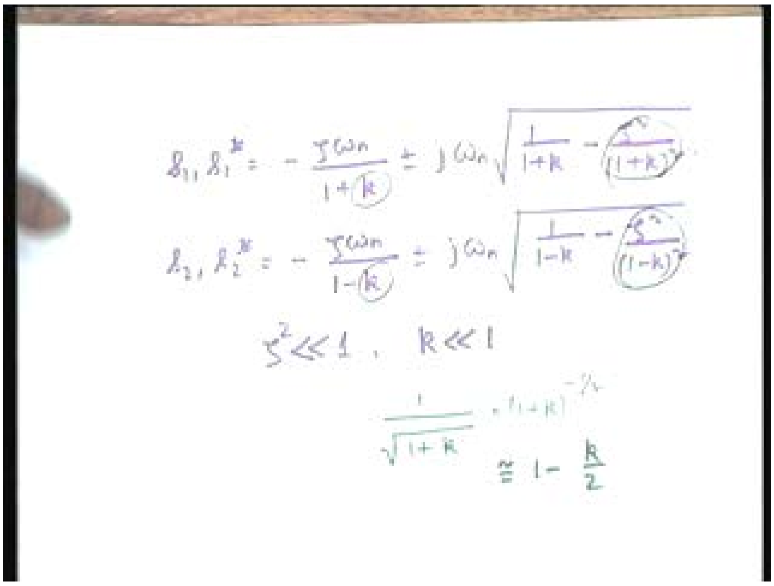
To understand the approximations that we shall be using a little later, let us look at s 1 and s 1 star. Now s 1 and s 1 star are the roots of the equation s squared plus twice zeta omega n divide 1 plus k s plus omega n squared divided 1 plus k equal to 0, agreed. Therefore, s 1, s 1 star would be equal to minus 2 zeta omega n divided 1 plus k plus minus j, square root of s which term shall I have first?

Student: (…)

Sir: omega n squared divided by 1 plus k minus, now 4 times this. 4 zeta square omega n square divided by 1 plus k whole square, divided by 2, you must not forget that. Now this 2 can be cancelled out, 2 with this 2 this 4 and this 4. So I get, equal to minus zeta omega n divided by 1 plus k plus minus j. I can take omega n out, omega n from omega n square, I can take this out. Square root of 1 by 1 plus k minus zeta square divided by 1 plus k whole squared, agreed.

i have not made any approximation so far. I have only made simplification and I made the assumption that the poles are complex. If they are not complex, then you see zeta must be greater than the critical value. There shall be no peaking. Even if there is peaking, it shall be horrible, the q shall be large. q is 1 by 2 zeta, I am sorry, q shall be small and therefore, the circuit shall offer no selectivity. It is only for the purpose of selectivity that we take the trouble of combining energy storage elements of different kinds. Similarly, if you, let me right this again s 1, s 1 star is equal to minus zeta omegas n divided by 1 plus k plus minus j omega n square root of 1 by 1 plus k minus zeta squared divide by 1 plus k, whole squared.

(Refer Slide Time: 26:16)

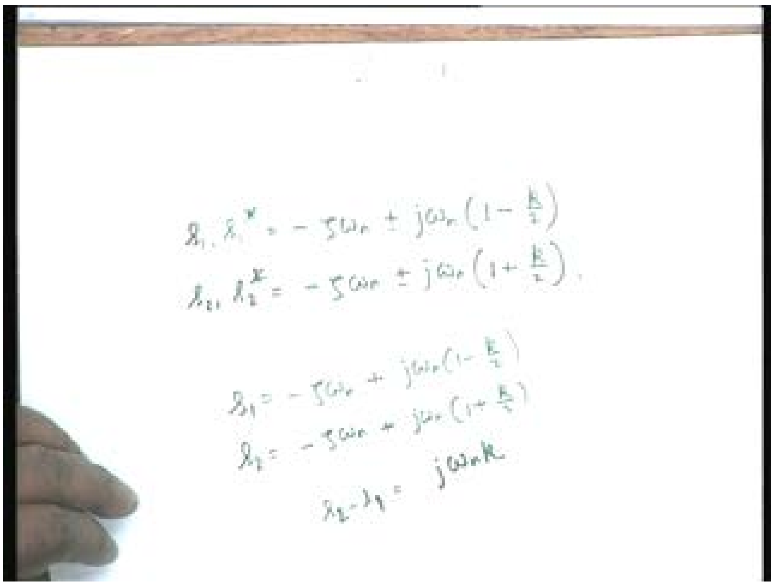


Similarly, I can write s 2, s 2 star, the only change will be k shall be replaced by minus k, that is all. So minus zeta omega n divided by 1 minus k plus minus j omega n square root of 1 by 1 minus k minus zeta square divided by 1 minus k, whole squared. I now make some approximations. I now argue that a circuit of this complexity, where there are 2 coils which are been brought close to each other, magnetically coupled, there are capacitances is both the circuits which shall be useful only if it is a high q situation. That is, if individually the tuned circuit, there are 2 tuned circuits identical tuned circuits, we assumed for simplicity, the tuned circuit themselves must be high q. That is, zeta must be small.

We do assume the zeta square is much less compare to unity. We assume the zeta squared is much less compared to unity and we also assume that the couple that the coils are loosely coupled, that is, we also assume that k is much less than unity. They are loosely coupled. If that is the case, then I can ignore this k. Let me indicate the simplifications. I will ignore this k and this k in the denominator, I will take them out. I will ignore this term, as compared to 1 by 1 plus k. 1 by 1 plus k would be slightly less than unity but because of zeta squared here, this should be very small compared to the first term, so I will ignore these 2 term also.

In addition, I will make the simplification that 1 by square root of 1 plus k is equal to 1 plus k to the power minus half, approximately equal to 1 minus k by 2. Mind you, the simplifications are based on the assumption of high q circuits, that is, low resistance. Capital R must be low, number 1. Number 2 is that the coils are not too critically coupled, too tightly coupled, they are loosely coupled. If that is so, then you notice that s 1 s 1 star becomes a very simple expression.

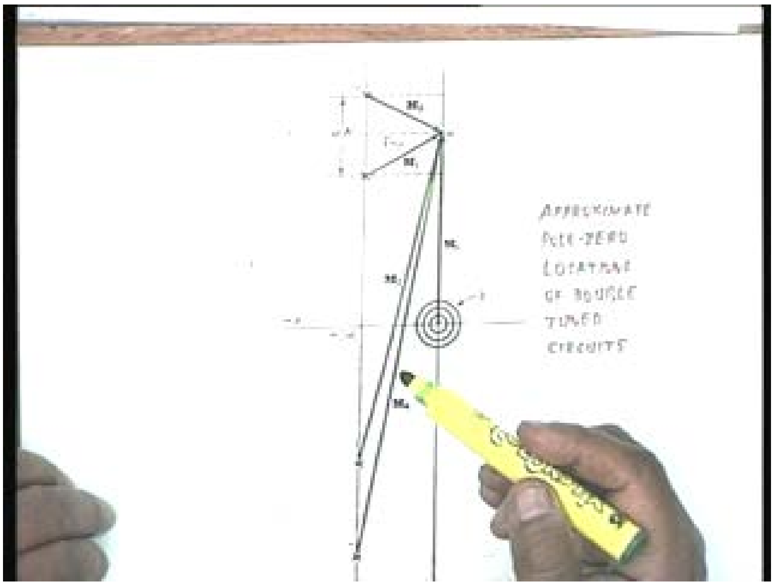
(Refer Slide Time: 29:07)



It simply becomes minus zeta omega n plus minus j omega n 1 minus k by 2 and similarly, s 2, s 2 star that becomes minus zeta omega n plus minus j omega n, he only thing that happens is k changes its sign, so 1 plus k by 2 and the situation with this approximation, the location of the roots, you see that they occur, there is quad of roots, quad, 4 of them. And the real parts of all the roots are the same. That is, all the poles lie on a laying parallel to the j omega axis. The real part is same minus zeta omega n in the left half plane.

However, on this line, which is parallel to the j omega axis, there are 2 in the upper quadrant and 2 in the lower quadrant. 2 in the second quadrant and 2 in the third quadrant and the 2 in the second quadrant are symmetrical, with respect to omega n. Is not that right? One is j omega n 1 minus k by 2, the other is j omega n 1 plus k by 2 and the other 1 is negative, that is, in the third quadrant. So the situation is like this.

(Refer Slide Time: 30:48)



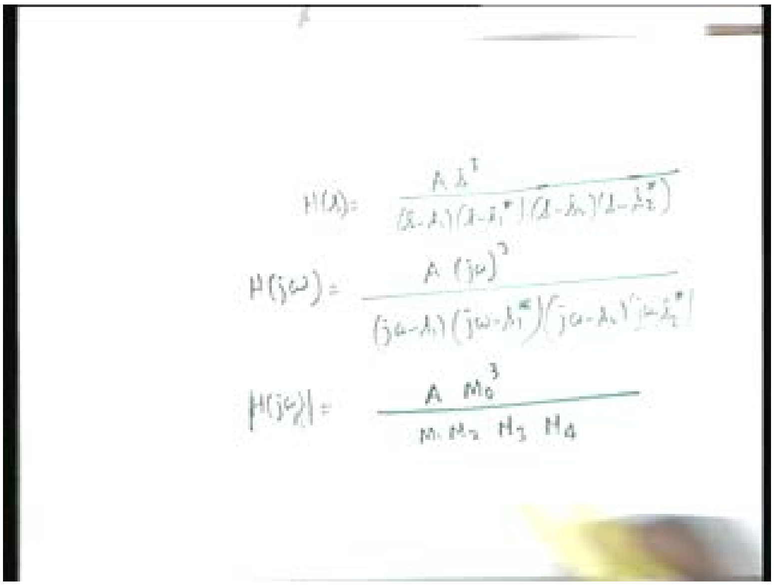
Let me explain, these are the 3 zeros at the origin and we indicate this by 3 concentric circles, 3 zeros. There are 4 poles with this approximation. This is 1, this 1, this 1, this is 1. 2 in the third quadrant, 2 in the second quadrant and 2 in the third quadrant, the real part is minus zeta omega n, for all of them. So this root is minus zeta omega n plus j omega n 1 minus k by 2. So this is omega n, you see, at a distance of zeta omega n. So it is half way below omega n, so this distance is omega n k. The distance between 2 poles, is that clear?

No, shall we go back to this? You see, what I am saying is, s 1, s 1 is, let us say, minus zeta omega n plus j omega n 1 minus k by 2 and s 2 is minus zeta omega n plus j omega n 1 plus k by 2. So what is s 1 minus s 2? It is simply, or s 2 minus s 1, it is simply j omega n k. So the difference between the 2 poles, it is simply, the distance is omega n k and it is on the j omega axis so j omega by n k. This distance is omega n k and you see that the simplification that it achieves is that, the 2 poles in the upper quadrant becomes symmetrical about the natural, undamped natural frequency of the tuned circuits about omega n.

Similarly, these 2 in the lower quadrant, I am going to make further simplifications, so it is essential that you understand this simplification and the only thing that we assumed is that zeta is much less than 1 and k is much less than 1, actually zeta squared. You see zeta, it is very easy to satisfy zeta squared much less than 1 because if zeta is, let us say, point 5, zeta squared is point 2 5. If zeta is point 1 and zeta squared is point 0 1,1 hundred. So under these conditions the poles and zeros of the double tuned circuits assume this particular form.

Now here we are going to make further simplifications, further simplifications like this. Suppose, I want to find out the magnitude responds at this frequency omega n. Then as you know, my transfer function is A s cubed divided by s minus s 1, s minus s 1 star , s minus s 2, s minus s 2 star.

(Refer Slide Time: 33:55)

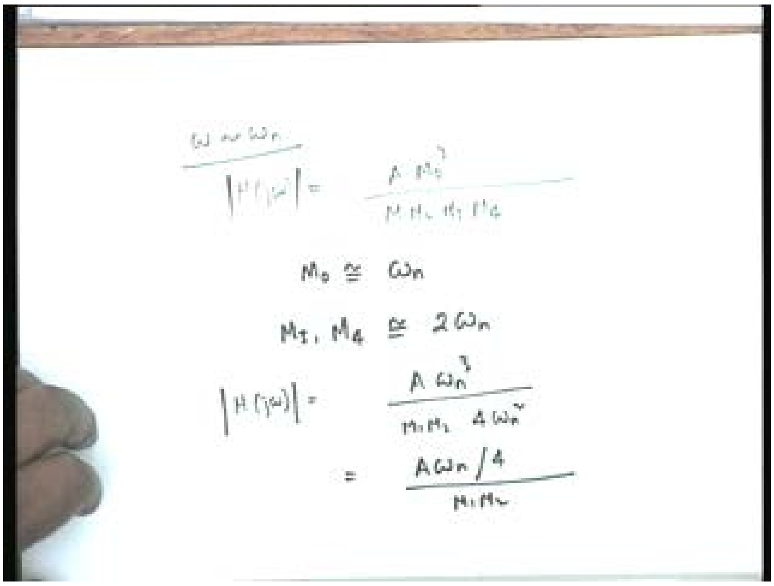


Therefore, H of j omega would be equal to A j omega cubed divided by j omega minus s 1, j omega minus s 1 star, j omega minus s 2, multiplied by j omega minus s 2 star and as far as magnitude is concerned, all we do is, we draw a vector from the point j omega to the zeros and to the poles. We draw the vectors. These are the vectors, from the poles, from the pole to the point at which you want find out the magnitude response. From these 2 poles to the point M 1 M 2 M 3 M 4, these are the magnitudes of these vectors and M 0 is the vector drawn from the origin to from this zeros to omega n and therefore, my H of j omega n, under this condition, shall be A M 0 cubed, 3 zeros divided by M 1 M 2 M 3 M 4, the magnitude. I can also find the phase from this diagram. Let us consternate on the magnitude at the moment, any question?

Student: Sir, what is this M 0 cubed?

Sir: M 0 cubed comes from j omega cubed M 0 is this vector, from the 0 to omega n, it is the 0 vector and since it occur 3 times, we are taking m 0 cubed. Now I want you look at this diagram very carefully. You see, when omega, when our frequency, this is the situation to omega n, now when our frequencies around omega n, what does it mean, a narrow band centered around the undamped natural frequency? Then M 0, even if omega shifts a little on the upper side or lower side, M 0 can be approximated as omega n, agreed. When omega, let me write this again, I will come back to this diagram again and again.

(Refer Slide Time: 36:58)



So H of j omega, omega around omega n, this is what, this is how we indicate it. Omega around omega n, H of j omega which is equal to A M 0 cubed M 0 is now a function of omega, divided by M 1 M 2 M 3 M 4. When omega is around omega n, M 0 can be approximated by omega n, agreed? When the frequencies close to omega n. When the frequencies around this point, it can be approximated by omega n. Not only that, if zeta is low, this figure is an exaggerated figure. Zeta is low, the lines j omega axis and this line are very close to each other. If they are close to each other, naturally, M 3 and M 4 should also be close to each other.

These poles, you see, k is a small quantity, so lose coupling and therefore, this poles are also very close to each other, which means, that when frequencies around omega n, both M 3 and M 4 can be taken to be approximately equal and approximately equal to twice M 0, is that clear? So I do this approximation M 3 and M 4 both are approximately equal to twice omega n. If I do that then H of j omega becomes equal to magnitude becomes equal to A omega n cubed divided by M 1, M 2, 4 omega n squared. That is equal to A omega n divided by 4 divided by M 1 M 2. I hope you realize what I have done.

Student: Excuse me sir, in that you are assuming omega n also to be small?

Sir: No.

Student: Sir otherwise what would happen M3 and M 4 will not be equal to 2 n, 2 omega n?

Sir: Why not? You see, this line, this line is very close. Just a minute, I understand your confusion. You see, first we say because k is small omega is not necessarily small because k is small, these 2 poles are very close to each other, so M 3 and M 4 are approximately equal. Then I argue that these 2 lines are very close to each other.

Student: Sir, but this is zeta times omega n.

Sir: Which is zeta times? So correct, zeta is a small quantity and therefore, they are very close to each other, therefore, M 3 and M 4 are approximately twice M 0. Is not it approximation and a fairly good approximation, in practice, as far as design of this circuit is the concerned.

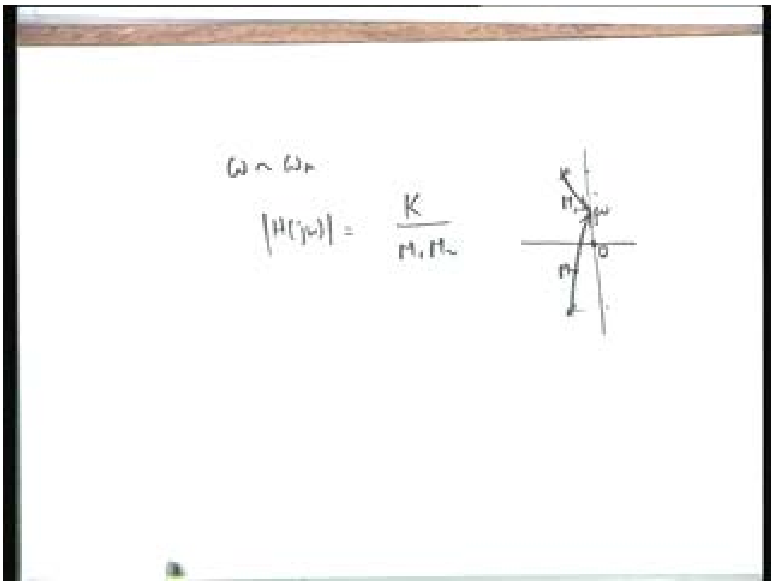
Student: Sir that means, we have approximated that s 2 star and s 1 star equal to j omega

Sir: Quite so, quite so.

Student: Both the poles are at j omega.

Sir: Quite so, that is correct and therefore, if you notice carefully what we have done is, this is constant, let us call it a K, and therefore, for the double tuned circuit, with omega approximately equal to omega n around omega n, the magnitude function has been approximated as K by M 1 and M 2, which physically means, I go back to this diagram, which physically means that it is these 2 poles which are close to omega, approximately equal to omega n, affect the magnitude response much more than the zeros and the other 2 poles.

(Refer Slide Time: 40:45)



This is all that I have expressed in terms of this mathematical approximation, which is obviously true. If omega shifts a little bit, M 1 and M 2, the relative change in M 1 and M 2 are much greater than the relative change in M 3 M 4 or M 0. This is what we have done. No, I have done something else. If you recall this single tuned circuit, in the single tuned circuit, we had 2 poles like this and we had expressed the magnitude response as K sine psi over M 1 M 2.

Student: (…)

Sir: Oh, it was simply K sine psi because M 1 and M 2 are included but there also the magnitude response if this is omega. If this is j omega and this is M 1 and this is M 2, magnitude responses is simple a constant divide by M 1 M 2 and therefore, virtually, the double tuned circuits has been converted to a single tuned circuits, with 1 difference. In a single tuned circuit, this was 0 and therefore what we have done is we have raised the origin or the real axis.

We have raised it by an amount omega n and therefore, the totals, the response of the double tuned circuits at frequencies around omega n, which are all concerned, will be basically determined, exactly in the same manner as that of a single tuned circuit. In other words, I can now bring in the concepts of the peaking circle. The peaking circle, for example, would be a circle, what would be the center?

Student: Sir, omega n.

This should be the center and this as the radius, you draw a circle. If it cuts the j omega axis, then there shall be peaking. If it does not cut the j omega axis, then there shall be no peaking. That is not correct. Unfortunately, there shall be peaking. You see, this is not 0, this is not 0, this is omega n.

Student: Sir, what did you say about raising the?

Sir: Raising, you see, in this single tuned circuit, this center was 0, was the origin of the complex plain. Whereas the center here is omega n. So it is as if the real axis has been raised by a level, has been transformed to omega n. Once that is established, this behaves exactly like single tuned circuit. However, whether the peaking circuit cuts the j omega axis or does not cut the j omega axis, it does not matter. There shall still be a peaking and this can be understood with reference to another diagram, which we shall project next time. It is 2 0 1, so we continue on Thursday.