Circuit Theory
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Lecture - 31
Scattering Parameters of a 2-Port

This is thirty-first lectures on Scattering Parameters of a 2 port. We first recall a few results from 1 port.

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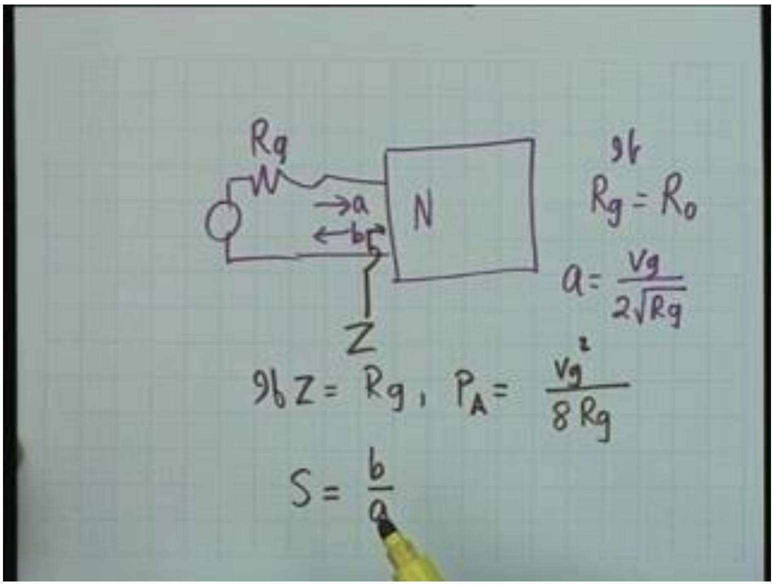

We had proved that, if you have a1 port network N and it is driven by a source of internal impedance Rg and you choose Rg equal to the reference resistance R naught. If this is the case, then the incident wave a is simply equal to Vg by 2 square root Rg. This we proved the other day, we also proved the reflected wave is b, we also proved that if the input impedance of this network is Z and if Z equals to Rg, then the power delivered to the network is the maximum available from the source and is given by Vg squared divided by 8 Rg.

This to results shall be utilized today in the discussion on 2 port, 2 ports Scattering Matrix. We also start a discussing the properties of this Scattering Parameter for a1 port which is simply the ratio of the reflected wave to the incident wave. And we said several things mainly that, the magnitude of S also we started from this expression.

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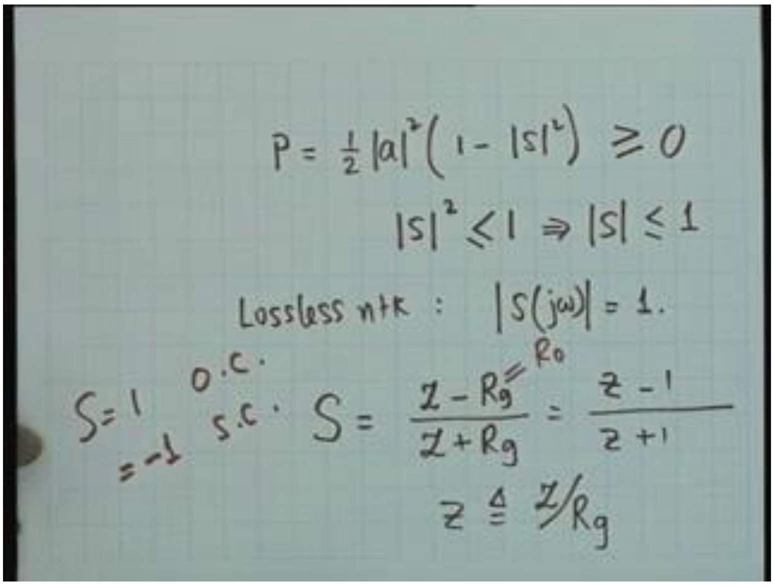

That P is equal to half mod a squared 1 minus mod S square. And for a passive network this should be greater than or equal to 0. And therefore, for a passive network mod S squared must be less than equal to 1. That is: mod S must be less than or equal to 1 all right. That is the Scattering Parameter is upper bounded by unit. And this equality sign can be attained when the network is lossless. So, for a lossless network magnitude S j omega is equal to 1.

We also showed from the definition of S another property. We said that this is reflection coefficient Z minus Rg divided by Z plus Rg and we wrote this as in the normalized form z minus 1 divided by z plus 1, where z is defined as Z divided by Rg. And we are good that, obviously for open circuit that is, if z goes to infinity S is equal to 1 for open circuit and S equal to minus 1 for short circuit. If z.

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That is correct and we have tacitly assumed here, that R naught is equal to Rg. We have assumed that, R naught equal to Rg. It is a good point you will see that, this is always done because it facilitates calculations. The terminating impedance is taken as the reference impedance. This is the general rule it facilitates computations; however, there are situations where this is not done there are situations, where this is not done there are situations, where this is not done. But we shall not encounter such situations in this class you might do. So, in a course on microwave circuits at a later stage in the curriculum, but up to this we did discuss in the previous class.

The further properties of the Scattering Parameter S for a1 port network shall be obtained after we give 2 definitions.

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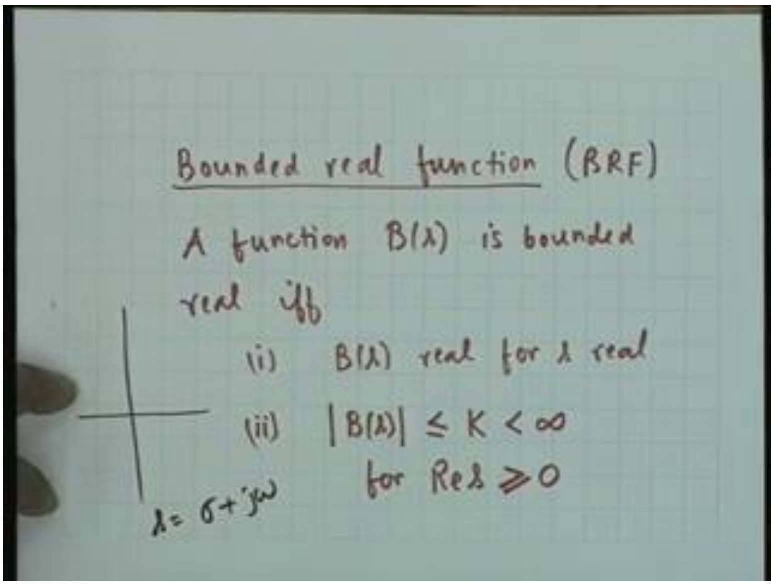

1 is that of the so, called bounded real function or which we abbreviate as BRF. And the definition is this very simple, a function B of s is bounded real is defined to be bounded real if and only if 2 conditions are satisfied, number 1 that B of s should be real for s real. That is B of s in B of s all coefficients must be real quantities; is that clear? Then if s is real, if the variable is real the function would be real this is very simple to apply it is obvious it should be obvious by instruction.

The second 1 is not obvious that is the magnitude of B of s must be bounded; that means, it should be less than or equal to some quantity K; obviously, K is positive all right which is bounded; that means, K is less than infinity for the right half of the s plane, that is for real part of s greater than equal to 0 this is the definition, you see in the s plane in the s plane, the right half is real part of s greater than or equal to 0.

If you write s equal to sigma plus j omega complex quantity then sigma is greater than equal to 0. In the right half plane the magnitude of the function must be bounded it cannot go to infinity which means: that B of s cannot have a pole in the right half plane is that clear? B of s cannot have a pole in the right. If it has a pole then the magnitude goes to infinity all right or B of s is regular there are many other kinds of expressions in the in the theory of complex variables to describe this but, this is, what this is what a bounded real function is. And we shall show that S the reason for bringing this definition is we shall show that S is bounded real, but, before we do that we want to give another definition of another kind of function which are called positive real functions.

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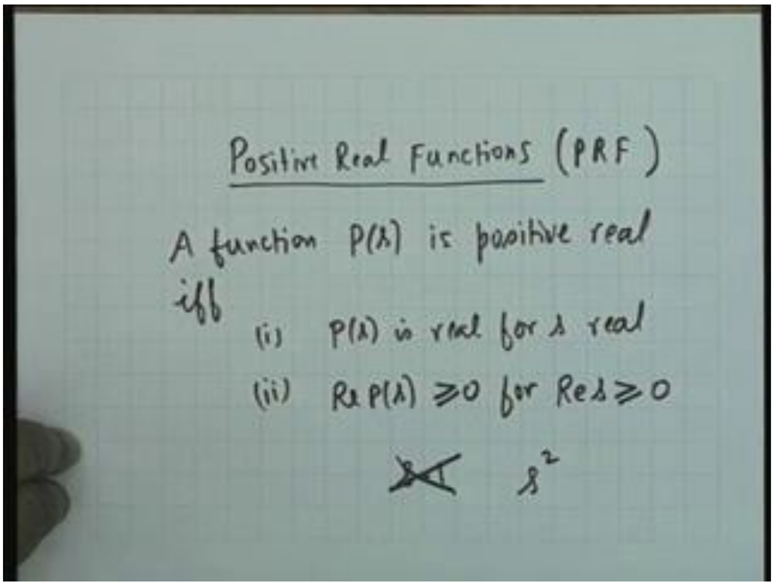

These definitions, will be useful in our further studies on network synthesis and it is a good point to give this definition. This is abbreviated as PRF and the definition is this a function P of s is positive real, if and only if 2 conditions are satisfied once again. First is that P of s is real for s real, P of s is real for s real. And the second is that the real part of P of s is non negative. It is it has nothing to do with the magnitude now, it is just the real part. Real part is non negative for the right half plane that is: for real part of s greater than or equal to 0.

In the right half of the s plane the real part of the function should be nonnegative. Now, is that point clear for example, the function s minus 1 is this positive real? No it is not positive real because it is real part a sigma minus 1 can be negative in the right half plane all right. What about s squared?

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Positive real no, it is not necessarily, it is not positive real. No as far as positive realness is concerned a function is either positive real or not positive real.

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It is not all right.

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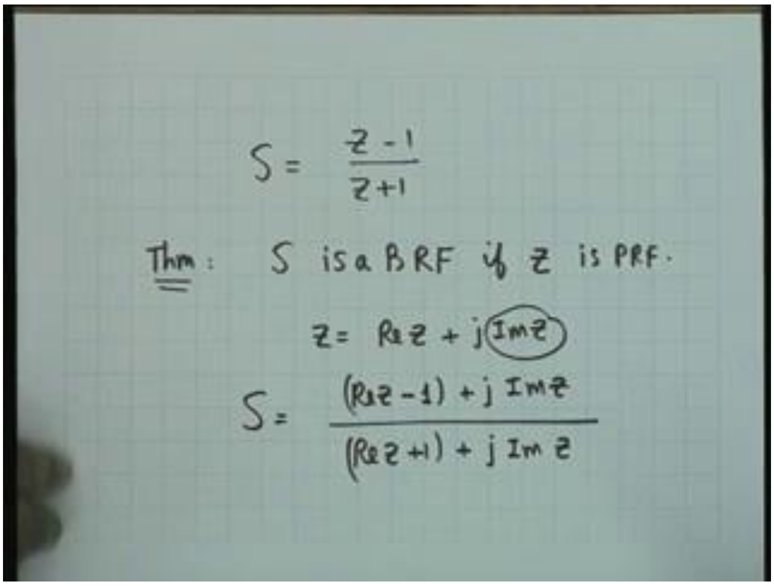

Now, the reason for bringing these 2 functions is that you saw that S for a 1 port is the normalized input impedance Z minus 1 divided by Z plus 1. And the next property can be stated in the form of a theorem that is: the Scattering Parameter S is bounded real, is a bounded real function is a BRF if Z which is the input impedance divided by a constant is PRF. If the impedance if the input impedance is a positive real function then the Scattering Parameter is a bounded real function and the proof is very simply.

Now, I can write z as equal to real part of z plus j times imaginary part of z all right. Let's call this as no let us leave it like that, then S is equal to real part of z minus 1 plus j imaginary part of z. And in the denominator I shall have real part z plus 1 plus j imaginary part of z agreed. I have just written z equal to real z plus j imaginary part of z, please recall that this is a real quantity imaginary part of z is a real quantity by multiplying by j I make it imaginary all right any question?

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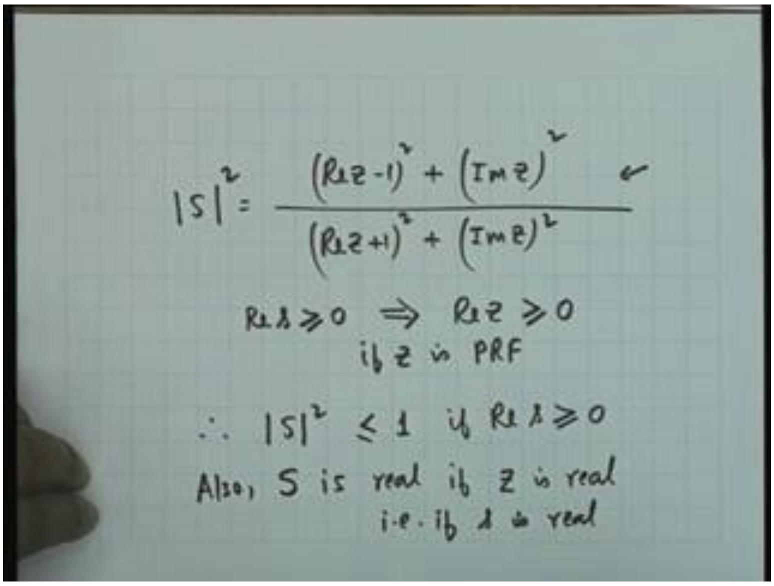

Therefore magnitude s squared is equal to real z minus 1 squared plus imaginary z squared real z plus 1 squared plus imaginary part of z squared. If real part of s is greater than or equal to 0 this implies that real part of z, if z is PRF if z is PRF then this should be greater than or equal to 0 agreed, if z is PRF all right. Now, if real part of z is greater than or equal to 0 then; obviously, the numerator of this function shall be less than the denominator, is that clear? Yes or no. The numerator here.

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This is the definition of positive real function that, if z is positive real then real s greater than or equal to 0 shall imply that real z is greater than equal to 0. Therefore all right. Let I was explaining this point that under this condition under this condition which happens when real s is greater than or equal to 0 that is in the right half plane, in the right half plane real part of z is nonnegative. And therefore, the numerator of this must be less than the denominator.

The only way the numerator can exceed the denominator is, if real z can become negative all right, if real z can be become. Now, therefore, magnitude S squared is less than equal to 1, if real part of s greater than or equal to 0 all right. And obviously, S also S is real, if z is real because S is z minus 1 divided by z plus 1, if z is real then S is real that is when z real z is a positive real function and therefore, it is real when S is real if S is real.

Therefore I get 2 things about S that S is a real is real, if S is real and the magnitude of S is upper bounded by unity therefore, we have proved that is a bounded real function all right that proves the theorem agreed, I have gone very slow I have gone very slow we will come back to positive real functions in great details at a later stage, but, this is just the tip of the iceberg, if positive real functions are icebergs. They are very useful and a constant companion of electrical engineers whether they do control or circuits or whatever it is. Any question?

You see we have proved by mapping that is: real part of S greater than equal to 0 maps into real part of z greater than or equal to 0 that maps into mod S squared less than equal to 1. It is by mapping from S plane to z plane and z plane to S plane. And this mapping also many a times in your career you shall have to do yes.

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This is because z is PRF…

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s this is s equal to sigma plus j omega this is the variable, S is the Scattering Parameter all right.

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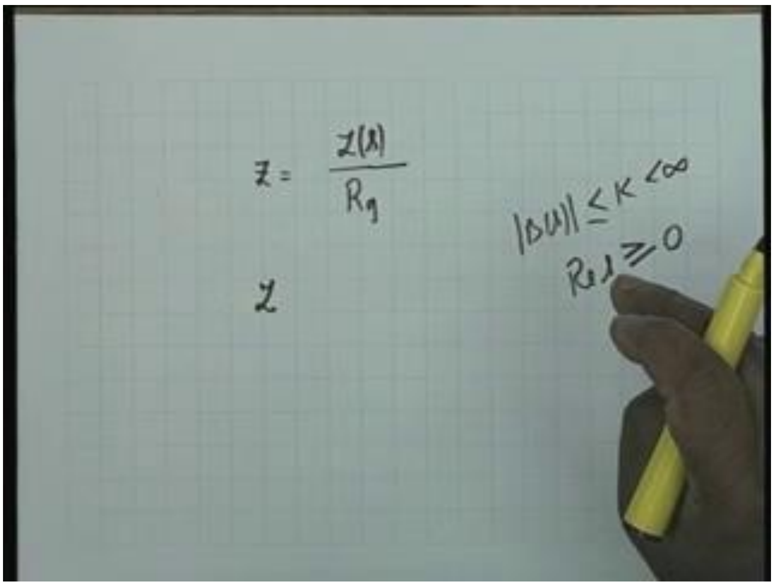

Of course, z is equal to Z divided by Rg therefore, z is a function of s I did not write it specifically z as well as z are functions of S. Yes.

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Both of them cannot have a pole in the right half plane good question.

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How do I get that s is because Z is a positive real function by hypothesis and 1 of the conditions is that it, must be real for s here.

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There is no bound in positive real functions yes.

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We will come to this, fine distinction at a later stage the difference between positive real functions and bounded real functions is in bounded real function the constraint is in magnitude only Whereas, in a positive real function the constraint is on the real part which certainly implies that, there is a bound on the magnitude also is not that right? It does imply but.

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That will come as a result of this as a result of the restriction in the real part, it will come as we will show at a later stage. A bounded real function cannot have a pole on the imaginary axis let me point out at least 1 distinction it cannot have a pole on the imaginary axis that is for a bounded real function B of s shall be less than equal to K less than infinity. For real part of S greater than equal to 0 all right it cannot have a pole on the j omega axis on the other hand a positive real function can have a pole on the j omega axis.

The 2 are not the same they have differences all right. We shall come back to positive real functions at great details after the second miner.

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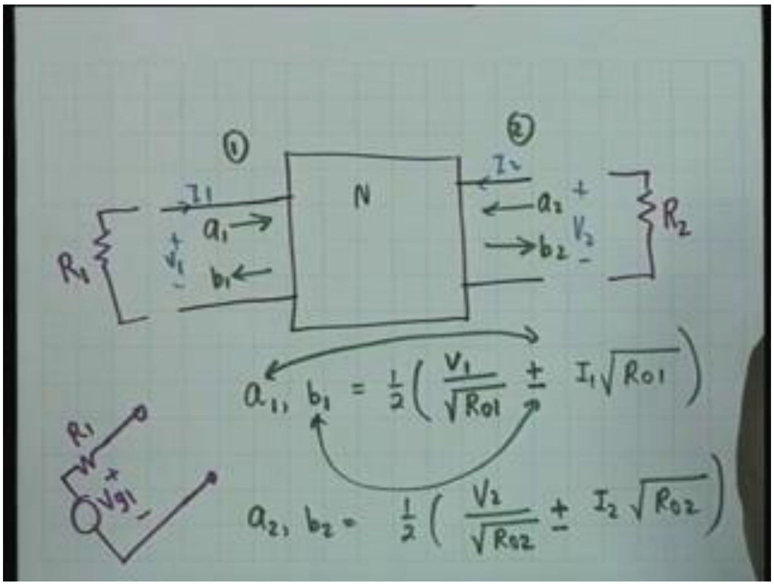

Now, we the time is appropriate for going to a description of a 2 port in terms of scattering parameters. Let us suppose that N is a 2 port, port 1 and port 2 the parameters the variables are taken as a1 b1, a1 is: the incident wave, b1 is: the reflected wave at port number 1. And at port number 2 a 2 is the incident wave and b 2 is the reflected wave we require 2 incidents waves and 2 reflected waves for a 2 port description.

And the definitions of the incident and the reflected parameters are the same that is a1 and b1 would be equal to half if I call this voltage and current I require more colors V1 I1 V2 I2. If the voltage is in currents are V1 I1 and V2 I2 then my a1 V1 is V1 divided by square root R 0 1, obviously we require 2 reference impedances now.

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Correct you see it is at port that we are concerned with a1 b1 V1 I1 this port number 1.

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No we are considering a lumped network no it is not a lumped network the network could be distributed, but, we are considering at a specific value of x at a specific location in space we are considering the incident wave reflected wave voltage and current all right therefore, they are not functions. We are not going inside. We are not going inside the network you see all these 2 ports descriptions are external descriptions they are the descriptions of a black box.

What is inside it does not concern us at the moment all right. We will see we will take an example of a transmission line later how to find the scattering parameters. So, the definition is if you take a1 then it shall be plus, if you take b1 then it shall be minus. V 1 corresponds to minus a1 corresponds to plus, plus I1 multiply by square root of R01 the usual definition. R01 is an arbitrary positive resistance which is called the reference resistance for port number 1.

Similarly, a2 and b2 is half V2 divided by square root of R02 plus minus I2 square root of R02 the usual definition. And I will be using introducing new terms 1 by 1. We introduce 2 terminating resistances that is the ports are not left open they are terminated 1 is terminated in a resistance R2 port number 2. And port number 1 is terminated in a resistance R1. Now, where I have not shown it exactly connected the reason is the following. That if you excite the network the 2 port network at port number 1 then R1 shall be the internal resistance of the source all right.

If you excite the network at port number 2 then R2 shall be the internal resistance of the source Vg 2 that is, a source connected to terminal number port number 2. So, termination can be a load termination can be the resistance connected between the terminals of the port or it can be the source resistance. Now, why do we call it a termination. Suppose R1 is the internal resistance of a source Vg 1 at port 1 then actually port 1 is terminated in a source Vg 1 and the resistance R1. What is the resistance that it is terminated in total resistance; obviously, R1.

And this is the reason why we call R1 and R2 as terminating resistances and we show them separately to indicate the fact that, R1 and R2 could as well b the internal resistances of voltage sources connected at these 2 ports is it point clear?

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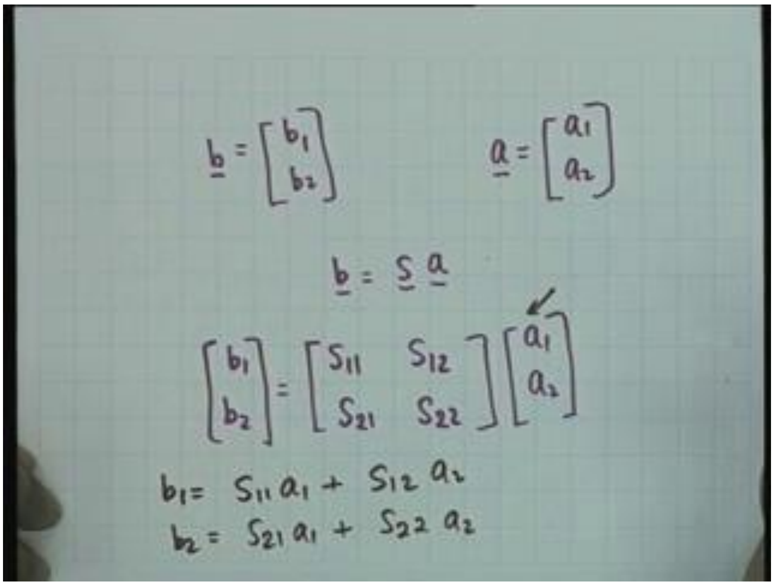

Now; obviously, we have gone back to a 2 port in which there are if a is a voltage current description 2 voltages, 2 currents. Here there are 2 incident waves and 2 reflected waves and this Scattering Parameter description is in terms of the reflected variables b1 b2 the matrix b in terms of the incident variables a1, a2. And in terms of a matrix we write b as equal to S a. S is now, a matrix because we require 4 parameters in S.

We require a 2 by 2 matrix to relate a 2 by 1 matrix to another 2 by 1 matrix that is: if I write specifically the reflected parameters are written as a 2 by 2 matrix multiplied by the incident parameters a1, a2. We call this S11 in the usual manner, we call this as S12, this is S21 and S22 all right. Since, we have encountered this for the first time let us, expand this and look at the interpretations you see b1 is equal to S11 a1 plus S12 a2 and b2 is equal to S21 a1 plus S22 a2 all right.

This is exactly the same type, the same type of description as z parameters or y parameters or a b c d parameters or h parameters. Here instead of taking voltages and currents we have taken combinations of them to define an incident wave and a reflected wave. And what we are relating is reflected wave is being related to the incident wave variables they are not parameters, they are variables.

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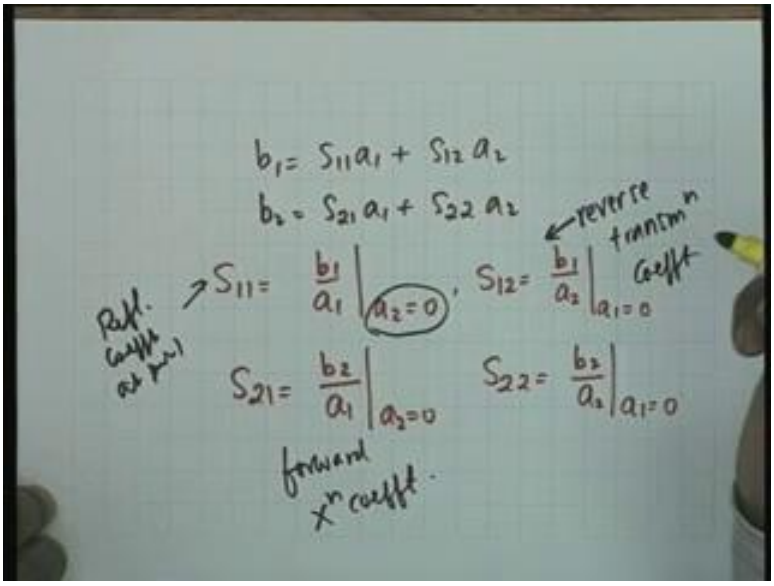

If we look at this, you let me write this down again b1 equal to S11 a1 plus S12 a2 b2 is equal to S21 a1 plus S22 a2. And you notice that S11 from this is simply b1 by a1 under the condition a2 equal to 0. We have to find out now, what a 2 equal to 0 means we will have to find out we will do that you notice, S12 is equal to b1 divided by a2 under the condition a1 equal to 0. S21 is b2 divided a1 under the condition a2 equal to 0 and similarly, S22 is b2 divided by a2 under the condition a1 equal to 0 of this 4 parameters

I would like you look at first the 1 1 and 2 2 parameters that is, where the 2 subscripts are equal Si parameter you see S11 relates the reflected wave at port 1 to the incident wave at port 1 in other words S11 is something like a reflection coefficient. This is how we define the S parameter the Scattering Parameter of a1 port. Now, this is a 2 port and therefore, there must be some condition on port number 2, if you wish to measure the reflection coefficient at port number 1 it depends on what conditions are there at port number 2 and a2 equal to 0 is the condition at port number 2.

We shall see, what this means in a minute excuse me therefore, I can call S11 as the reflection coefficient at port 1. And in a similar manner S22 is the reflection coefficient at port 2 with port 1 terminated in such a manner that a1 equals to 0 all right. Now, to interpret S12 and S21 we shall have to look at the figure again S12 all right. Let us first look at S21; S21 is b2 by a1 under the condition a 2 equal to 0. Let’s look at here b2 by a1 under the condition a2 equal to 0 all right.

Now, if a2 equal to 0 you see the only wave going out is the reflected at b2 and it agrees with the direction of a1 ar. So, it is as if due to the incident wave a1 how much wave goes out, how much is transmitted out of port 2 and therefore, b2 by a1 under the condition a2 equal to 0 is actually called the forward transmission coefficient. Forward transmission coefficient S21 is therefore, the forward transmission coefficient and in a similar manner S12 is what do you call S12? It is the…

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Reverse transmission coefficient. These are the names for the 4 parameters. Now, we shall try to see, what the interpretations are.

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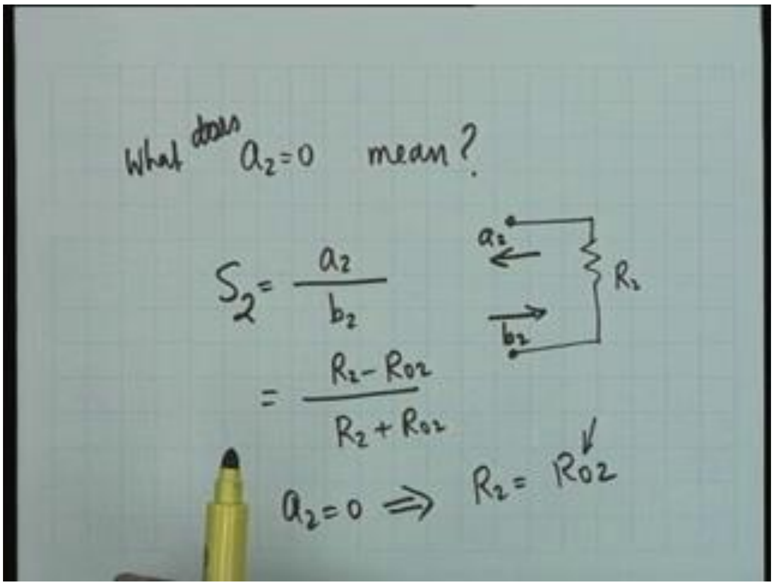

First let us consider let us find out what a 2 equal to 0 means. Let’s look at this terminating resistance R2 terminating resistance R2. Now, we had an incident wave a2 and a reflected wave b2 for port number 2; if you look at R2 if you forget about port 2. As far as R2 is concerned b2 is it is incident wave, is not that right? And a2 is it is reflected wave. This is a1 port R2 is a1 port in which b2 is the incident wave and a2 is the reflected wave is the point clear? So, we can define the Scattering Parameter of R2 S2 as equal to yes.

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a2 by b2 and in terms of the reference resistance this is yes the Scattering Parameter input impedance minus reference So, R2 minus R02; R02 is the reference resistance at port number 2 divided by R2 plus R02. Now you notice, that a2 equal to 0 what does this imply? This implies that R2 equal to R02 is this point clear? If R2 equal to R02 which is the other way round that is, if the reference resistance at port number 2 is taken as the same as the terminating resistance then a2 becomes equal to 0 all right.

So, a2 equal to 0 simply means; that R2 the terminating resistance is equal to the reference resistance or it is the other way round that is: the reference resistance is chosen as the terminating resistance.

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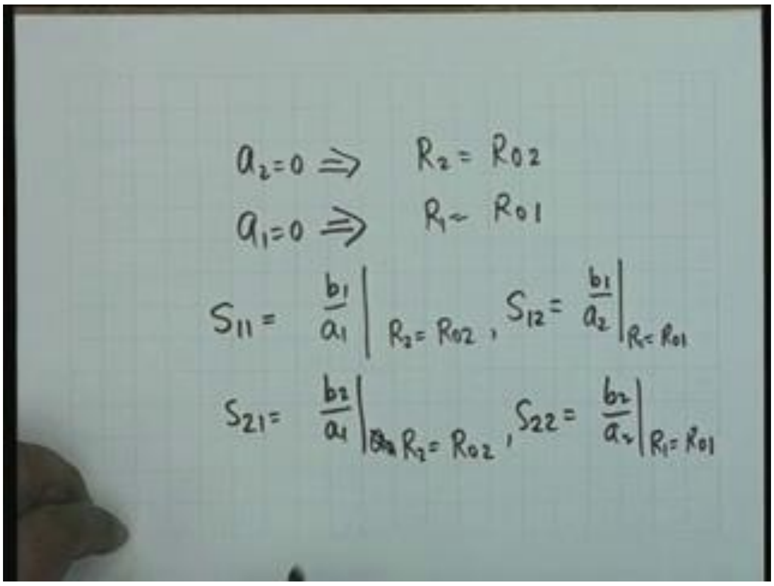

Therefore what I have is a2 equal to 0 implies that R2 equal to R02. And similarly, a1 equal to 0 shall imply that R1 equal to R01. And therefore, it makes sense for a 2 port to use the references resistances as the same as the terminating resistances. And under that condition our S11 shall be redefined as b1 by a1 under the condition that R2 is equal to R02 S12 will come in a minute V1 by a2 under the condition.

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R1 equal to R01 S21 is equal to b2 by a1 under the condition a2 R2 equal to R02 and S22, shall be equal to b2 by a2 under the condition R1 equal to R01 all right. And this is the logic of the rationale behind choosing the reference resistances to be equal to the terminating resistances then the definition becomes absolutely clear.

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This is unmatched termination yes, yes.

It is not necessarily matched matching will occur when R1 is equal to the input impedance that is V1 by a I1 all right others not otherwise.

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Shall I explain reference resistance once again; reference resistance is an arbitrary positive constant having the dimension of impedance all right. Now, reference resistance is left arbitrary as long as possible then at a certain point of time when you are going to calculate this Scattering Parameter you have to fix the reference resistance we have to choose them. And this choice for a 2 port is to be equal to the terminating resistances the terminating resistances, are not under your control.

It is fixed some 1 gives you a source, whose power is to be transferred to a given load some1 gives you a source for example, a microphone. The power of the microphone is to be transmitted to a loud speaker then in between you design a network all right. So, the terminating resistances here are the microphone resistances and the loud speaker resistances. And in order to calculate the scattering parameters you fix R01 equal to R1 and R02 equal to R2.

So, long as you have the flexibility you keep them flexible you keep them arbitrary, but, when actually required to calculate and make use of the network, then you choose the scattering you choose the reference resistances.

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No, we are not matching right away matching should occur only, if we should transfer maximum power.

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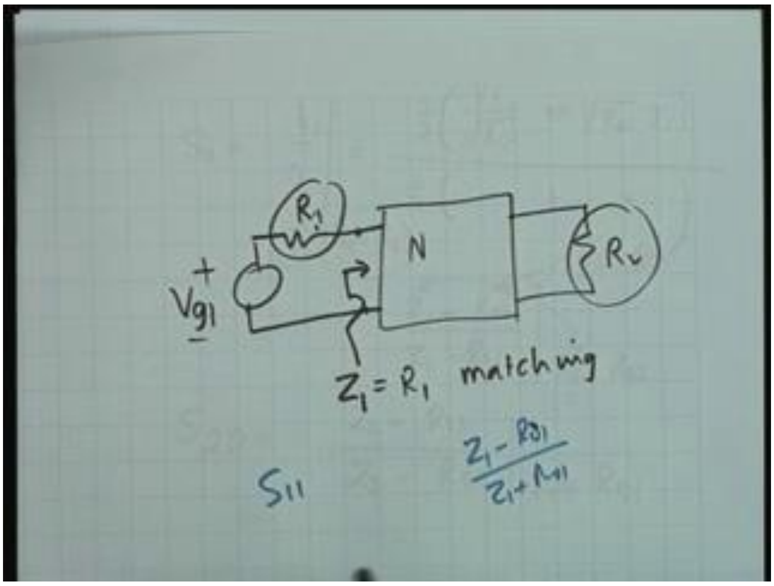

Matching shall occur suppose R2 is fixed suppose R2 is this is a loud speaker. We want to transfer maximum power from the source Vg 1 from the source Vg 1 2R2. Then you must design N in such a manner that Z1 which is equal to V1 by I1 is equal to R1 this is the condition for matching, that means; maximum available power shall be transferred we have not brought this into consideration here. All we have done so, far is in the definition; we have fixed the terminating we have fixed the reference impedances to be equal to the terminating resistances.

We will see what happens, when matching occurs at a later stage, but before that.

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Well it is not necessarily matched because R1 and R2 may not be under our control. And all that is under our control is N you may able to design N. In fact, that should be your aim your aim should be design N in such manner that the input impedance is as close to R1 as possible all right then N shall be called a matching network, any other question?

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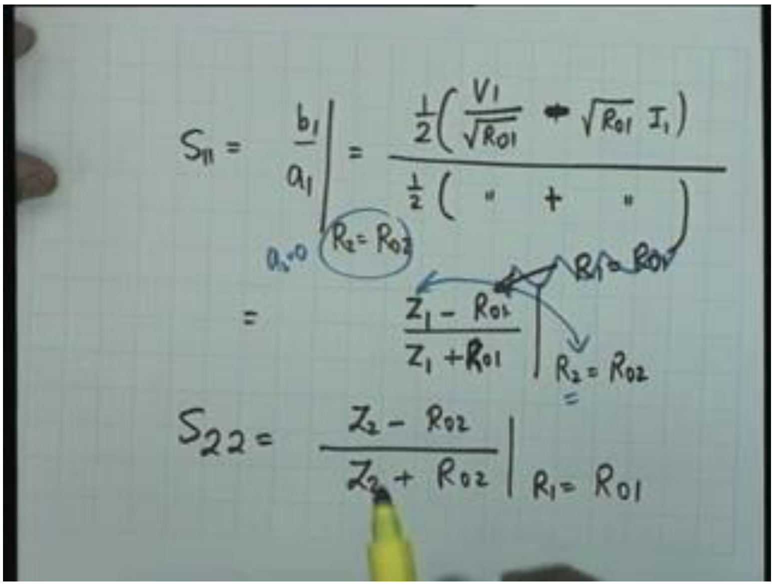

Let us look at the interpretation of this parameters S11 and S22 to start with S11 is equal to b1 by a1 under the condition a2 equal to 0 that is R2 is equal to R02. And if you take the if you recall the definition this is half V1 divided by square root of R01 plus square root of R01 I1 divided by half this quantity minus this is minus and this is plus. And this is minus and this is plus and you notice that this is simply equal to Z1 minus R01 divided by Z1 plus R01 under the condition that R2 is equal to R02.

It is indeed a reflection coefficient that is: the input impedance minus reference resistance which you have chosen to be equal to R1 all right divided by Z1 plus R01. And you notice what happens when the terminals when the resistances are matched. That is if the input impedance Z1 is equal to R01 which is equal to R1 then; obviously, maximum possible power shall be transferred to the network agreed.

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We had, we have put R1 equal to R01, if I want to match not otherwise. Similarly, S22 is equal to S22 is equal to Z2 minus R02 without going into this derivation divided by Z2 plus R02 under the condition that R1 equals to R01. And once again you can see that; S22 shall be equal to 0 that is, there have been no reflection then the absolute there will be all absorption of power in the resistance R2 if Z2 is equal to R02 that is, if the port number 2 is matched.

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This is under the condition R2 equal to R02 that is a 2 equal to 0.

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How does it affect? This has to be measured you see you have to measure the input impedance Z1 with the output terminals terminated in R2 Z1 is affected by R2. Is that clear? The input impedance depends on R 2 therefore, you must specify the terminating condition.

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Well this is the general expression.

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Yeah, but, then you have to fixed what is your R01 because R01.

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It does not mean anything R01 has to be related now, to the terminating impedance you see as soon as you say that S11 is the reflection coefficient at port number 1. Now, reflection coefficient you define as Z1 minus R01 divided by Z1 plus R01 as long as R01 is arbitrary Z1 equal to R01 it does not mean anything. It does not indicate how much power is actually being transmitted from Vg 1 R1 to the network. Therefore, to get a quantitative estimate of that; you must relate R1 to R01.

If R1 is equal to R01 then maximum available power from the source Vg R1 is being transmitted to N all right.

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Well in the a practical situation when you are calculating S11, S11 you require a choice of R01 you require a choice of R02 as I said R02 is chosen to be equal to R2, but, R01 must also be chosen otherwise how do you get a quantity? How do you calculate S11. So, R01 under that condition is taken to be equal to R1.

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And this would be more clear.

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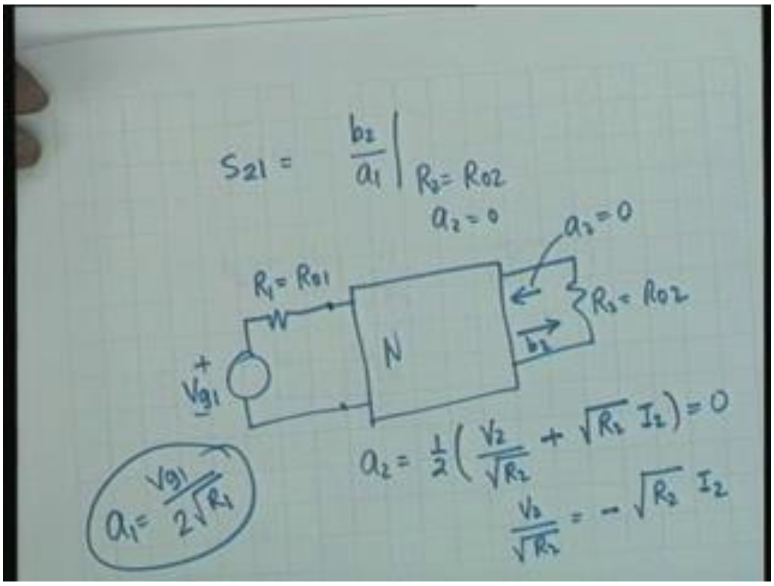

If we calculate S21, b2 upon a1 under the condition R2 equal to R02. Now, b2 upon a1 which means a2 equals to 0. And our situations, for calculation of this please do follow this carefully. The situation is that we have R1 which is now chosen equal to R01 a Vg 1 and network N and a resistance R2 which is equal to R02. And do you want a2 we want only b2 we want a2 to be equal to 0. If you look at a2, if you look at a2 the definition it is half V2 by square root R2 plus square root R2 times I2.

If this is equal to 0 what does this mean? It means that: V2 by square root R2 should be equal to minus square root R2 multiplied by I2.

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That is right Z2 equal to minus R2 yes not quite, does this mean Z2 equal to minus R2.

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This is a constraint on V2 and I2 V2 by I2 is equal to minus R2 because of the termination it is nothing to do with the impedance.

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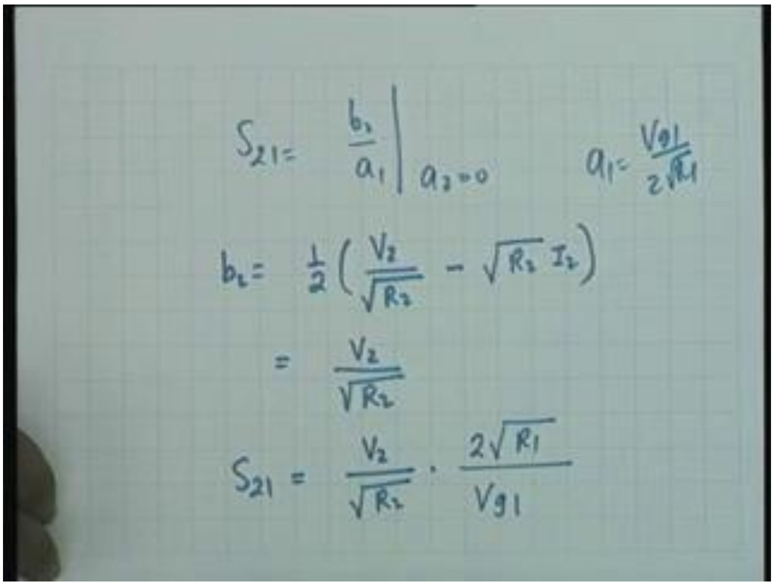

Now, I want you to follow this carefully S21 is b2 by a1 under the condition a2 equal to 0 and what is b2. If you go back to the definition half V2 by square root R2 minus square root R2 I2, but, we have already proved that this is equal to minus this therefore, this is simply equal to V2 by square root R2 agreed. Now I go back to the to the 1 port result you recall you recall that, if consider this now as 1 port. If the internal resistance is chosen as the reference resistance then what is a1; a1 is Vg 1 divided by twice square root R1 all right.

I shall utilize this result here and therefore, I get S21 as equal to V2 divided by square root R2 multiplied by twice square root R1 divided by Vg 1 is it. I have substituted for a 1, a1 is Vg 1 divided by 2 square root R1 I have substituted this.

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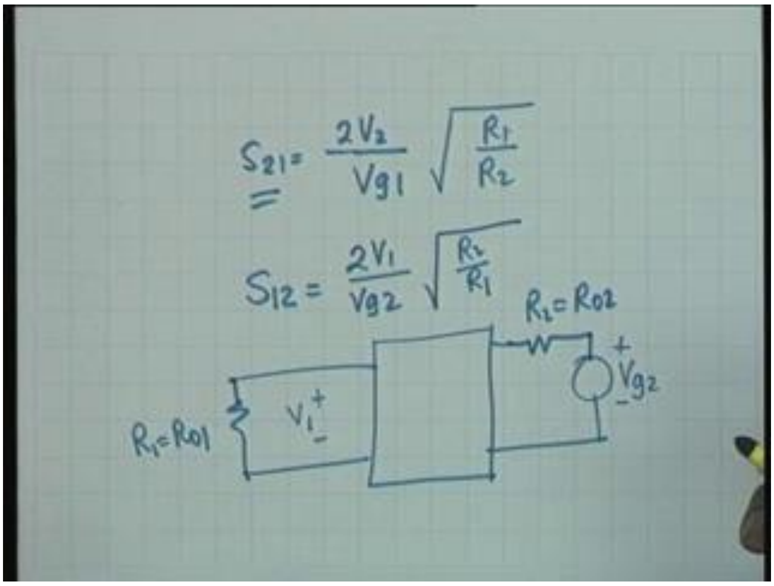

And the end result that I get is S21 equal to V2 by Vg 1 multiplied by twice multiplied by square root R1 divided by R2. And this gives you a physical interpretation of S2 voltage transfer function multiplied by a constant. And if R1 and R2 are equal, if it is a network with equal termination then it is simply twice a voltage transfer functions. So, to calculate S21 all that you have to calculate is a voltage transfer function. And in the problem session today we shall, take examples of such calculations.

In a similar manner for calculating S12 what shall you do you will terminate port number 2 in a source in series with a resistance R2, R2 and measure the voltage across port number 1 V1 with a termination of R1 which is equal to R01. And under this condition S12 shall be twice V1 divided by Vg 2 multiplied by square root of yes. R2 by R1 let us close, this class with a simple example. Is there any question?

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Why did we put R2 because this shows our choice of reference impedance.

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No S21 had, the condition a2 equal to 0 S12 request 1 equal to 0 R naught 1.

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We can have any other value or R02, but, we calculate with R02 equal to R2 with the specific value and this is how it is done. That is you see once, a network is chosen once a network is there, if you want to calculate the 4 parameters; obviously, you shall have to fix your R01 and R02. It is not that S12 only shall be useful you want to characterized the network completely, and therefore it is the what is done is we first fixed R01 fixed R02 equal to R1 and R2 and then calculate the parameters.

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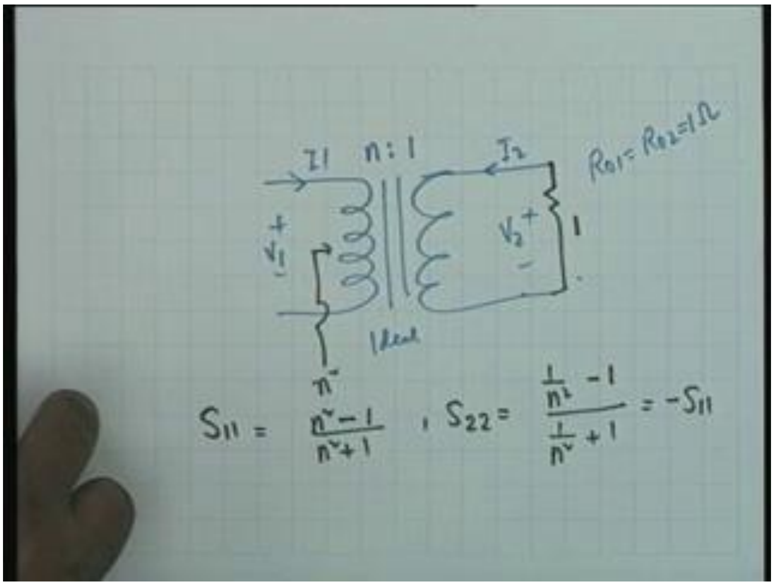

Now, I want to conclude this class with an example, which I can do by inspection an ideal transformer.

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It is not maximum power. R2 equal to R02 is not the condition for maximum power, it only makes a2 equal to 0 under which condition you have to measure S11 and S21 all right. This is an ideal transformer V1 I1 V2 I2. And it is given that R01 equal to R02 equal to 1 Ohm for simplicity all right. And you are required to calculate the four parameters. To calculate S11 I am going very slow to calculate S11 you have to terminate this in 1 Ohm.

If you terminate this in 1 Ohm then what is the impedance here? N squared. Therefore S11 shall be equal to n squared minus the reference impedance 1 divided by n squared plus 1. Can you tell me what would be S22? If you call if you terminate this.

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This will be 1 by n squared.

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Don’t you see that this is exactly minus S11 is not right? This is minus S11.

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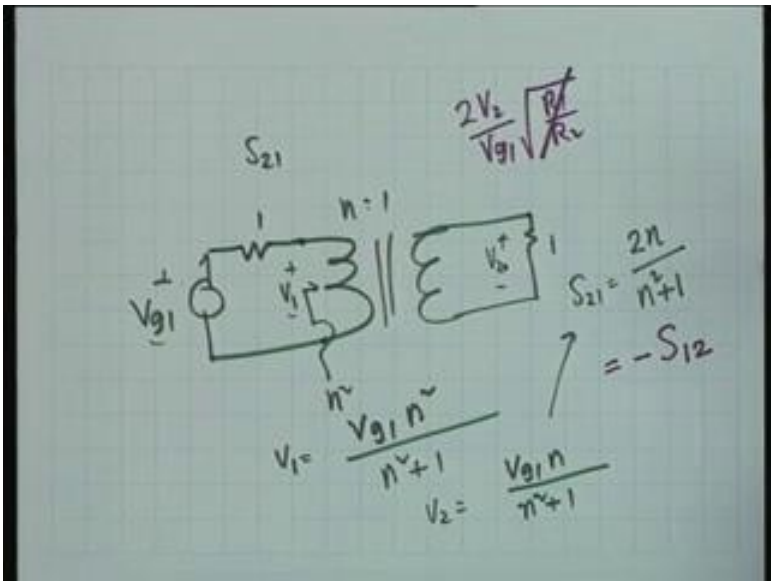

Now, calculation of S21 requires. Now, you have to terminate this have a Vg 1 n is to 1. And then terminate this in 1 Ohm and this is V2 all right. So, what you have to calculate is V2 by Vg 1 actually twice V2 by Vg 1. R 1 and R 2 are equal. So, the ratio is equal to 1. Now, to calculate this; what is this impedance here? This is n squared therefore, V1 this a style of calculation by you can do it by inspection V1 is therefore, the potential division of Vg 1 into 1 Ohm and n squared Ohms therefore, V1 is Vg 1 n squared divided by n squared plus 1 agreed yes or no.

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Not n times divided by n. So, it is Vg 1 n divided by n square plus 1 therefore, S21 would be simply equal to twice n, you understand why this factor 2 come? because, it is 2 V2 by Vg 1. So, twice n by n squared plus 1 And I leave it to you to show that; this is equal to minus S12.

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S21 by definition is twice V2 by Vg 1 squared root R1 by R 2. R 1 and R 2 are equal. So, we have calculated 2 V2 by Vg 1. One can show that S21Ii am not too sure about this whether the minus sign is there or not, we will clarify this in the next class.

Thank you.