Circuit Theory

Prof. S.C. Dutta Roy
Department of Electrical Engineering
IIT Delhi

Lecture 32

Problem Session 8: Two - Port Parameters

second lecture and we are going to discuss problems on scattering parameters

the problem that i started in the last class that is that of an ideal transformer

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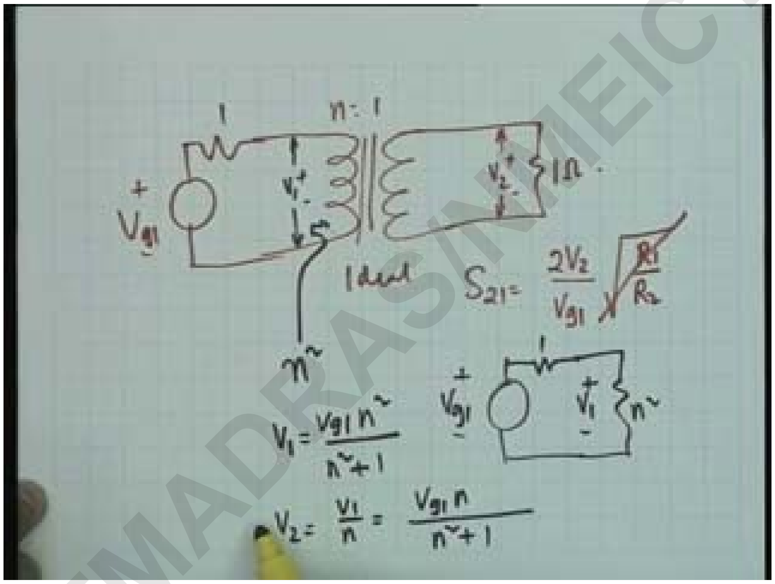

we had left with the question of whether S one two and S two one were equal and opposite of each other or not [Noise]

well to calculate S two one we had terminated in one ohm Vg one terminate in one ohm and this is V two

we have to calculate S two one by definition is twice V two by Vg one square root of was it R one by R two or <a_side> ((R one by R two)) <a_side> R one by R two and R one and R two are equal here one ohm and one ohm and therefore this factor goes to one

all we have to calculate is V two and for that we first calculate V one [Noise] that is this voltage and since the impedance looking here is n squared obviously V one the equivalence circuit is this in case you have not followed this one what you see here is a resistance n squared and this V one

therefore V one is equal to Vg one times n squared divided by n squared plus one okay and V two is because the transformer is ideal it is V one divided by n so this is Vg one multiplied by n divided by n squared plus one

so that S two one which is two V two by Vg one shall be given by [Noise]

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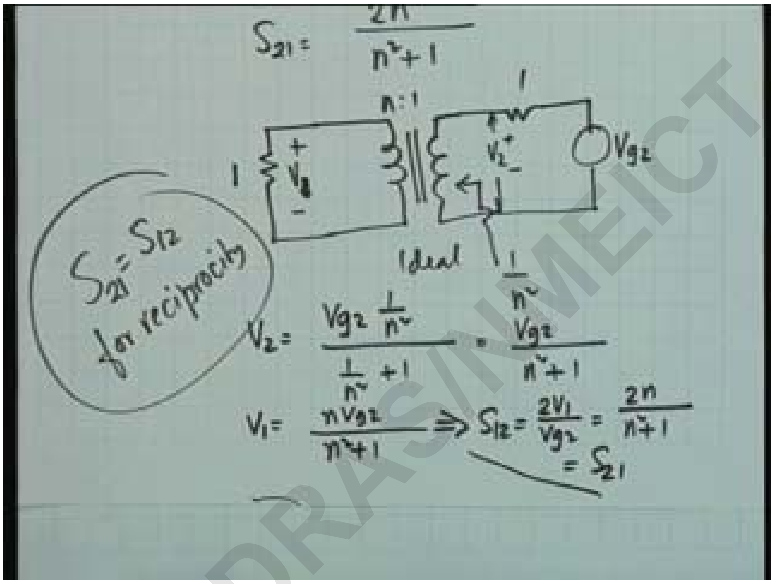

S two one shall be given by twice n divided by n squared plus one okay

now to calculate S one two what we have to do is terminate the primary in a one ohm secondary the source yeah this is n is to one ideal this should be the same they turn out to be the same yeah this is Vg two and this is V two no i ((beg your pardon)) (00:02:51) this is V one okay

so all that you have to calculate is V two here and you notice that the impedance here is one by n squared therefore V two is equal to Vg two one by n squared divided by one by n squared plus one which is Vg two divided by n square plus one okay and V one is n times this so n times Vg two divided by n square plus one which means that S one two {whe} (00:03:30) which is twice V one divided by Vg two is equal to twice n divided by n square plus one same as S two one [Noise] now this should not surprise you because the transformer the ideal transformer is a reciprocal device and all we have done in calculating S one two and S two one is to interchange the source and the response the cause and effect points and the ratios should be the same

this is indeed so from which we take the lesson that S two one must be equal to S one two this is the condition for reciprocity [Noise] okay

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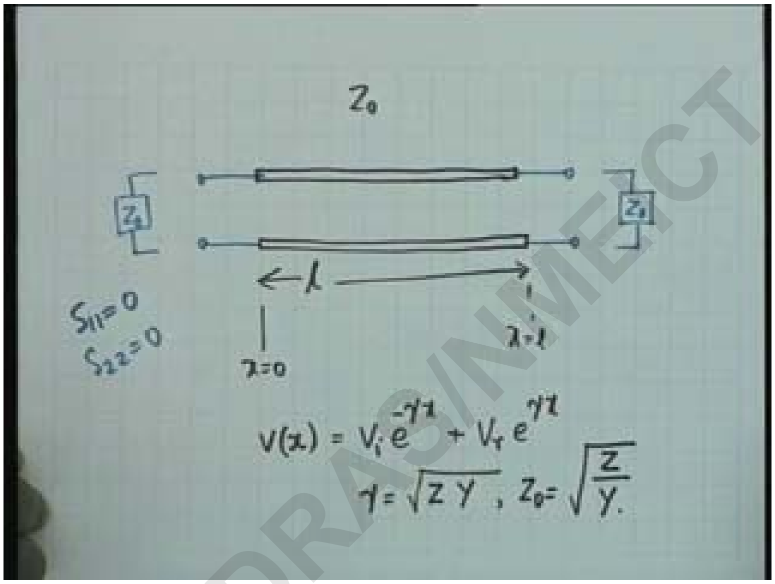

let's take another example in which the the device is a distributed device we have a a transmission line we have a transmission line of characteristic impedance Z zero and length l

then in general you know that if excited at x equal to zero this is [Noise] x equal to l if excited at the x equal to zero then the voltage at any point on the line shall be of the form V of x equal to Vi e to the minus gamma x plus Vr e to the gamma x where gamma is the propagation constant of the line it is the square root of ZY where Z is the impedance per unit length and Y is the admittance per unit length

the characteristics impedance is square root of Z by Y okay

now let's find the scattering parameters of these two port considered as a black box okay these are the terminals and [Noise] let us say did the terminating impedances both are Z zero the characteristic impedance then the reference impedances at both ports are Z zero all right

so [Noise] if we want to calculate S one one we terminate this in Z zero all right and find the impedance here what is the impedance here Z zero right and therefore the reflection coefficient driven from a source of impedance Z zero reflection coefficient would be zero there is no reflected wave

so S one one shall be equal to zero and there is no reason why S two two shall not also be equal to zero all right for the same under the same arguments is this point clear that a ah transmission line terminated in this characteristics impedance produces an impedance equal to Z zero at its input [Noise]

so the only thing that is to be calculated is S two one or S one two the two have to be equal because the transmission line itself is a reciprocal device and therefore we need to calculate only S two one or S one two

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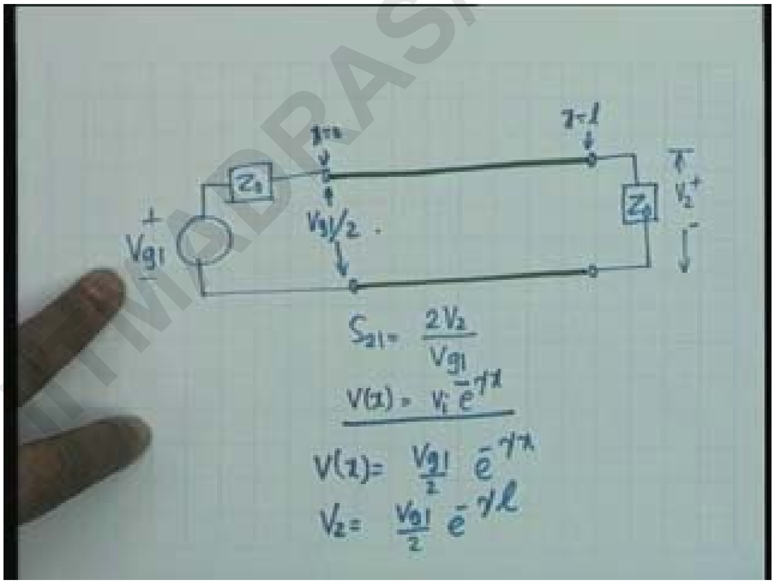

let's calculate S two one the forward transmission coefficient for that we terminate the line in a source Vg one Z zero then this is the line [Noise]

<a-side> ((sir what if the termination goes on by Z zero)) <a_side> then it will be different you would have a non zero S one one non zero S two

<a_side> ((sir is the source could also be different)) <a_side> yes it could be different we have the taken the situation in which the terminations are equal and both are equal to Z zero if they are not then there will be reflection coefficient at the input there will be reflection coefficient at the output okay

let’s say the green coloured one is the line and this is terminated in Z zero

what we have to calculate is V two okay what we have to calculate is V two and related to Vg one S two one if you recall would be twice V two by Vg one all right

now [Noise] also we also know that V of x in general is equal to Vi e to the power minus gamma x plus Vr e to the gamma x but Vr would be equal to zero therefore this will be the general relationship at any point x this is x equal to zero and this is x equal to l at any point x this would be the voltage there is no reflected way all right this has been made sure by terminating is Z zero both sides okay

so all that we have to find out now okay is uh this voltage what is this voltage <a_side> (( )) (00:09:01) <a_side> this is Vi how is it related to Vg one <a_side> ((sir Vg by two)) <a_side> that is correct because the input impedance is Z zero and therefore this is Vg one by two and this gives out Vi

what is Vi <a_side> ((Vg one by two)) <a_side> Vg one by two agreed Vi at x equal to zero V of zero should be equal to Vi is equal to Vg one by two and therefore the V two the voltage at x equal to l is simply Vg one by two e to the minus gamma l okay which means that S two one will be equal to will be equal to simply e to the minus gamma l

there is a mistake in the text book this is the correct answer e to the minus gamma l the text book says twice of this value which is a mistake okay

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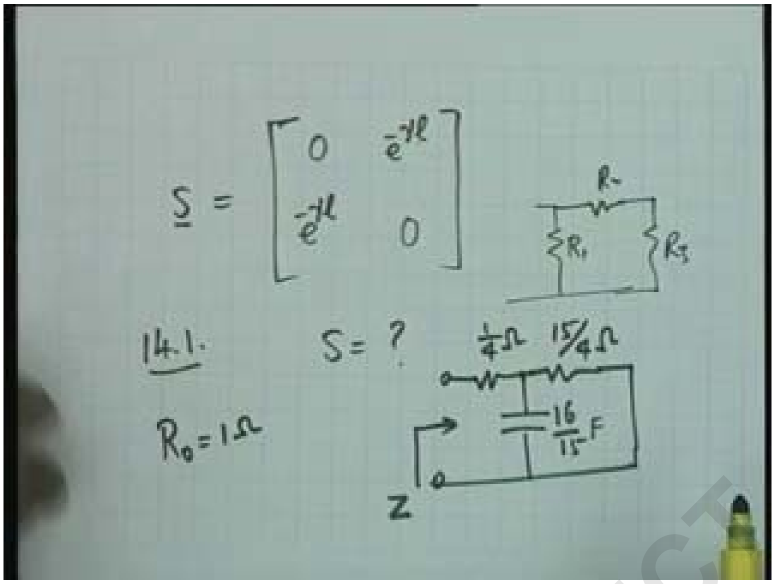

therefore the [Noise] scattering parameters of the transmission line terminated in characteristic impedance R zero zero diagonal elements e to the minus gamma l e to the minus gamma l

the ah equality of this two comes from <a_side> ((reciprocity)) <a_side> reciprocity and [Noise] anything else <a_side> ((just reciprocity)) <a_side> just reciprocity no symmetry does not guarantee just reciprocity okay

so this is the scattering matrix to denote that it's a matrix i will put a bar ah at the bottom i will underline the letter [Noise]

now let's take some examples from the example sheet [Noise] we take fourteen point one fourteen point one says determine the reflection coefficient S for the one port network shown in the figure

ah the reference impedance is not given okay reference impedance has to be given otherwise you cannot find the reflection coefficient so if it is not given then you assume it to be one ohm all right or you can assume anything one ohm is a is a very nice figure right

we are assuming one ohm ah because the element values that are given well two henry half farad one henry and so on suppose there are given in milli's then i could have assume ah reference impedance is a in K kilos and so on okay all right we shall workout the third problem third network only it says find the reflection coefficient s <a-side> ((excuse me sir sir do we take it as one ohm for second part or b part)) <a_side> for all of them for none of them it is given oh for the b part okay [Laughter] for the b part you assume an R naught because the symbols what is given in terms of symbols

so you assume some value and ((work)) (00:12:27) okay and express in terms of R one <a_side> ((sir we could uh might as well take this values as normalized impedances)) <a_side> which which values

<a_side> ((all of these impedances R one R two)) <a_side> as one ohm each <a_side> ((normalized can be divided by R naught)) <a_side> okay we could do that we could do that then one ohm would be good enough okay

if the reference impedance is not given it is your choice if it is a two port and the terminations are given then the choice is made all right the choice is that the reference impedances are equal to the terminating impedances

now as i said you have to find for the third network the network is one quarter ohm fifteen by four ohms and sixteen by fifteen sixteen by fifteen farad

so all that you have to do is to assume a reference impedance we assume this as one ohm and find the input impedance scattering parameter S would be equal to Z minus one divided Z plus one let's look at Z

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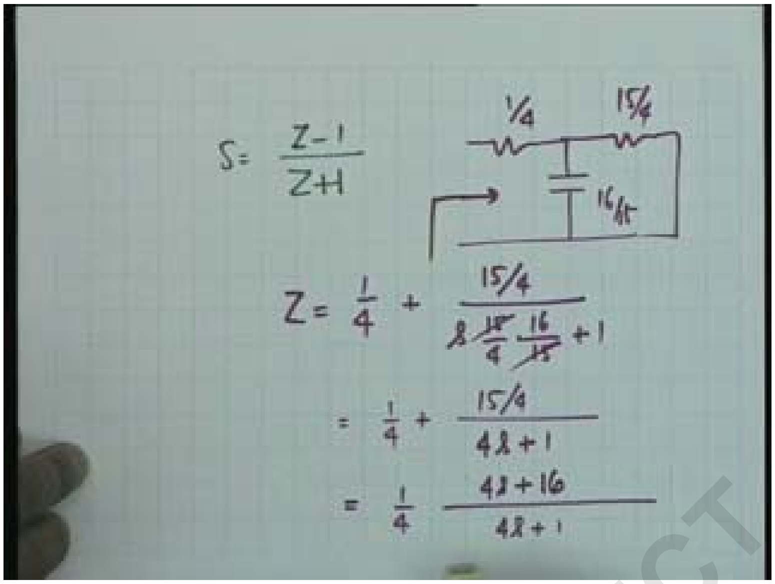

we have one quarter sixteen by fifteen and fifteen by four Z obviously is equal to one quarter plus the parallel combination of resistance and capacitance and i have already told you R divided by sc R plus one

so fifteen by four divided by s fifteen by four multiplied by sixteen by fifteen plus one which is equal to four s plus one

so it is one quarter plus fifteen by four four s plus one which is equal to i can take one quarter common right four s plus one and here four s plus sixteen that is correct

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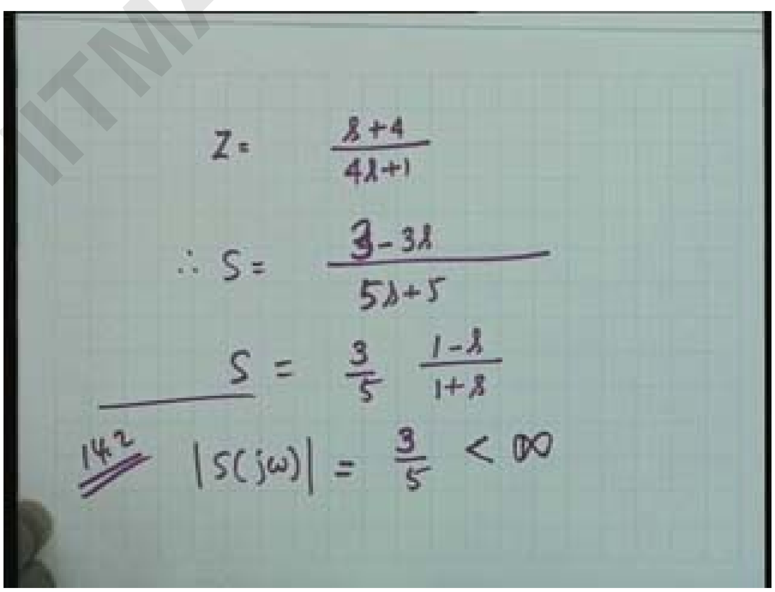

therefore i get Z as equal to s plus four divided by four s plus one agreed therefore capital S is Z minus one by Z plus one

so four minus three s divided by <a_side> ((three minus three s)) <a_side> okay three minus three s divided by <a_side> ((five s plus five)) <a_side> so five s plus five which is equal to three by five one minus s divided by one plus s all right

similarly you can find out the other two networks

we shall skip fourteen point two we go to fourteen point three it says for this network for the network of problem fourteen point one for the network it should have been networks there are three of them determine capital S of j omega obviously for this third network fourteen point two we are considering the third network this is the value of capital S

so magnitude S j omega is it obvious that this is equal to three fifth <a_side> ((yes sir)) <a_side> one minus j omega one plus j omega magnitude would be equal to one and you see that this is indeed indeed bounded by upper bound unity three fifth is less than four fifth correct it is bounded it is less than infinity so it is bounded show that this scattering elements s for the networks in problem fourteen one are bounded real functions okay you have to show that capital S is a bounded real function

this is not enough because this is on the j omega axis we must understand this okay a bounded real function is a function defined in the total S plane not just on the j omega axis

so in order to show that this function is bounded real you have to write S of S as equal to three fifth

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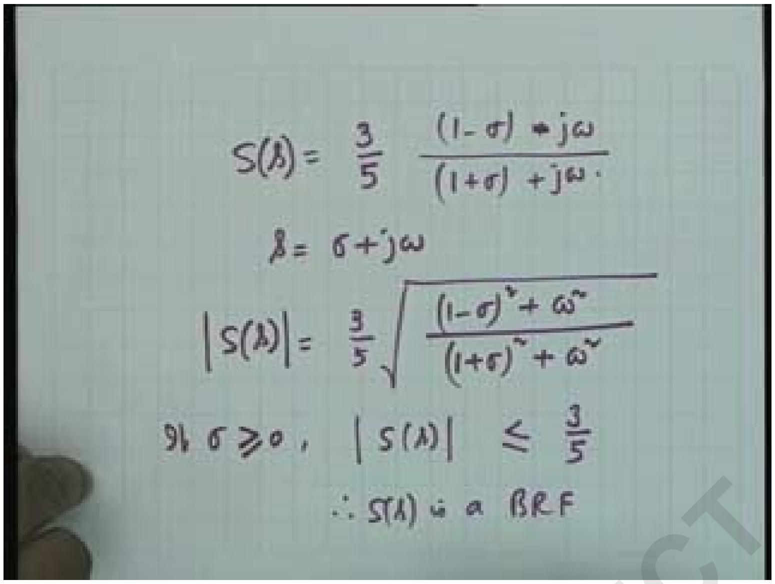

put small s equal to sigma plus j omega then you have one minus sigma [Noise] plus no minus j omega all right divided by one plus sigma plus j omega

so magnitude s of s is equal to three fifth square root of one minus sigma whole squared plus omega squared divided by one plus sigma whole [Noise] squared plus omega squared and if the real part [Noise] bounded real function definition contains the real part of s greater than equal to zero

so if sigma is greater than or equal to zero then obviously the magnitude of s of s is upper bounded by three fifth and therefore s of s is a bounded real function is that quite okay no that is one step that we have to skipped

you must bring in that step what have you skipped <a_side> (( )) (00:18:26) <a_side> that capital S is real for S S real you must make the statement that by looking at this we see that if small s is real then capital S is real and therefore capital S is indeed a bounded a real function

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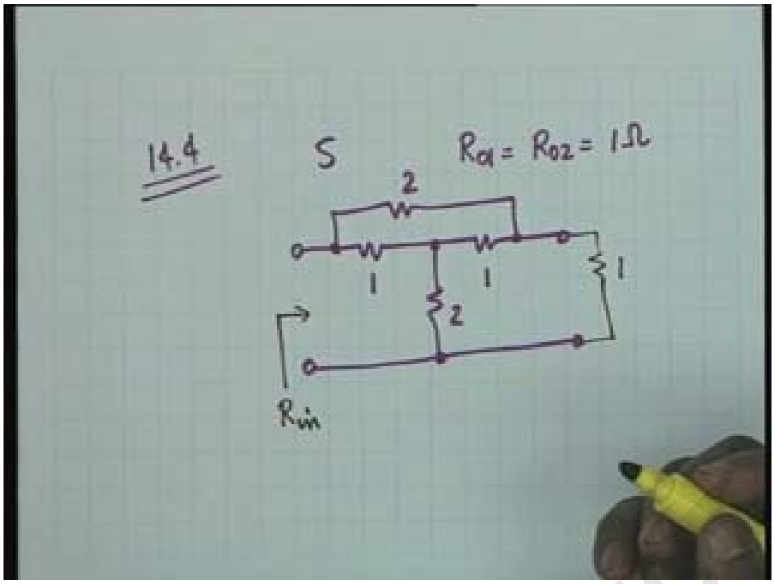

consider fourteen point four and we shall calculate for a couple of them a couple of this let's take the second network which looks simple

what you have to do is for each of the network shown find this scattering matrix scattering matrix for a two port given that the references impedances at both ports are one ohm R zero one and R zero two are both one ohm and the network is this one one i am not showing the dimensions then you have two and two

this is the network it is a three terminal network and now to calculate S one one S two two S one two and S two one you will have to terminate for example for S one one you have to terminate this in one ohm and find the input impedance here let say R in is resistive

obviously S one one S and S two two shall be equal is that correct because of the symmetry of the network because of the symmetry with respect to the ports S one one and S two two shall be equal you don't have to calculate again but calculation of S calculation or R input is not cannot be done by inspection because there is a bridging here we have to write either kcl or kbl or use one of those transformations well as i said you don't have to commit to memory any formula you can do it ab initio

you will look at this as the parallel connection of this t and the other one other network is a two ohm and a short circuit

so let's let's convert this t to a pi and i shall do it ab initio without remembering any formula

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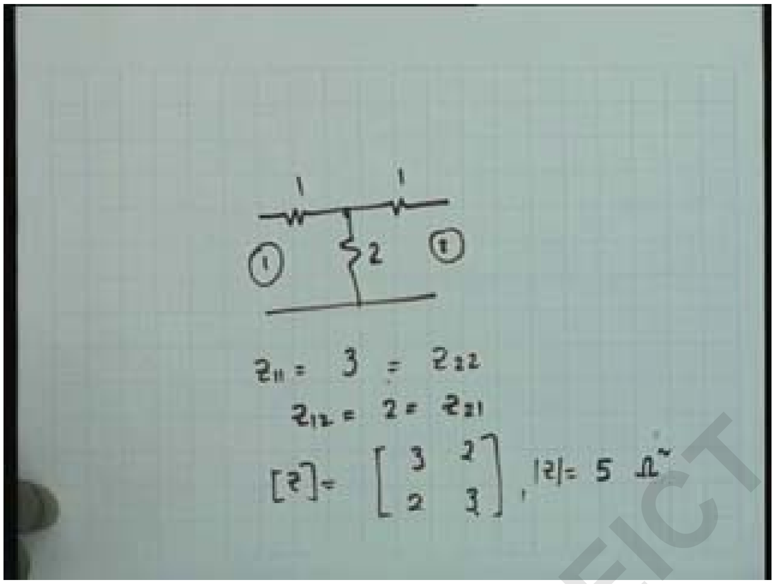

let’s take this network one one and two okay

this is port one port two Z one one is equal to three ohms is equal to Z two two because of symmetry Z one two is equal to two ohms this is equal to Z two one

therefore the Z matrix is simply three three two two and the determinant of Z is equal to nine minus four that is five

what is the dimension [Noise] dimensionless <a_side> (( )) (00:21:45) <a_side> ohm squared isn’t it right Z two two Z one one minus Z one two Z two one therefore okay

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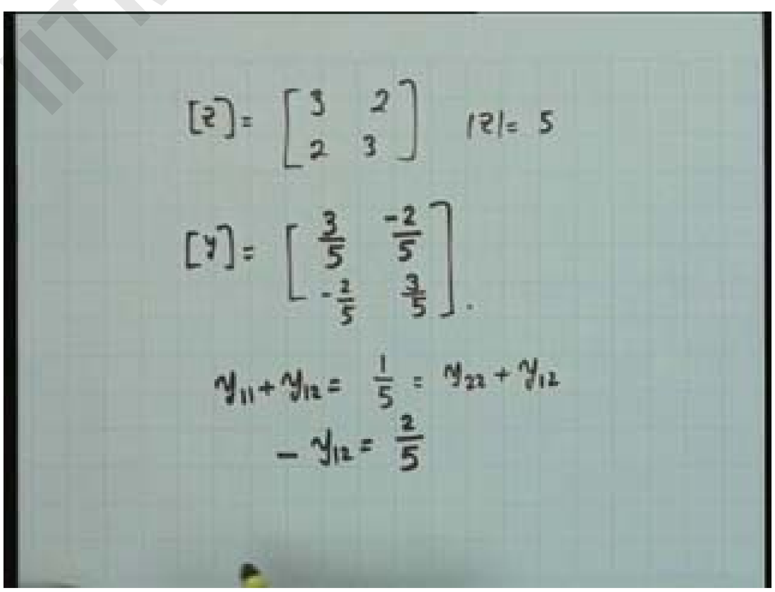

they did while z matrix is three two two three the determinant z is five therefore the y matrix which is the inverse of this would be z two two by determinant z so three fifth then <a_side> ((minus two by five)) <a_side> minus two by five and the rest is known minus two by five three fifth

therefore y one one plus y one two we need this for the equivalent pi network y one one plus y one two one by five this is equal to y two two plus y one two and y one two is equal to ((pardon me)) (00:22:46) <a_side> ((minus two)) <a_side> minus two by five what i need is minus y one two

so it is two by five <a_side> ((sir instead of sir instead of doing this directly we can add the y parameters)) <a_side> directly we can add <a_side> ((the y parameters of the second circuit that will be one by two minus one by two minus one by two)) <a_side> we don’t want to do that you see the things that are simpler than that

we don’t our problem is not to find the y parameter our problems is to find the equivalent network okay so what we will do is we shall draw the equivalent of this

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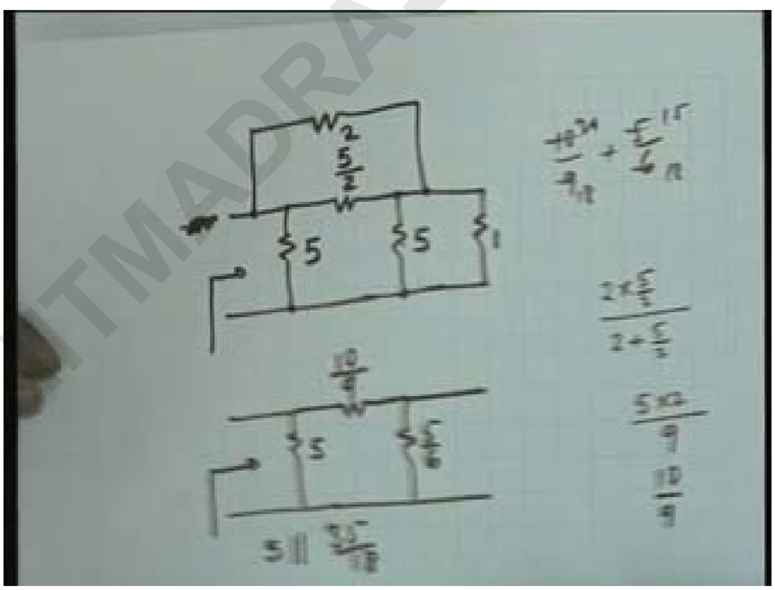

obviously that would be this resistance would be <a_side> ((one by five)) <a_side> no <a_side> ((five ohm)) <a_side> five ohms one fifth is the is the admittance this resistance would be five by two and this resistance would be five okay what did we have for the bridging two ohms and we terminated this in one ohm all right so the equivalent network equivalent network is this now [Noise] there is a reason why i don’t want to add the y parameters or simplify it further simplify it further only means that i shall combine these two and i shall combine these two

let us see what does it become five [Noise] how much is this two times five by two divided by two plus five by two this is equal to five multiplied by two divided by nine equals to ten by nine is that okay two times two point five is five divided by two plus two point five that’s correct ten by nine and this resistance how much is this

<a_side> ((five by six)) <a_side> five by six okay so this impedance this resistance therefore is five parallel you see this combination of these two how much is that ten by nine plus five by six you know how i do things is i make this eighteen eighteen so fifteen twenty thirty-five by eighteen okay ah hum thirty-five by eighteen

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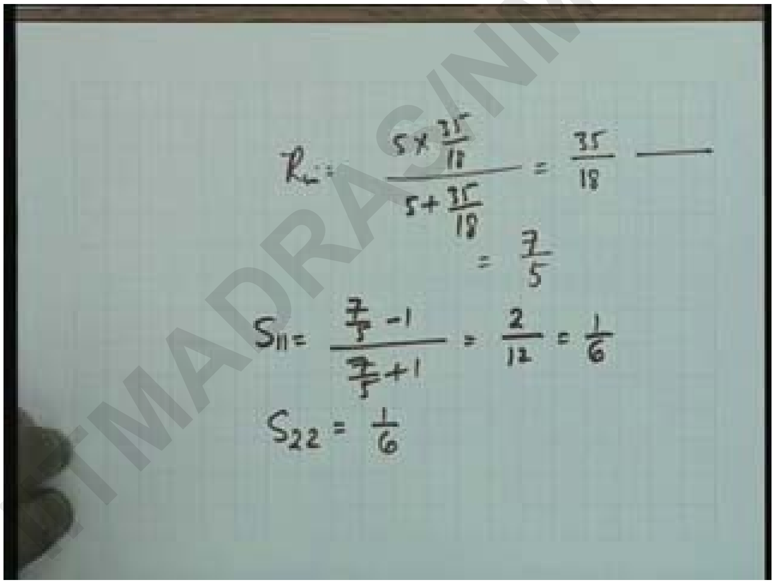

which means R in is equal to five multiplied by thirty-five by eighteen divided by five plus thirty-five by eighteen

there is no reason why i cannot divide by five so thirty-five by pardon me thirty-five by eighteen you guarantee this is seven by five <a_side> ((yes sir)) <a_side> okay fine

so capital S one one would be seven by five minus one seven by five plus one this is equal to two divided by twelve which is one sixth all right what is the dimension of S one one <a_side> ((dimensionless dimensionless)) <a_side> okay good and this should also be equal to S two two agreed S two two is also equal to one sixth because of symmetry okay

now the problem of calculation of S two one calculation of S two one is good enough because the network is reciprocal okay

so for S two one what i have to do is i have to connect a one ohm

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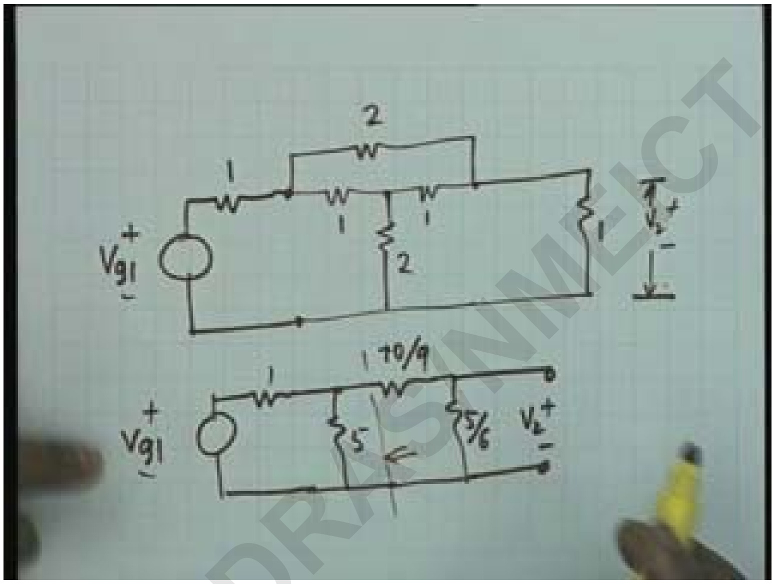

Vg one then this network one one now you will understand what i didn’t do all that manipulation earlier i have a bridge elements of two ohms i have a two ohms here and then i have to terminate this in one ohm and find out V two okay

now this network i have already simplified so let’s take that simplified form Vg one one ohm yes what did i had <a_side> ((five)) <a_side> five then here <a_side> ((ten by nine)) <a_side> ten by nine and here does this change V two no you must see that it doesn’t change so this is V two

how do i solve this do i apply uh no node equations or mesh equations no all i have to do is to find V two by Vg one so i apply (( )) (00:27:41) to the left of this okay

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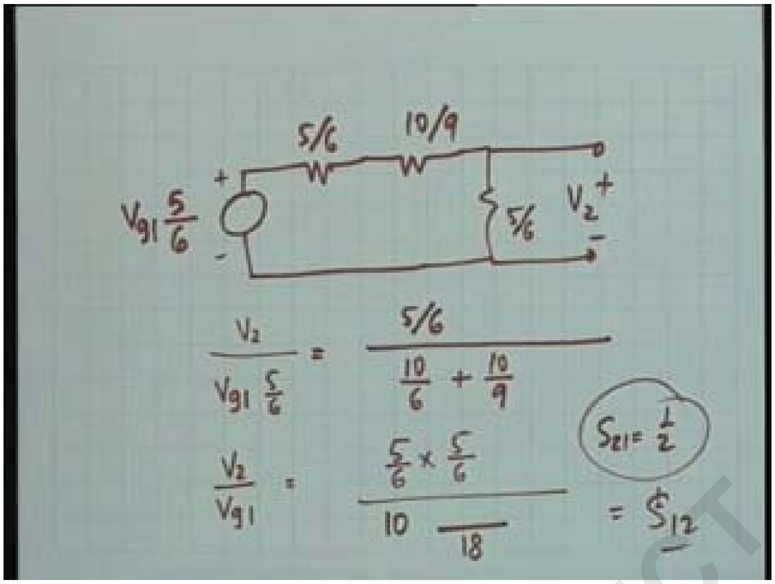

what do i get i get Vg one five by six open circuit voltage five ohms divided by five plus one six and what is the equivalent resistance of five ohms and one ohms yes five by six then i had ten by nine okay and five by six this is V two

therefore V two by Vg one five by six is equal to five by six divided by this plus this plus this which is ten by six plus ten by nine therefore V two by Vg one is equal to five by six into five by six [laughter] five by six plus five by six is ten by six okay

so this would be ten how much is this eighteen <a_side> ((half sir the whole thing is half sir)) <a_side> whole thing is half how did you do this so quickly

<a_side> ((sir this ten by six is thirty by eighteen and that is twenty by eighteen so another fifty by eighteen up steps we have twenty-five by twenty-six sir)) <a_side> [Noise] you have calculated the S two one directly

<a_side> ((yes S two one will be half)) <a_side> S two one is half is it <a_side> ((yes sir)) <a_side> hold it you are pretty smart i can't do things so quickly

yes you are right it agrees with my results S two one is half and this is also equal to S one two okay

you understand that given a problem given a simple problem very simple solution okay if you start writing mesh equations and node equations you are going into a mess all right and it may be difficult to come out of that fortunately the numbers are not too nasty here one two and three [Noise] so but what i am ah emphasizing is given a problem if you can solve this by inspection do that because inspection it is very difficult to make a mistake you have to make a positive effort to make a mistake in solving a problem by inspection if it is solvable

if it is solvable for example that bridging bridge network there is no way you can do it you have to make a conversion either you have to make a conversion or you have to write the mesh equations or node equations okay

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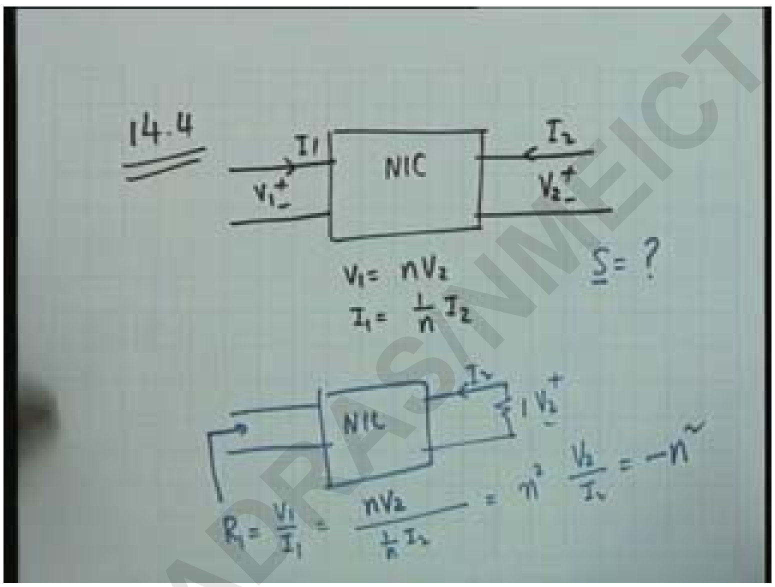

let's take the [Noise] third problem in fourteen point four that is a negative impedance converter that is we have a two port NIC NIC in which V one I one V two I two and the NIC is defined by V one equal to n V two I one equal to one by n I two

can you tell me what is the difference between an NIC and then ideal transform what is the difference <a_side> ((minus sign)) <a_side> there is a minus sign in the ideal transform it is simply a minus sign that's all okay all right

now let's see [Noise] how to calculate the S parameters

for now we have to we cannot go we cannot calculate one hence say the other would be equal or opposite or whatever it is isn't it because this NIC is is not a reciprocal device okay i didn't add necessarily it is necessarily a non reciprocal device isn't that right you see if the sign here was minus then it was a reciprocal device since the sign is not minus it is necessarily a non reciprocal device and therefore we cannot rely on calculation of one parameter to calculate the other [Noise] we have to do all the four

now what we do is to calculate well R zero one and R zero two both are one ohm so we terminating one and find out the [Noise] the impedance here R one which is equal to V one by I one V one is n V two this is the style of calculation i don't go to anything else i go to the fundamentals divided by I one is one by n I two

so this is equal to n squared V two by I two now you recall the signs <a_side> ((minus one)) <a_side> so this is minus n squared and if that is so

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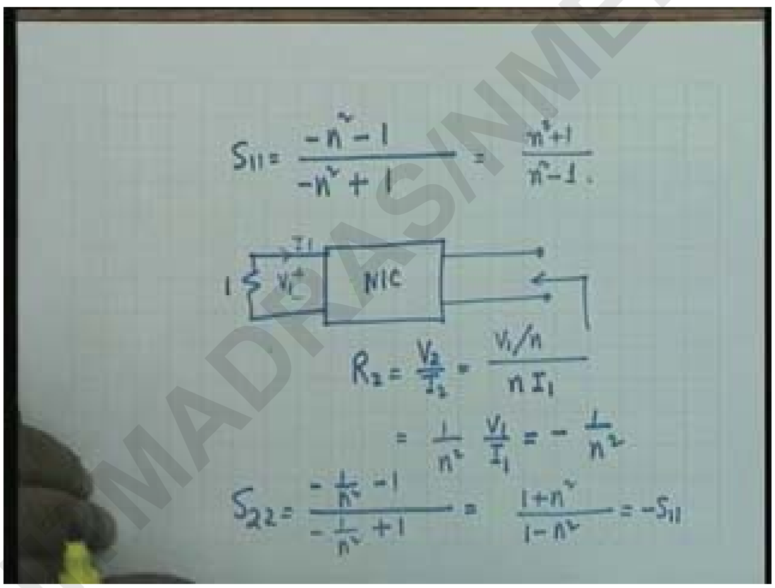

then obviously S one one would be minus n squared minus one divided by minus n squared plus one agreed Z minus one divided by Z plus one that is equal to n squared [Noise] plus one divided by n squared minus one

<a_side>) (( )) (00:33:30) <a-side> no n squared that's what i wrote n squared minus one okay for S two two now my termination will come on the other side please be careful this is one this is V one and this is I one i have to calculate the impedance here R two is equal to V two by I two and the definition says V two shall be V one by n divided by I two shall be n times I one

so this is one by n squared V one by I one and V one by I one is <a_side> ((minus one)) <a_side> minus one so this is minus one over n squared

therefore S two two is equal to minus one over n squared minus one divided by minus one over n squared plus one which is equal to if i multiply by n squared if i multiply by minus n squared one plus n squared divided by one minus n squared and you notice that this is exactly equal to minus S one one all right

this is not true in general in general any nonreciprocal device this is not correct okay this is true only for this specific case of NIC

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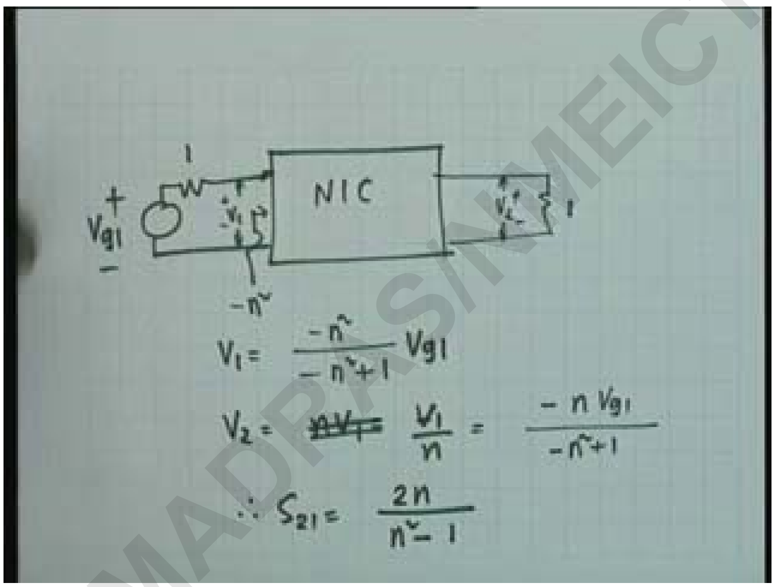

now let's look at [Noise] S two one and S one two we have the NIC for S two one we have to calculate [Noise] we have to connect a source Vg one one ohm terminate in one ohm and this is V two okay

how do i proceed now i proceed in the same manner

<a_side> ((sir Vg one minus I)) <a_side> pardon me <a_side> ((sir Vg one minus I is equal to)) <a_side> no i don't go that way that is simpler way all we have to calculate is V two <a_side> (( )) (00:35:45) <a_side> that's right i calculate V one <a_side> ((sir we have minus N square)) (00:00:00) <a_side> this is minus n square we have already calculated this and therefore V one is equal to minus n squared divided by minus n squared plus one and V two is equal to <a_side> ((sir Vg one sir V one is equal to

multiplied by Vg one)) <a_side> multiplied by Vg one okay <a_side> ((and V two is n V one V one)) <a_side> V two is n times V one <a_side> ((V one by n)) <a_side> oh dear minus no plus V one by n agreed that is equal to minus n <a_side> ((minus n)) <a_side> Vg one divided by minus n squared plus one therefore S two one is equal to <a_side> ((minus two n by one minus n square)) <a_side> or two n divided by n squared <a_side> ((one minus n squared n squared)) <a_side> okay [Laughter] i have taken care of the negative sign all right

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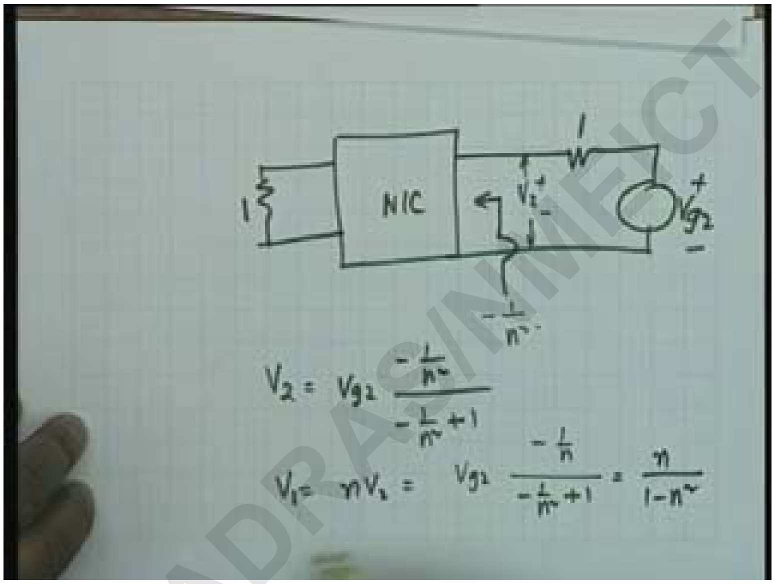

i have to calculate next the finally i have to calculate S one two and for that one should not jump into conclusions with non reciprocal devices one has to be absolutely careful one ohm and this is Vg two

the calculation procedure is the same this impedance is as we know minus one over n squared therefore V two V two is this voltage is Vg two minus one over n squared divided by minus one over n squared plus one and V one is

<a_side> ((n times V two)) <a_side> n times V two therefore this is equal to Vg two minus one over n minus one over n squared plus one let's multiply by minus one over n is minus n squared then i get n divided by [Noise] one minus n squared

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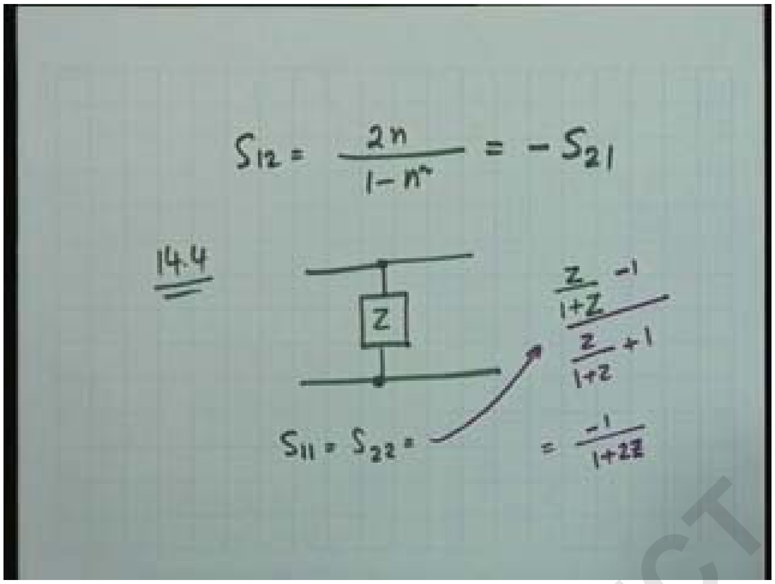

therefore S one two is equal to twice n divided by one minus n squared which is exactly <a_side> ((S two)) <a_side> minus S two which shows the non reciprocal nature of the device okay

unfortunately we cannot calculate we cannot do fourteen point five or the rest of the problems because we have not done it in the class let us conclude this class with the last example under fourteen point four which is an extremely simple network just a shunt element Z okay can one

<a_side> ((sir reciprocal network)) <a_side> it is a reciprocal network it is also a symmetrical network therefore calculation of S one one suffices can you tell me what this is

well the input impedance would be Z divided by one plus Z no <a_side> ((yes sir)) <a_side> if i terminate this in one ohm then the input impedance is Z divided by one plus Z

so this minus one divided by Z divided by one plus Z plus one this would be my S one one or S two two which is equal to minus one divided by <a_side> ((one plus two Z)) <a_side> [Noise] one plus two Z minus one does it bothering if it is one ohm it is minus one divided by two plus two okay

what about the what about the ah S two one

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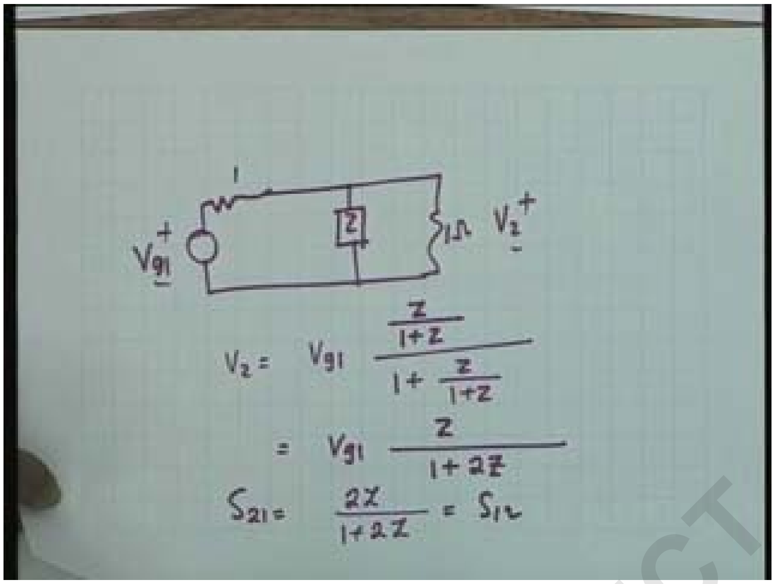

we have one ohm Vg one Z and one ohm this is V two

so V two is equal to Vg one Z divided by one plus Z equivalent of this two divided by one plus Z divided by one plus Z that is equal to Vg one Z divided by divided by one plus two Z okay

therefore S two one is equal to <a_side> ((two Z by one plus two Z)) <a_side> two Z divided by one plus two Z

a couple of question before we close this class this is of course equal to S one two because the network is reciprocal okay

is it possible to have a network in which the S parameters do not exist you see there are networks in which z parameters do not exist y parameters do not exist h parameters do they always exists

ABCD parameters always exists there is no network in which ABCD parameters do not and what about h <a_side> ((no sir)) <a_side> pardon me <a_side> ((no sir)) <a_side> there is no network in which h parameters do not exist is that correct double negative [Laughter] don't use another then it becomes a problem

what about S parameters can you think of a network in which S parameters do not exist if not why not <a_side> ((actually resistance is equal to negative one)) <a_side> resistance is negative that is perfectly all right <a_side> ((V naught is equal minus R naught but zero)) <a_side> say it again <a_side> ((sir the reference ah the input impedance is negative of the reference)) <a_side> input impedance is the negative of the reference impedance just a minute just a minute what you are saying is Z minus R zero divided by Z plus R zero

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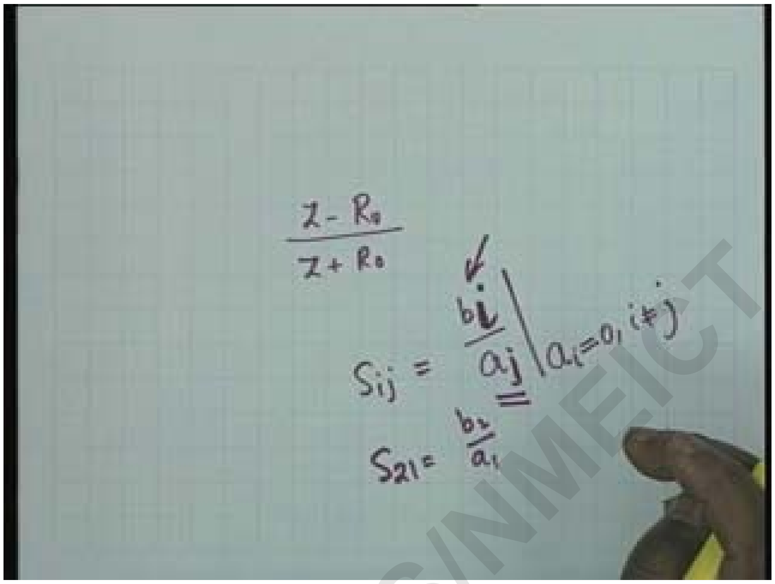

<a_side> ((Z equal to minus R zero)) <a_side> okay so it is possible <a_side> ((is that exist at that point)) <a_side> why not no we have already have this is why i asked the question we already had an example in which the parameters may not have existed yes [Noise] no

you see look at this uh NIC example S one one and S one one was n squared plus one divided by n squared minus one so when n is equal to one Z doesn't exist

similarly you see S two two for that case should also not have existed what about S one two and and S two one same and therefore scattering parameters belong to the same category as Z or y they may or may not exist

they are conditional okay but one thing is sure about scattering parameters that the numerator of all the parameters is a reflected wave isn't that right the numerator either it is one one or two two or one two or two one the numerator is a b parameter and the denominator can you tell me what is S two one

<a_side> ((sir V two by a one)) <a_side> V two by a one so it is Sij that is bi divided by aj okay pardon me S two one b two by n correct under the condition <a_side> ((all other i values)) <a_side> all other ai's are zero i not equal to j agreed

now here ah all other i is not equal to zero we are assuming a multi port more than two which also shows the scattering parameter description is equally good for more than ah two port it could be ah large number of ports doesn't matter but the point that i wanted you to notice is the [Noise] numerator is a reflected parameter

reflected parameter may or may not exist all right may or may not exist if it doesn't exist it means that the reflected variable is zero but what hurts it is if if an aj is zero how can an aj be zero

if the reference impedance is taken equal to <a_side> ((terminate)) <a_side> the {termin} (00:44:50) no ah the terminating resistance you shouldn't forget so quickly a two equal to zero what was the interpretation R two was equal to R zero

so it was a choice and therefore right from this definition without going into that example you should have been able to say that scattering parameters may or may not exist but if they exist at high frequencies it is a boom the description becomes very very simple

we close at this point.