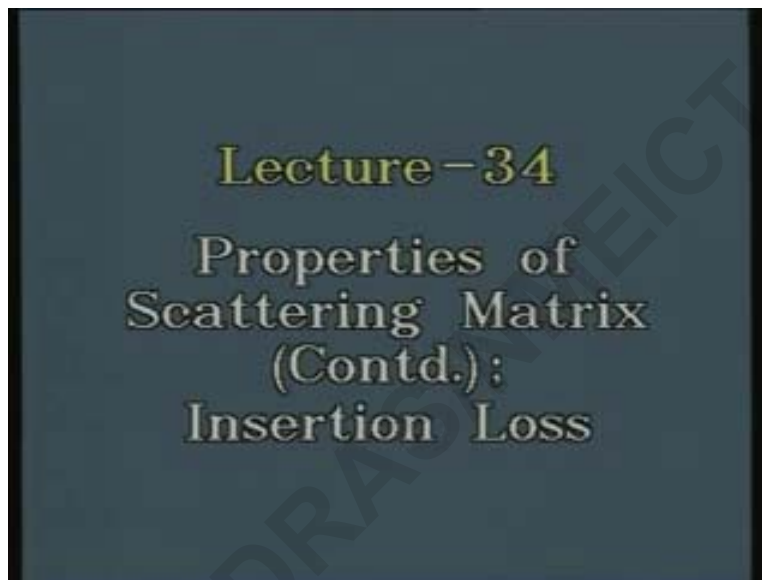


Circuit TheoryProf. S.C. Dutta RoyDepartment of Electrical EngineeringIIT DelhiLecture 34

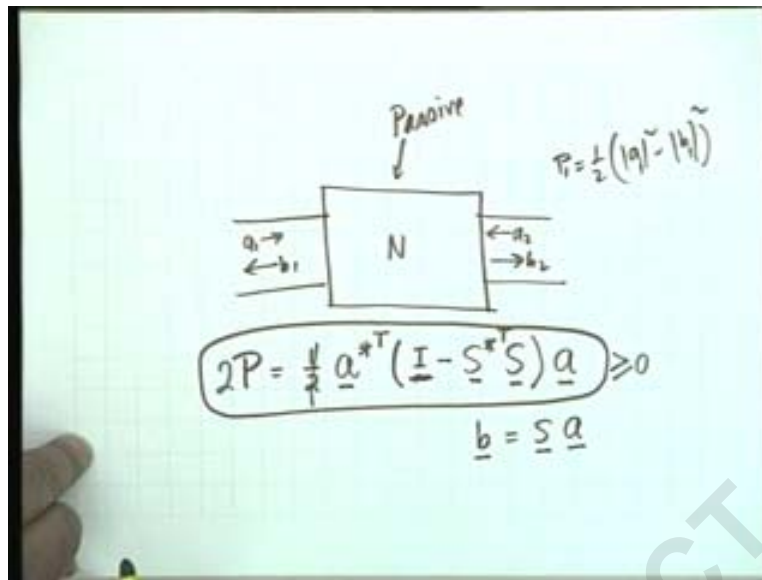
Properties of Scattering Matrix (contd.); Insertion Loss

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this is thirty-fourth lecture [Noise] and we are going to uh continue our discussions on properties of scattering matrix and if time permits insertion loss synthesis

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last time the point were we stopped was the following

we had considered a two port N [Noise] a passive one not necessarily reciprocal a passive one with the incident parameters a_1 and a_2 at the two ports and the reflected parameters b_1 and b_2 at the two ports

then we showed that the power [Noise] that is observed by the network is half a star transpose if you recall a star transpose then identity matrix minus $S^* S$ is that okay the these are all matrices multiplied by post multiplied by the matrix a where we utilized the relationship that the the power capital P is the sum of two powers P_1 and P_2 P_1 is half $|a_1|^2 - |b_1|^2$ and P_2 similarly is half the factor half comes becomes a is considered as a ((peak)) (00:02:05) vector and P_2 is half $|a_2|^2 - |b_2|^2$ and [Noise] magnitude $|a_1|^2$ is a one magnitude $|a_1|^2$ is a one a one star

we also utilized something else in this $b = S a$ that the reflected parameters reflected parameter matrix is equal to this scattering parameter matrix multiplied by a

so this was the final result and [Noise] if the network is passive let's write the final result in a slightly different form let's bring this two here two P and if the network is passive well this is obviously a scalar quantity the power is a scalar quantity is not a vector is not a matrix [Noise] the power must be greater than equal to zero if the network capital N is passive and therefore the result is that for a passive network

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Quadratic form

$$\underline{a}^{*T} (\underline{I} - \underline{S}^{*T} \underline{S}) \underline{a} \geq 0$$

$$\downarrow$$

$$\det(\underline{I} - \underline{S}^{*T} \underline{S}) \geq 0$$

Passive + Lossless

$$\underline{a}^{*T} (\underline{I} - \underline{S}^{*T} \underline{S}) \underline{a} = 0$$

$\underline{x}^{*T} \underline{B} \underline{x}$

~~\underline{x}^T~~

the scattering parameters shall obey this particular relation that is capital I minus S star transpose S a greater than equal to zero and the necessary and sufficient condition for this to be true ((or)) (00:03:23) incidentally we also introduced the term quadratic form quadratic form any ah form like uh $\underline{x}^{*T} \underline{B} \underline{x}$ this form is called a quadratic form because where \underline{x} is of course a vector \underline{x} is a vector because all terms in this shall contain product of two components of \underline{x} two components these two components may be the same

for example this will contain \underline{x} one squared it will contain \underline{x} two squared \underline{x} one \underline{x} two and if there are other components \underline{x} two and \underline{x} three and so on and so forth okay

so this is a quadratic form if this quadratic form is to be greater than or equal to zero then the necessary and sufficient condition is that for the coefficient matrix this is the coefficient matrix I minus S star transpose S and these are the variable matrices \underline{a} and \underline{a} these are the variables

the coefficient matrix must have a determinant which is non negative okay this is the condition

now for the special case for the special case that the network is passive and lossless passive and lossless well i argued that the determinant must be equal to zero and in addition in an addition each term in the matrix shall be equal to zero that is not obvious from determinant consideration but if we take this relation that for a passive network the

passive and lossless network the relation that should be valid is a star transpose I minus S star transpose S a should be equal to zero

if we take this relation and a this should be true for all possible vectors a all possible variable vectors a the only condition under which this can be true is that this matrix itself should be the null matrix okay does it stand to reason

a is a variable vector is a vector a variables and therefore for all possible values of the vector a the quadratic form should be equal to zero and the only way that it can be done is that the coefficient matrix itself should be a null matrix

the proof is simple and i would very much encourage you to try to prove it taking a as a two by one no it's a yeah that's right two rows and one column

that is taking a

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$$\underline{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad \underline{B} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

Prove

$$\underline{a}^* \underline{B} \underline{a} = 0$$

$$\Downarrow$$

$$\underline{B} = \underline{0}$$

as equal to a one a two and take B as B one one B one two B two one B two two all right prove that if a star transpose B a is equal to zero then this implies that B itself is a null matrix okay prove

this proof is very instructive and i strongly encourage you that you try to do this now in our case in our case

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The image shows a whiteboard with handwritten mathematical equations. The first equation is $\underline{I} - \underline{S}^{*T} \underline{S} = 0$. The second equation is $\underline{S}^{*T} \underline{S} = \underline{I}$. Below these, it is written that \underline{S} is unitary. A hand holding a yellow marker is visible at the bottom right of the whiteboard.

therefore the result that we shall have is that $\underline{I} - \underline{S}^{*T} \underline{S}$ should be equal to zero that is $\underline{S}^{*T} \underline{S}$ there is a small hitch in the proof which has to strike you

there is a small point which has to strike you in the proof and it depends on the value of \underline{B} that \underline{B} is of this form okay but it's in general also true all right

therefore $\underline{S}^{*T} \underline{S}$ is equal to the identity matrix that is [Noise]

now before we go to the next step matrices which obey this property that its transpose its conjugate transpose pre multiplying the matrix gives you an identity matrix not hermitian hermitian is a slightly more {spec} (00:18:17) special case this is called the unitary matrix unitary therefore \underline{S} is a unitary matrix the scattering matrix is therefore a unitary matrix

if we write this in an expanded form

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$$\begin{bmatrix} S_{11}^* & S_{21}^* \\ S_{12}^* & S_{22}^* \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$S_{11}^* S_{11} + S_{21}^* S_{21} = 1$$

$$\left. \begin{aligned} S_{11}^* S_{12} + S_{21}^* S_{22} &= 0 \\ S_{12}^* S_{11} + S_{22}^* S_{21} &= 0 \end{aligned} \right\}$$

$$S_{12}^* S_{12} + S_{22}^* S_{22} = 1$$

then i get S one one star what will be the next term S star transpose

so it would be S two one star then S one two star and S two two star this would be equal to no this multiplied by S one one S one two S two one S two two this should be equal to the identity matrix that is one one zero zero

if i multiply out if i if i write down that the four equations now you see what we shall get is S one one star S one one plus S two one star S two one should be equal to unity all right then S one one star S one two plus S two one star S two two this should be equal to zero

the third equation would be multiply this by this so S one two star S one one plus S two two star S two one should be equal to (()) (00:09:56) zero and finally the equation would be S one two star S one two plus S two two star S two two should be equal to one and if you see {care} (00:10:12) [Noise] carefully you notice that these two equations are compliments of each other are conjugates of each other

that is if you take the conjugate of this S one one star comes as S one one S one two star comes as S one two star similarly this term is the conjugate of this and the conjugate of zero is zero

therefore all though there are two equations they do not express anything new they are same equations

so ah in actuality we have three equations all right is that clear we have three equations and if you look at the first and the fourth that you have written here you notice that the left hand sides are real quantities isn't that right left hand sides are real quantities S_{11} one star multiplied by S_{11} is simply magnitude of S_{11} one whole squared okay however we cannot say this about the left hand sides of this curly bracketed equations we can't say that they are purely real all right

they will have real part as well as imaginary part the real part shall be zero the imaginary part shall also be zero but the fact of the matter is that these two equations are not different they are the same equations

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$$|S_{11}|^2 + |S_{21}|^2 = 1$$

$$|S_{22}|^2 + |S_{12}|^2 = 1$$

$$|S_{11}|, |S_{22}|, |S_{12}|, |S_{21}| \leq 1$$

Passive, lossless & Reciprocal:
 $|S_{11}| = |S_{22}|$

$S_{21} = 0$ at $1 - j\omega_0$

$$\frac{2V_2}{V_{g1} \sqrt{R_2}}$$

the first and the last equations that gives us S_{11} one squared plus S_{21} one squared is equal to one and the other one is S_{22} two squared plus S_{12} two squared is equal to one all right which shows two very important characteristics of this scattering parameters namely [Noise] that since magnitude squared cannot be negative and the sum of two magnitude squared makes a one each of them therefore must be less than unity

that is S_{11} one S_{22} two S_{12} one and S_{21} one each of them should be less than or or equal ((to unity)) (00:12:27) one could have one of them equal to one the other one shall be zero okay

we we will come to this this is a very interesting phenomena physically it's very simple it's very simple to see but notice another thing that if the network is passive lossless

which we have assumed and in addition if the network is reciprocal okay if it is reciprocal then S_{12} and S_{21} should be equal and if S_{12} and S_{21} are equal then obviously S_{11} magnitude is simply equal to S_{22} magnitude isn't that right

that is the reflection coefficient magnitude at the port number one and at port number two should be equal agree

these are the various interesting properties of the scattering matrix parameters

in addition in addition we have already stated that if one of them is zero suppose S_{21} suppose S_{21} is equal to zero at some frequency let's say at $\omega = \omega_0$ let's say

what does this indicate it indicates that the forward transmission scattering parameter forward transmission scattering parameter is zero

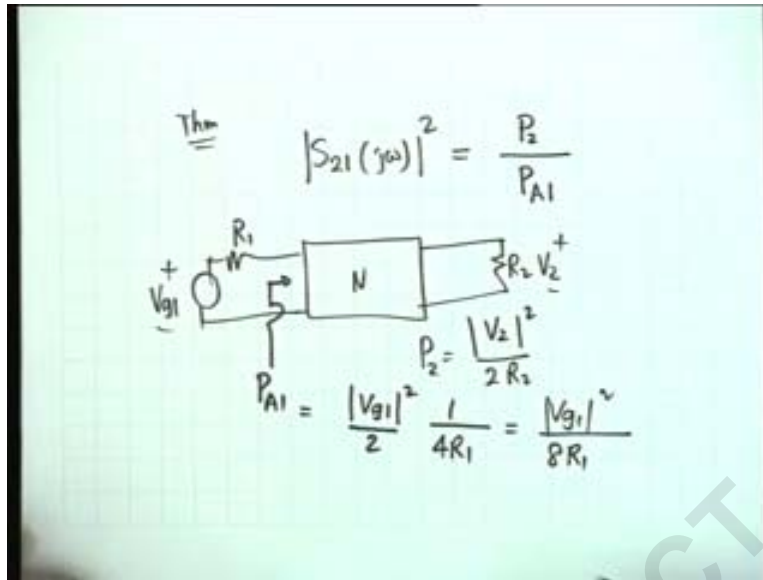
what does that mean you also recall that S_{21} is directly proportional to the voltage transfer function isn't that right S_{21} is twice V_2 by V_1 square root of R_1 by R_2 this is S_{21}

if this is zero what does it indicate it indicates null transmission that is the output voltage at this particular frequency shall be a zero okay

output voltage is zero therefore there is no power that goes out of the network and it stands to physical reasoning that if no power goes out of the network all the power must therefore come back to the source why because the network itself is lossless it doesn't absorb any power and therefore it follows that magnitude S_{11} should be equal to one that is all the power that is fed from port number one comes back to port number two now such a frequency such a frequency at which S_{21} is equal to zero is called a zero of transmission this is called a frequency of zero transmission or simply called a zero of transmission

so at a zero of transmission the magnitude S_{11} is equal to one all right any question now establish a relationship the next one is a theorem

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which says that for a two port S two one j omega magnitude squared is equal to the power that is absorbed by the load divided by the power that is available at port one okay which [Noise]

let me clarify what this means we have a network N which is driven from a source V_g one and a resistance R one and you know that in a two port we take the reference resistances as equal to the terminating resistances and this is R two

P two is the power if this voltage is V two then P two is the power that is actually absorbed by the load all right so P two is equal to V two magnitude squared

<a_side> ((by two R two)) (00:16:37) <a_side>

by two R two the factor two comes because V two is taken as the peak value okay V two by square root of two is the RMS value square of that divided by R two

now this is the power P two and P_{A1} is the power that is available at port number one that is the maximum power that can be drawn from the source V_g one R one and this shall be equal to this shall happen when input impedance is equal to

<a_side> (()) (00:17:09) <a_side>

R one okay and therefore P_{A1} one would be V_g one squared divided by two first okay divided by four times R one is this clear why it happens okay

we can find out the current and current squared multiplied by the resistance okay <a_side> ((the resistance P one has maximum power)) (00:17:35) <a_side> that is

correct maximum power and it is called the available power maximum word sometimes dropped so P_{A1} is equal to V_{g1}^2 divided by eight R_1 all right now to prove the theorem all that we have to recall is that

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$$S_{21} = \frac{2V_2}{V_{g1}} \sqrt{\frac{R_1}{R_2}}$$

$$|S_{21}(j\omega)|^2 = \frac{4R_1}{R_2} \left| \frac{V_2(j\omega)}{V_{g1}(j\omega)} \right|^2$$

$$= \frac{|V_2|^2 / (2R_2)}{|V_{g1}|^2 / (8R_1)}$$

$$= \frac{P_2}{P_{A1}}$$

S_{21} is equal to twice V_2 by V_{g1} multiplied by square root of R_1 divided by R_2 okay we had established the relation (00:18:09) this relation

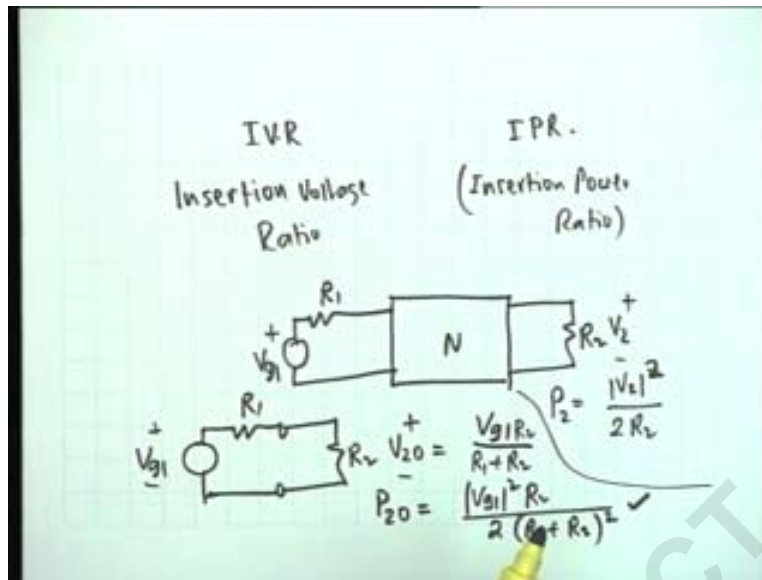
therefore $|S_{21}(j\omega)|^2$ is equal to four R_1 by R_2 and V_2 squared divided by V_{g1} at $j\omega$ squared all right

this can be written as V_2 squared divided by twice R_2 divided by V_{g1} squared divided by eight R_1 is that okay eight R_1 divided by two R_2 because four R_1 by R_2 and V_2 by V_{g1}

and therefore we have proved that the forward transmission scattering parameter which is $|S_{21}|^2$ is simply a power ratio P_2 by P_{A1} all right

next we consider

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two other quantities one is called the insertion voltage ratio IVR they are very simple ah [Noise] quantities and the insertion power ratio IPR

let me give the definitions first okay i have a situation this usual situation that we are considering is that the network N driven by a source V_{g1} one internal resistance R_1 and terminated in R_2 the voltage here is V_2 and the power that is absorbed by the load is P_2 which is equal to V_2 squared divided by twice R_2 this is the situation

now one argues is suppose the network was not there suppose the load is connected directly to the source then what difference does it make this is what is answered this is what is measured by the two quantities insertion voltage ratio and insertion power ratio that is what one does is suppose V_{g1} one R_1 that is the source instead of the network instead of inserting a network inserting a network between the source and the load suppose we connect it directly R_2 let the voltage across R_2 be V_{20} V_{20} zero means without the network and let the power absorbed by the network by the load B equal to P_{20} zero all right

then what is V_{20} obviously V_{g1} divided by R_1 plus R_2 is that okay
<a_side> () (00:21:27) <a_side>

into R_2 okay this is V_{20} and what is P_{20} (00:21:33) P_{20} zero P_{20} zero would be V_{20} squared divided by two R_2 okay which means that V_{g1} squared R_2 divided by twice R_1 plus R_2 have i done it correctly

<a_side> () (00:22:00) <a_side>

$R_1 + R_2$ whole squared okay R_2 cancels one R_2 cancels is that okay we shall require this expression so be careful whether this is correct or not R_2 which was yeah this is correct okay

so let me write this again

(Refer Slide Time: 00:22:24 min)

The image shows handwritten mathematical derivations on a grid background. The equations are as follows:

$$V_{20} = \frac{V_{g1} R_2}{R_1 + R_2}$$

$$P_{20} = \frac{|V_{g1}|^2 R_2}{2(R_1 + R_2)^2}$$

$$V_2$$

$$P_2 = \frac{|V_2|^2}{2R_2}$$

The Insertion Voltage Ratio (IVR) is defined as:

$$IVR \triangleq \frac{V_{20}}{V_2} = \frac{V_{g1} R_2}{V_2 (R_1 + R_2)}$$

The Insertion Power Ratio (IPR) is defined as:

$$IPR \triangleq \frac{P_{20}}{P_2} = \frac{R_2^2}{(R_1 + R_2)^2} \left| \frac{V_{g1}}{V_2} \right|^2$$

if the network is not inserted then V_2 zero is $V_{g1} R_2$ divided by $R_1 + R_2$ and P_2 zero the power absorbed by the load is $V_{g1}^2 R_2$ divided by twice $R_1 + R_2$ whole squared

on the other hand if the network is inserted then we get the voltage V_2 across the load and the power is P_2 which is V_2^2 divided by twice R_2 all right

the insertion voltage ratio IVR is then defined [Noise] as the voltage V_2 zero divided by V_2 that is the ratio of the voltage across the load without and with the network this is called the insertion voltage ratio and obviously this is equal to V_{g1} divided V_2 multiplied by R_2 divided by $R_1 + R_2$ okay this is the insertion voltage ratio ah keep this in a prominent place because we shall require this expression while working out some problems and IPR insertion volt insertion power ratio is given by P_{20} zero divided by P_2 that is equal to ah [Noise] how much shall it be

<a_side> ((sir $R_1 R_2$ divided by $R_1 + R_2$ whole squared)) (00:23:59)

<a_side>

$R_1 R_2$ divided by $(R_1 + R_2)^2$

<a_side> () (00:24:05) <a_side>

let's see V_{g1} by V_2 whole squared this would be uh

<a_side> ((greater than the R_2 squared sir R_2 square)) (00:24:18) <a_side>

R_2 square by $(R_1 + R_2)^2$ is that okay

<a_side> ((yes sir)) (00:24:26) <a_side>

let me right it again

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The image shows handwritten mathematical derivations on a whiteboard. The equations are as follows:

$$IPR = \frac{R_2^2}{(R_1 + R_2)^2} \left| \frac{V_{g1}}{V_2} \right|^2$$

$$S_{21} = \frac{2V_2}{V_{g1}} \sqrt{\frac{R_1}{R_2}}$$

$$|S_{21}|^2 = \frac{4R_1}{R_2} \left| \frac{V_2}{V_{g1}} \right|^2$$

$$(IPR) |S_{21}|^2 = \frac{4R_1 R_2}{(R_1 + R_2)^2}$$

It is noted that if $R_1 = R_2$, then $(IPR) |S_{21}|^2 = 1$.

IPR is equal to R_2 squared divided by $(R_1 + R_2)^2$ times V_{g1} by V_2 whole squared is that okay

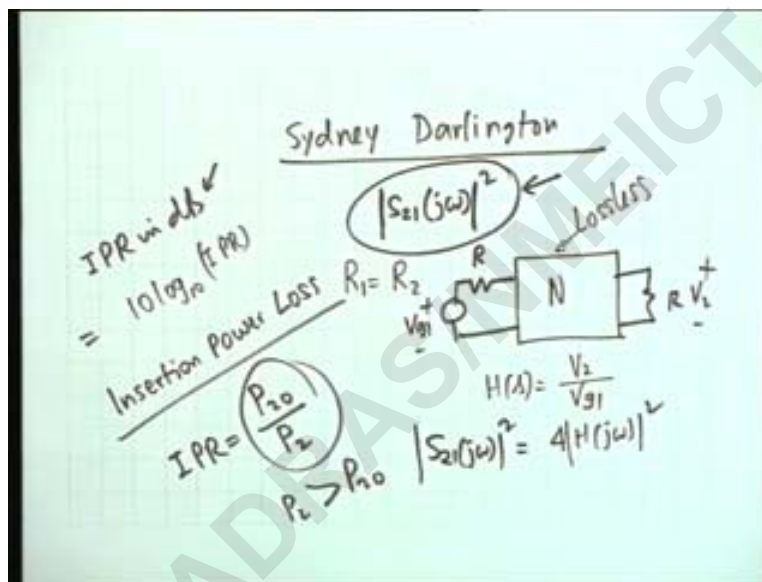
now you also notice you also recall that S_{21} is given by twice V_2 by V_{g1} square root R_1 by R_2 so what is S_{21} squared that is equal to four R_1 by R_2 V_2 by V_{g1} whole squared agreed and you see that these two are very intimately related to each other

the insertion power ratio and the magnitude squared of S_{21} they are very intimately related to each other in fact if we multiply IPR by S_{21} squared if we multiply the two what do we get we get four $R_1 R_2$ divided by $(R_1 + R_2)^2$ is that okay

this and this cancel and by multiplication R_1 and R_2 cancel so $\frac{R_1}{R_2} \frac{R_2}{R_1}$ divided by this all right and the interesting result is that the product is a constant product only depends on the terminating resistances at any frequency at any frequency and you also notice that if it is an equally terminated network that is if R_1 is equal to R_2 then this product IPR and $|S_{21}|^2$ would be equal to one so one is the reciprocal of the other all right

this relationship was utilized by a gentleman named Darlington

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who died at the age of 99 a few years ago Sydney Darlington he was Sydney Darlington utilized this result to synthesize networks and this is our first glimpse into network synthesis

he synthesized network from the given $|S_{21}(j\omega)|^2$ that is from the given forward transmission scattering parameter he was able to synthesize networks which had equal terminate to be to be to be specific while he did for the most general case but for our purpose at this moment

let us say that we are considering equally terminated networks that is our problems is to synthesize a network N and [Noise] let's make it lossless what does that mean it only contains pure inductors and pure capacitors

R_1 equal to R_2 so equal terminations if this R this is also R and this is V_{g1} and this is V_2 suppose this is specified S_{21} $j\omega$ squared

know usually this is not what will be specified what will be specified will be the voltage ratio V_2 by V_{g1} the transfer function but if you know what is V_2 by V_{g1} then you also know S_{21} isn't that right because S_{21} is simply twice V_2 by V_{g1} so S_{21} $j\omega$ squared will be four

suppose this is given H of S equal to V_2 by V_{g1} if this is given then what is S_{21} $j\omega$ squared magnitude squared this is simply yes

<a_side> (()) (00:28:36) <a_side>

not twice four times H of $j\omega$ magnitude squared

so if the [Noise] if this is given S_{21} $j\omega$ squared or the voltage transfer function is given or something else if the insertion power ratio is given then you know insertion power ratio and S_{21} $j\omega$ squared they are reciprocals of each other and therefore in any of these circumstances you can find out what S_{21} $j\omega$ magnitude squared is and from this Sydney Darlington was able to synthesize the network and the steps are like this

oh let me also mention [Noise] that sometimes the insertion power ratio most of the times the insertion power ratio is given in decibels and therefore insertion power ratio in decibels IPR in dB would be equal to what

<a_side> ((ten log ten)) (00:29:40) <a_side>

ten log ten ten log ten the IPR that is P_2 by P_2^0 by P_2 okay insertion power ratio is P_2^0 ((that's the)) (00:29:54) power without the network divided by the power with the network

so if this is given in decibels then you have to find out the IPR by ah taking the by taking the ordinary ratio that is decibel divided by ten and then e to the power of that and this is sometimes in the literature is called the insertion power loss insertion power loss

why is it a loss if you recall IPR is equal to P_2^0 by P_2 why is it a loss because usually P_2^0 is less than P_2 can P_2^0 be greater than P_2 the question yes it can be pardon me

<a_side> (()) (00:30:49) <a_side>

no even other ways i can make a mess

<a_side> (()) (00:30:53) <a_side>

i can make a mess of the network i can design it such that is not but that's not the purpose the purpose of an insertion network insertion network costs money it contains inductors and capacitors why should one use it

one uses it only to increase the power transfer that is usually P_2 is greater than P_2^0 and therefore this quantity is less than one taking log it will give a negative sign minus and that's why it is a loss is that okay is that clear that's why it is a loss all right

so insertion power loss is defined as $10 \log_{10} \text{IPR}$ insertion power ratio

<a_side> (()) (00:31:39) for a system R_1 for which R_1 is equal to R_2 we will have $|S_{21}|^2 \text{ mod } |S_{21}|^2 = 1 - \omega^2$ always greater than one in this case if IPR is less than one <a_side>

no say it again

<a_side> sir IPR is less than one as you were saying just now then from that equation $|S_{21}|^2 \text{ mod } |S_{21}|^2 = 1 - \omega^2$ IPR is equal to one <a_side>

correct

<a_side> sir $|S_{21}|^2$ will be always greater than one <a_side>

[Laughter] has anybody can answer to this question

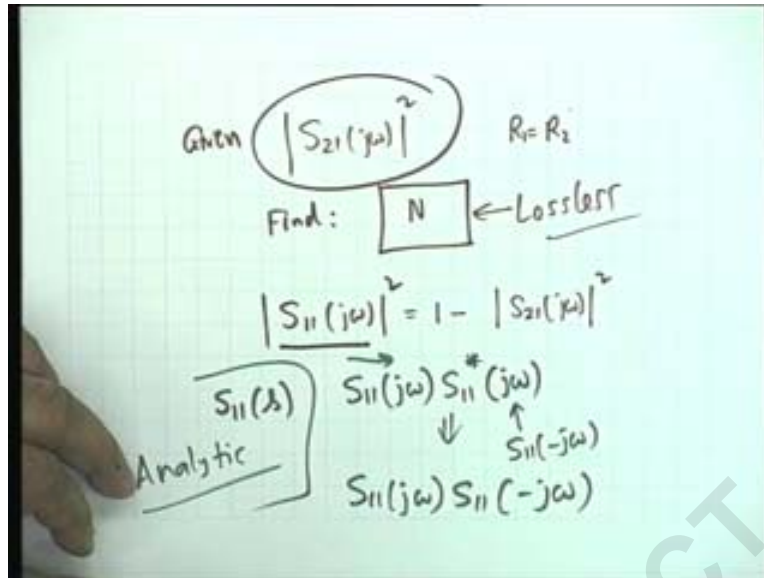
<a_side> (()) (00:32:11) <a_side>

you see the network itself doesn't absorb any power and therefore what ever power is fed must go to the load the rest of it shall comeback this is so it is a maximum condition that $|S_{21}|^2$ magnitude squared can be equal to one all right it's a maximum otherwise it will be always be less than one okay all right

let me take an example of synthesis then you will understand this problem yes your question is still not fully answered but i think you would get the answer as i go ahead

[Noise] well the question is

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$|S_{21}(j\omega)|^2$ is given. Let me set the problem. This is given as a function of ω as a function of frequency. Okay, and it is given that $R_1 = R_2$. The terminations are equal. The problem is to design the lossless network N . Find N . This is a synthesis problem. All right.

This relates to the response to the excitation. It is proportional to V^2 by V_g one magnitude squared. This is given, and the terminating conditions are given. You are required to find the network N .

So the first thing that you do is first thing that you do is you recall that $|S_{11}(j\omega)|^2$ and N we wanted to be lossless. We don't want N to absorb power because that then the power available to the load shall be reduced by the power absorbed by the network. So you want a lossless network.

<a_side> ((we have to find the transfer function of that network)) (00:34:04) <a_side>

no we don't want we don't find any transfer function

<a_side> ((we want to realize the network)) (00:34:11) <a_side>

We want to realize the network. We want to find out what network shall realize this $|S_{21}(j\omega)|^2$ okay.

So we recall that for a lossless network $|S_{11}(j\omega)|^2 + |S_{21}(j\omega)|^2$ magnitude square is equal to one.

therefore this is equal to one minus S^2 one $j\omega$ magnitude squared so if this given then you can find out the input reflection coefficient magnitude squared all right and at this point comes a big [Laughter] leap forward and that is the following

you see S one one $j\omega$ squared can be found out and this is a parameter if you take the total complex plane it is the values are given on one of the axis that is the imaginary axis okay can i find out the value of the parameter S one one at any other value of S

it turns out it turns out this is the partial specification specification only $j\omega$ axis the question is can you find out the value at some other frequency which is not on the $j\omega$ axis can you do that

it turns out from the theory of complex variables that this can be done provided the function S one one S is what is the word for it have you done a course on complex variables

<a_side> (()) (00:35:47) <a_side>

yes analytics what is an analytic functions

<a_side> ((sir you have all differential differential all order at every point)) (00:35:55)

<a_side>

except

<a_side> except similarities <a_side>

except at a finite number of similarities okay

so without going into details our network functions are very well behaved otherwise engineers would not have liked them engineers would not use them if they are not well behaved okay

it turns out that S one one S is indeed well behaved that is it is analytic and therefore for an analytic function one can go from partial specification to complete specification which means that i write this expression i write this expression as S one one $j\omega$ S one one star $j\omega$ all right the left hand side this is there is nothing nothing ah hidden in this this is obvious isn't it magnitude squared is equal to the quantity multiplied by its complex conjugate and then what i do is i take a big leap forward that is from the imaginary axis i jump to any other point in the S axis in the S plane which means that i

replace $j\omega$ by S but before that if it is an analytic function what is the relation between this and this

<a_side> (()) (00:37:19) <a_side>

it is only $j\omega$ is replaced by minus $j\omega$ that's all okay and therefore what i get from this is S one one $j\omega$ multiplied by S one one minus $j\omega$ and therefore my final relationship is

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$$S_{11}(j\omega) S_{11}(-j\omega) = 1 - |S_{21}(j\omega)|^2$$

$\downarrow j\omega = s$ (Analytic Continuation)

$$S_{11}(s) S_{11}(-s)$$

$$S_{11}(s) \left(= \frac{z_1 - R_1}{z_1 + R_1} \right)$$

that S one one $j\omega$ S one one minus $j\omega$ is equal to one minus mod S two one $j\omega$ magnitude squared and from this relation i know this now from this relation i put $j\omega$ equal to S

this procedure as i have said again and again is valid only if the function S one one of {capit} (00:38:08) of small s is an analytic function and this procedure is therefore given the name of analytic continuation and the meaning of the two word should be obvious it it can be done only if the function is analytic and it is a continuation of the value of the function from the $j\omega$ axis to any other point in the S plane and analytic continuation and therefore you can find out S one one S multiplied by S one one of minus S and the next step is to find out what is S one one of S okay

what is given is S one one of S multiplied by S one one of minus S how do you find out S one one of S

now if you recall the definition of S one one of S what is the definition

<a_side> ((input impedance)) (00:39:09) <a_side>

input impedance minus R_1 one divided by input impedance plus R_1 one okay

now [Noise] let me go slow what is the property of input impedance Z_1 one where should it's poles be poles of an input impedance it is a passive lossless network terminated in a resistance can it have poles in a right half plane no it's poles must be on the left half plane

now Z_1 one plus R_1 one if you add a constant obviously poles will change but can they go to the right half plane yes or no what is Z_1 one plus R_1 one it is a series combination of resistance R_1 one and an impedance Z_1 one it is still an impedance and therefore <a_side> ((adding would change the poles)) (00:40:11) <a_side>

adding would change the poles yes of course of course it will change the poles suppose i had a function like this S^2 plus ω^2 not squared you add a resistance a adding won't change the poles you are quite right do you see that his claim is correct if you add a resistance the poles would not change the zeros would change okay

therefore the poles of Z_1 one plus R_1 one are the same as the poles of Z_1 one but it's zeros will change zeros of Z_1 one plus R_1 one shall change where can the zeros be can they be in the right half plane this we have done again and again you see for an impedance there can be neither zeros nor poles in the right half plane poles and zeros must be in the left half plane why because the reciprocal of an impedance is admittance an admittance poles must also be in the left half plane okay things will become a bit more clear when you take an example but stay with me stay with me the problem here was to identify what is S_1 one one of S obviously S_1 one one of S shall have poles which are in the left half plane and S_1 one one of minus S then shall have poles which are in the right half plane isn't that right therefore from this decomposition you shall choose that part which has poles in the left half plane and once you do that once you are able to identify

(Refer Slide Time: 00:42:14 min)

$$S_{11}(s) = \frac{Z_1 - R_1}{Z_1 + R_1}$$

$$\rightarrow Z_1 = R_1 \frac{1 + S_{11}(s)}{1 - S_{11}(s)}$$

Synthesize Z_1

So one of S since this is equal to $Z_1 - R_1$ divided by $Z_1 + R_1$ you then find out Z_1 this will be equal to R_1 multiplied by $1 + S_{11}(s)$ divided by $1 - S_{11}(s)$ this is the componendo and dividendo okay and the final task is to synthesize Z_1 all right this is the total procedure but let me go through the steps once again [Noise]

before I take up an example the steps are let me go back steps are that given $S_{11}(j\omega)$ or any other version for example IPR or the insertion loss or the voltage transfer function anything that is given you first find out this subtract this from one to get magnitude squared of the input reflection coefficient and then you make an analytic continuation

that is you write this as $S_{11}(j\omega) S_{11}^*(j\omega)$ and replace $j\omega$ by s okay after you do that you get a product of terms which can be identified as $Z_1 - R_1$ and $Z_1 + R_1$ and $S_{11}(s)$ and $S_{11}^*(s)$ at this point we recall the property of $S_{11}(s)$ or the definition and the definition says that its poles must be in the left half plane if one part has poles in the left half plane $S_{11}(s)$ or $S_{11}^*(s)$ must have poles in the right half plane so you take that part which has poles in the left half plane and from there you construct the input impedance Z_1 and then you synthesise Z_1

let's take an example

(Refer Slide Time: 00:44:29 min)

$$|S_{21}(j\omega)|^2 = \frac{1}{1+\omega^6}$$

$$|S_{11}(j\omega)|^2 = \frac{\omega^6}{1+\omega^6}$$

$$S_{11}(s)S_{11}(-s) = \frac{\left(\frac{s}{j}\right)^6}{1+\left(\frac{s}{j}\right)^6} = \frac{-s^6}{1-s^6}$$

Butterworth LPF of order 3

suppose S_{21} follow this example carefully suppose $S_{21}(j\omega)$ is $\frac{1}{1+\omega^6}$ this is what is given as a function of ω what kind of a filter is this what kind of a filter is this obviously a low pass filter isn't that right

it's a low pass filter ω equal to zero the value is one and as ω increases it continuously decreases what is the what is the frequency at which the value is half at ω equal to one all right [Noise]

as you will know later this is called a Butter-worth filter a Butter-worth low pass filter Butter-worth low pass filter of order three the power of ω in the magnitude squared is six the order of S in H of S the transfer functions will be three

therefore it's a butter-worth low pass filter of order three and the first thing we do is $S_{11}(j\omega)$ magnitude squared we subtract this from unity so we get ω^6 divided by $1+\omega^6$ all right

then we argue that $S_{11}(s)S_{11}(-s)$ shall be equal to the right hand side with $j\omega$ replaced by s $j\omega$ replaced by $-s$ that is ω replaced by s by j if ω is replaced by $-s$ by j can you tell me what the denominator shall be numerator $1-s^6$ and the denominator $1+s^6$ okay

now what you have do is to decompose this into $S_{11}(s)$ and $S_{11}(-s)$ that is in the denominator you have to identify those zeros which are in the left half plane now it's not too bad

(Refer Slide Time: 00:46:56 min)

$$\begin{aligned}
 S_{11}(\lambda) S_{11}(-\lambda) &= \frac{-\lambda^6}{(1-\lambda^3)(1+\lambda^3)} \\
 &= \frac{-\lambda^6}{(1-\lambda)(1+\lambda+\lambda^2)(1+\lambda)(1-\lambda+\lambda^2)} \\
 S_{11}(\lambda) &= \frac{\lambda^3}{(1+\lambda)(1+\lambda+\lambda^2)}
 \end{aligned}$$

our situation is $S_{11}(\lambda) S_{11}(-\lambda) = -\lambda^6 / ((1-\lambda^3)(1+\lambda^3))$ minus s to the six then one minus s to the six you can write as one minus s cube multiplied by one plus s cubed and this you can write as let's find out the zeros minus s six one minus s one plus s plus s squared then one plus s one minus s plus s squared agreed i have factorized one minus s cubed and one plus s cubed

now [Noise] which of this factors will have left half plane zeros obviously this factor has a right half plane zero one minus s the zero is at s equal to one this has a left half plane zero at s equal to minus one and this agreed whole

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okay i'm talking of zeros of the denominator all right in the denominator you look at those factors which create which create a left half plane pole of s one one all right and therefore for $S_{11}(\lambda)$ our choice should be one plus s multiplied by one plus s plus s squared is that clear

the other factor one minus s and one minus s plus s squared obviously should go to $S_{11}(-\lambda)$ one of minus S all right the next question is what do we take for the numerator should we take plus s cubed or minus s cubed

<a_side> sir plus <a_side>

why not minus

<a_side> ((minus S cube is normalized)) (00:48:51) <a_side>

minus S cubed zeros are at the origin

<a_side> ((we can take either)) (00:48:58) <a_side>

we can take either excellent we can take plus or minus accordingly we shall get two networks accordingly we shall get two networks okay once we have found out

(Refer Slide Time: 00:49:13 min)

$$R_1 = 1 \Omega$$

$$S_{11}(s) = \frac{s^3}{s^3 + 2s^2 + 2s + 1}$$

$$Z_1 = \frac{1 + S_{11}}{1 - S_{11}}$$

$$Z_1 = \frac{2s^3 + 2s^2 + 2s + 1}{2s^2 + 2s + 1}$$

S one one S let us say this is plus s cubed divided by let's let's simplify this it becomes s cubed plus two s squared plus twice s plus one then you find out Z one as equal to R one let's take R one as one ohm specifically let's take R one as one ohm then this would be one minus S one divided by no it's the other way round it is one minus S one plus S one one divided by one minus S one one okay

so this is equal to twice s cube plus twice s squared plus twice s plus one and in the denominator we shall have simply twice s squared plus twice s plus one this is my input impedance and the last problem last task in the process is to synthesise this impedance Z one okay and it's a beauty of Darlington synthesis that the constraint of a termination of one ohm the constraint that Z one shall be a lossless network terminated in a resistance which is equal to the resistance at port one that is one ohm has already been taken care of and how to do this i shall demonstrate next time okay