Circuit Theory

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Lecture 37

Positive Real Functions

this is the thirty seventh lecture and we talk about positive real functions today

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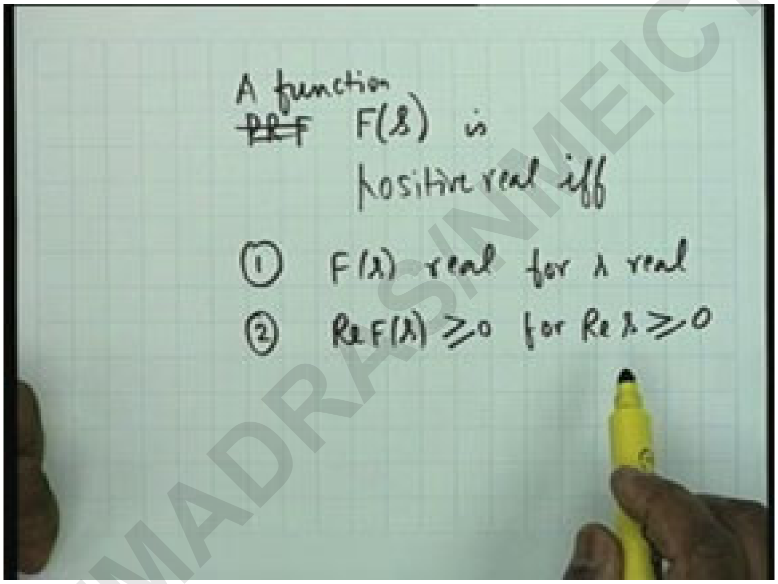

positive real function if we denote it by F of s is defined as a function F of s is PRF a function F of s is positive real if and only if the definition very simple definition number one F of s is real for s real it must be a real function and number two the real part of F of s real part of F of s should be non-negative for real part of s non-negative okay

F of s is real for s real and real part of F of s non-negative for real part of s non-negative

real part of s non-negative means the right half plane isn't that right including the j omega axis that is it is the closed right half plane

therefore what it means is that in the closed right half plane F of s should also be in the closed right half of the F of s plane All right in other words we can view this in terms of mapping of one complex plane that is s plane to F of s plane

do not be scared with the word mapping what it simply mean is the following

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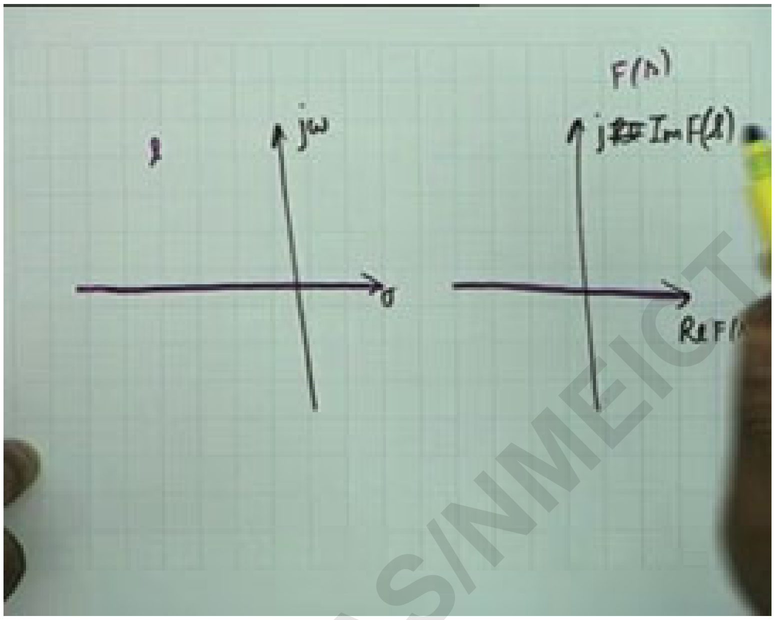

that if we have an s plane sigma j omega and a F of s plane j real i am sorry j imaginary part of F of s and this is the real part of F of s then it what what it means is the is that um when when s is real this is the s plane and this is the F of s plane when s is real that is when s is on the real axis the pink line F of s should also be real therefore F of s must lie on this line this is the mapping that is the s plane this line maps into this line in the F of s plane that's the first part of the definition

and the second part says that if if real part of s is non-negative which means that we take the closed right half s plane when the real part of s is non-negative then real part of F of s should also be non-negative

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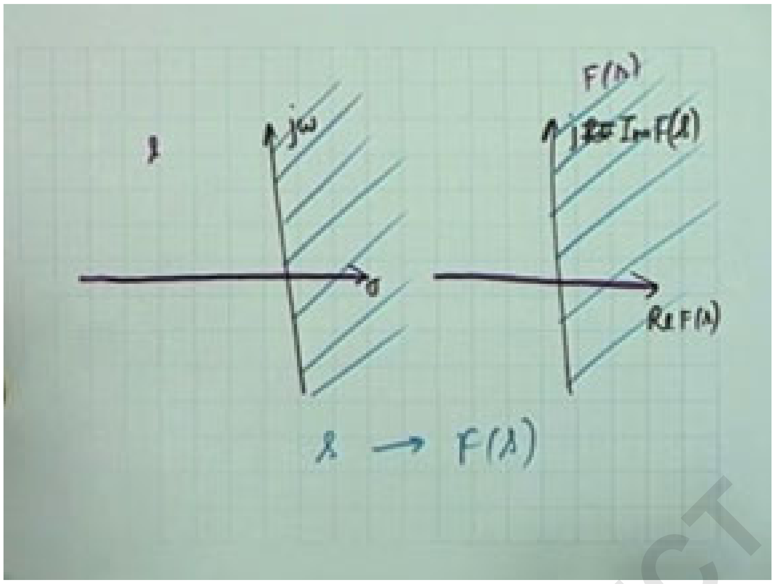

in other words the shaded area here maps on to the shaded area in the F of s plane this is what is meant by mapping All right

it is an s to F of s mapping in such a manner that the real axis real axis of the s s plane maps on to the real axis of the F of s plane and the right half of the s plane maps on to the right half of the F of s plane that's the definition of a positive real function

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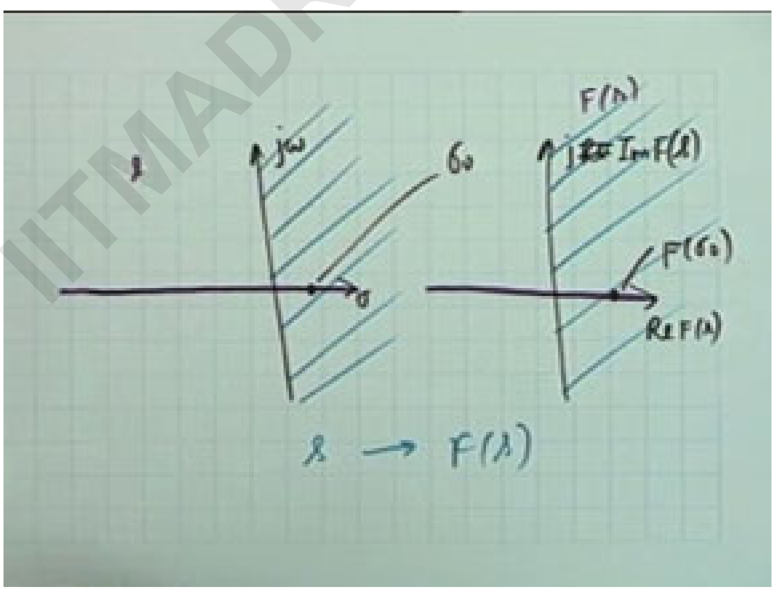

ah a a question suppose i have a point let's say here let's say this point is sigma zero what would be the possible location of on the F of s plane [Conversation between student and professor – Not audible ((00:04:05))] real axis positive half positive half so it would be somewhere here let's say F of sigma All right

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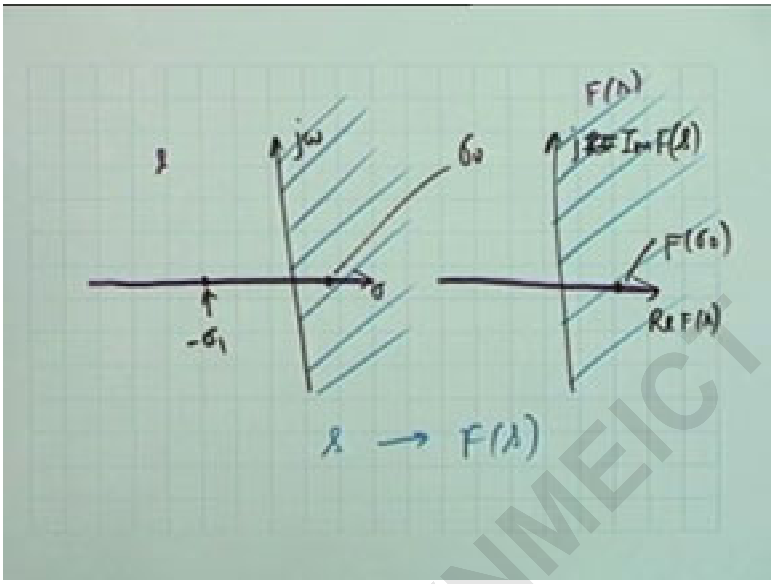

another question if you have understood the definition suppose i have a point here let's say minus sigma one what would the corresponding F of minus sigma on b [Conversation between Student and Professor - any point on the real axis((00:04:29 min))] [Laughter-any point] on the real axis that is correct

so it is a one to one mapping of the positive real axis of the s plane on to the positive real axis of the F of s plane it does not mean that the negative real axis maps on to the negative real axis no is the point clear [Conversation between Student and Professor – Not audible ((00: 04:52 min))] [Laughter] why not because the definition only says that if real part of s is non-negative then real part of F of s is non-negative it doesn't say that if real part of s is negative what will happen it doesn't say [Noise] it only puts a constraint on the real part of s non-negative if it is negative than F of sigma zero it has to be real isn't that right

if s is on this point then s is a real quantity and F of s has to be real

and therefore it must be somewhere on this line but it doesn't specify where it could be in the right half of the uh real axis or in the left half of the real axis it could even be at the origin it does not matter is the point clear all that it says is that real part of s is non negative [Noise] that is the positive half of the real axis in the s plan maps on to the positive half of the real axis in the F of s plane okay

this constraints must be kept in mind

a simple mathematics but the concepts are important and one if one loses sight of one small concept one is able to make a Himalaya [Noise] okay a mistake which is of the dimension of Himalayas

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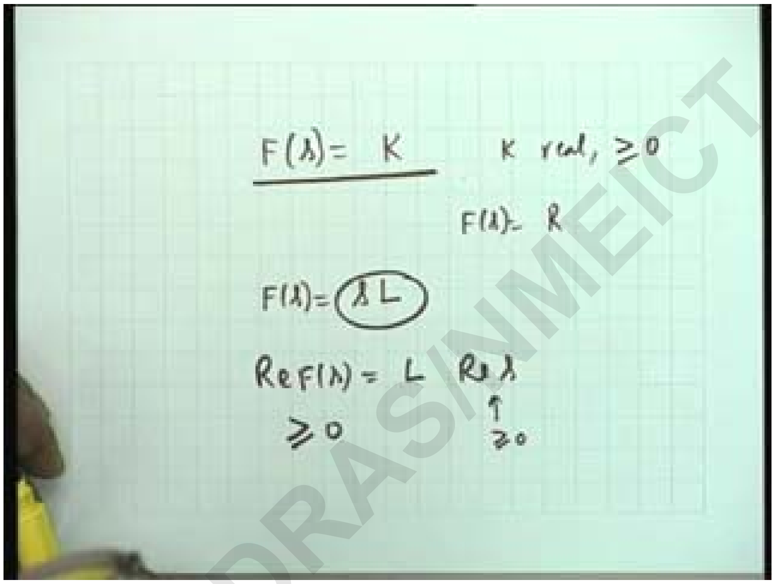

now very simple example

suppose F of s is a constant let's say K where K is real and non negative K is real and non negative

obviously this function is real why because when s is real well it's always real is always real so s is real it's also real when real part of s is non negative K it is a constant that's always non negative so it is a therefore a resistance for example F of s equal to a resistance [Noise] or a conductance is a positive real function a resistance is a positive real function a conductance is also a positive real function

the impedance of an inductor F of s is equal to s times L this is a impedance of an inductor this is also positive real because when s is real it is real and when real part of s is non negative the real part of F of s after all real part of F of s is equal to L times the real part of s and therefore then this is non negative obviously this will also be non negative

therefore an inductor the impedance of an inductor is also a positive real function okay

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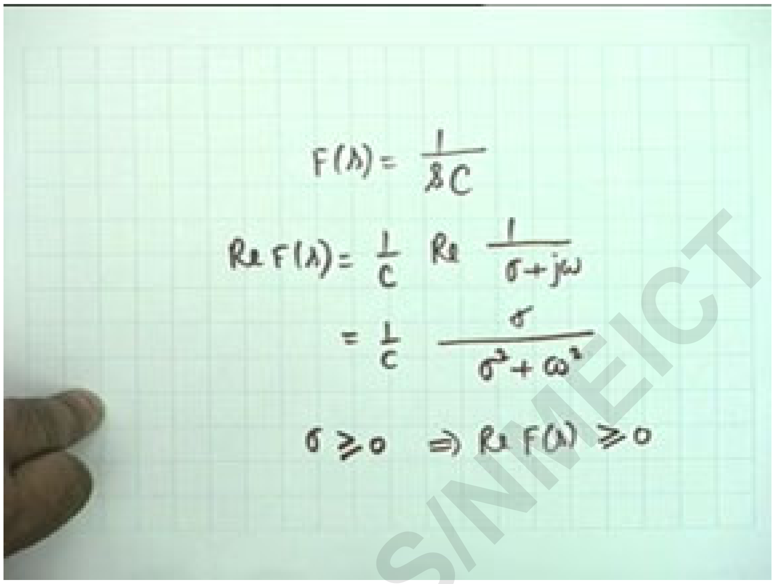

let's take the impedance of a capacitor let's say F of s is equal to one over s C C is a real positive quantity All right

and you see that the first the first part of the definition is obviously met that one s is real this quantity is real

now let's look at the real part of F of S this is equal to one over C then real part of one over sigma plus j omega All right and this is equal to one over C real part of this is sigma square plus omega square and the numerator you get sigma

and obviously if sigma is greater than equal to zero what is sigma this is the real part of s if sigma is greater than equal to zero then obviously real part of F of s is greater than equal to zero the sign depends on sigma right

and therefore if the sigma is non negative then this the whole quantity is non negative in other words one by s C impedance of a capacitor is also a positive real function

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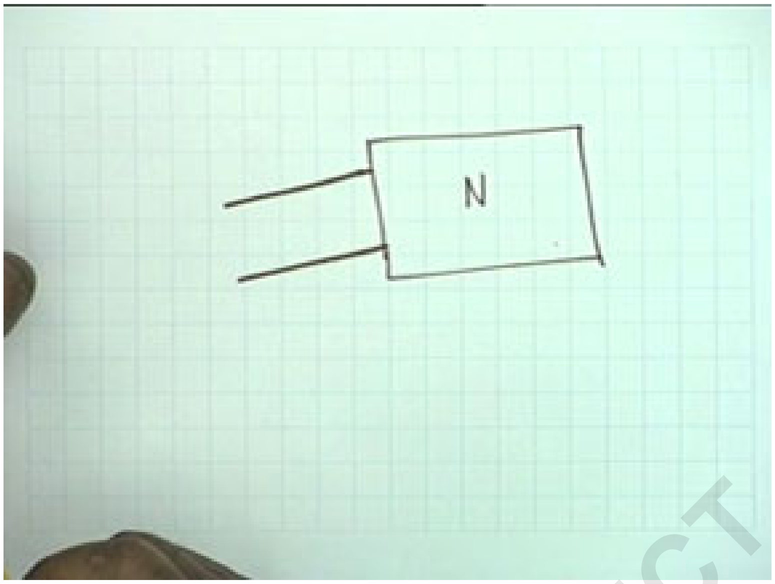

and in general one can conclude one can conclude that any network can [Noise] any network can which consist of passive elements like resistance inductance and capacitance shall be shall have an input impedance and driving point impedance which is positive real and this is why our interest in positive real functions the driving point functions of passive networks are positive real

now um we have not yet proved it we have only said that of an inductance is positive real impedance of a capacitor is positive real impedance of a resistor {capa}((00:09:22)) is positive real and after it transform and contains three inductances and therefore the impedance should also be positive real a combination of them which is contained in N it appears to reason it appears to {intrusion}((00:09:40)) that it should also be positive real but let's prove it lets make a rigorous proof and look at the proof very interesting proof

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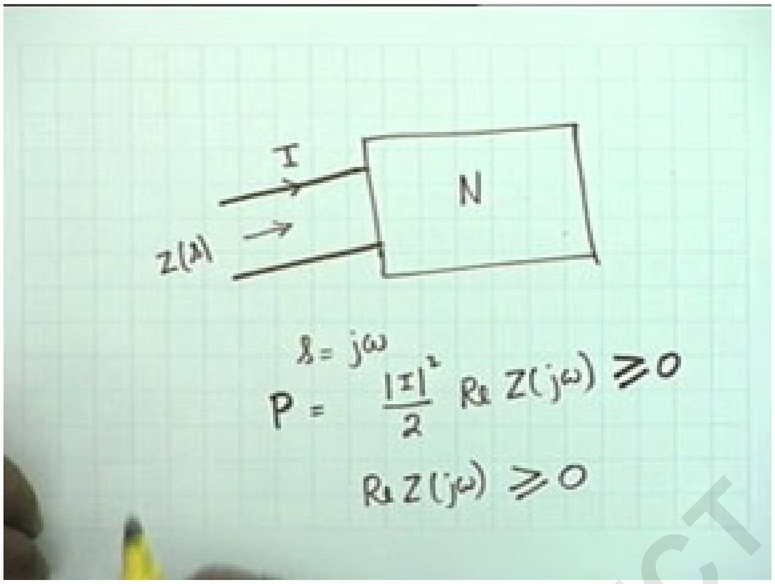

what we do is let the input impedance be Z of s okay let the input impedance is Z of s and let the current going in be capital I agreed then for sinusoidal excitation that is s equal to j omega for sinusoidal excitation the power that is absorbed by the network the power that goes into the network shall be magnitude I square divided by two capital I is the peak value peak vector magnitude I square divide by two this is the root mean square current multiplied by [Conversation between Student and Professor ((00: 10: 32 min))] no the real part it is the resisted part which absorbs power the reactive part is the reactive part so this will be the real part of Z of j omega and since the network is passive this would be greater than equal to zero which means that the real part of Z of j omega is greater than equal to zero agreed this is the first part of the [noise] first part of the proof

the second part well we have not proved anything we have simple said that the real part on the j omega axis should be non negative real part of the j omega axis should be non negative that doesn't mean that it should be non negative ah over the whole right half plane it doesn't mean that

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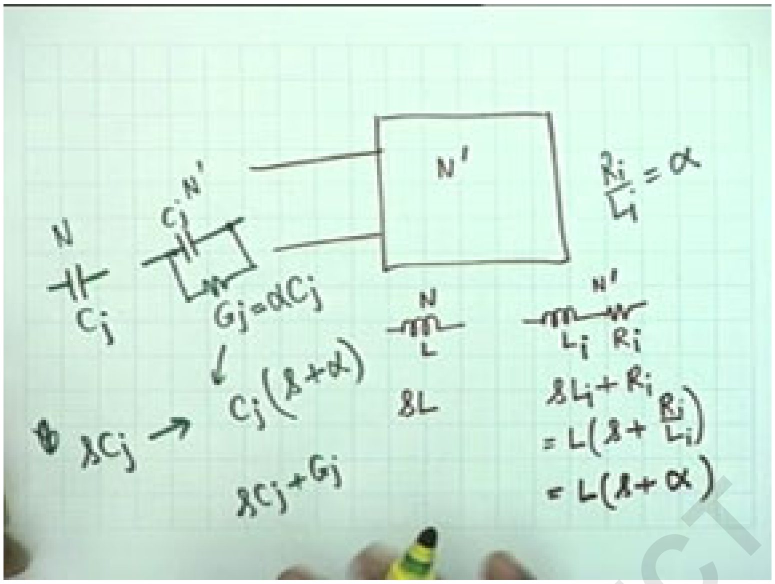

now to prove that part that part this is only a partial proof to prove that part let us let us modify capital N let us modify capital N to N prime such that such that what was an inductance in N is now what was a pure inductance N [Noise] let us add in N prime an inductance L in sinusoidal resistance R okay that is N prime we'll make it into a more losing network All right

in sinusoidal every inductance we put a resistance such that the impedance of this element impedance of this element in N was s L here it is s L plus R and i can write this is as L s plus [Noise] R by L agreed i can write this is as L s plus alpha let's say

and let let the value of the alpha be the same for each inductor in other words what I am saying is each inductor Li where i can go from zero to where i can go from one to n let's say each inductor Li is augmented by resistance Ri in such a manner that Ri by Li is a constant equal to alpha

we can do that every inductor that i find i add a resistance alpha times Li that's all that it means in series i add a resistance All right

similarly every capacitor C in N is augmented every capacitor let's say Cj is augmented in N prime by a shunt conductance Gj such that that is i make each capacitor losing each capacitor in the N and a lose is introduced in such a manner that Gj is equal to alpha times Cj All right

this alpha and this alpha is the same we can do that okay we are taking we take a box of resistor and just go on adding resistor across each capacitor and serializing each inductor we can do that now then the capacitance the admittance of this capacitor which was S Cj now becomes now becomes Cj times S plus alpha is that clear it becomes S Cj plus Gj S Cj plus Gj i take Cj common therefore I get Cj times S plus alpha is that okay

resistors in N will deep in-tagged that is resistor R K in N goes as resistor R K in N prime

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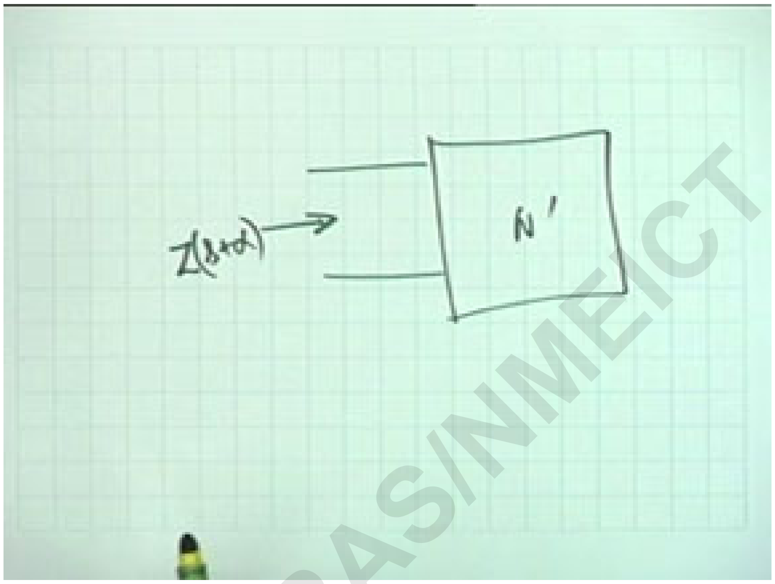

then what happens to the input impedance of N prime what happens to input impedance to N prime

you see the only change that has happened is wherever s occurred in N s should be replaced s plus alpha and therefore is this point clear whenever s occurs the impedance of inductor it is L times s plus alpha in admittance of a capacitor it [Noise] is C time s plus alpha therefore [Conversation between Student and Professor – Not audible ((00: 15:20 min))] [Laughter-our objective will be clear] in a moment

we are trying to derive another network N prime whose impedance now instead of s it shall be[Conversation between Student and Professor – Not audible ((00: 15:34 min))] s plus alpha this is what we wanted to derive that is from the given network N whose {impi}((00:15:41)) input impedance [Noise] is Z of s we want to derive a network whose input impedance is Z of s plus alpha

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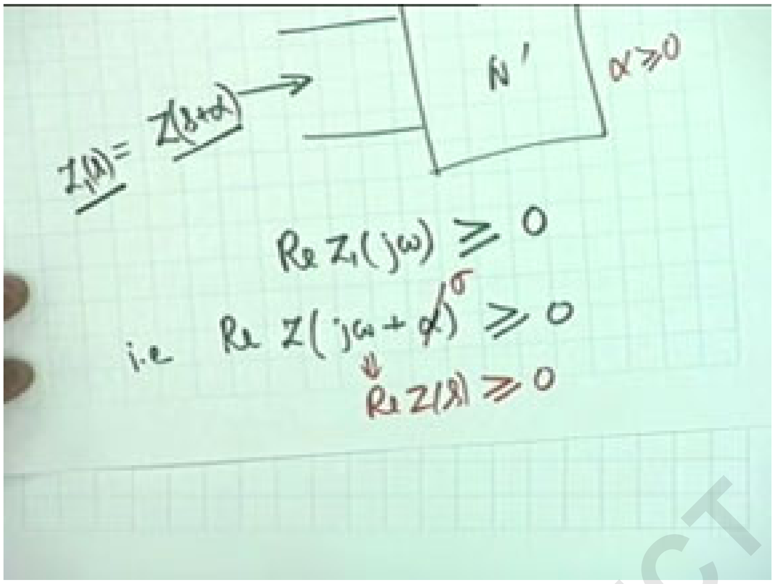

let's call this as Z one of s All right then by the same token by the same token as was applied to capital N we must have real part of Z one of j omega as greater than equal to zero is that right for N the real part on the j omega axis was greater than equal to zero there is no reason why N prime should not also have the same property all we have done is added a few resistors we have not changed the character of the network right

therefore the real part of Z one of j omega is greater than equal to zero which means that the real part that is real part Z of j omega plus alpha is greater than equal to zero clear because what is Z one of s it is Z of s plus alpha therefore this means that the real part of Z of j omega plus alpha is the is greater than equal to zero

i argue now that alpha was a purely arbitrary quantity what was the constraint on alpha that alpha should be non-negative i can not add a negative resistor

{alphaf}((00:17:07)) the resistor that I add with parallel with a capacitor or in series with an inductor the only constraint is that alpha should be greater than equal to zero

if alpha is equal to zero we get the original network if alpha is non-zero we get a derived network and therefore alpha is not a separate notation we could as well replace this by sigma where the only qualification of the sigma is that sigma should be real and non-negative

but what is sigma plus j omega isn't that precisely [Conversation between Student and Professor – Not audible ((00: 17:43 min))] s therefore what we get from this derivation is that real part of Z of s should be greater than equal to zero is that clear is a very beautiful uh derivation originally due to cover we will have cover a (jarment jentlepen) ((00:17:59 min))

now what does it mean it means that Z of s is real for s real and real part of Z of s is greater than equal to zero and therefore Z of s which is [Noise] the driving point impedance of an arbitrary passive network we have specified nothing we only said it contains resistors capacitor and inductor nothing else

it could also contains transformers because transformers have combinations of three inductors and therefore we have proved that the driven point impedance of a passive network is positive real and this is why we are interested in positive real functions the connection between the circuit theory and positive real function is through driving [Noise] point impedance [Noise]

now it takes a moment to justify that this property of positive realness also [Noise] applies to a driving point admittance function that is[Conversation between Student and Professor – Not audible ((00: 18:56 min))] yes [Conversation between Student and Professor – Not audible ((00: 18:58 min))]would you please explain the proof [Laughter-okay]

what we did was let's go back uh what we did was we took an arbitrary network N which contains resistors conductors and capacitors and we argued that the input impedance Z of s its real part on the j omega axis must be positive why because the power absorbed by the network must be positive must be non-negative okay

so the real part of Z of j omega [Noise] is greater than equal to zero from N when we derive a network from N we derive a network okay by adding resistors in serial with inductor and resistor in parallel capacitor in such a manner that instead of Z of s we get Z of s plus alpha okay

that's what we did here in N prime we added conductances in parallel capacitor and resistor in series inductors in such a manner that each s L now becomes L times s plus alpha and each s C becomes C times s plus alpha and therefore all that changes in N prime is that s is replaced by s plus alpha

[Conversation between Student and Professor – Not audible ((00: 20:26 min))] pardon me [Conversation between Student and Professor – Not audible ((00: 20:28 min))] if the network is not passive than of course ((we cannot)) (00: 20:32 min) this argument shall not be valid if capital p is non is can be negative then this argument is not valid yes

[Conversation between Student and Professor – Not audible ((00: 20:40 min))] that's right [Conversation between Student and Professor – Not audible ((00: 20:46 min))] why cannot oh because the power in a sinusoidal excitation the power is absorbed normally in the resistive components the reactive power is of no concern the power you see the reactive power can be positive as well as negative and there is an exchange between inductor and capacitors overall the average [Noise] reactivity [ ]((00:21:11)) but the average power absorbed in the resistor will be non-negative okay so the it is on the passivity is on the basis of power absorbed by the network All right and ((theref)) (00: 21:27 min) that's why it was it was on the j omega axis

[Conversation between Student and Professor – Not audible ((00: 21:30 min))] um-huh a non sinusoidal input can always be broken up into [Conversation between Student and Professor – Not audible ((00: 21:36 min))] sinusoidal input and therefore it would be a sum of such terms okay

any other questions

so what we did was we argued that the real part of Z of j omega plus alpha is non-negative and we argued that the alpha was purely our choice [Noise] alpha could be anything any non-negative value is admissible and therefore we could replace it by sigma where sigma is greater than equal to zero when sigma is greater than equal to zero then sigma plus j omega is [Noise] [ ]((00: 22:08))

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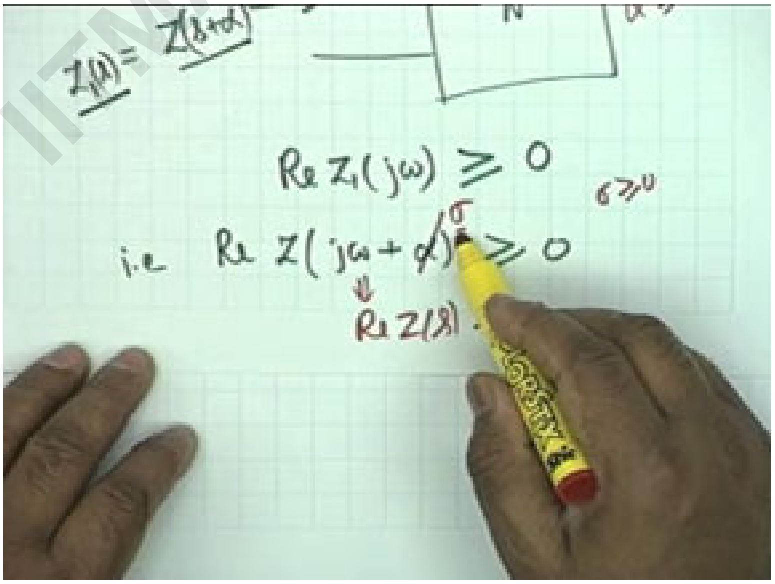

and therefore real part of it is s it is s with sigma greater than equal to zero is the right half plane omega could be positive or negative it doesn't matter

and therefore when s is in the right half plane then the sigma greater than equal to zero obviously the real part of Z of s will greater than equal to zero which shows

[Conversation between Student and Professor – Not audible ((00: 22:31 min))] no it didn't say

[Conversation between Student and Professor – Not audible ((00: 21:37 min))] only in the right half plane not all values

you see a sigma here alpha was positive quantity we could not add a negative resistor okay so we prove that Z of s the driving point impedance of a passive network is always positive real

now we want to extend it to driving point admittance

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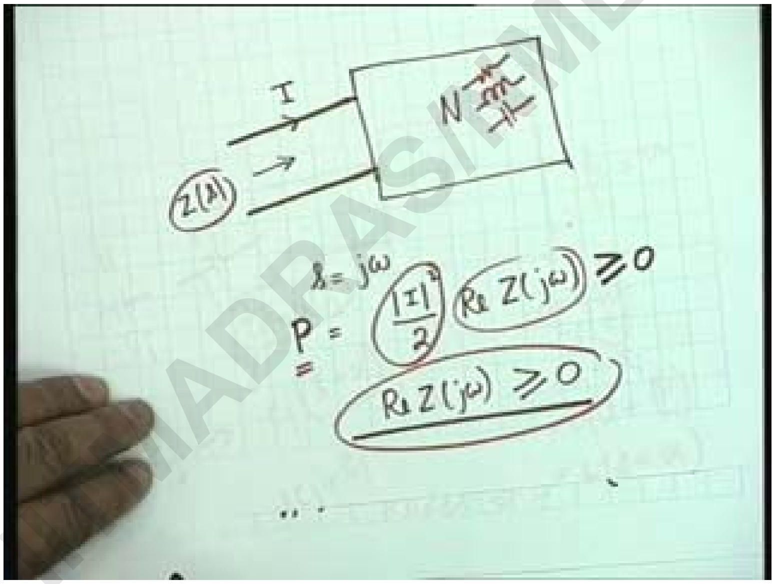

and all that require all that required is to prove a theorem that if F of s is P R [Conversation between Student and Professor – Not audible ((00: 23:09 min))] yes [Conversation between Student and Professor – Not audible ((00: 23:11 min))] two things [Conversation between Student and Professor – Not audible ((00: 23:14 min))] okay [Conversation between Student and Professor – Not audible ((00: 23:16 min))] oh we don't have to prove it because Z of s after all is made up of a sum one by s C and R each of them are real for s real and it'll only be a series-parallel and multiple series-parallel combination so all of them all of these quantity would be real for s real okay that is trivial but this is not trivial that real part of Z of s is non-negative is not a trivial proof

any other question

[Conversation between Student and Professor – Not audible ((00: 23:49 min))] why did you say that real part of Z one j omega is {great}((00: 23:56 min)) than [Conversation between Student and Professor – Not audible ((00: 23:57 min))] is the first case because the power has to be non-negative okay

let's go back this has to be non-negative and this is the positive quantity magnitude square divided by two and therefore this must be non-negative [Noise]

anything else [Noise]

i appreciate these questions because first time one {make}((00:24:20)) synthesis synthesis does appear to be tough but it's a very simple once you get over this fear of complex variables and this {func}((00:24:28)) the functions are extremely simple form very nicely behaved well behaved functions nothing to be scared of okay

All right as i said Z of s the driving point impedance function is positive real we want to extend it to driving point admittance function all that is needed is to prove this theorem

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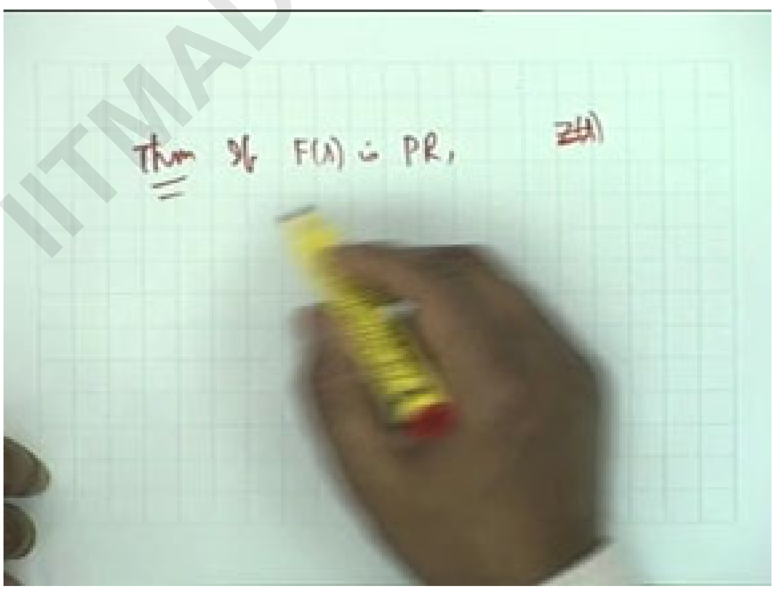

that the F of s is P R then so is one by F of s that is if impedance is P R then the admittance also be PR and it is extremely simple to prove

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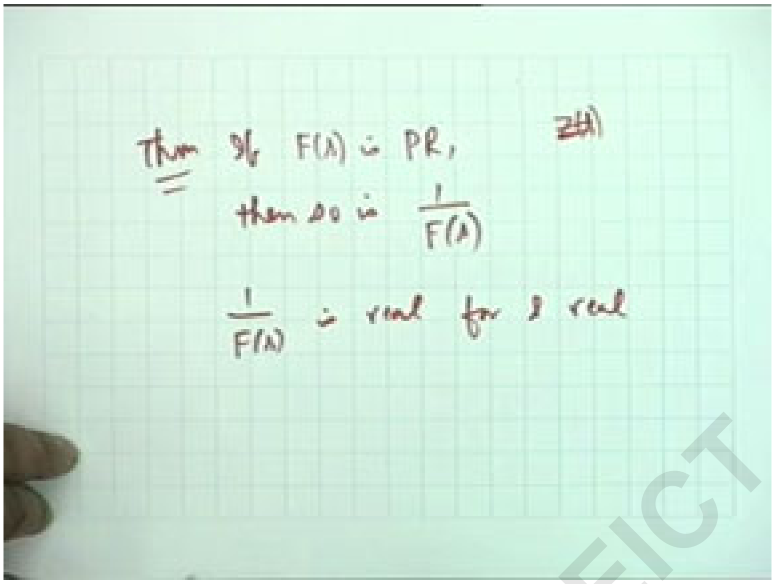

you see if F of s is PR then obviously one by F of s is real for s real agreed because F of s is real for s real

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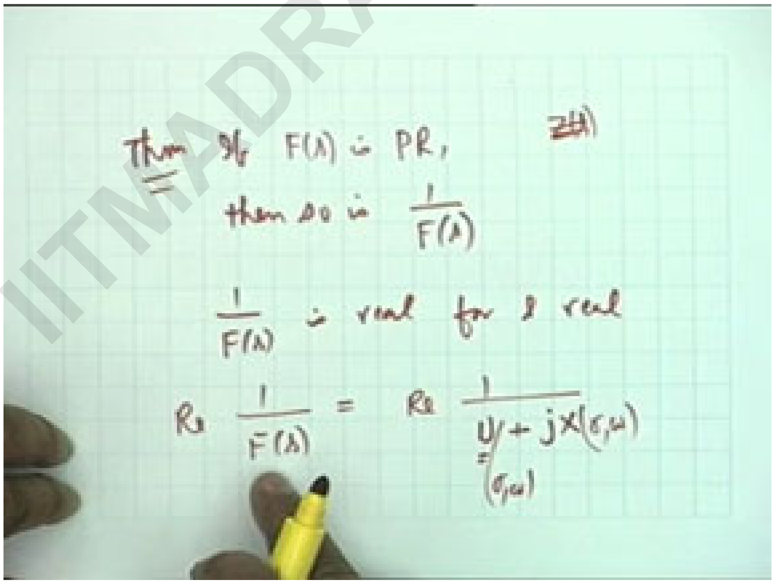

and the the other thing that we have to prove is real part of one by F of s okay

suppose [Noise] suppose F of s is U plus j X U is its real part and X is its imaginary part

since it's a function of s U is a function of sigma and omega it's a two variable function similarly X is a function of sigma and omega you see F of s s is sigma plus j omega I break this up this complex function into a real part and an imaginary part

the real part shall be a sigma and omega imaginary part shall also be a function of sigma and omega

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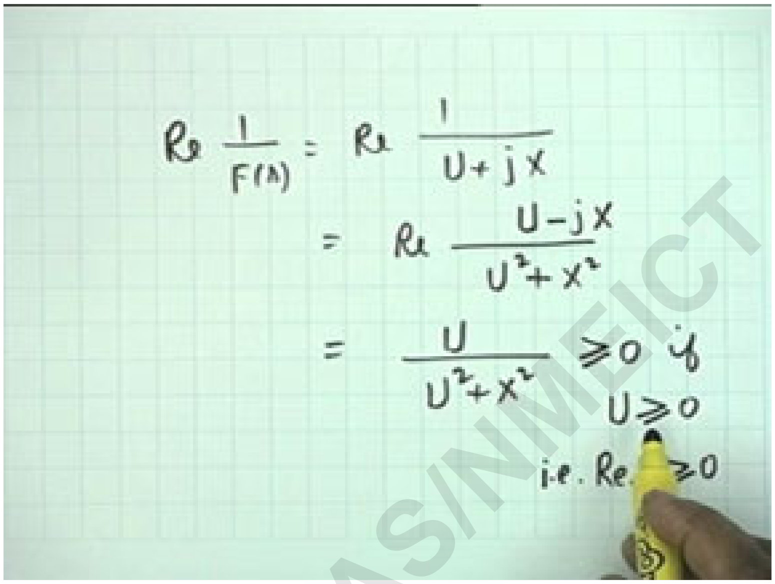

so what is the real part [noise] real part of one over F of s it is real part of one over U plus j X which is equal to real part of U square plus X square and in the numerator [Noise] it is U

[Conversation between Student and Professor – Not audible ((00: 26:24 min))] pardon me [Conversation between Student and Professor – Not audible ((00: 26:26 min))] okay U minus j X this is equal to U divided by U square plus X square All right

and obviously this is non-negative if U is non-negative

when is U non-negative that is real part of s non-negative is that clear because by hypothesis F of s is P R so its real part is non-negative for real part of s non-negative okay

therefore this quantity is non-negative if U is non-negative but U is non-negative in turn when sigma is non negative [Noise] therefore we prove that the real part of this reciprocal function is non-negative for the right half plane and therefore one by F of s is P R

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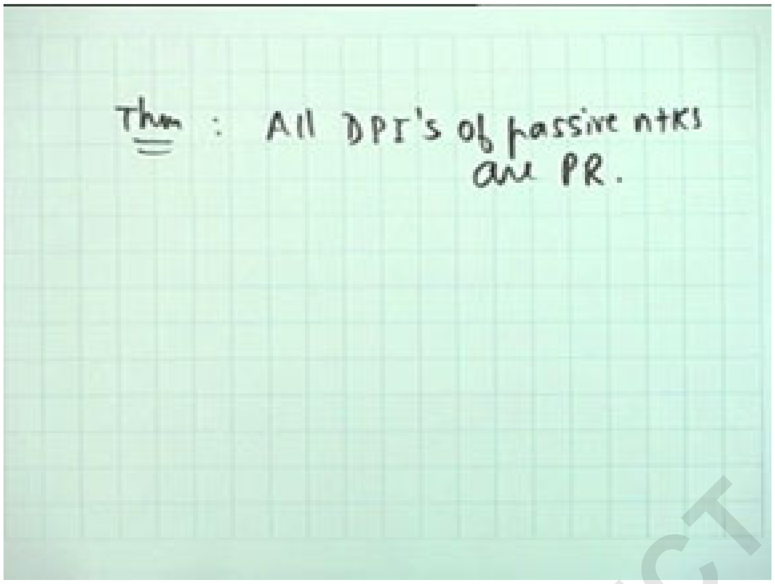

and therefore our theorem now states that all DPIs and this I is now an immittance i m m i t t a n c e it is impedance as well as admittance all DPIs of passive networks are positive real okay

now the question of testing that before i test let's look at some useful properties

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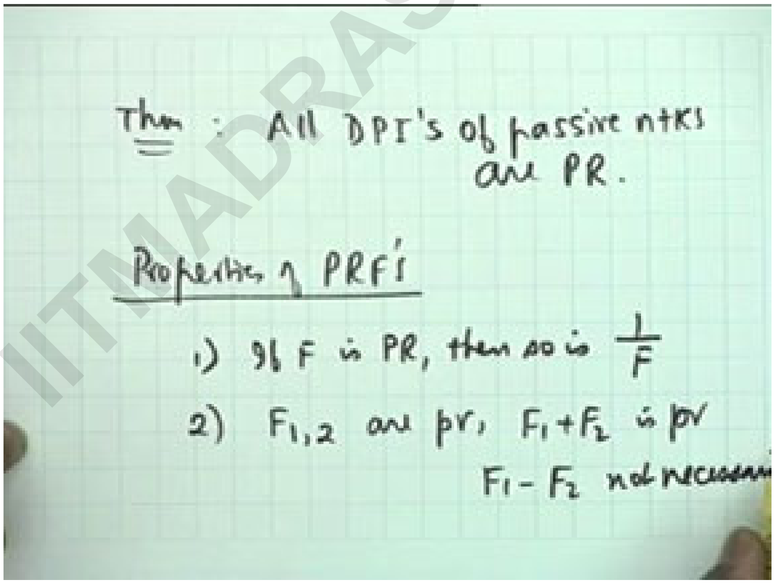

properties of positive real functions some useful properties

one of them that i have seen is if F is P R then so is one over F this we have already proved that's how we we stated the theorem that all DPIs are all DPIs of passive networks are P R suppose F one and F two are P r then what can you say about F one plus F two [Conversation between Student and Professor – Not audible ((00:28:46 min))] that is also P r the proof is trivial because real part of F is equal to real part F one plus real part F two and both of them are individually [Noise] non-negative so the sum is non-negative

what can you say about F one minus F two [Conversation between Student and Professor – Not audible ((00: 29:03 min))] not necessarily P r

because the real part due to subtraction may become negative not necessarily P r

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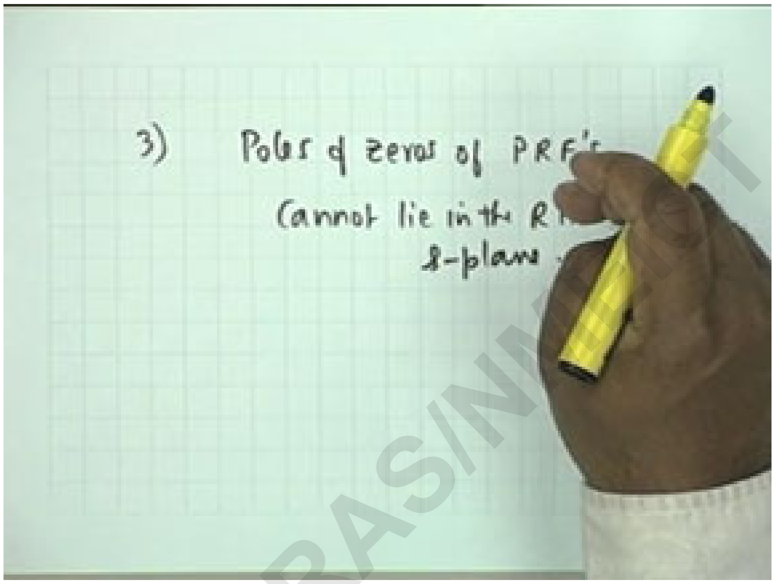

All right next poles and zeros of positive real functions

where are they okay cannot lie in the in the right half plane why not [Noise] right half s plane why not poles and zeros of positive real function cannot lie in the right half of the s plane what is the reason

[Conversation between Student and Professor – Not audible ((00: 30:00 min))] pardon me [Conversation between Student and Professor – Not audible ((00: 30:03 min))] which polynomial it has something to do with the real part it is a passive network you see all positive real functions this is interchangeable impedance admittance or positive real functions all driving per {limittences}((00:30:20)) are PRFs it can be shown that all PRFs can be realized by driving point {limittences}((00:30:26)) of passive network

now if there is pole in the right half plane what does it mean it means that it means instability that it's a real part real part is no longer non negative in the right half plane what does instability mean that the output can grow indefinitely without an input that is the unstable condition All right

that is possible only when the when the network is able to generate power where as a passive network the power has always to be non-negative so they cannot lie in the right half right half of the s plane

[Conversation between Student and Professor – Not audible ((00: 31:07 min))] no that is not correct because for a transfer function which is not a P r not necessarily a P r that can be zero in the right half plane for a transfer function there can be

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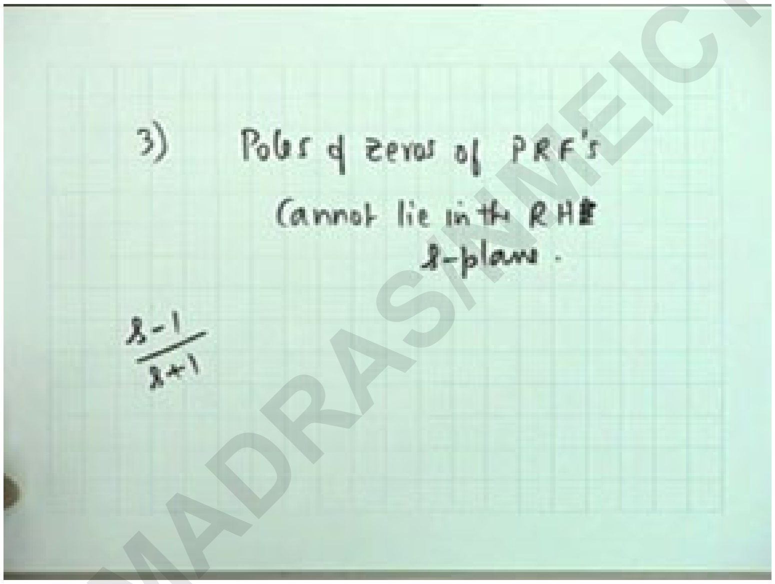

for example i can have a function s minus one [Noise] by s plus one it is an all pass function as you know the poles and zero are in mirror image symmetry did you do this in the class [Conversation between Student and Professor – Not audible ((00: 31:37 min))] we have okay

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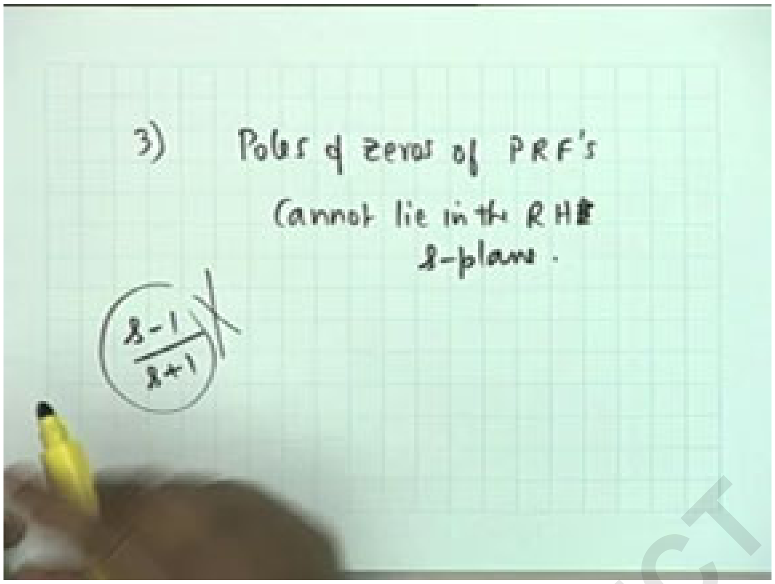

but it is not P r if it is a positive real function then the function as well as its reciprocal have to be P r and therefore we cannot have poles or zeros in the right half plane okay

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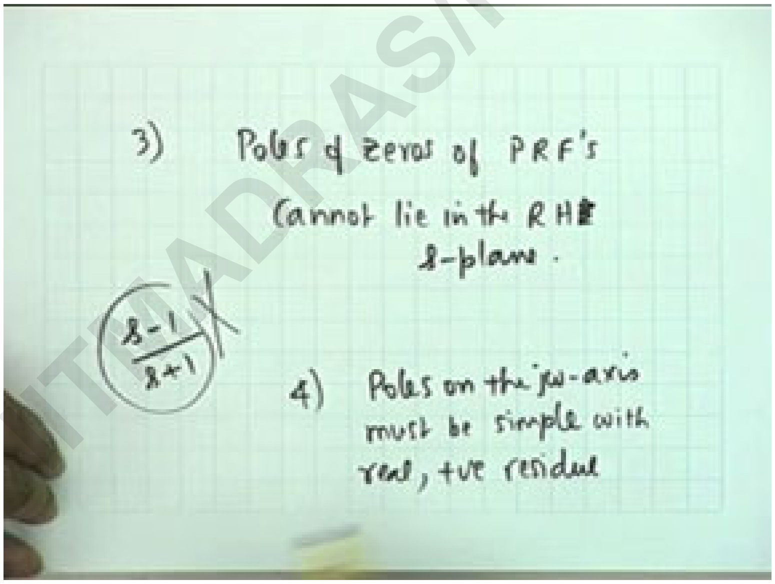

number four poles on the j omega axis this is important and we shall we shall go slow on this

poles on the j omega axis must be simple why simple because if they are multiple then it leads to instability instability okay poles on the j omega axis must be simple now i'm adding something to this for a positive real function with [Noise] real positive residual with real positive residual

poles on the j omega axis must be simple with real positive residual okay

now why is it so let us look at this

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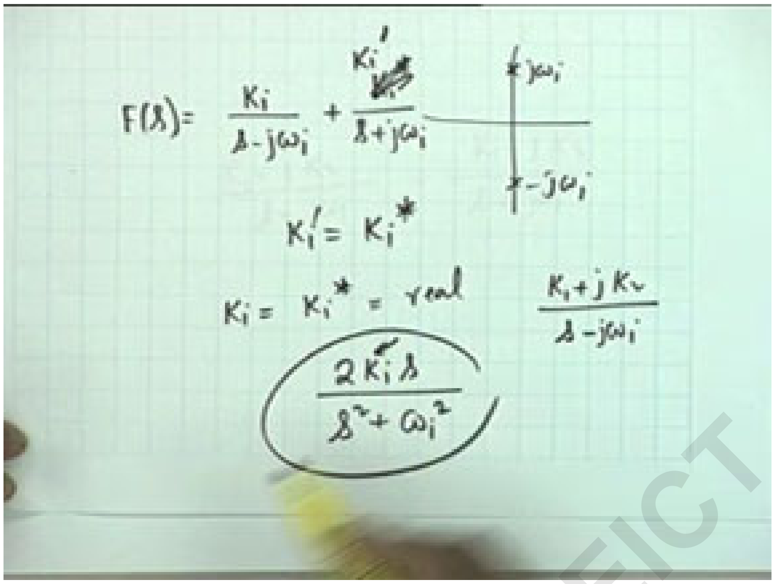

first we say why the residue should be real

but first let us see if we have a function F of s with let's say two poles one at j omega i and the other at minus j omega i while complex poles must occur in conjugate pairs why [Conversation between Student and Professor – Not audible ((00: 33:01 min))] because its function is real for s real okay

so if j omega i is a pole minus j omega it should also be a pole and in F of s if i expand in partial fraction we shall have Ki divided by S minus j omega i that would be the partial fraction corresponding to j omega i

and for corresponding to minus j omega i it should be s plus j omega i then a residue which has to be complex conjugate of this [Noise] residue why is it so that is this residue must be Ki star

why

is the point clear[Conversation between Student and Professor – Not audible ((00: 33:48 min))] okay i could let's let's write this as Ki prime

let's write this as Ki prime now you add this two terms [Conversation between Student and Professor – Not audible ((00: 33:59 min))] pardon me [Conversation between Student and Professor – Not audible ((00: 34:02 min))] when you add them up the function must be real function and it cannot be real unless Ki prime is equal to Ki star is this point clear [noise] therefore therefore and in addition it says in addition it says that Ki must be real if it is real i will tell you why it why it cannot be complex in a minute [Conversation between Student and Professor – Not audible ((00: 34:29 min))] uh-huh if Ki is real obviously both the residue should be same identical let's assume for a moment that there we are

if they are real then obviously if Ki is equal to Ki star equal to real then obviously the sum of this two terms would be twice Ki s divided by s square plus omega i square

and this shows that Ki the residual must be real otherwise it cannot give rise to a real function okay is that clear [Noise]

suppose for argument sake let we do have the residue which is complex let's say we have K one plus j K two divided by divided by s plus let's say [noise] s minus j omega i

if i combine this with K one minus j K two will be sum be here

[Conversation between Student and Professor – Not audible ((00: 35:31 min))] pardon me [Conversation between Student and Professor – Not audible ((00: 35:33min))] it will be real [Conversation between Student and Professor – Not audible ((00: 35:35min))] pardon me [Conversation between Student and Professor – Not audible ((00: 35:42min))] okay in other words i can have a residue like this K one plus j K two divided by s plus j omega i plus K one minus j K two divided by s plus j omega i [Conversation between Student and Professor – Not audible ((00: 36:03min))] no some shall be negative [Conversation between Student and Professor – Not audible ((00: 36:07min))] it cannot be negative why not [Conversation between Student and Professor – Not audible ((00: 36:15min))] okay

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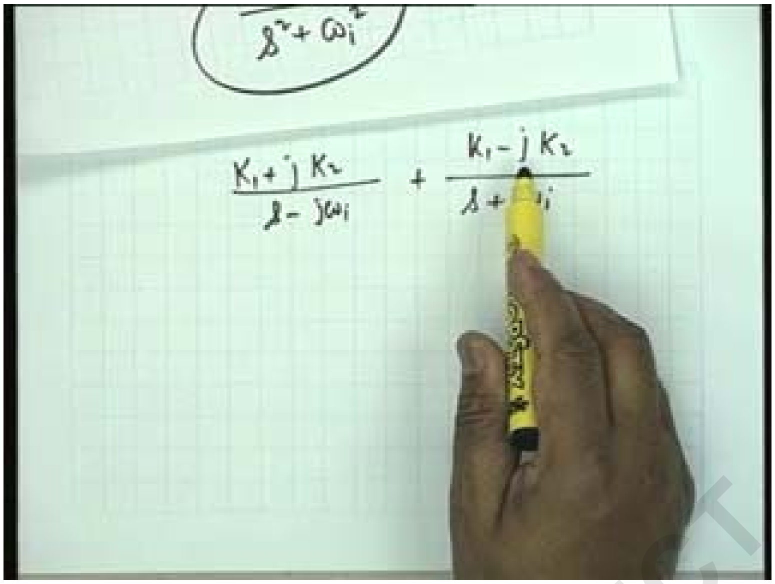

same as the last [Noise] now the sum of these two term whether you take it in this form or in this form the sum of these two term shall be this

this Ki can this be complex [Conversation between Student and Professor – Not audible ((00: 36:38 min))] no it cannot be complex okay for a pair of poles on the j omega axis Ki is the residue okay Ki is the residue that is Ki is equal to the function let me write in this next page [Conversation between Student and Professor – Not audible ((00: 36:58min))] uh this point is not clear okay

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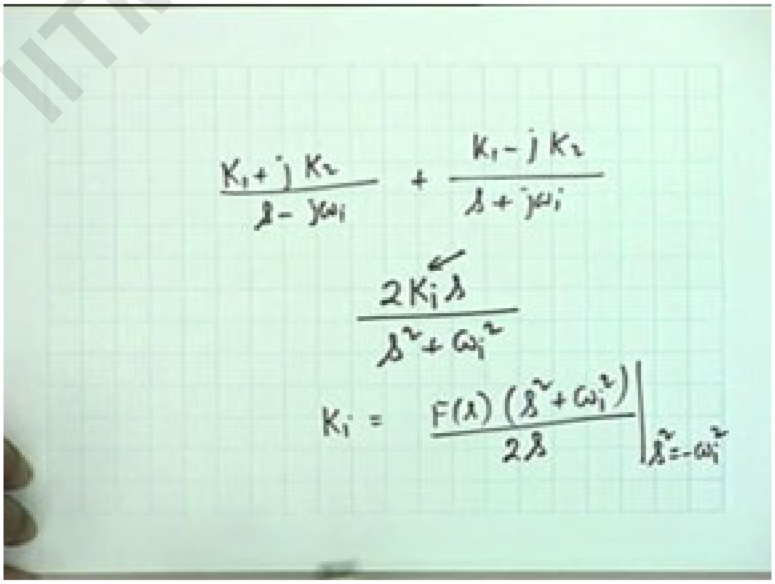

if you add this two you can get a real quantity and the quantity must be of these form two Ki s divided by s square plus omega i square isn't that right

where this Ki cannot be complex it must be real if it is be complex then the whole quantity will be complex it cannot be a real function

and Ki as you see is given by F of s multiplied by s square plus omega i square divided by s twice s at s square equal to minus omega i square now this point not clear [Conversation between Student and Professor – Not audible ((00: 37:48 min))] no [Conversation between Student and Professor – Not audible ((00: 37:51 min))] yes [Conversation between Student and Professor – Not audible ((00: 37:52 min))] okay [Conversation between Student and Professor – Not audible ((00: 37:59 min))]

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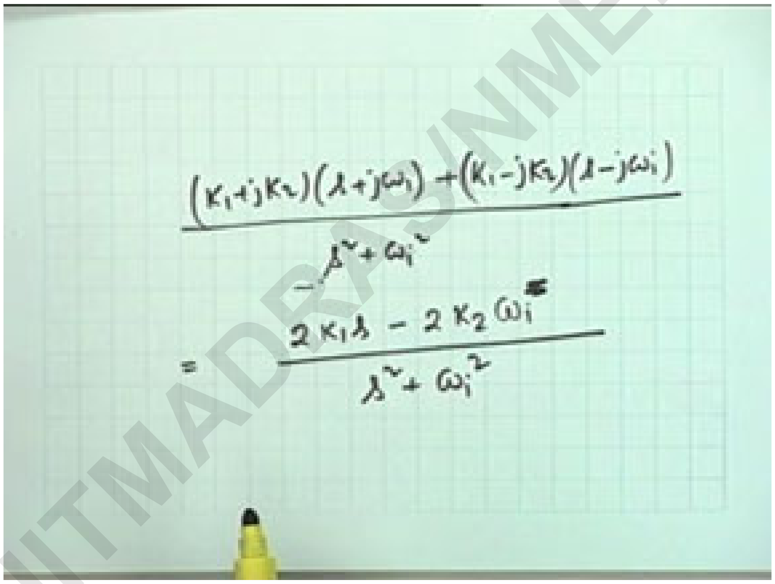

fine let's let's clear this s square plus omega i square i get K one plus j K two s plus j omega i plus K one minus j K two s minus j omega i okay

this is equal to K one s [Noise] K one s twice K one s [Conversation between Student and Professor – Not audible ((00: 38:27 min))]

minus twice K two [Conversation between Student and Professor – Not audible ((00: 38:29 min))] omega i square [Conversation between Student and Professor – Not audible ((00: 38:34 min))] no only omega i divided by s square plus omega i square

[Conversation between Student and Professor – Not audible ((00: 38:43 min))] [Noise] k when s is real the function is real okay but [Conversation between Student and Professor – Not audible ((00: 38:55 min))] the real part [Noise] real part real part of this function can be negative

when s equals to j omega for example look at this when s equals to j omega this contributes to an imaginary part this contributes to a real part and depending on K two it can be positive or negative [Conversation between Student and Professor – Not audible ((00: 39:21 min))] [laughter] pardon me [Conversation between Student and Professor – Not audible ((00: 39:23 min))] then {property}((00:39:24)) is not satisfied

so the this cannot be allowed in other words the residue K one [Conversation between Student and Professor – Not audible ((00: 39:32 min))] should be positive and real

this is the proof we did it in a slightly round about fashion but [Conversation between Student and Professor – Not audible ((00: 39:40 min))] K one must be positive and real it cannot be [Conversation between Student and Professor – Not audible ((00: 39:44 min))] no it is not sufficient i coming to the [Conversation between Student and Professor – Not audible ((00: 39:49 min))] oh this is what i was going to {describe}((00:39:52)) this is what i was going to

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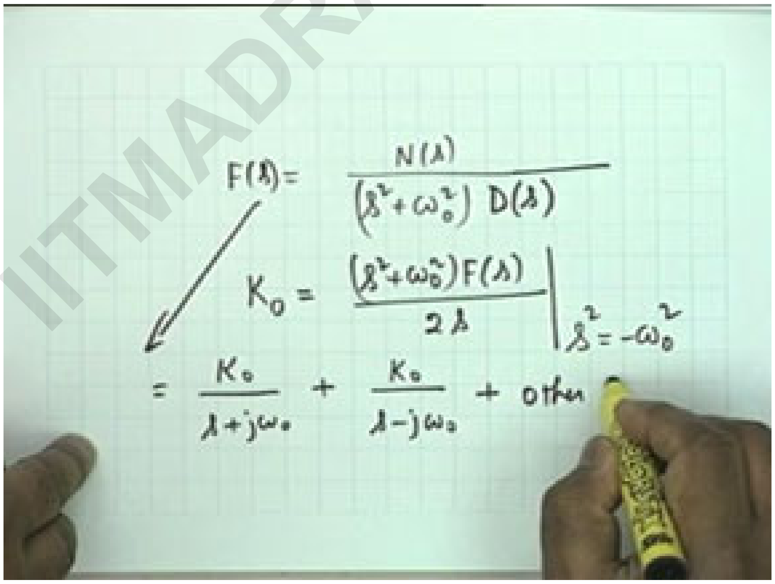

you see if F of s is let's say of this form some numerator divided by s square plus omega not square multiplied by let's say some polynomial D of s then obviously we have a pair of poles on the j omega axis and plus minus j omega not then the residue K not at this pole at either of the two poles is defined as s squared plus omega not squared multiplied by F of s divided by twice s at s squared equal to minus omega not square

which simply means that what we have done is that we had expanded F of s in partial fraction we have expanded this in K zero by s plus j omega not

plus K zero divided by s minus j omega not plus other terms that is what K zero K zero is called the residual at the pole at plus minus j omega not and this residue must be real and positive why positive

you are you are convinced about the realness of it okay why positive suppose K zero is negative suppose K zero is negative

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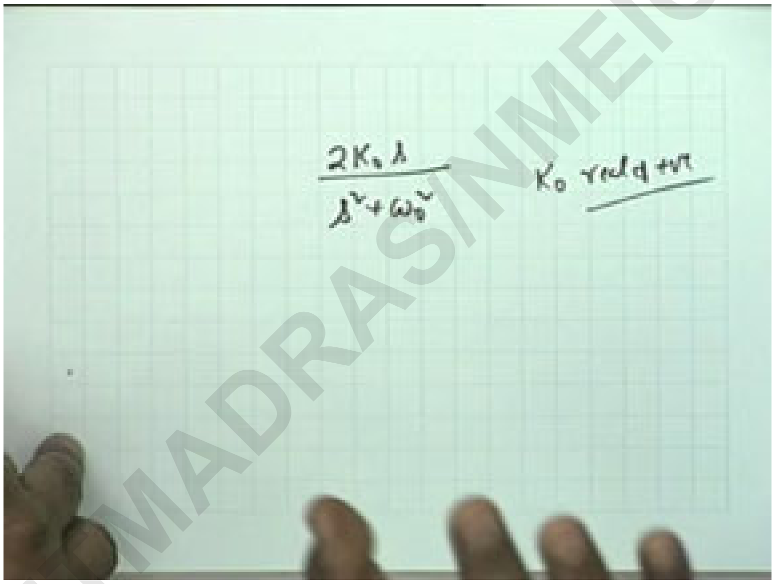

after all the term that comes is twice K zero s divided by s square plus omega not square [Conversation between Student and Professor – Not audible ((00: 41:34 min))] [Noise] I didn't follow [Conversation between Student and Professor – Not audible ((00: 41:46 min))] yeah [Conversation between Student and Professor – Not audible ((00: 41:49 min))] that is correct the real part condition should be validated and therefore K zero must be real and positive All right

this point must {so keen}((00:42:07)) that if a positive real {functio}((00:42:09)) [Conversation between Student and Professor – Not audible ((00: 42:09 min))] pardon me [Conversation between Student and Professor – Not audible ((00: 42:10 min))] oh what is the real part of this real part in the numerator ((00:42:18)) twice K zero multiplied by sigma and if K zero is negative obviously sigma non-negative will make the real part negative okay and therefore K zero must be real and positive [Noise]

the sum of this discussion sum and substance of this discussion is that if a positive real function has a pole on the j omega axis this pole cannot take any liberty it must be constraint

the constraint is that the pole must be simple number one and the residual at this pole must be real and positive okay if F of s if a positive real function has a pole on the j omega axis it must be simple and the residual there it must be real and positive

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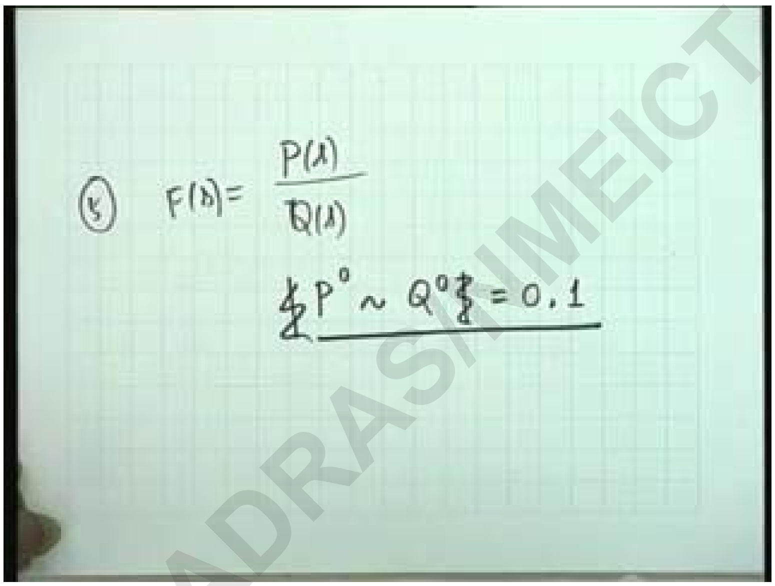

next suppose F of s [Conversation between Student and Professor – Not audible ((00: 43:19 min))] fifth property yes

suppose F of s now we we bring you a further restriction that our positive real function that we shall deal with are all rational if you have a finite number of resistors capacitors and inductors obviously the impedance function or admittance function or any function any network function has to be rational what is a rational function [Conversation between Student and Professor – Not audible ((00: 43:45 min))] what is P s and what is Q s [Conversation between Student and Professor – Not audible ((00: 43:50 min))] both are what is a polynomial [Conversation between Student and Professor – Not audible ((00: 43:53 min))] polynomials [Conversation between Student and Professor – Not audible ((00: 43:58 min))] that's it a polynomial you are not saying the key thing [Conversation between Student and Professor – Not audible ((00: 44:05 min))] what is the number of terms in the polynomial [Conversation between Student and Professor –

Not audible ((00: 43:11 min))] so it is a finite series that is the essential [Laughter-part] of the definition it's a finite series containing positive integer [Noise] powers of the variable okay

now the question is what can be the difference in degree between P and Q we use the simple [Noise] definition

degree of P is simply written as P and the temperature degree okay

what can be the difference between the degrees of P and Q [Conversation between Student and Professor – Not audible ((00: 44:40 min))]

the difference so i don't have to prove this side this is this is equal to what are the possible values [Conversation between Student and Professor – Not audible ((00: 44:47 min))] can it be zero can this degree be equal [Conversation between Student and Professor – Not audible ((00: 44:51 min))] yes if the degrees are equal then what is the behavior at infinity [Conversation between Student and Professor – Not audible ((00: 44:57 min))] a constant okay

now can the degree be differ by one [Conversation between Student and Professor – Not audible ((00: 45:01 min))] yes then what is the behavior at infinity a pole or a zero now what is a pole of F of s is a zero of one over F of s

and what is a zero of F of s is a pole of one over F of s

therefore a difference in degree of one is permitted what about two

no because then we shall have multiple pole or zeros at infinity so the degree difference between P and Q can be at the most one zero or one All right

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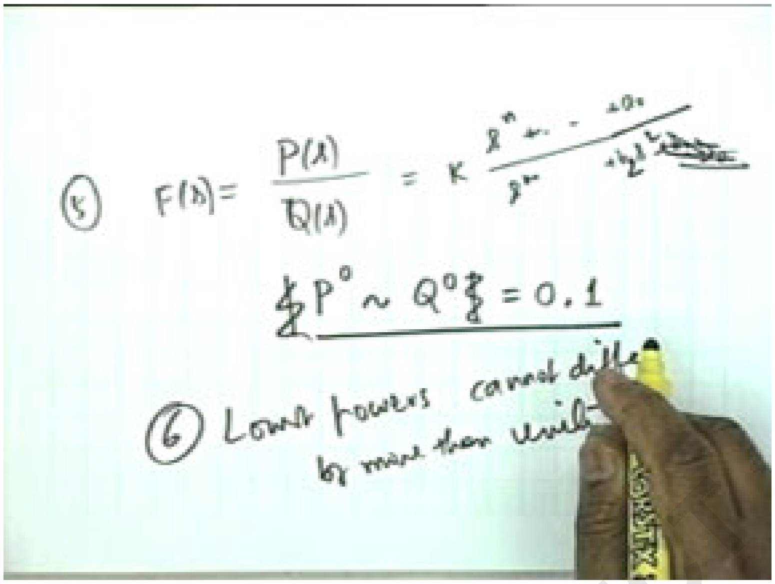

what does this mean this means that the highest powers if i write this as s to the n plus etcetera and s to the n k then it means that the highest powers can differ only by zero or one All right they cannot differ by more than one

now m can be greater than n or n can be greater than m it doesn't matter

what can you say about the lowest powers suppose the constant term here s zero and there is no constant term here suppose the final term is b one s is this permitted is it permitted to have b zero equal to zero [Conversation between Student and Professor – Not audible ((00: 46:26 min))] constant

suppose b zero is zero then we should have a pole at the origin if s zero is zero and b zero is non-zero then there is a zero at the origin can we have in the denominator b zero and b [Conversation between Student and Professor – Not audible ((00: 46:44 min))] that is correct [Conversation between Student and Professor – Not audible ((00: 46:46 min))]

what does this mean that the lowest powers in numerator and denominator cannot differ by more than one okay so this is the sixth property lowest powers in numerator and denominator cannot differ by more than unity All right

so if i combine the two if i combine the two i can say that if the function is positive real these are all necessary conditions they are not sufficient isn't that right even if these properties are true the function may not be positive real because we have said the mapping of the real part yet therefore these are all necessary condition not sufficient but we can say that the lowest power as well as the highest power cannot differ by more than one for example [Conversation between the Student and Professor-Not audible ((00: 47:43 min))] pardon me [Conversation between the Student and Professor-Not audible ((00: 47:45 min))] why it is so okay let's take some example

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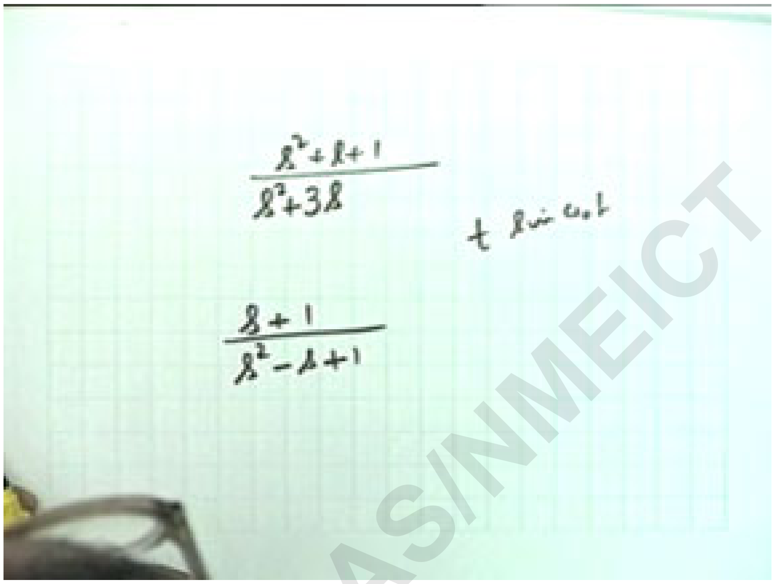

suppose I have s square plus s plus one divided by let's say three s square s squared plus three s then obviously at s equal to zero the function is a pole

on the other hand if three s is also absent then also it is a pole [Conversation between Student and Professor – Not audible ((00:48:15 min))] double pole and therefore the lowest powers cannot differ by more than one

is the point clear

let's take some example [Conversation between Student and Professor – Not audible ((00: 48:24 min))] oh that we have already [Conversation between Student and Professor – Not audible ((00: 48:29 min))] elaborated if it is not

okay what does the pole on j omega axis cause in the time domain time domain it is either a sine function or a cosine function if there a multiple poles for example pole of double order then this is multiplied by t [Conversation between Student and Professor – Not audible ((00: 48:47 min))] from definition of PRF well instability means that the real part condition shall be valid

instability means that the {rea}((00:48:57)) that’s the power can be negative okay All right

now let’s take a few examples [Noise] suppose i take s plus one divided by s square minus s plus one can this be P r [Conversation between Student and Professor – Not audible ((00: 49:17 min))] this can be P s [laughter]

then tricky question is it the denominator there is a negative sign what does it mean there must be poles in the right half plane that means this this polynomial is not {high rates}((00:49:34)) okay that is the point that i wanted to bring

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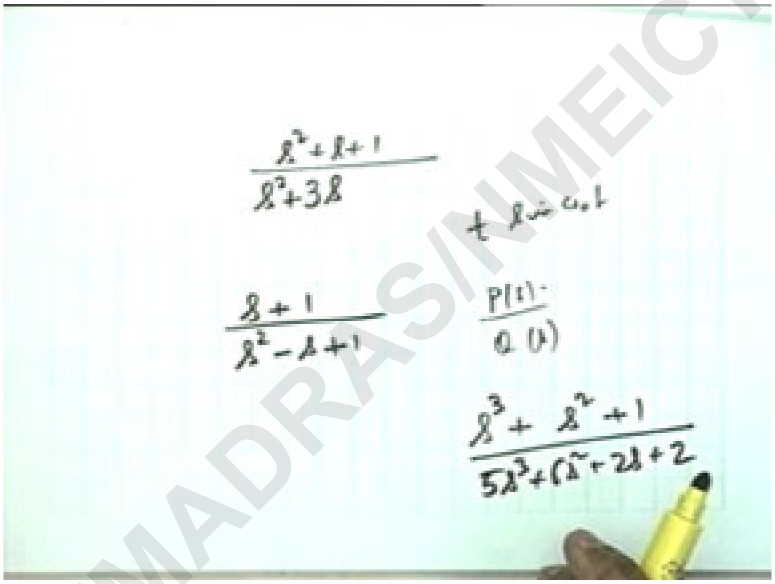

if i write a network function as P of s by Q of s one of the necessary property is that both P and Q must be {high rates}((00:49:47)) okay must be {high rates}((00:49:49)) All right

suppose i have s cubed plus s squared plus one divided by let’s say five s cubed plus six s squared plus twice s plus two [Conversation between Student and Professor – Not audible ((00: 50:04 min))] [Noise] numerator is not {high rates}((00:50:06)) because there is a missing term

okay we will have more fun next time on examples and going into more details of positive real functions