

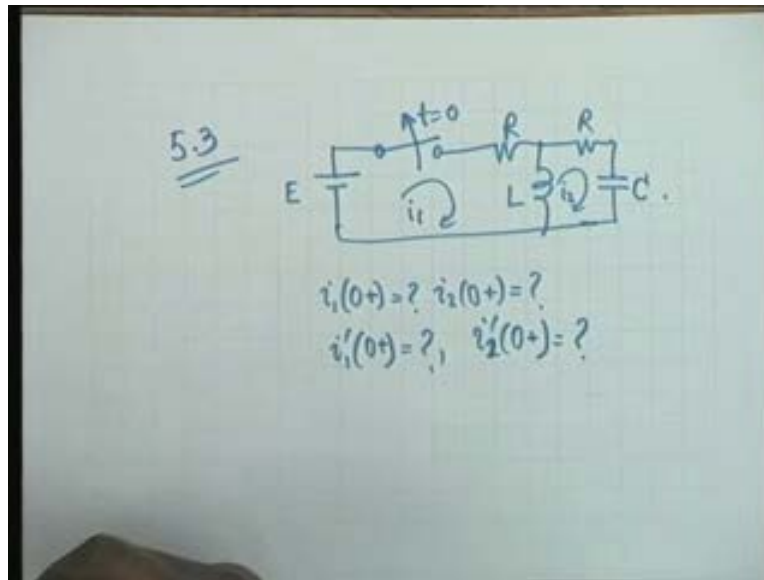
Circuit Theory
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Lecture - 4

Problem Session 1: Initial & Final Conditions, Analysis of Circuits

Welcome to the problem session one and in each problem session, I will circulate a set of problems, 8 to 10 problems. We will not have time to work out all of them. So I will work out selected ones. The rest of them, I would expect that each of you will work out, and most of the problems I have solved myself. So solutions are available but I will not give you the solutions till I see you, yourself working out the problems and these are the Tuesdays and Fridays that I shall be available. You can come and check the solutions. You can come and discuss the solutions and so on. We will start with 5 point 1 and 5 point 2. I have already done in the lecture class so you start with 5 point 3.

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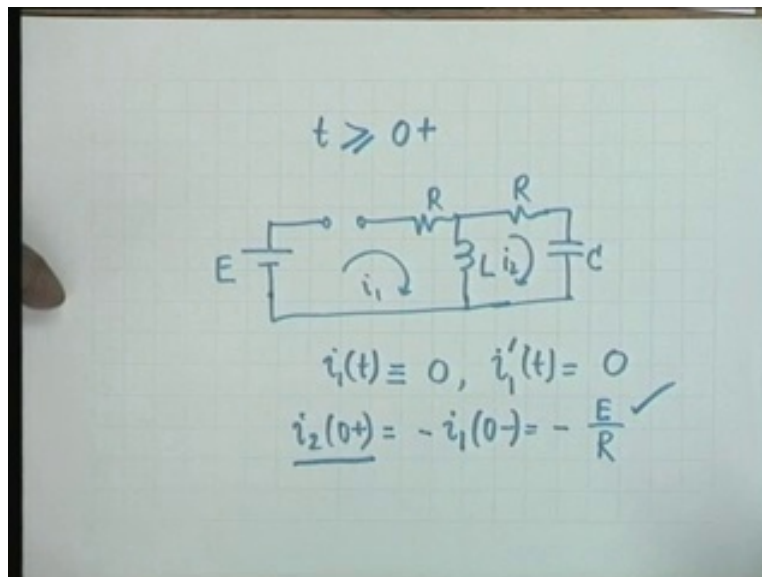


The problem is a network is given like this E, switch s is thrown out at t equal to 0, that is, it was closed earlier, it have been opened now and then you have a resistance and inductance Lm another resistance R and a capacitance C. This is the network. This current is i 2 and this current

is i_1 . The switch was initially on and it has reached a steady state and the switch is opened at t equal to 0. It is thrown out. Determine the initial conditions for the currents $i_1(t)$ and $i_2(t)$, that is, $i_1(0^+)$ and $i_2(0^+)$. You have to find out this and their derivatives. That means $i_1'(0^+)$ and $i_2'(0^+)$, these are the 4 quantities to be found out from the equation, from the network.

Now obviously, before the switch is thrown open, the conditions have reached steady state and therefore, $i_1(0^-)$ and $i_2(0^-)$ can be found out from physical inspection of the circuit. At t equal to 0^- , the inductor will act as a short circuit and the capacitor will act as an open circuit and therefore, if this is short circuit, then obviously $i_1(0^-)$, $i_1(0^+)$ would be equal to E divided by R . Is that clear? From physical considerations and $i_2(0^-)$ because the capacitor is open it has to be 0.

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Now regarding the primes, regarding the primes, what we have to do is, you have to find out the circuit at t greater than equal to 0^+ . Let us look at this circuit, t greater than equal to 0^+ . There is this voltage source E and then the switch has been thrown open, then a resistance R and inductance L . This is i_1 , and then another resistance R and the capacitance C and this is i_2 ,

obviously, $t \geq 0^+$, because the switch is opened, i_1 of t would be identically equal to 0 because of open circuit.

If i_1 of t is identically equal to 0, what can be i_1 prime of t ? Obviously, that is also 0, because it is a constant, so that is no problem. However i_2 0^+ , i_2 0^+ would be found out from considerations of continuity of current in an inductor. You see, the current in this inductor L has to be continuous from 0^- to 0^+ . Now, therefore i_2 0^+ would be equal to, minus, see i_1 and i_2 are in opposite directions, so minus i_1 0^- and you know what this is, minus E by R is this point clear.

Student: Excuse me sir, (...) sir, we should also write i_2 0^+ .

Sir: Yes, this is what I have written, i_2 0^- , no i_2 0^- , we have already said it is 0

Student: In case it is 0, then we would not have written?

Sir: Then we would have had something else here. You see, because it is a short circuit at $t = 0^-$, so the current through the inductor at 0^+ , which is in this direction, from bottom to up, is the negative for i_1 and therefore, it is minus i_1 0^- 0^- equal to minus E by R . The other part of the equation, other part of the problem, namely, to find out i_2 prime 0^+ , we shall have to write the differential equation. We shall have to write the differential equation for this mesh. Let us see what the differential equation is.

It is $R i_2$ plus, $R i_2$ plus $\frac{1}{C} \int i_2 \tau d\tau$ plus V_{c0} minus. Then plus $L \frac{di_2}{dt}$, that should be equal to 0. Is it clear? At $t \geq 0^+$, this current is 0 and therefore, I have only 1 mesh, $R i_2$ plus the drop in c which is $\frac{1}{c} \int i_2 dt$ plus V_{c0} minus, plus the dropping L which is simply $L \frac{di}{dt}$.

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$$Ri_2 + \frac{1}{c} \int_{0^-}^t i_2(\tau) d\tau + v_c(0^-) = 0$$
$$+ L \frac{di_2}{dt} = 0$$
$$\left. \frac{di_2}{dt} \right|_{0^+} = - \frac{R}{L} i_2(0^+)$$
$$= \frac{E}{L} \checkmark$$

So if I put in this equation, t equal to 0 plus, this is, 0 minus is? 0 and this would be from 0 minus to t , if I put t equal to 0 plus then, obviously, this will also be equal to 0 . This is also 0 for t equal to 0 plus and therefore all I have is $d i_2 / d t$ at t equal to 0 plus would be equal to minus R divided by $L i_2$ of 0 plus and i_2 of 0 plus. We have already found out to be equal to minus E by R and therefore, it would be simply E by L . Is it okay? Yes.

Student: Sir, why E c 0 minus is 0 ?

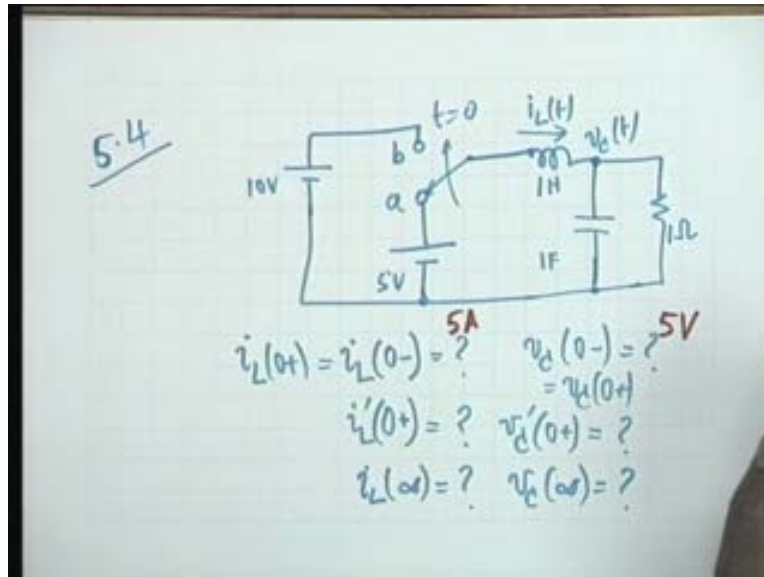
Sir: E c 0 minus is 0 , it is a good question.

Student: Sir, is this steady state?

Sir: Steady state, this is a short circuit and therefore, i_2 is identically 0 , c cannot be charged. See, for charging of c , there is required a current, but since L is identically equal to 0 , no voltage can be supported from this point to this point. To start with, is a different story that is, t equal to minus infinity, the battery starts charging. But in the steady state, when steady state is reached even if the capacitor had acquired a charged, it would have discharged because it finds a very

easy path here, short circuit. What we are considering is the steady state condition that takes care of 5 point 3, next we consider 5 point 4, 5 point 4.

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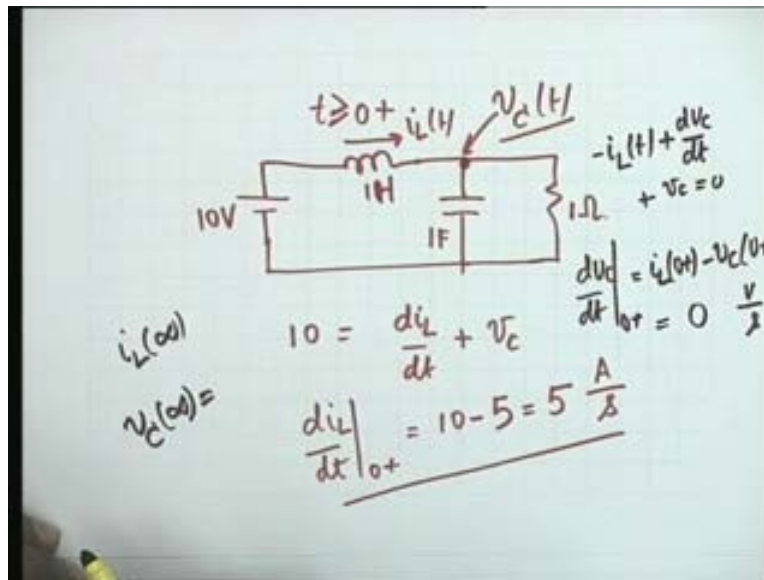


The problem in 5 point 4 is the following. The network is this, let me draw it. 10 volt, I will draw a simplified diagram, I do not want to draw many lines for battery. There is a switch which moves from a to b, that means it was like this, then it moves in this direction t equal to 0 at t equal to 0. The switch s moves from a to b and there is a voltage source 5 volt here. Then we have a 1 Henry inductor, the current through this is $i_L(t)$. We have a 1 Farad capacitor. The voltage here is $V_{sub c} t$ and it is shunted by resistance of 1 ohm. This is the circuit, the circuit has reached steady state in the position a and then at t equal to 0, it is the switch is moved to position b, determine the initial conditions for $i_L(t)$ and $V_{sub c} t$.

That is, you have to find out $i_L(0^-)$ $V_{sub c} 0^-$ and their first derivatives, that is, $i_L'(0^+)$ well, obviously, $i_L(0^-)$ will be the same as $i_L(0^+)$. By initial conditions, we mean conditions at t equal to 0 plus because there are no impulses they would be the same. So write down $i_L(0^+)$ and this is also equal to $V_{sub c} 0^+$. what you have to find out is $i_L'(0^+)$ and $V_{sub c}'(0^+)$ and also the final values, that is, $i_L(\infty)$ and $V_{sub c}(\infty)$.

Let us look at the circuit carefully, first is, when the circuit reaches the steady state at position a, the inductor is a short circuit, the capacitor is an open circuit and therefore, $i_L(0^-)$ or $i_L(0^+)$ can be written down by inspection and this would be 5 amperes, 5 volt. This is short, this is open, 5 volt across 1 ohm and therefore, the current must be 5 ampere and since this is a short, $V_{sub c}(0^-)$ would also be equal to 5 volt, this is clear. Now to find out the derivatives, to find out the derivatives, we have to find out the conditions at t equal to 0^+ .

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At t equal to 0^+ the circuit is, this the battery. Now comes as 10 volt, then you have the inductor 1 Henry this current is $i_L(t)$, then you have the capacitor 1 Farad, this voltage is $V_{sub c}$ of t and then you have the 1 ohm resistance. This is the condition at t greater than equal to 0^+ . Now let us write to find out the derivatives of i_L and V_C at t equal to 0^+ . Let us write a couple of equations. First, you notice that, for this loop, now in the matter of calculation of initial condition, a beat of ingenuity and there is no blind mechanical role. In fact, the electrical engineering is quite different from mechanical engineering. There is nothing much of mechanical here. You have to apply your ingenuity and your intelligence. Now in writing this equation, you see, we write 10, the driving source equal to 1 Henry then $d i_L / d t$, you see the $d i_L / d t$ has come in the equation.

We have to stick those equations which give you the desired quantity, $\frac{di}{dt}$ plus, now this voltage I will simply write as V_c , simply write as V_c . I do not write in terms of the element equation. Then I put $t = 0$ plus, obviously, $\frac{di}{dt} \big|_{t=0}$ plus would be equal to 10 minus $V_c \big|_{t=0}$, plus the $V_c \big|_{t=0}$ minus was 5 volt, therefore, $V_c \big|_{t=0}$ plus is also 5 volt and therefore, this would be 10 minus 5 and the quantity is 5 the unit need to be amperes per second.

Student: What about the current in the capacitor?

Sir: Oh, you do not care, because this potential, we are writing $k v l$, that is, the ingenuity. You see, we are while writing $k v l$, so this the driving source, is equal to this drop across 1 Henry plus drop across 1 Farad, fine and the drop across 1 Farad is V_c . We do not care what the current is. Then, that takes care of $\frac{di}{dt}$. Now we have to find out $\frac{dv_c}{dt}$. Obviously, if you, if you want to find out $\frac{dv_c}{dt}$, we have to take care of the currents because $\frac{dV_c}{dt}$ is the current and therefore, what we do is we write a node equation here. If we write a node equation here, let me write in a different color.

You see, the current is $\frac{di}{dt}$, the current going out plus 1 Farad $\frac{dV_c}{dt}$ plus V_c by 1 ohm. This current, the sum of these three currents shall be equal to 0 , agreed? And therefore, in this equation, you have to write that equation. You see, you are not writing loop equation from mesh equation, both of them, no. We are writing 1 mesh equation and 1 node equation because that is what gives us the desired quantity and therefore, $\frac{dV_c}{dt} \big|_{t=0}$ plus would be equal to $\frac{di}{dt} \big|_{t=0}$ plus minus $V_c \big|_{t=0}$ plus.

Students: (...)

V_c by 1 V_c by 1 ohm that is why, it is V_c . Of course it this also looks ridiculous, is not it? As a current, a voltage is been subtracted from a current. But the suppressed factor is 1 ohm division by 1 , so actually we are differencing two currents. So and this, as you see, $\frac{di}{dt} \big|_{t=0}$ plus is 5 numerically $V_c \big|_{t=0}$ plus is 5 , so this is 0 volts per second. The quantities have been found out finally. Finally, what we have to found out is, $\frac{di}{dt}$ of infinity and V_c of infinity. Obviously, $\frac{di}{dt}$ of infinity would be

Students: (...)

Sir: Pardon me, this part?

Student: Yes sir.

Sir: I will repeat this. What I wrote was, I want to write a node equation here. The current going out through the inductor is minus i_L , that is, what I write here. Then the current that goes out through 1 Farad capacitor is c which is $1, dV_c/dt, c dV_c/dt$ and the current that goes out through 1 ohm resistance is simply V_c by 1, so V_c . The sum of this three currents equal to 0 and then I say dV_c/dt at t equal to 0 plus is, I take this quantity and this quantity to the right hand inside and substitute their values. As far as infinite time value is concerned, when this circuit reaches steady state, once again, the inductor become short, the capacitor becomes open and therefore, the currents at the circuit would be 10 volt by 1 ohm. So it would be 10 amperes.

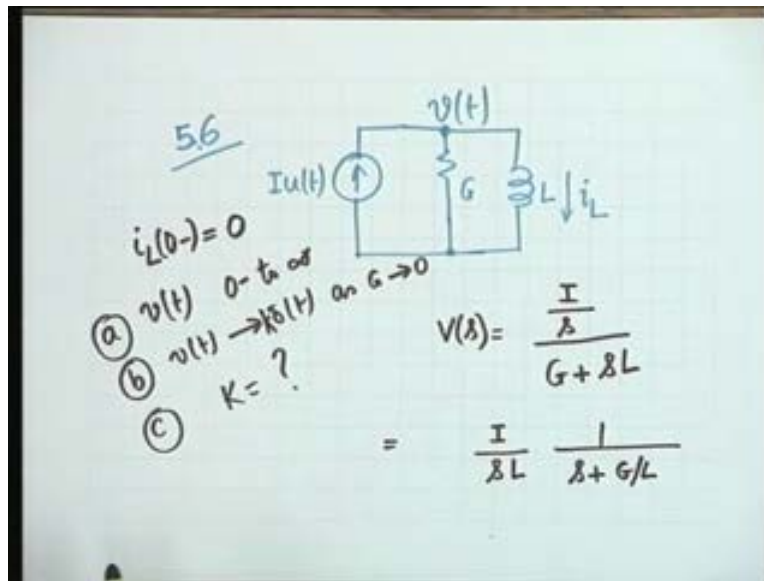
You see, what has happened, initially, initial condition was 5 ampere i_L plus, $i_L(0)$ plus. So from 5 amperes, it raises to 10 amperes. Obviously, exponentially, determined by the time constant of the circuit and V_c infinity, after infinity is again, it is again equal to 10 volts. It started from 5 volt, now it has risen to 10 volts and that is the total solution of the problem.

You see, a bit of a bit of thinking is needed. It is not routine, it is not mechanical that you write this and solve this solution and in the solution also, you have to find a simpler way. Was there a question? No. the next problem that is solved, we will skip now one problem at least. Next problem we will solve this, 5 point 6 on the other side. The remaining ones are to be done by you.

5 point 6, the problem is this, this is an interesting problem. There is an excitation of $I_u(t)$, that is, a current source is switched on, to a parallel combination of a G . G is a conductance and L , the current through L is i_L , and this voltage is $V(t)$. This is the circuit, given, and for the network shown, it is given, the initial conditions are that $i_L(0)$ minus is equal to 0, and what you have to find out is $V(t)$ for 0 minus to infinity. This is part a, this is to be found out. Second, you have to

show the $V(t)$ approaches $\delta(t)$ as G tends to 0, as the conductance tends to 0. Which means, the resistance becomes infinite, you have to show that the voltage appearing across $V(t)$ tends to $\delta(t)$, a unit impulse and then find the, not unit sum k . Part c is, find k , the strength of the impulse. That is the question.

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Now we can solve it by differential equations, but it would be easier to solve it by Laplace transform. Why? Because there is no problem of initial conditions, even if initial conditions are there, Laplace transforms can be used but the situation will dictate to you what should be used. Whether time domain or frequency domain. If we use the time domain, then you see V of s , let us take Laplace transform V of s , that is, the Laplace of $V(t)$ shall be equal to the current multiply by the impedance. The current has a transform I by s , $Iu(t)$, the Laplace transform is I by s multiplied by impedance which is simply reciprocal of admittance and the admittance is, because of a parallel combination, G plus sL . And therefore, V of s is equal to I divided by sL , s plus G by L . Is that all right? This is V of s and therefore, let me repeat it.

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The image shows a whiteboard with handwritten mathematical work. At the top, the Laplace transform of voltage is given as $V(s) = \frac{I}{L} \frac{1}{s(1 + G/L)}$. Below this, it is simplified to $V(s) = \frac{IL}{LG(s + \frac{1}{LG})} = \frac{IR}{s + \frac{R}{L}}$. A box highlights the time-domain solution $v(t) = IR e^{-Rt/L} u(t)$. To the right, a graph shows the voltage response over time, starting at IR and decaying towards zero. The time constant $T = \frac{L}{R}$ is indicated, along with the value $\frac{IR}{e}$ on the y-axis.

V of s equal to I by L 1 over s. s plus G by L was this is step function. Yes, which I can write as, have I made a mistake?

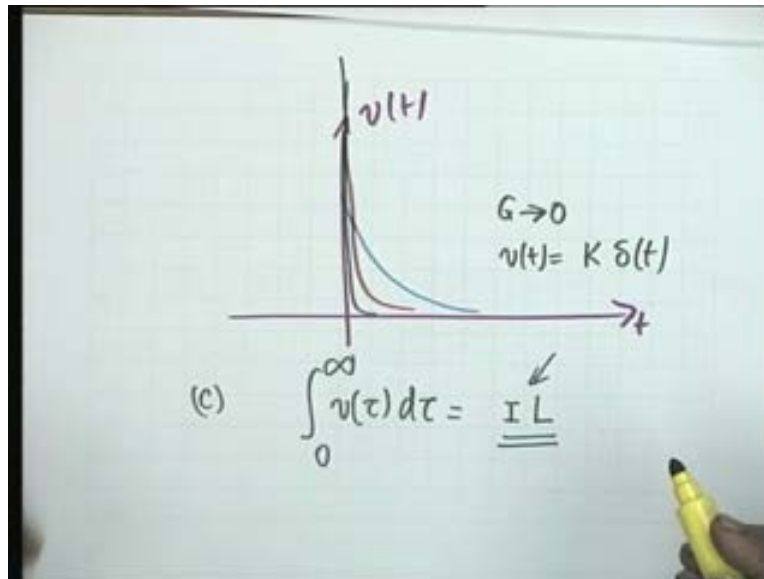
Student: Yes sir, we cannot write G and s L.

Sir: Yes, why did not you tell me earlier? 1 upon s L, therefore, it would be, let me simplify, this would be I by s, s L, 1 plus s L G. Therefore, V of s is equal to, this s and s cancels. s and s cancels, so I get i times L, i times L divided by, let me take L G common, then I get s plus 1 over L G. s plus 1 over L G. This is equal to I divided by G, which is simply I R. Capital G is the reciprocal of the resistance divided by s plus R by L and therefore, I can take the Laplace inverse of this to get V of t as I R, then e to the minus R t by L. This is not complete, we must write multiplied by u of t. This is the voltage solutions and you can see, that the voltage starts from I R and decays with time.

You also know, is there any question up to this point? No? Do not allow me to make a mistake. You also know that the time constant of the circuit is the time required for the voltage to drop to 1 by eth of its value at t equal to 0 and therefore the time constant of the circuit is simply, L by R.

So this is L by R and this value is $I R$ divided by e . This is the time constant. Now part b, part b says, show that the voltage response approaches an impulse, as capital G tends to infinity, that is, I am sorry, 0 that is, capital R tends to infinity. Now you see, that if capital R increases, the height at t equal to 0 increases and capital T which is L by R decreases and therefore, with variation of G the situation would be like this.

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If I plot V versus t , this may be for 1 time constant, if the time constant is increased or decreased, capital G decreases, means time constant decreases, then the curve would be like this. If we further, if we further increase capital R , that is, if we further decrease capital G , may be the curve is like this and obviously, capital G tends to 0, V of t tends to an impulse function $k \delta(t)$. This is from physical listing to establish it on a sound basis, on an analytical reasoning. What we do is we take part c, part c, if we integrate V versus t from 0 to infinity, if we integrate this, then you use the expression and integrate. You can show that this expression is simply I times L .

I carried out the integration here. This expression is simply I times L , now what does it show? It shows that capital G varies, the area under the curve remains same and that is the property of a delta function. A delta function, the area under the curve remains the same, but the duration keeps on decreasing and decreasing. Duration, we define here by the time constant capital T . As

capital T goes to 0, obviously, the curve tends to an impulse function, whose strength instead of unity, it is i times L, so that answers part c and also strengthens the physical argument given in part b. That is what we have showed.

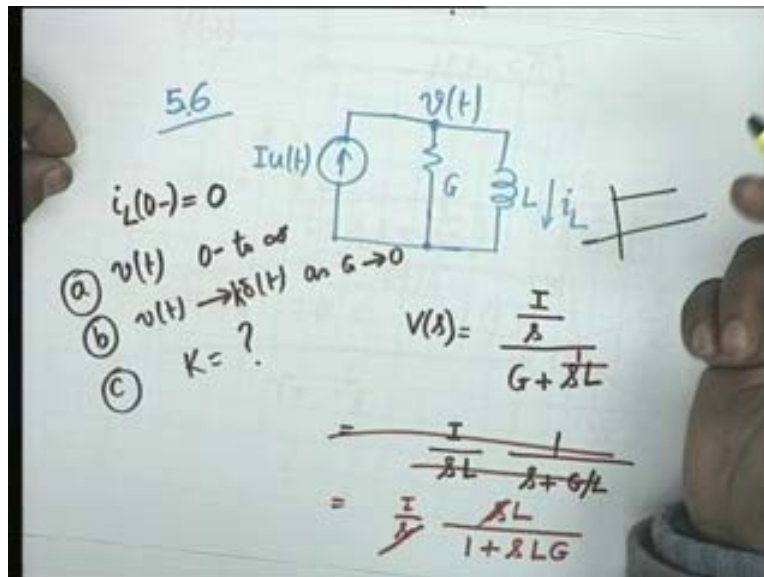
Student: No sir, from the expression, area under the curve is constant. Do not we need to rate at when G tends to 0 is whole of the area is present?

Sir: At equally 0 because the time constant is 0, capital T is 0. So in 0 time, it falls to 1 by eth of its value in 0 times, it falls 0 at 0 of its value also. There is nothing sacred about 1 by e. It could be 1 by hundred, it could be 1 by 2 hundred or it could be 1 by infinity even then, but I agree it is still a physical argument, a physical argument.

Student: Sir, can we establish this fact from the formula V t from the expression that we have obtained for V t?

Sir: Can we establish this fact from the expression? Yeah, this is what I am going to tell you too. Yes. It is precisely what yet no, first let me take the physical situation.

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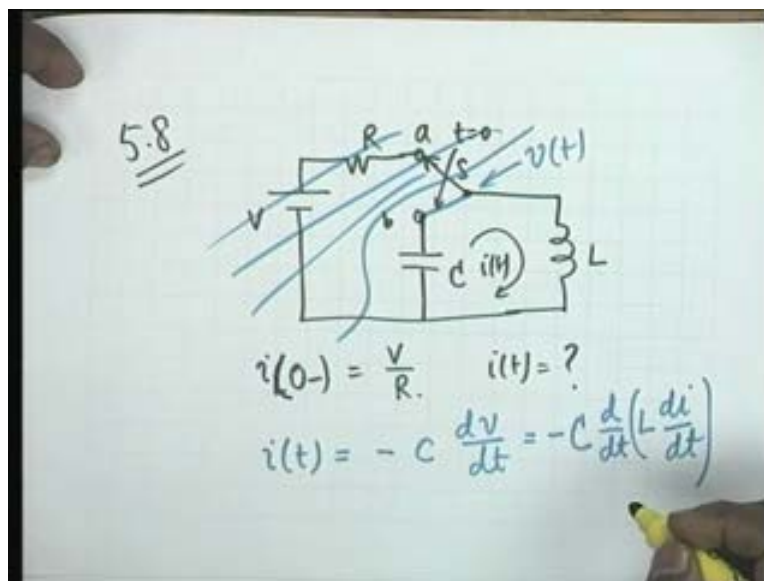
In a physical situation as capital G, as capital G tends to 0, it becomes an open circuit. So what we are doing is, you are applying a unit step.

Students: Open circuit.

Sir: Open circuit capital G equal to 0 is R equal to infinity. So it becomes an open circuit. Now I L 0 minus is 0 and therefore, initially, when we apply unit step, it is an open circuit. On the other hand, unit step stabilizes immediately. So what happens is, this current capital I is being forced into L despite the fact that it tries to resist, and this is a singularity condition, that is, the step current from 0, it is being forced to become capital I. The derivative therefore, is a delta function. Derivative is infinitely large. Derivative of a unit step is a unit impulse function and therefore, the current the voltage which is L d i d t.

You see, small i establishes from 0 to capital I instantaneously, and therefore, the voltage must be a unit impulse function and this argument can now be extended to the equation and I leave that to you. This physical argument can be translated in terms of a mathematical argument in terms of the equation. The next problem that we solve unless, there is a question here, the next problem would be 5 point 8.

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I have skipped 5 point 7, 5 point 8 that is, a very interesting problem. There is a voltage source V , a resistance R and a switch S , which goes from the point a to the point b at time t equal to 0 , and to b is connected a capacitor c , when, to S is connected, an inductor L . This current is i of t , this is the circuit and the conditions of the problem are, before the switch moves from a to b steady state condition prevailed and therefore, i of 0 minus, i of 0 minus is simply equal to V by R . agreed. Then you have to find out current i t , obviously, for t greater than or equal to 0 plus. i of 0 minus is V by R . How does the current behave when the switch is moved from a to b ?

Now let us argue physically, first, physically. You see a current V by R is established in L and when the switch moves from a to b , all we have is an initial magnetic energy in the inductor. The capacitor is initially uncharged, there is no electrostatic energy. There is no energy stored in the electric field. Now this current in the inductor, this current in the inductor, the flux in the inductor, will try to decay because it finds a short circuit at t equal to 0 plus, capacitor is a short circuit. So this current tries to flow through this and in the process, charges the capacitor. Any current flowing through a capacitor charges the capacitor. So this current will start charging till the voltage across the capacitor, as a result of it charging, is the same as the voltage across the inductor.

You see, voltage across the inductor is $L \frac{di}{dt}$ till the two voltages equalize. If the voltage is equalize, then there should be no current and therefore, what happen after that the capacitor now is the steady state. The capacitor, immediately, starts discharging through L and this process continues. It is a see saw situation, in other words, you will get pure oscillations. Now as I told you this part of the circuit is a passive circuit. How come it is able to generate oscillations? How can it behave like an active, while in practice, it cannot, because an ideal inductor is a dream element, so is an ideal capacitor, there would be resistances and therefore, what you will see in practice is a decaying sinusoid. But in theory it can be a pure oscillation. Now let us look at it mathematically.

Mathematically, the current, let me write it here because the circuit is here, the current i of t at t equal to 0 plus, the situation is that the switch has come here. So this part has become invalid, at

t equal to 0 plus and i of t, if this voltage is v, then i of t is equal to minus c d v d t. Do you understand the meaning of the negative sign? Why it is so, because V is considered positive here and negative here. So the current to the capacitor is in this direction c d v d t is in this direction, whereas, the current i t has been taken in the opposite direction. Is that clear?

You have to be very careful about the sign. So i of t is minus c d v d t. Now this is minus c d d t of, what is V in terms of L, it is L d i d t and therefore, this is L d i d t and that does it. That gives you the equation i of t equals to minus L C d 2 i d t 2 and it is I that we have to find out.

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Handwritten mathematical derivation on a whiteboard:

$$i(t) = -LC \frac{d^2 i}{dt^2}$$

$$LC \frac{d^2 i}{dt^2} + i(t) = 0$$

Initial conditions:

$$i(0^+) = \frac{V}{R}$$

$$i'(0^+) = 0$$

Using Laplace transform:

$$L\left(\frac{di}{dt}\right) = V$$

$$LC[s^2 I - s i(0^+) - i'(0^+)] + I = 0$$

$$I(s^2 LC + 1) = \frac{LC s V}{R}$$

Therefore, what we have is L C d 2 i d t 2 plus. It is equal to 0. Now, you can solve this differential equation subject to the condition that i of 0 plus is equal to V by R. This is a differential; this is a second order differential equation and therefore one, 0 minus is the same as 0 plus, one initial condition does not surprise. You also require i prime of 0 plus, is not it? This is a second order differential equation.

Now let us look at this. What is i th prime of 0 plus physical consideration or mathematical consideration? Can someone tell me what would be i th prime of 0 plus?

Student: It will be exponential.

Sir: Exponential? No.

Student: At 0.

Sir: Why is it 0? 0 is the correct answer, but why? Because?

Student: Current cannot change its response time.

Sir: No, that is not the reason. How do you know? At 0 plus the current could be raising or falling.

Student: Sir, but the decreasing inductance is same as rising inductance.

Sir: That, I, no. Pardon me.

Student: Sir, current cannot change through the inductor so.

Sir: So, current cannot change, that is why $i(0^-)$ is equal to $i(0^+)$.

Student: Sir, voltage across the capacitor.

Sir: That is correct, that is the clue. You see, voltage across the capacitor $V(t)$ is simply equal to $L \frac{di}{dt}$ and $V(t)$ at t equal to 0 plus is 0 and therefore, $\frac{di}{dt}$ at 0 plus must be 0. Do you see the argument? You must be able to justify the, justification is at t equal to 0 plus. This is the equation which is valid and V_c of 0 plus is equal to 0 and therefore, $\frac{di}{dt}$ at 0 plus must be equal to 0. So we have 2 initial conditions, now you can solve this. Instead of doing it in the time domain, even the initial conditions are there. It is instructed to do it in the frequency domain.

So let us take the Laplace of this equation. Let us take the Laplace of this equation, obviously, what we get is LC , the Laplace of second derivative is $s^2 I$ minus $s i(0)$ plus $i'(0)$. Do you remember this? Then plus i would be equal to 0 and therefore, what we get is $s^2 LC + 1$, which takes care of this term and this term, this is 0 would be equal to LCs and $i(0)$ plus $i'(0)$ is V/R .

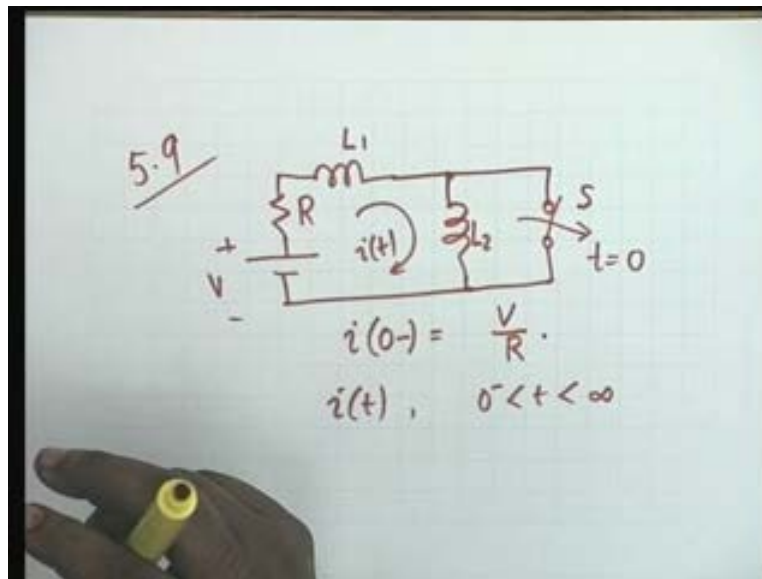
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$$\begin{aligned}
 I &= \frac{VLC}{R} \frac{s}{s^2 LC + 1} \\
 &= \frac{VLC}{R} \frac{s}{LC(s^2 + \frac{1}{LC})} \\
 &= \frac{V}{R} \frac{s}{s^2 + \frac{1}{LC}} \\
 i(t) &= \frac{V}{R} \cos \frac{t}{\sqrt{LC}} u(t)
 \end{aligned}$$

So that is it. That is what it is, which means, the capital I would be equal to VLC divided by R , s divided by $s^2 LC + 1$ and I can write this as VLC by R , s divided by $s^2 + 1/LC$ and then you have to multiply by LC . This cancels with this, so it is equal to V/R s divided by $s^2 + 1/LC$ and in any table of Laplace transforms will show you that under this condition, $i(t)$ is equal to V/R cosine of square root of this. So t divided by square root LC multiplied by $u(t)$ and so there is a sinusoidal oscillation in this circuit.

It is instructed to do this because we had we get a chance to review our knowledge or Laplace transform and the second derivative of the Laplace transform. It also shows how $i'(0)$ plus has to be determined. So this problem is very instructive. Any question on this? Finally we work out 5 point 9.

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5 point 9 is also a peculiar problem. 5 point 9 says that you have a resistance R , a battery V plus minus, an inductor L_1 and then an inductor L_2 . Across L_2 there is a switch and the switch was initially closed and thrown opened at t equal to 0. For the circuit shown the switch s is opened t equal to 0 after this circuit had been in steady state, which means that, if this current is i of t then i of 0 minus would be simply equal to V by R .

What you are required to do is, to find out i of t for 0 minus less than t less than infinity. You have to find out the current i t , something peculiar happens. Let us look at it from physical point of view. For an electrical engineer, physical concept has no replacement. You can solve differential equations, you can consult Laplace tables, you can find the spectrum by Fourier transform and so on. You can do hundreds of different things, you can write algorithms on the computer, you can simulate equation, you can find, however complicated, there is no replacement for physical concept.

Let us look at the physical concept. You say t equal to 0 minus, this inductor was shorted, this inductor was shorted and therefore, L_1 carried all the current and the magnetic flux was generated by L_1 only through L_2 the current was 0. At t equal to 0, this switch is open and therefore, whatever current was flowing through L_1 is now being forced to pass through L_2

because otherwise, the current cannot flow and the current requires a path and therefore, this is a case of singularity. This is a case in which, the current in L 2 has to change instantaneously, in order to establish a flux and provide a path, provide a closed path for the current. This is a case of singularity and one has to be extremely careful in solving this problem.

Student: Sir, why not we can change the environment instead?

Sir: Current has to change in L 1 also.

Student: But instantaneous change?

Sir: Yes, you will see how it changes. The physical situation, you would be clear now. Now the question is to find i for $t < 0$ to $t = 0$ to $t = \infty$. One of the things that I confused you, intentionally with, is the question of current through L 2 at $t = 0^-$. Is it really 0? It is 0. Well, at $t = 0^-$ L 2 is a short circuit, s is also a short circuit, so there are 2 short circuits in parallel.

Student: Sir, L2 is not a short circuit.

Sir: At $t = 0^-$, it is a short circuit, because the current has established the steady state. There is no change of current.

Student: When the switch s was closed and there would be no current in L 2 .So since it has been shorted, there will never be any current.

Sir: All that has been infinite time ago.

Student: Sir, since then, sir, through the short switch again but not through L 2 sir?

Sir: But L 2 also provides a short at $t = 0$

Student: Initially some current goes but.

Sir: Initially some current goes. No, this is a degenerate situation again, because it is being shorted because the switch is being thrown open. It is a degenerate situation and we do not know at t equal to minus infinity what happen. We do not know. We are concerned at t equal to 0 minus when s is closed and v has been there for a very long time.

Student: Sir, we can have two situations but since then cannot short unless a current flows through it and since s is already a short, so no current start flowing in it therefore, the situation of it being starts again at 0 minus.

Sir: This current after coming here when this is short, this is also a short L_2 is also a short. the current has established

Student: sir whatever current will be there, that we will discharged.

Sir: Who will oppose it? once the current has established, it becomes steady, you agreed that the current, the steady state current is V by R .

Student: Yes sir.

Sir: The current is steady. There is no $d i d t$. So the voltage across this shall be 0. If voltage is 0, obviously, it is a short circuit.

Student: Sir, it depends when the switch was closed.

Sir: Switch is closed after t equal to infinity. From minus infinity to 0 minus the duration is infinity.

Student: At t equal to 0, initially L_1 and L_2 are relaxed.

Sir: Initially, L 1 and L 2 are not relaxed.

Student: Suppose, in a case and we have closed the switch such that this has short, L 2 is shorted. Now whenever even a slight bit of current will tend to flow through L 2, it will be acting as a open circuit because it will oppose the current and that current will be tending to flow through the short. This situation prevails until infinity. At infinity, this is now the same thing that the current at infinity would (...) at 0.

Sir: That's not this case.

Student: Sir, there is no magnetic energy in L 2.

Sir: Where?

Student: in L 2.

Sir: Oh! there will be a magnetic energy

Student: Sir, there is no current passing through.

Sir: That is, what I am not agreeing with you

Student: At this point, we are assuming that, sir, the battery was installed before the switch was closed.

Sir: Battery was installed before the switch was closed, yes, is it a history is not known. How battery was, but you were looking at a situation after infinite amount of time. You see, after let us say 47 years of independence, you are not concerned with who fought whom, you are only concerned with your independence? Who are the people? Most of the young generation does not know who are the people, for whom you are now with? Your high head high and all that, so we do not know the history.

What we know is that at t equal to 0 minus, the current driven from a battery, drawn from a battery V by R . The current after reaching here, finds two short circuits. One is through the switches, L_2 , is no better than a short circuit because for a steady current L_2 cannot develop a voltage across it, therefore, it is a short circuit and if it is a short circuit, identically short circuit, there should be different between short circuits. Also, there can be incremental resistance but these are ideal cases and therefore, the current through L_2 is half of V by R . And $i_2(0^-)$, no, not i_2 , just a minute, just a minute, $i_2(0^-)$ is equal to V by $2R$. Yes.

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Handwritten equations on a whiteboard:

$$i_2(0^-) = \frac{V}{2R}$$

$$i(0^+) (L_1 + L_2) = L_1 i_1(0^+) + L_2 i_2(0^+)$$

$$t \geq 0^+ \quad V = Ri + (L_1 + L_2) \frac{di}{dt}$$

Student: why is it the case that we considered the current at L_2 depending upon our convenience?

Sir: No, it is not a convenience. You make a circuit and allow it. The best test, is experimental test. Allow it to stand why not?

Student: This is an ideal situation?

Sir: No. We have a resistance here, nothing will burn. You take a battery, connect a resistance and connect 2 inductors, short one of them and allow the circuit to stabilize. Measure the current through the inductor and through the short circuit, you do exactly how.

Student: Sir, but the inductor is not short circuit but the resistance is here.

Sir: There will be slight difference, but that can be attributed to the resistances.

Student: Sir, like, if we have the resistance then we can say that half current will flow?

Sir: This resistance?

Student: No sir, these, these are some resistances.

Sir: No, this is, as I said, it is a degenerate case. It is an ideal case. You take R_1 and R_2 and allow R_1 and R_2 both to go to 0. You will see that current will be exactly half. No, current division, R_1 by $R_1 + R_2$ and R_2 by $R_1 + R_2$ as in the limit it will be exactly half. This is a degenerate situation and one has to consider. That is how flux shall be instantaneous case established in L_2 , not otherwise. So the condition, I have a few minutes now, I will allow you to solve the problem yourself, but the condition for finding i_0 plus would be the principle which we have not utilized so far. What is that principle? Continuity of flux, flux cannot be increased or decreased instantaneously and therefore, what we have is, at $t = 0$ plus i_0 plus L_1 plus L_2 would be equal to $L_1 i_{L_1 0}$ plus, plus $L_2 i_{L_2 0}$ plus. This would be the condition for finding $i_{L_1 0}$ plus and, I am sorry, i_0 plus i_0 plus and your equation at $t \geq 0$ plus, or this gives the initial condition of i_0 plus.

Student: Should we not have 0 minus on the right hand side?

Sir: Yeah, I am sorry. I beg your pardon, yes, thank you. Here, $i_{L_1 0}$ plus and $i_{L_1 0}$ minus are not the same, is not that right? $i_{L_1 0}$ plus, how is it related to i_0 plus? Identical, similarly $i_{L_2 0}$ plus would be identical to i_0 plus. Had I written $i_{L_1 0}$ plus and $i_{L_2 0}$ plus, here it would

have been a Himalayan mistake. I can effort to do that, you cannot. So from here you find $i(0)$ plus then at t greater than or equal to 0 plus, obviously, the equation is V equal to $R i$ plus L_1 plus L_2 .

Now you can combine the two because $d i / d t$ does not involve the initial conditions. That is why you can combined the $2 L_1$ plus $L_2 d i$ by $d t$ and you solve this equation. You solve this equation, obviously, the solution to this equation without doing anything else, you can take Laplace transform and do it, or you can do a first order differential equation solution. But in any case the solution would be of the form, i equal to some constant K_1 plus another constant $K_2 e$ to the power minus, time constant would be. R divided by L_1 plus L_2 times u of t from which, well, K_1 can be found out immediately, by allowing t to go to infinity.

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The image shows a hand holding a yellow marker pointing to a whiteboard. The whiteboard has the following handwritten equation:

$$i = K_1 + K_2 e^{-\frac{Rt}{L_1 + L_2}} u(t)$$

An arrow points from K_1 to $\frac{V}{R}$.

If t goes to infinity, what is the, what is the current again, in the circuit? Again V by R ? So we know K_1 would be equal to V by R for K_2 . Now you have to put t equal to 0 and find out K_2 and then the total solution well, I will, I will leave the rest to you, fine. Thank you.