Circuit Theory

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Lecture 40

More on PRF's and Their Synthesis

the this is the fortieth lecture and the topic today would be more on positively and functions and their synthesis

yesterday we had open this topic on testing a positive real functions and we saw the testing requires

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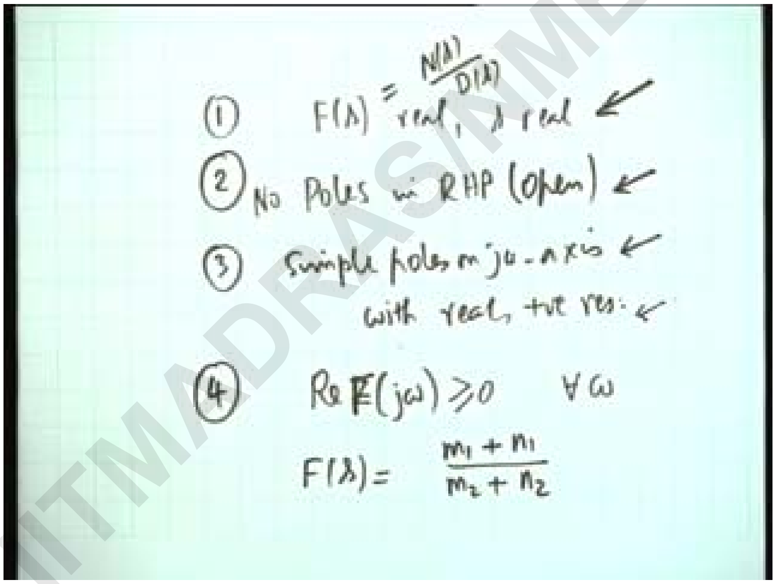

first to test whether F of s is real for s real this this is an obvious inspection test you can you can conclude by looking at the function

second is that you ah do (( )) (00:00:52) test on the denominated you F of s is M of s by DFS then you test whether D of s has any roots in the right have plain

so poles no poles in RHP open your closed

<a_side> (( )) (00:01:10) <a_side>

open [Laughter] open because poles in the j omega axis are permitted if you discover poles on the j omega axis they must be simple simple poles on j omega axis with real positive residue with real positive residue and if it satisfies all the three then the fourth one would be the test of the real part of capital F on the j omega axis real part on the j omega axis here to find out whether this is true or not for all omega and we saw the mechanization mechanization is this is by inspection this is obtained by (( )) (00:01:57) test on D of s on and if there are poles on the j omega axis they will be revealed in the (( )) (00:02:03)test

so you find whether how it is last deviser which will reveal if there are poles on the j omega axis and then test whether these poles simple or not and if they are simple if they are multiple then you don't go further if they are simple then you test their residues and find out whether they are real and positive or not and finally if all of them are satisfied then we will go to last one and the last one as you saw we had simplified in to the following

if F of s is equal to m one plus n one divided by m two plus n two then all that you have to test is a polynomial [Noise]

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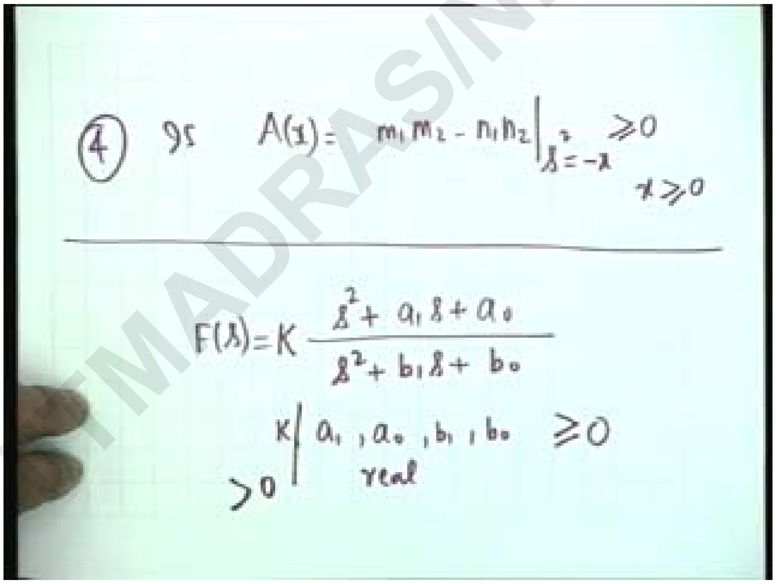

a polynomial which is A of x which is equal to m one m two minus n one n two under the condition s squared equal to <a_side> ((minus x)) <a_side>

minus x you have to test whether this is non negative for x is omega squared so omega negative as well as positive are taken care of

so it (( )) (00:03:06) to do for x greater than equal to zero okay this is what the fourth test is all about and with this we had considered one simple example of a linear polynomial divided by a quadratic polynomial

today we will consider a bi quadratic polynomial that is a quadratic divided by another quadratic and you know that all such functions can be written in this form s squared plus b one s plus b zero and and you can have a multiply with constant k

we can write the numerator with leading coefficient equal to one denominator with leaving coefficient equal to one

then obviously the first ah criterion would be that K a one a one a zero b one b zero must be greater than equal to zero they must be real real all of them must be real and they must be greater than equal to zero with one exception

what is the exception what is the exception K cannot be equal to zero if K is zero it’s a trivial isn’t it

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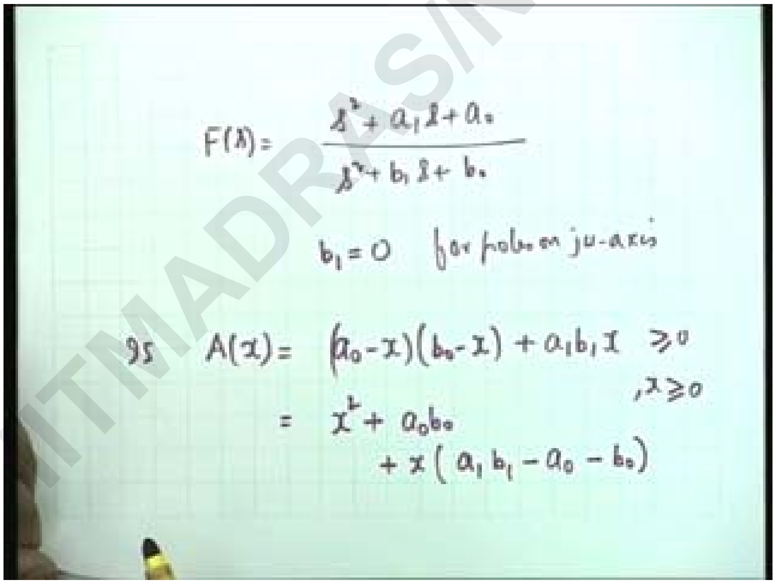

so K has to be strictly greater than zero K cannot be equal to zero all right that is a trivial thing or that’s the first thing

now therefore we can we can forget about K we can simply concentrate on on all the rest of it if you want you can call it F one or F prime or whatever it is or you normalize K two one because K is at no concern okay K has to be a real positive quantity so let’s consider this by quantity s square plus b one s plus b zero the second thing ah um the second test says that you carry out ah (( )) (00:05:04) test in the denominator

obviously if b one and b zero are real and positive there is no need the real part of the root shall be negative okay but there is one ah um ah the third one says are there poles on the j omega axis

well if there are poles on the j omega axis obviously you require b one is equal to zero isn’t that right if there have to be poles on the j omega axis poles on j omega axis and under that condition under that condition what would be what would be the ah [Noise] the poles obviously simple okay

we have s square plus b naught and therefore the roots are at plus minus j squared root of b they are obviously simple you have to test the residue

now is the residue half a one whether it is or not i think i left that as a question ah as a question because it's a fuzzy discussion

if b one is zero you will see that for the function to be positive real there are constraints on the other constraints let us see what this constraints are

so at this point whether the poles [Noise] whether the residues on the j omega axis are real and positive or not we will leave it for the present we skip this to the fourth test that is we find out what is A of x

A of x obviously is m one would be a zero minus x all right m two would be b zero minus x is this step clear why it is s zero minus x will be zero minus x and [Noise] minus a one b one s squared with s squared equal to minus x it would be a one plus b one x is that is that okay plus a one b one x

so ah i what i have to test whether this is non negative is this non negative for x x non negative okay

now the right hand side can be written as ah [Noise] x squared minus x minus x plus a naught b naught a naught b naught that is the constant term and the coefficient of x would be a one b one minus a naught minus b naught is it okay all right agree all right

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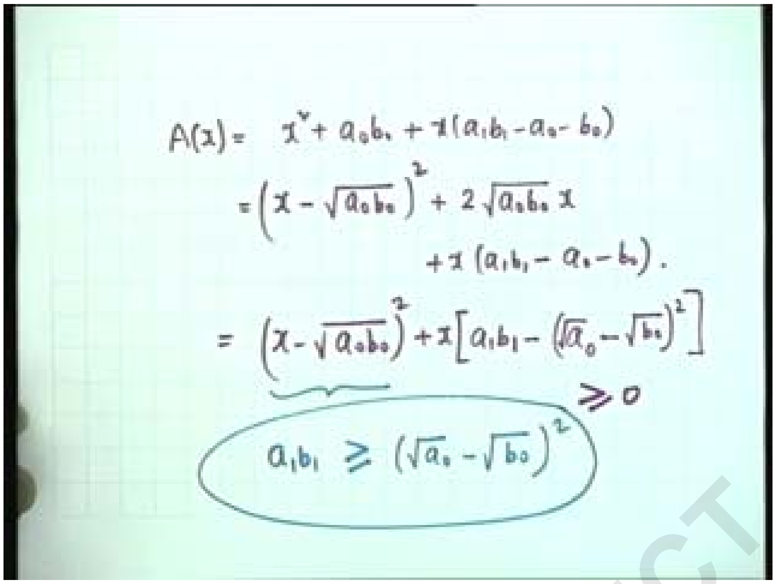

so we we repeat this here A of x is equal to x square plus a naught b naught plus x a one b one minus a naught minus b naught i have to test whether this is non negative or not

now i do a very simple thing i have ah x squared and i view a naught b naught x square root a naught square root b naught whole square that is what i want to do is to write this is as x minus square root a naught b naught whole square okay then i have to add plus twice square root a naught b naught x isn't that right

what i done is i have taken minus twice square root a naught b naught x here and i have added it so there should be no change plus x into a one b one minus a naught minus b naught which means that this is equal to x minus square root a naught b naught whole square plus x into a one b one minus no <a_side> root a naught <a_side> root a naught plus or minus <a_side> plus and minus <a_side> minus because when this comes inside it is plus

so square root a naught minus square root b naught whole square agreed and under what condition will this be non negative you see a square term a square term cannot be negative [Noise]

so the minimum value of this is zero A of x should be non negative even under that condition which means that the coefficient of x here must be non negative that is the conditions is that a one b one should be greater than or equal to square root a naught minus square root b naught whole square agreed

this is the condition on the coefficient this is the restriction on the coefficient [Noise] all right

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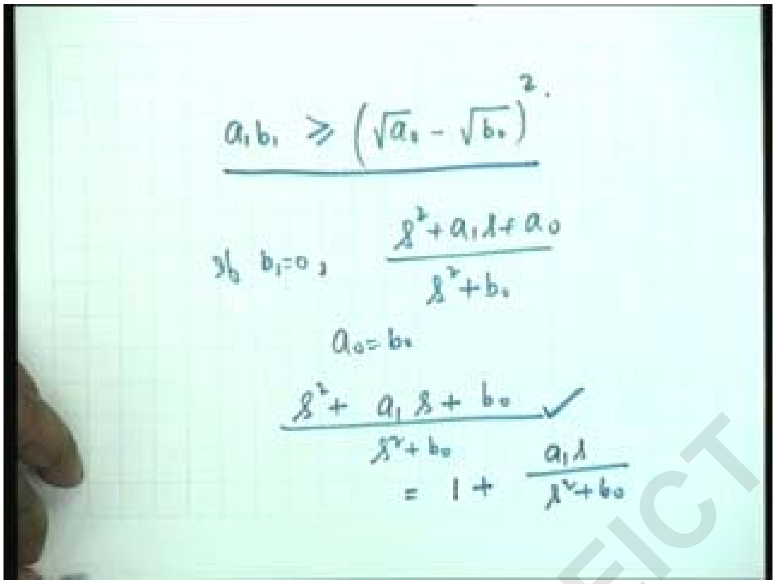

let me repeat a one b one should be greater than equal to square root a naught minus square root b naught wholes square

now we had ask the question in the third part of the test that is under the condition if b one equals to zero if b one equals to zero then only we had poles on the j omega axis right then the denominator becomes s squared plus b naught numerator becomes s squared plus a one s plus n this is my F of s say b one by zero

now b one is zero and this condition is to be satisfied a zero must be equal to be zero therefore a zero equal to b zero which means which means that our function now becomes s squared plus a one s plus b zero divided by s squared plus b zero you understand this is it okay

if b one is zero left hand side is zero now zero has to be greater than equal to this whole square and therefore the only condition is that the two sides must be equal because right hand side cannot be negative zero is of course greater than a negative quantity but the right hand side being a whole squared cannot be negative and therefore the only condition under which it is satisfied is that a naught should be equal to b one

in other words my function becomes this which obviously i can right as one plus a one s divided by s square plus b naught and now i am in the conditions to state what the residue is

residue is obviously half a one okay all right and therefore we have we have found out the conditions that a general quadratic

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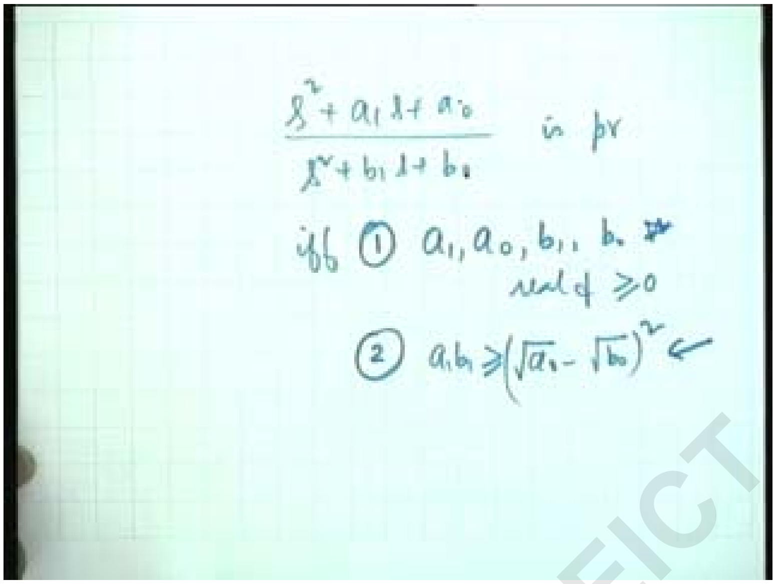

general bi quadratic that is s squared plus a one s plus a zero divide s square plus b one s plus b zero is pr if an only if just two conditions are necessary what are they

number one a one a zero b one b zero should be greater than should be real and greater than equal to zero all right

number two just one more condition that is a one b one should be greater than equal to square root a naught minus square root b naught whole square

i don't need to state third conditions because the third condition that is if b one equal to zero b one equal to zero then s zero has to be equal to this zero this is contained in this just two conditions okay

let's take some examples [Noise]

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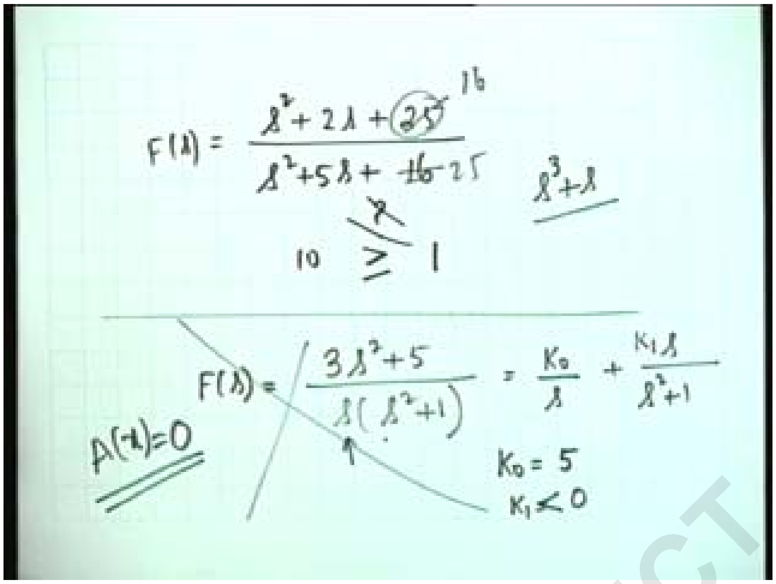

let’s say s squared plus two s plus twenty-five divided by s squared plus five s plus sixteen F of s is this pr all that we have to test it a one b one which is it greater than equal to the question mark square root

<a_side> (( )) (00:13:06) <a_side> okay

so it is one now {sep} (00:13:10) therefore it is pr we remove this question marks out suppose suppose these two are interchanged that is instated twenty-five i have sixteen here and this is twenty-five does it change <a_side> no sir <a_side> no

therefore a naught and b naught can be interchanged in this function whether it is s square plus a one s plus b naught or s squared plus b one s plus a naught it doesn’t matter but of course

<a_side> (( )) (00:13:35) <a_side> in every bi quadratic not in every other bi quadratic but the synthesis that is the network we analyzing this will of course be different

if if the constant in the numerator is taken to the dominator and ah vice versa obviously the network shall be different but the function shall remain positive okay

what about ah [Noise] if this ah no okay that’s one

let's take another function three s square plus five divided by s times s square plus one [Noise] F of s now how do you test whether this is positive real or not

<a_side> (( )) (00:14:34) <a_side> do i have to yeah okay i will do that [Noise] i will do that i have to cheek the (( )) (00:14:42) poles are one is at the origin and the other is at pr on the j omega axis

so what is have to do is to write this s K zero by s plus twice K one s okay i can i can replace two K one by k one how does it matter okay so let’s write it as K one s by s square plus one [Noise] can you say what k zero is

<a_side> three or five <a_side> three or five k zero is five [Noise] and k one is minus

<a_side> minus two a square < _side> I don’t care whether minus two or minus five the sound minus is good enough for me okay

K one is less than zero so the function is not clear okay

the actual value is not important

<a_side> (()) (00:15:38) <a_side>

denominator is not a (( )) (00:15:42) polynomial why not <a_side> (( )) (00:15:46) <a_side>

it’s a [Laughter] clearly ah it is a (( )) (00:15:50) okay

do i have to test the real part no because it is [Noise] but suppose i had to [Noise] suppose i had to what is the real part of this what is A of x [Noise]

<a _side > (( )) (00:16:08) <a_side> identically equal to zero why because it’s an odd rational function okay [Noise]

then some ah some more examples let say we have ah two positive two positive real functions F one and F two

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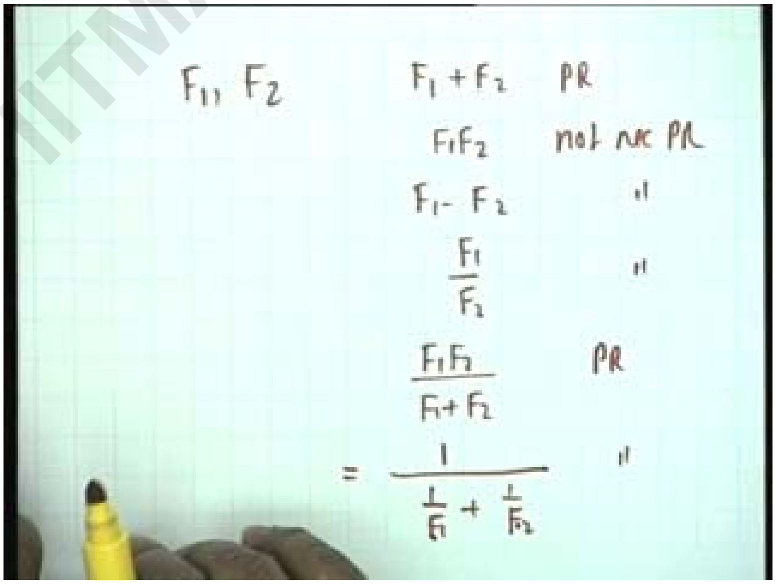

you know that F one plus F two is PR okay ah F one F two not necessarily PR may or may not F one minus F two belongs to the same category F one by F two belongs to the same category not necessarily PR

what can you say about F one F two divided by F one plus F two it is PR because

<a_side> (( )) (00:17:03) <a_side> because it can be written as one over F one plus one over F two PR plus PR [Noise] so the total thing is PR and one by PR is also PR okay all right

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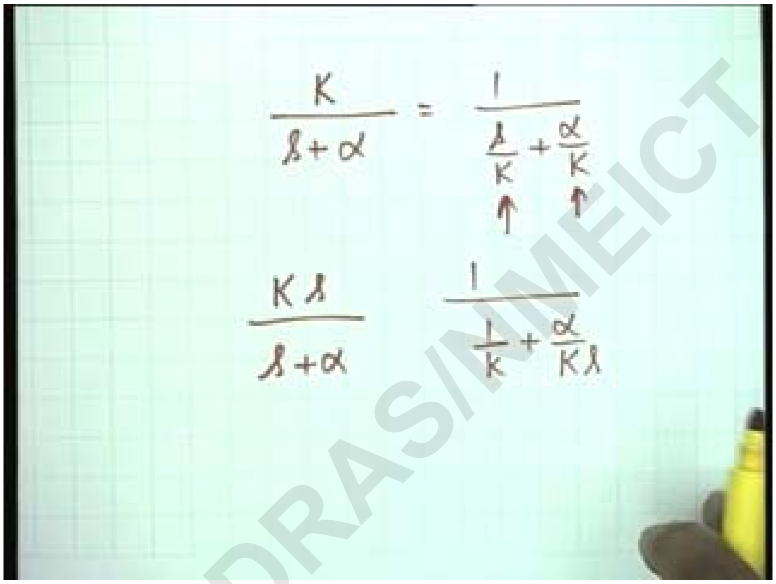

now let's take some specific examples of this let us say you have K divided by s plus alpha is this PR K and alpha are real and positive is this PR yes or no

i can write this is S by K plus alpha by K this is PR this is PR PR plus PR is PR one by PR square

<a_side> (( )) (00:17:47) <a_side> division of two positive real functions but this is [Noise] division of two positive real functions is not necessarily PR but in this case it is PR

you should view it like this instead of a division of two positive real function

suppose i multiply this by S [Noise] is this also clear

<a_side> ((yes sir)) <a_side>

yes of course because i can write this as one over K plus alpha by KS this is also PR okay next [Noise]

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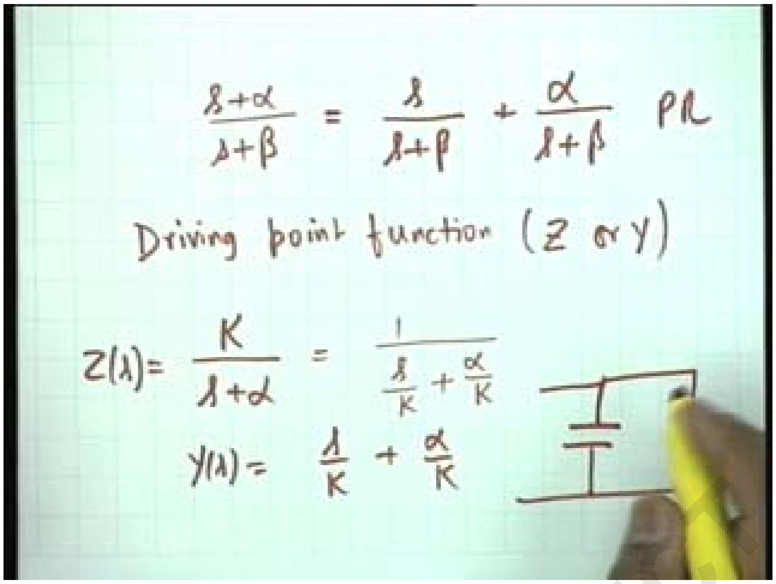

suppose we have s plus alpha divide by s plus beta where alpha and beta are real and positive is this PR

<a _ side> yes sir <a _ side> yes of course because i can write this is s by s plus beta plus alpha divided by s plus beta so this is also PR [Noise] all right

this is some of the some of the elementary functions which are PR

now let’s look at ah we had done yesterday some problems on synthesis some very simple problem let’s look at the general principles of synthesis of a driving point function [Noise] by this remain either an impedance or an admittance a driving point remittance and to be specific let us say we consider impedance

ah for example ah if the impendence is let’s K by s plus alpha when then what we do is we could do it like this if we write this is one over s by K plus alpha by K this is admittance therefore Y of s is equal to s by K plus alpha by K which means that it is a parallel combinations of a capacitor and a resistor and the value of the capacitor is one by K or K

one by k all right then the resistor [Noise]

<a_side> k by alpha <a _ side> k by alpha do not make a mistake we never write conductance value we writes resistance value

this is a conductance so the resistance is k by alpha [Noise] all right

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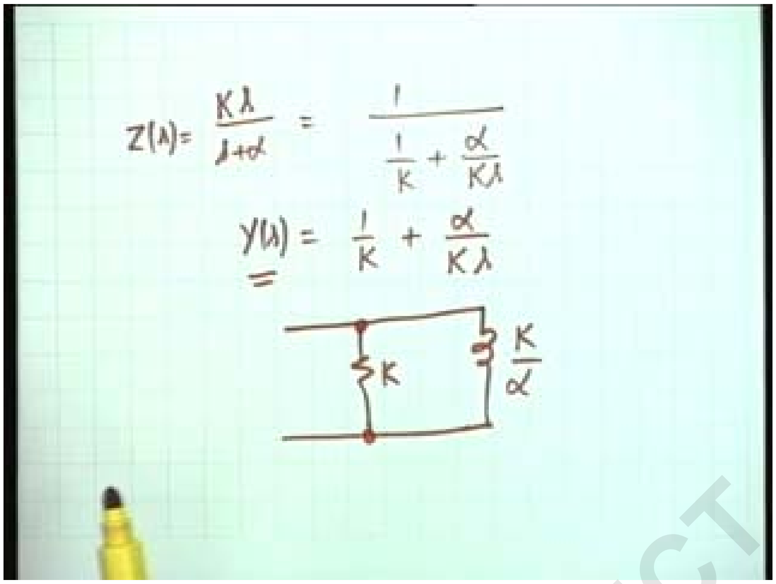

if this was let say K s by s plus alpha if that was my Z of s then i write this as once again one over one by K plus alpha by K s and therefore the admittance is one over K plus alpha by K s which means that i have a resistance in parallel with inductor and the value of the inductor is

<a_side> k by alpha <a_side> all right and the resistance is not one by k all right do not make a mistake [Noise]

this is the convention that we follow that element values are resistance capacitance and inductor not their reciprocals okay

the general principle appears to be [Noise] even in this prevail cases that we express the given function as some of simpler functions well in this case i [Noise] could not express Z of s is some of simpler functions

so what i did was i took Y of s it's reciprocal and express this a some of two functions each of which i could synthesis

similarly here also here also what i did was ah i found at the admittance could be composed in to a some of simpler functions

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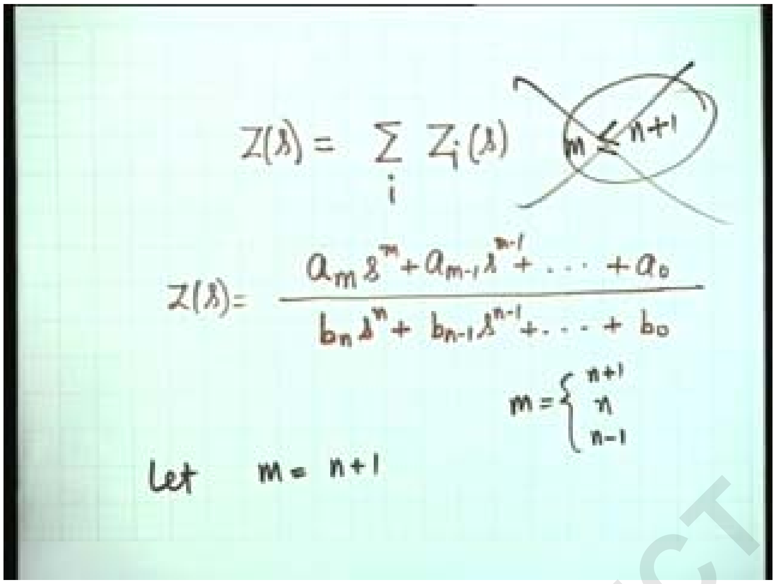

the general principle in driving point function synthesis is if possible decompose Z of s in to a some of simpler functions Z is some (()) (00:21:48) if not possible than take the admittance and decompose that into simpler functions okay it's one of the two either you work with the impendence or the admittance

now in between after you do that you might find that one of these components can be decomposed when you take its admittance so you do that and it go on doing this till you get elementary ah elements elementary elements namely resistance capacitance and elements that is the principle general principle it doesn't always work because you may not be able to decompose

let's takes some uh some examples [Noise]

suppose Z of s is equal to let's say an no am s to the m plus am minus one s to the m minus one plus etcetera plus a zero divided by bn s to the n [Noise] plus bn minus one s to the n minus one plus etcetera plus b zero

suppose this is your function now we know that m could be equal to n or m could be one greater than n or m could be one less than n right this is the different this is the restrictions of m that m must be the restriction is m must be less than or equal to n is this correct

<a_side> no sir <a_side> no i write a wrong thing and you all agree can i can n be zero for example and n can be three can that be can that be true [Laughter] the powers

<a_side> (( )) (00:23:42) <a_side> difference is either zero or not period it cannot be more [Laughter] than by one and therefore m

<a_side > (( )) (00:23:53) <a_side>

the possibilities are m can be equal to n can be n plus one or n minus one that's it one of them all right [Noise]

suppose [Noise] suppose to be to be specific suppose m is ah equal to n plus one suppose let m be equal to n plus one then the function Z of s will have a pole at infinity isn't that right

if the numerated degree is one greater than the denominator degree then obviously as s tends to infinity the function tends to infinity and this pole [Noise] is on the Z omega axis it is a simple pole and the function shall qualify for further PR testing if the resistance at this pole is real and positive

what is the residue at this pole on at infinity can you tell me by the inspection it is <a_side> ((am by bn)) <a_side> simply am by bn

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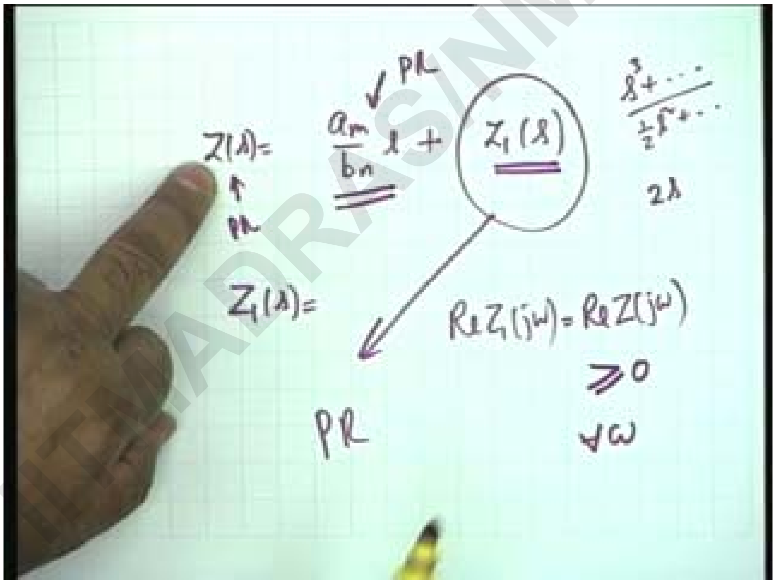

and therefore i can write Z of s is equal to am by bns plus the rest of the function Z one of s [Noise] the step that we have done is called poled removal at infinity

obviously that's what we have done and since all coefficients are to be real and positive obviously m by bn that is the residue at infinity is also real and positive okay

so this is PR now Z of s is PR Z of s is PR this is PR

<a_side> (( )) (00:25:42) <a _ side >

oh we allow s to go to infinity only the highest powers come only the highest powers come and therefore we get [Noise] <a_side> (( )) (00:25:51) <a_side> m is c that is what we have seen m is equal to n plus one

so as s tends to infinity Z of s s tends to infinity this tends to am by bn times s the rest of the terms to (( )) (00:26:08) what is the problem okay

suppose we had let's say s cubed plus etcetera divided by half s squared plus etcetera at infinity obviously this behaves as twice [Noise] and therefore two is the residue

this residue is obvious for instruction at big question now is [Noise] the remainder function Z one of s which is a difference between two positive real functions is not necessarily PR okay

let us see whether it is PR or not let's see is that necessarily PR it could be PR also

now what are the conditions of positive realness if s is real Z one of s should be real this obviously must be true [Noise] because Z of s is PR therefore when s is real Z of s should be real which is obviously true this is real this must also be real some of a real and complex quantity cannot be real all right okay

so first condition is satisfied second condition does it ah can it have poles in the ((writer)) (00:27:21) plane can Z one of s had poles in the writer plane obviously not because if it is poles in the writer planes the same pole shall belong to Z of s also but Z of s by hypothesis is PR therefore Z one cannot to be pole in the writer plane

next can you tell me pole on the j omega axis is yes it can it can but can any of this poles be multiple no because that pole also belongs to Z of s by the same token the residue at a simple pole is ((ablaze)) (00:27:55) to be real and positive because that residue also belongs to a positive real function Z of s

all that remains for us to test is the real part of Z one of j omega whether it is non negative for all omega

well don't you see that the real part of this term is zero for s equal to j omega this term is purely imaginary and therefore real part of Z j omega Z one of j omega is the same as real part of Z of j omega and this has to be non negative for all omega

therefore Z one s is PR agreed can i explain okay [Noise] the the problem was that we are removing the positive real function from a positive real function is the reminder positive real that was the question and we have shown by systematic argument that Z one of s the reminder function has to be PR

there were [Noise] three arguments one was that if s is real it has to be real if it is complex then of Z of s will not be PR if it is a pole in the writer plane that pole should also belong to the Z of s

so it cannot have a pole if it is a multiply pole on the j omega exist that also belongs to z of s

so it cannot be multiple powers the residue at a at a simple pole has to be real and positive because it's also a residue to Z of s and finally the real part of Z one of j omega because Z one is Z minus am by bn s and if you put s equal to j omega this term becomes purely imaginary

so the two real parts are exactly equal and real part of Z of j omega is non negative so real part of Z one of j omega is also non negative

in other words what we have proved this by removing a pole at infinity you do not destroy the positive real character of the remainder function

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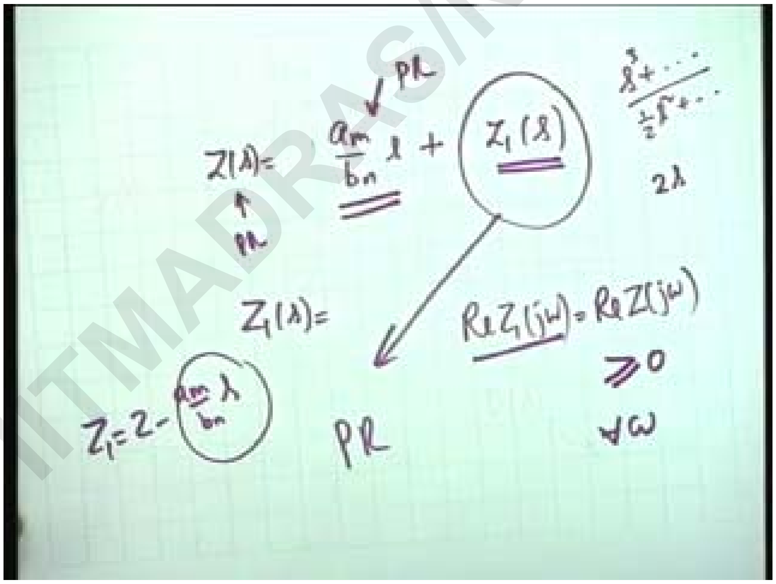

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in other words what we have got is a partial synthesis of Z of s that is you have got an inductance of value am by bn [Noise] and the rest of the function is Z one rest of the function is Z one

then you can look at the Z one okay partly it has been synthesized the pole at infinity has been removed then you look at Z one does Z one have a pole at can Z one have a pole at infinity no because pole at infinity has already been removed

so possible Z y one may have a pole at infinity in other words Z one may have the zero at infinity then y one shall have a pole at infinity [Noise] then (( )) (00:30:54) by taking the reciprocal you may be able to {rem} (00:30:57) remove that pole

<a_side> (( )) (00:31:00) <a_side> no it is not necessary by removing the pole at infinity you don't equalize the order you don't necessarily equalize the order

<a_side> (( )) (00:31:11) <a_side> oh doesn't matter you might cancel more than two trans more than one trans it's possible we will show examples all right

<a_side> ((sir this is not clear how we can cancel it)) <a_side> how can how we can <a_side> (( )) (00:31:28) <a_side> yeah <a_side> (( )) (00:31:30) <a_side>

why the degrees are [Laughter] (( )) (00:31:36) suppose the degrees were unequal with start with they were unequal <a_side> ((yes sir)) <a_side> okay

suppose we have an odd polynomial s cube plus etcetera divide by even polynomial if you remove the pole at infinity you were still left with a difference in one in the degrees okay

suppose M of s by D of s suppose this is of degree three and this is of degree two by removing the pole at infinity you will be left to it a polynomial whose numerator shall be either even or odd and the denominator shall be the opposite we will take examples okay we will we will we will be convinced when you takes examples the degrees i am not necessary said yeah

<a_ side> (( )) (00:32:27) <a_side> oh because [Laughter] <a_side> (( )) (00:32:34) <a_ side> because Z of s had the pole at infinity which has been completely removed if Z one of s also is a pole at infinity this must be added to this

we have removed the pole at infinity from here [Noise] so the given their function cannot br the pole at infinity no [Noise] okay

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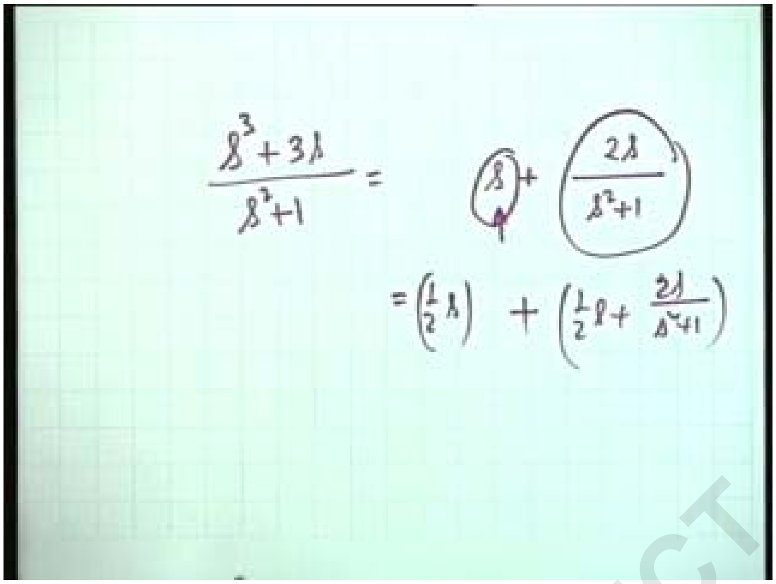

let's take an example suppose we have s cube plus three s plus divided by s square plus one this has a pole at infinity and the pole at infinity has a residue one

so it is s plus [Noise] the remainder now what you think the reminder would be the denominator is s plus one and <a _ side> (( )) (00:33:23) <a _ side> this would be two s

so the degrees are node necessarily equal degrees are not necessarily equal number one and ths function does not pole at infinitive (( )) (00:33:35) pole at infinitive because we have removed this poles

if the remainder also is a pole at infinitive that means we have not be able to calculate the residue property oh you can argue like this i will take half s here and the other half s i will add with this but what's this fun [Laughter] if the reminder function is equally complex as the original one you have not made any procures

so you don't remove partially you remove completely you remove pole at infinitive from this and [Noise] talk about the remainder of functions okay all right

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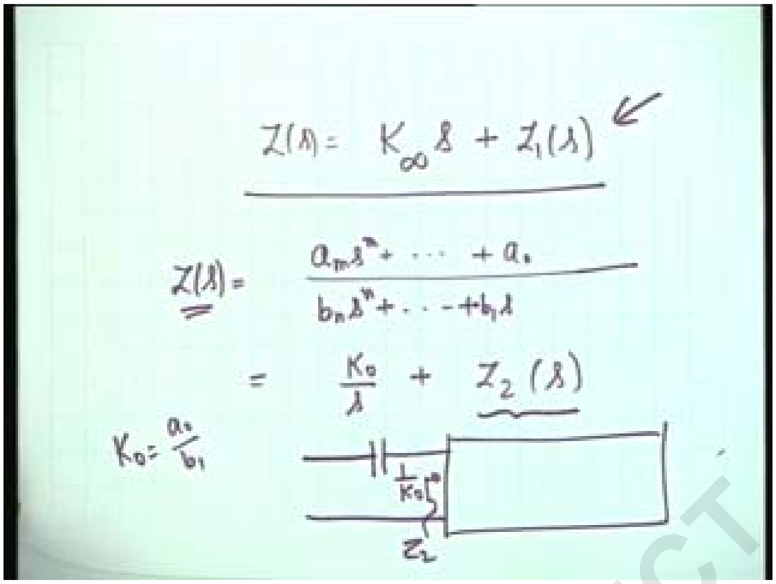

so what we have done is Z of s we [Noise] have said now let let's denote the residue at infinity as K infinitive plus Z one s K infinitive we had we are making a symbol instead of am by bn and writing by kn infinitive

now let's go back to the original function original function was am s to the m plus etcetera plus a zero divided by bn s to the n plus etcetera plus b one s

suppose the b zero time is not there the lowest powers [Noise] cannot differ by more than unity that is lowest powers could be equal if b zero was there lowest power could differ by one that is the demaninator could have a power of one the numerator as zero power all right or it could be the other way round we may have missed s zero and b zero is there i am taking one spcefic case is there a problem were in to put a pieace of cloth there is some problem okay (( )) (00:35:42) can go ahead would be recorded okay regarding ah is okay

now if that is the case is this a permissible thing does this necessary make Z of s non PR no we can have a degree difference over if this is the case then what can you say about Z of s its behaviour at orgin at s equal to zero it has a pole at the orgin [Noise] which means that i can write this is K zero by s plus some other function let's say Z two of s agreed

so if you discover that there is the pole at the orgin you can remove this now what is K zero K zero is obvious by inspection it would be a zero by <a_side> ((b one)) <a_side> b one

a zero by b one agreed

so ah and then by the same arguments that we did in this case we find that Z two s is also PR by the same arguments Z two s is also PR in other words what we have obtained is we have obtained a partial synthesis this a capacitor of value

<a_side> (( )) (00:36:54) <a_side>

one upon cannot yes <a_side> (( )) (00:36:57) <a_side> no we have not assumed anything n and m could be equal

this is not of importance the relative degree difference whether m is equal to n or n plus one or n minus one doesn't matter what we are saying is the behaviour at the orgin that is when s goes to zero all other terms disappear from this scene it is only s equal to zero

<a_side> (( )) (00:37:23) <a _ side> no we are not considering whether the function has a pole at infintive or not [Noise]

<a_side> ((pole at zero)) <a_side > pole at the orgin and that's what we are removeing

so the rest of the function is Z two [Noise] this is also a partial synthesis so we can remove a pole at infinitive we can remove the pole at the orgin we can also remove a pole on the Z omega axis

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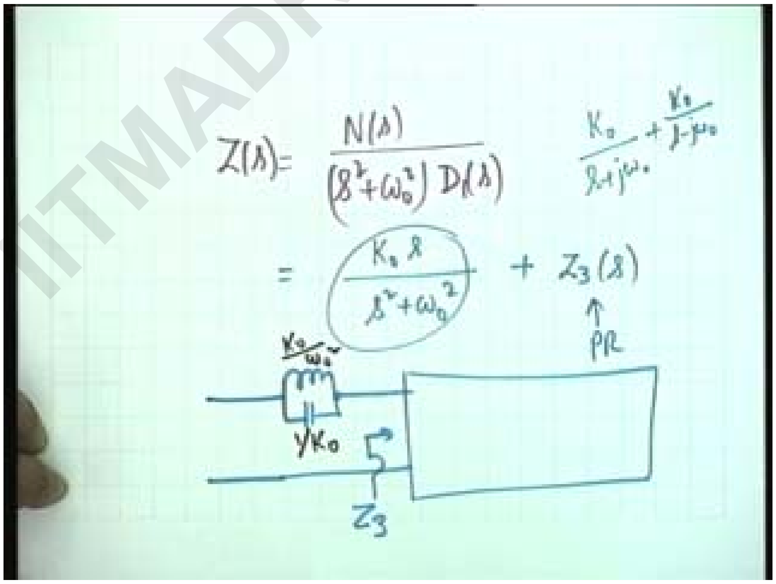

if we had a pole on the Z omega axis that is we find that Z of s is n of s divided by s square plus omega naught squared multiplied by let's say D one s suppose we discover that there is indeed a such a factor well ah what we can do is we can write this as K zero s divided by s squared plus omega naught squared plus some other function that's Z three of is all right where the residue at this pole is how much

<a _ side > ((k zero by two)) (00:38:30) <a _ side >

k zero by two we should have getting twice K zero but we are saying twice K zero is K zero

<a_side> (( )) (00:38:37) <a_side>

that is correct it has [Laughter] to be like that otherwise the function will be not clear isn't it we have said that the residue is real and positive

so what all we are doing is here we are adding a term like this to a term like s minus Z omega and this is exactly this that is a condition imposed by positive realness character of the function and by the same token by the same arguments which is this is an odd function so on the j omega it is purely [Noise] imagenery

so it dosen't contebute to the real part (( )) (00:39:18) part shall be the same and other poles agruments is the same in other words Z three s is also PR which means that what we have been able to achieve is that we have been able to remove and in that tends in the capacitor in parellel and the rest of the impedent is Z three

can you tell me what these values are the inductance for an the capacitiences one by one by K naught is that is that is it correct okay and then this one K naught by omega naught sqaure wonderful

so we have removed the series we have removed in series a parallel resonance circuit okay

these are the various steps that one can imply if one discovers that the function given function has a pole at the origin remove it if it is at a pole at infinity remove it if it is a pole on the j omega axis a pair of poles remove it all right

now you can do this either with the impedance or with admittance if we had removed from admittance a turn like this that would have meant that we have removed a series resonant circuit in parallel agreed

if this was instead of Z if it was Y if this was Y and this was also Y then instead of s instead of a series connection we would have a shunt connection a parallel connection of an inductance and a capacitor in series okay these things would be illustrated with the examples i would not go into the theory because the theory is exactly the same

the fourth step these are three [Noise] steps of a pole removal pole at origin pole at infinity pole [Noise] on the j omega axis

<a_side> (( )) (00:41:17) <a_side>

not necessary you can do it any order and that that adds variety and spice to life in an every turn you get a different network and then you choose [Noise] if you have a multiplicity of a realization you choose which one ah um according to your choice okay

you may like the shunted one or you may like the series one for different results okay

the fourth step that can be done is to look at

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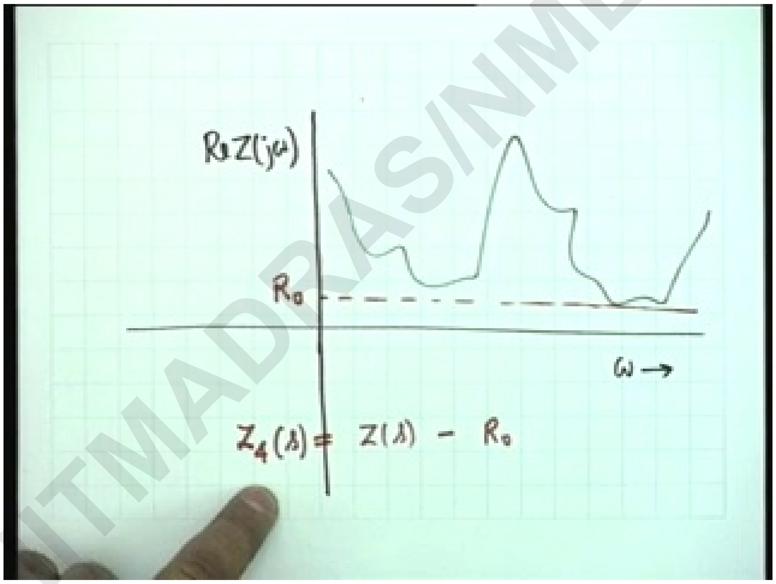

this we have already illustrated yesterday with an example is to look at the real part of Z of j omega it has to be non negative for all omega and it suffixes to look at in the first quadrant right it suffixes to look at because real part of Z of j omega is an even function [Noise] it contains a numerator and denominator [Noise] which contain only even parts of omega

so it suffixes to look at in the first quadrant and suppose that this variation is something like this okay it is maxima and minima then you look at absolute minima that is the [Noise] global minimum this is the global minimum and suppose this value is R naught okay obviously from Z of s if you remove this resistance R naught [Noise] the remainder function let's say Z four of s will this be positive here will this remain positive here

yes it will because the the minimum value of the real part of Z four of j omega shall now be zero instead of one okay

this is another step that can be performed if you [Noise] remove a constant from a given rational function the rational function may simplify okay

this is the fourth step that one can perform

<a_side> (( )) (00:43:16) <a_side>

you get a simpler function the decreased attitudes okay let in synthesis because in synthesis means you must remove something which simplifies the {re} (00:43:28) rest [Noise] of the function then you are making progress otherwise you are not making progress okay

now let's take a number of examples to illustrate this [Noise]

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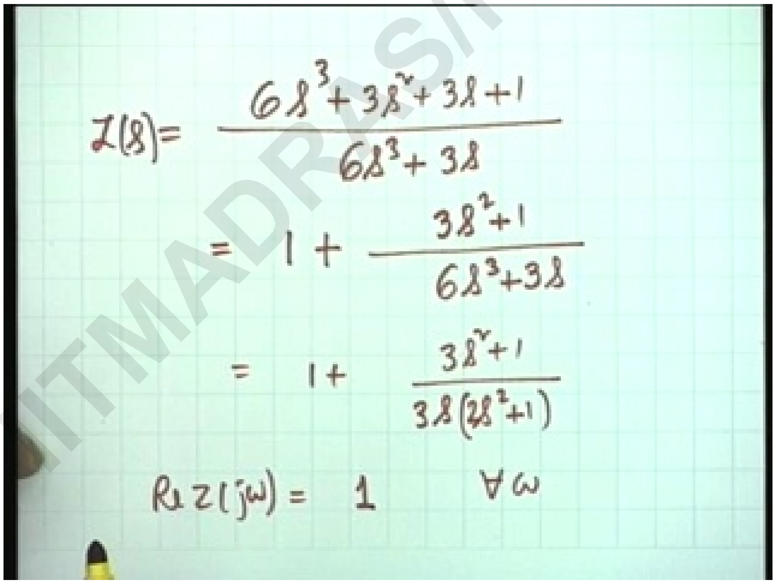

ah first we take let's say six s cube plus three s squared plus three s plus one divided by six s cube plus three s suppose this is a Z of s all right the degrees are equal it has a pole at the origin there is no pole at infinity it is a pole at the origin you could remove this but before that an intelligent eye intelligent pair of eyes should discover that the denominator is contained the numerator which means that we could remove a constant of one [Noise] there is no guarantee that by removing the constant one the rest of the function shall be PR there is no guarantee but let's do this let's do this and see if the rest of the function is PR then we had made progress

you see the rest of the function is ah three s squared plus one divided by six s cubes plus three s okay which contains

<a_side> (( )) (00:44:51) <a_side> will not be possible not be possible not possible

<a_side> (( )) (00:44:58) <a_side> it will not be possible provided one provided the constant that we removed is less than or equal to the minimum value of the real part then the remainder function shall be PR okay

if one is greater than the minimum real Z of j omega then obviously the rest of the function will not PR but let's look at it and not finding the minimum value of the real part formally and just saying that since the denominator is containing the numerator let us see what happens if i do this

so what i get is three s square plus one divided by s three s multiplied by

<a_side> two s plus two square <a_side> two s square plus one agreed

so the remainder function remainder function you see is purely odd not imaginary

when in terms of s it is not imaginary when we put s is equal to j omega the function will be imaginary which means that real part of Z of j omega is equal to [Noise] one for all omega because this is purely imaginary is it clear and therefore we should we shall be we should be permitted to happily remove this one ohm from resistance agreed

one ohm is the minimum value also the maximum value we can removed that now if i do that if i do that

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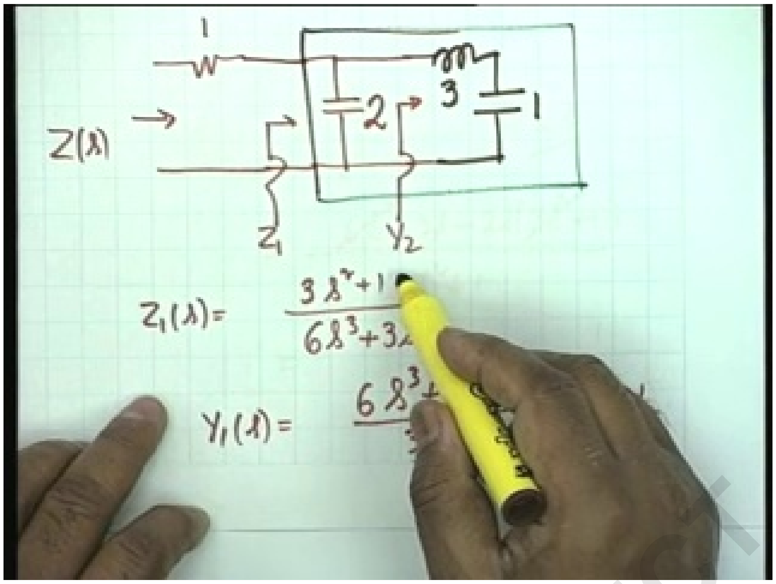

our synthesis has made progress Z of s is one ohm plus the rest of it which is [Noise] let's use a different colour let we call this as uh as Z one then Z one of s is equal to three s squared plus one divided by

<a_side> (( )) (00:47:01) <a_side>

i will write as six s cube plus three s and i shall keep my life simple what i will do is i could have written three s times two s square plus then removed the pole at the origin i could do that there is a pole at the origin

<a_side> yes <a_side>

yes i could do that but i want to keep my life simple i want to make as few calculations as possible

so what i do is instead of this is the variety i could do that i could also do something else what i do is i find the admittance i find this much simpler then the rest i find the admittance the admittance is the pole at infinity right and the residue is obvious residue is two

so it is two s plus let's say y two agreed which means that i have removed a what a capacitor from an admittance we removed two s so the capacitor value is two and the rest of the admittance the rest of the admittance is y two

let us see what y two is [Noise]

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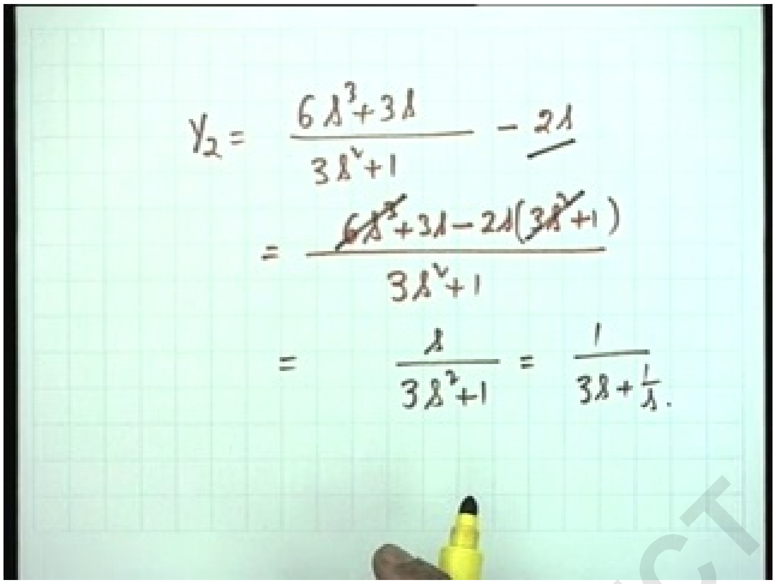

y two [Noise] is six s cubed plus three s divided by three s square plus one minus two s which is equal to three s square plus one six s cubed plus three s minus two s times three s square plus one which means there is a cancellation of this term or this term so it is simply equal to s divided by three s squared plus one agreed

<a_side> (( )) (00:48:48) <a_side>

wait a second before [Laughter] before i do that before i do that please do observe that when i remove the pole at infinity the remainder function cannot have a pole at the infinity because it has been removed okay

the remainder function in fact has a zero at infinity that is not always the case but here it has a zero

now i can i can look up on this [Noise] as if the synthesis problem is over isn't that right because i can write this as one by three s plus one over s which means that in the network in the network a series inductance of value three and uh

<a_side > (( )) (00:49:31) <a_side> and a capacitor of value

<a_side> (( )) (00:49:34) <a_side> one why one upon three three s plus one upon s so three plus one and the synthesis is complete this is an RLC method [Noise] is it okay this is an RLC network

now i also want you to notice that after the removal of that one ohm one ohm resistance we had a very specific ah very special type of impedance which was the numerator was purely even the denominator was purely odd and therefore the total rational function is odd is odd

it could be other way round also that is i could have an odd divided by even and i will now show rigorously that when such a function is encountered the synthesis is in terms of pure inductances and capacitance no resistance in the structure all right no resistances

in other words

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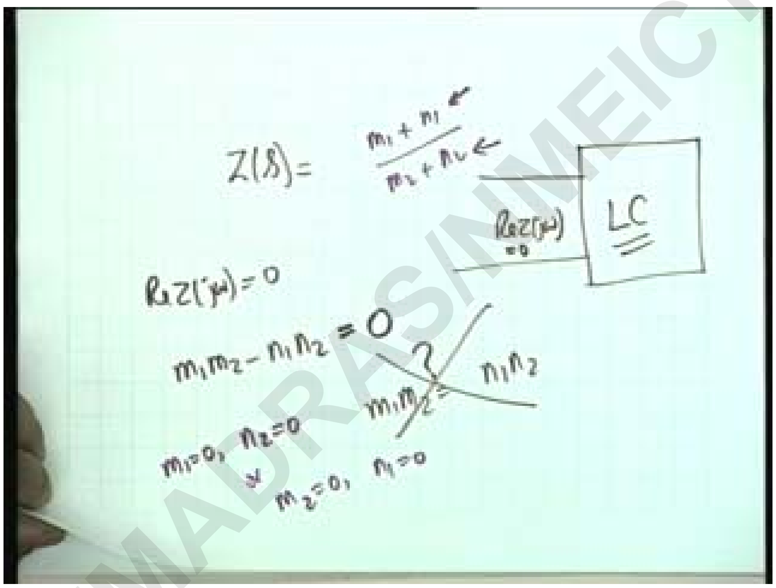

real part is zero on the j omega axis

<a_side> (( )) (00:50:54) <a_side>

real part is zero okay so

<a_side> (( )) (00:50:59) <a_side>

that’s right i am [Noise] going to be saying a very interesting thing [Noise] it says since the real part is zero if the real part is zero okay if real part of Z of j omega is equal to zero then the power observed by the network is zero from a sinusoidal (( )) (00:51:26) which means that there cannot be any resistance inside the network perfectly all right

in other words the network must be LC can you prove [Noise] that if the network is LC [Noise] then the drive in point impedance must be purely odd can you prove it the other way round

<a_side> (( )) (00:51:44) <a_side>

[Noise] yes we can let's do that

<a_side> (( )) (00:51:47) <a_side>

Let's do that you see that our condition is that the real part of Z of j omega equal to zero which means that m one m two minus n one n two should be equal to zero isn't that right

if the real part is zero then m one m two minus n one n two should be equal to zero how can this be possible can m one m two be equal to n one n two is that possible

<a_side> ((yeah)) <a_side>

[Noise] can m one m two i am asking a very trivial question a common sense question is this a possibility

<a_side> (( )) (00:52:32) <a_side>

no because m one m two as a constant term may have a constant term n one n two cannot have a constant term the minimum power in n one n two is s squared

<a_side> (( )) (00:52:43) <a_side>

no no no odd multiplied by odd is even but m one m two can have a constant term and n one n two cannot have a constant term and therefore this is not a possible [Noise]

what is the way that this can be possible

<a_side> (( )) (00:53:00) <a_side>

obviously there are yes

<a_side> (( )) (00:53:02) <a_side>

we don't have any constants okay then what are you taking of let's let's go back m two plus n two

<a_side> (( )) (00:53:14) <a_side>

okay if constant tem is zero then there is a factor of s in the numerator there is a factor of s in the denominator [Laughter] okay so

<a_side> ((sir out of two one of one m one or m two can one uh one of them can have a zero constant)) <a_side>

out of two i can then the uh then there will be difference in degrees the two degrees cannot be equal

you have a problem both right

<a_side> ((sir if the numerator has only ah odd or even and denominator has odd or even)) <a_side>

that is the only possible way that is i can have there are two possibilities i can have m one equal to zero and n two equal to zero or i can have m two equal to zero

<a_side> (( )) (00:54:07) <a_side>

why not both why not m one equal to zero and n one equal to zero

<a_side> (( )) (00:54:12) <a_side>

[Noise] that there is no function okay so an LC drive in point function

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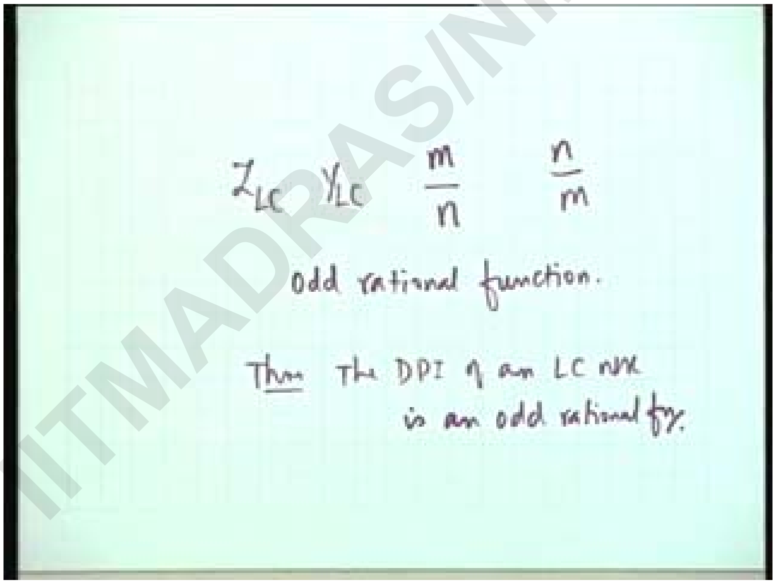

ZLC or YLC must be of the form [Noise] m by n or n by m agreed that is it must be either purely even by odd or odd by even in either case it is an odd rational function this is a big this is a great theorem that is the drive in point [Noise] the DPI of [Noise] of an LC network is an odd rational function okay

if it is a odd rational function [Noise] if the DPI is an odd rational function and what is the condition on m and n on the character this two polynomials if it is a positive real function isn't it what can you say about m and n

<a_side> (( )) (00:55:25) <a_side> that they must differ by

<a_side> (( )) (00:55:30) <a_side> can we differ by both than one

<a_side> (( )) (00:55:34) <a_side> then it will not be clear anything else that you can say is m (( )) (00:55:39) necessarily is the numerator (( )) (00:55:48)

<a_side> ((yes sir)) <a_side> any positive real functions must have a numerator as well as the denominator which has both (( )) (00:55:54) therefore m and n are both (( )) (00:55:56) if m a purely even polynomial is (( )) (00:56:00) where are its roots

<a_side> (( )) (00:56:04) <a_side> [Noise] on the j omega axis

n a purely odd polynomial a purely odd polynomial is s times and even polynomial agree it has it is (( )) (00:56:14) so its roots also must all on the j omega axis

origin included yes so we proof we proved that LC drive in point functions all poles and zeros are on the j omega axis and the rest of the s function the rest of the s plane is banned as for as the pole and zeros are concerned okay

i stated again the poles and zeros of an LC drive in point function are all on the j omega axis

we will start from here next time