

Circuit Theory

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Elementary RLC One - Port Synthesis and Introduction to Two - Port Synthesis

all right forty sixth lecture [Noise] elementary RLC one port synthesis and introduction to two port synthesis this is the topic

as i told you in the last class RLC one port in general possess a difficult problem and if at all solvable it is solvable by using a transformer

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Handwritten mathematical derivation on a whiteboard showing the synthesis of an impedance function $Z(s)$. The derivation includes a partial fraction decomposition step:

$$Z(s) = \frac{s^2 + 2s + 2}{s^2 + s + 1}$$

$$\frac{s^2 + 2s + 2}{s^2 + s + 1} = \frac{s^2 + s + 1}{s^2 + s + 1} + \frac{s + 1}{s^2 + s + 1}$$

$$\frac{s + 1}{s^2 + s + 1} = \frac{1}{s + 1} + \frac{s}{s^2 + s + 1}$$

The final result is $Z(s) = \frac{1}{s + 1} + \frac{s}{s^2 + s + 1}$.

if the use of a transformer is allowed then there is no problem or you use a non canonic synthesis of Bott & Daffin [Noise] or some of its modifications but as i said in some special cases if you are lucky it is possible to synthesize a an RLC {tran} (00:01:08) RLC impedance function and we shall take a few examples to check the uh the tricks of the tray if these tricks ah work perfectly all right if they don't work then they don't work that's it okay

ah one of the examples that i take first is s square plus two s plus two divided by s squared plus s plus one you see that the poles if this is an impedance functions

you see that the poles are uh neither on the uh imaginary axis nor on the real x axis they are complex and therefore if at all realizable it should be realizable as RLC [Noise] but before you check ah the RLC you must check the realizability condition

there is only one in this case because there are no poles on the j omega axis that is a one b one which is two in to one and square root a zero a square root two minus square root b zero square root one square this is two and this is two

so with greater than equal to it is equal to therefore the real part condition is satisfied

you understand the shortcut procedure because by inspection we see all all coefficients are real and positive there are no poles in the j omega axis so all that you have to do is to check whether a one b one is greater than equal to square root [Noise] of zero minus square root b zero

<a_side> ((sir greater than strictly greater than or equal to)) <a_side> greater than or equal to it can equal to

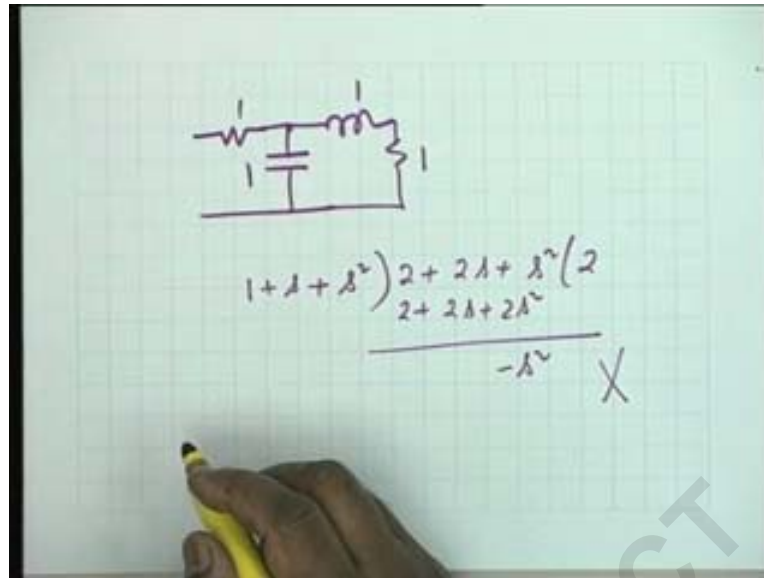
<a_side> (()) (00:02:41) <a_side> how come what did <a_side> ((point one sir)) <a_side> two into one [Noise] and square

<a_side> ((sir root two minus root one is point four point four square)) <a_side> oh yes i great (()) (00:03:01) [Laughter] root two minus root one is point uh <a_side> ((four one)) <a_side> four one four square so it is greater than okay i am sorry i make that obvious mistake but you see if you if you make a continued fraction expands let us see if it works

s squared plus s plus one blindly i mean i i don't no how else i could do it so we will try continue fraction s square plus two s plus two one this is an impedance s square plus s plus one

so i am left with s plus one divides s squared plus s plus one s s squared plus s [Noise] one s plus one let's see this is admittance then s s one one one one zero this is impedance and this is a admittance so it does work

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you have got all questions which are which are real and positive coefficients and therefore i can ah draw the network as uh one ohm then ah an admittance of s o [Noise] one farad then an impedance as that means a an inductor of value one Henry and then [Noise] an admittance one that is this

so obviously its an RLC network and that's why it was neither ah it didn't satisfy the condition of RC RL or ah LC [Noise] but suppose we do it the other way down that is starting with the lowest powers all right

ah one plus s plus s squared two plus two s plus s squared two so two plus two s plus two s squared and minus s squared we cannot go further

suppose we do it the other way down that is admittance we take the admittance let us see what happens [Noise]

<a_side> ((sir lowest power)) <a_side> again lowest powers okay

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$$Y = \frac{(2 + 2\lambda + \lambda^2) (1 + \lambda + \lambda^2)^{\frac{1}{2}}}{1 + \lambda + \frac{\lambda^2}{2}}$$

$$\left(\frac{\lambda^2}{2}\right) (2 + 2\lambda + \lambda^2) \left(\frac{4}{\lambda^2}\right)$$

so two plus two s plus s squared divides one plus s plus s squared half

so one plus s plus s square by two we are left with s squared by two this is an admittance divides two plus two s plus s square well four by [Noise] s squared we don't no how to realize this isn't that right so we don't proceed further

similarly you can try other procedures and see if any of them works you will have to try this there is no established procedure to do this

we take another another tricky problem [Noise] but you assure that if one uh synthesis exists the rather indefinite number so may be something else will succeed all right

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$$\text{Ex } Y(s) = \frac{(s+2)(s+3)}{(s+1)(s+4)} = \frac{s^2 + 5s + 6}{s^2 + 5s + 4}$$

$$= 1 + \frac{K_1}{s+1} + \frac{K_2}{s+4}$$

$$K_1 = \frac{2}{3} \quad \parallel \quad K_2 = -\frac{2}{3}$$

now this problem is slightly tricky $s^2 + 2s + 3$ $s^2 + 1$ $s^2 + 4$ suppose this is given as an admittance $Y(s)$

now the poles in zeros do not alternate first critical frequency is a pole but then you have a zero and other zero adjacent zero then a pole first is a pole last is also also a pole so it cannot be RC or RL it is not LC at all but let's see whether its positive real or not $s^2 + 5s + 6$ [Noise] $s^2 + 5s + 4$

a one b one is twenty-five a zero square root a zero minus square root b zero minus two whole squared how much is this [Noise] two point approximately two point four so obviously greater than sign is satisfied okay

so it is ((peer)) (00:07:20) there is nothing obviously in this which can make it non peer

therefore we proceed by let's say start with partial fraction $Y(s)$ well in partial fraction you take the infinite frequency value is one plus let's say k_1 divided by $s + 1$ plus k_2 divided by $s + 4$

let us try does this work k_1 multiply by $s + 1$ and put s equal to minus one you get two by three that is positive good two by three

then k_2 [Noise] minus two by three so this doesn't work if it is negative ah obviously we don't know how to do it okay so we give up

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$$\frac{Y(s)}{s} = \frac{(s+2)(s+3)}{s(s+1)(s+4)}$$

$$= \frac{2}{s} + \frac{k_1}{s+1} + \frac{k_2}{s+4}$$

$$k_1 = \frac{(s+2)(s+3)}{s} < 0$$

we try y of s by s whereas that might work let us see s plus two s plus three divided by s plus one s plus four s so it is k_0 by s six by four three by two s plus all right plus k_1 by s plus one plus k_2 by s plus four what about k_1 s plus two s plus

<a_side> ((minus there doesn't work)) <a_side> is minus <a_side> (()) (00:09:00) <a_side> okay doesn't work [Noise]

similarly you can try the reciprocal of this Z of s

in either case you see if Z of s works then we know ah we know how to ah how to find but Z of s also doesn't work one of the residues comes negative [Noise]

if Z of s by s work because if it is Z RL if it is RL then you know Z of s by s shall work it doesn't work there either

i give up i ah [Noise] would not ah would not repeat this procedure but then we ask if it doesn't work what is it that we are missing and the clue these are tricks of the tray there are no set patterns

the clue is the following let me uh show you this one yeah [Noise]

<a_side> (()) (00:09:49) <a_side> ah that also we will try we will try but let's exhaust my uh partial fraction

the clue is here you see k_1 is positive but k_2 is negative now [Noise] is it possible to combine a part of this is it possible to combine a part of this constant with this term which is a negative {cosh} (00:10:14) negative residual to make it positive that's the clue but if this was not there then of course nothing could be done but since there is a constant terms i could take a part of this and add here so that negative ah coefficients disappear

<a_side> ((sir that means here uh taking out the Z value which greater than)) <a_side> no [Laughter] it is not RC it is not RL either and therefore there those considerations do not apply here

what we can remove that consideration doesn't apply here because this is neither pure RC nor pure RL all right

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$$Y(s) = K_{20} K_1 + \frac{K_2}{s+1} + \frac{K_3 s}{s+4}$$

$$K_1 + \frac{K_2 s}{s+1} + \frac{K_3 s}{s+4}$$

$$K_3 = \frac{(s+4) Y(s)}{s} \Big|_{s=-4} = \frac{1}{6}$$

so we argue like this that perhaps what we are missing is because it cannot be RC because it cannot be RL it is not pure LC either it is RLC therefore perhaps what we have to do is we have let's say some constant k zero let's say or no let's let's put it something else ah something constant k [Noise] one plus k two divided by we found one of the uh one of the residues positive

so let's make it as plus one plus let's say some k three by s plus four k three cannot be negative why don't we try k three s by s plus four this is what i am trying here

<a_side> ((yes)) <a_side> a resistance a parallel combination of resistance and capacitance and this will be a parallel combination of resistance and inductors

now nobody told me nobody told me before hand whether k three will be positive or not and there is nothing sacred about assigning the pole at s equal to minus four to k three s by s plus four i could have done it the other way round

for example i could have written k one plus k two s [Noise] by s plus one let's do this first plus k three by s plus four why don't i do this

<a_side> ((k three)) <a_side> because to find k three the value of k three is found by the same procedure and k three was found to be negative so you don't try this do you follow this these are various steps one is to apply they said there is no set pattern you have to exercise your [Noise]

you see if i write it like this then how do you find k_3 you multiply the function by $s + 4$ and put s equal to -4 that you had already found to be negative so we don't try this

now [Noise] if i go by this then what is k_3 $s + 4$ y of s divided by s all right [Noise] at s equal to -4 and this ah comes out to be one sixth is that correct which ah is very hopeful which is very encouraging that it's positive therefore all my three ah constants k_1 k_2 k_3 are positive [Noise]

<a_side> ((instead of doing this procedure so we will replace one by ah say three $s + 4$ by three $s + 4$ and then we have added the two ah)) <a_side> uh {bes} (00:13:31) besides that's what i am doing you see taking a part of the constant and adding to the ah that negative coefficient negative residue term that's besidesly what i am doing but i am doing it in the more systemic manner i argue that one of the ah um ah parallel combinations perhaps is RL ah which which can it be obviously the one with a pole at s equal to -4 why obviously because otherwise we found k_3 to be negative

<a_side> ((sir how can we share about k_1)) <a_side> how can how can we be share about k_1 why why should k_1 be there

<a_side> (()) (00:14:07) <a_side> oh there is no guarantee you have to find out k_1 what is k_1

<a_side> ((one minus one plus)) <a_side>

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$$\frac{(s+2)(s+3)}{(s+1)(s+4)} = K_1 + \frac{2}{3} \frac{1}{s+1} + \frac{1}{6} \frac{1}{s+4}$$

$$K_1 + \frac{1}{6} = 1$$

$$K_1 = \frac{5}{6} \checkmark$$

so k one is now now wait wait a second there must be systemic procedure

s plus two s plus three divided by s plus one s plus four this i had said k one k two is the same as in the previous case how much was it

<a_side> ((two by three)) <a_side> two by three divided by s plus one then i find out one sixth s divided by s plus four so what is k one [Noise] and how do you find

<a_side> (()) (00:14:46) <a_side> put s equal to i could put s equal to zero also

<a_side> (()) (00:14:54) <a_side> [Laughter] what dear no it's not it's not that difficult you see k one if i put s equal to infinity becoming prefer that all right [Noise] we will do that if i put s equal to infinity this goes to infinity

so i get one sixth k one plus one sixth must be the infinite frequency value that means equal to one and therefore k one is five by six

i could put s equal to zero also [Noise] if i put s equal to zero then i get k one plus two third equal to three in to two six divided by four it would give the same result

so my final ah network is [Noise] five by six resistance then an RC don't you see the five by six that one sixth is now added to that negative residue term

<a_side> ((sir six by five resistance because it's the admittance)) <a_side> oh this is admittance yes six by five so it is not this fracture at all it is a faster two type structure

so i have six by five resistance then can you give me the values one RC can you give me the values

<a_side> ((there by two is L)) <a_side> three by two is L okay so i have R and L [Noise]
 why is three by two an L this is admittance okay you are right three by two is the inductance
 and the resistance is three by two

now what about the capacitance <a_side> ((one by twenty-four)) <a_side> capacitance is
 one by twenty four and the resistance is six that is a correct structure [Noise]

is there an alternative can you find an alternative another structure foster type why not take
 the impedance

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$$Z(s) = \frac{(s+1)(s+4)}{(s+2)(s+3)} = K_1 + \frac{K_2}{s+2} + \frac{K_3}{s+3}$$

$$= K_1' + \frac{K_2' s}{s+2} + \frac{K_3}{s+3}$$

take the impedance as Z of s equal to s plus one s plus four divided by s plus two s plus three
 and expand like this means sum k one plus k two by s plus two plus k three by s plus three
 will this work will this work and why not what is

<a_side> ((k two be negative)) <a_side> why should k two be negative the reason is obvious
 you have to see\

<a_side> (()) (00:17:39) <a_side> you see if you assume this form obviously you are
 assuming an RC network this is not an RC function and therefore one of the residues must
 be negative and which one is negative any one say

<a_side> ((k two is negative)) <a_side> k two is negative if k two is negative then you
 assume this to be of the form k prime plus k two prime s divided by s plus two plus k three
 by s plus three all right

you don't find k one to start with you find k three in the usual manner you find k two prime by multiplying by s plus two and dividing by s and putting [Noise] s equal to minus two then you put either s equal to zero or s equal to infinity to find k one prime is this point clear [Noise]

so there is the structure that you will get is [Noise] the faster one in which one ah parallel circuit is RC the other parallel circuit is RL you agree

now let's see [Noise] is there any question as i said this has to be trial and error but a bit of commonsense has to be applied then you can avoid some unnecessary unnecessary wastage of time and efforts [Noise]

now lets try uh continued fraction

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Handwritten mathematical work showing the continued fraction expansion of the impedance function $Z(s) = \frac{s^2 + 5s + 4}{s^2 + 5s + 6}$. The work shows the initial division and subsequent steps of the continued fraction process with various terms and remainders.

our function is s squared plus five plus plus four divided by s squared plus ah five s plus six all right this is by impedance function and blindly blindly let us start continued fraction expansion with highest powers

<a_side> (()) (00:19:34) <a_side> you will get negative so this term you are

what about starting with the lowest powers let's see y of s six plus five s plus six four plus five s

<a_side> (()) (00:19:50) <a_side> oh s square s squared four plus five s plus s squared let us see if this works [Noise] two third oh no three by two or two by three

<a_side> ((two by three)) <a_side> [Laughter] okay four plus ten by three s plus two by three s squared i am left with five by three s plus one by three s squared six plus five s plus s squared the quotient is eighteen by five s okay

good enough this is y this is z so i get six plus yeah [Noise] eighteen by fifteen is that right okay

<a_side> (()) (00:20:50) <a_side> which one is six by five s this is okay ah [Laughter] six by five all right all right [Noise]

so i get twenty-five minus six nineteen by five s plus s squared five by three s plus one third s squared now what

<a_side> ((sir uh sir how is the first one ah one)) <a_side> where is the first one because we started the oh no we didn't invert it this is Z thank you for the correction this is y [Noise] how much is this

<a_side> ((sir twenty-five by fifty-seven)) <a_side> twenty-five by fifty-seven so five by three s plus i need i need not go further because the remainder is negative okay

so sorry about it this doesn't work [Noise] and you can you can try taking [Noise]

<a_side> (()) (00:22:01) <a_side> oh it depends on where did you start with you see in here here my z of s was this and i started with lowest power six plus five s plus s squared in the denominator therefore the coefficient is independent

now [Noise] there comes now a a trick or exercising some ingenuity or trial and error

<a_side> ((excuse me sir)) <a_side> yeah <a_side> (()) (00:22:32) <a_side> this one <a_side> ((yes sir)) <a_side> we did try this isn't it did we try <a_side> ((yes sir)) <a_side> we did it didn't work

what happened to my well it didn't work

<a_side> (()) (00:22:50) <a_side> same page <a_side> ((on the top sir)) <a_side> on the top [Laughter] we did try you see the um ah obviously the {cosh} (00:23:02) the remainder will be negative

now what we ask is the following it is at this stage that the continued fraction expansion falls to ground suppose at this stage we reverse the order in other words we come up to this we

come up to this then i is i do continued fraction expansion starting [Noise] with highest powers let us do that

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$$\begin{array}{r}
 s^2 + \frac{19}{5}s \quad \left(\frac{1}{3}s^3 + \frac{5}{3}s^2 \right) \left(\frac{1}{3} \leftarrow \frac{1}{3} \right) \\
 \hline
 \frac{1}{3}s^2 + \frac{19}{15}s \\
 \hline
 \frac{28}{5}s^2 + \frac{19}{5}s \quad \left(\frac{5}{28} \leftarrow \frac{5}{28} \right) \\
 \hline
 \frac{19}{15}s \quad \left(\frac{19}{14} \leftarrow \frac{19}{14} \right) \\
 \hline
 \frac{2}{5}s \\
 \hline
 0
 \end{array}$$

s squared plus nineteen by five s divide one third s squared plus five by three s as i said there is no set procedure ah [Noise] the first quotient is one third one third s squared plus nineteen by fifteen s

<a_side> (()) (00:23:55) <a_side> um hum nineteen by fifteen s is it okay so i get fifteen

<a_side> ((six by fifteen)) <a_side> six by fifteen that's right thirty now how did you get six three times five fifteen okay six by fifteen s then s squared plus nineteen two by five s all right

then i get five by two s five by two s s squared five by two is okay all right then i get nineteen by five s two by five s it does work all right i don't have to proceed further i know i know it works all right [Noise]

so ah it is not necessary that you continue continued fraction expansion now why did you decide that we will do it at this stage at this stage

<a_side> ((because the next stage)) <a_side> because the next stage was coming negative

so we do something to the previous stage to correct it if it did not work if it did not work then what would you have done you have said sorry about it [Laughter] we can't do it

<a_side> ((one by three is)) <a_side> because the other way round it didn't work starting with the highest part of

<a_side> ((what about the values of impedance and admittance so the uh dimensions))

<a_side> oh dimensions are okay dimensions are okay starting with highest power or lowest power that does not change the dimension um dimensions are okay

there may be other variations inside if it doesn't work for example if we take the reciprocal and then start with highest power or lowest power then the dimension would have changed okay

<a_side> (()) (00:25:56) <a_side> we can <a_side> ((sir then we change the dimension))

<a_side> then we change the dimension the next term would not be impedance then it would be an admittance

<a_side> (()) (00:25:08) <a_side> by this one hum this is also possible that is at some certain stage you can uh either go from lowest to highest or highest to lowest or you can take the reciprocal and go but as i said purely trial and error there is nothing nothing that ensures that there exists a continued fraction

<a_side> (()) (00:26:30) <a_side> that's right we will try that again <a_side> (())

(00:26:38) <a_side> there are many possible cases many possible cases <a_side> ((sir even if the coefficient is ah ah positive then we can take the reciprocal or change that)) <a_side> still we can change the reciprocal if it works you see why where we encouraged to try reversing the ah order there must be a reason why were we encourage encouraged to try this procedure out that instead of

<A_side> ((s square cannot to be negative so we can reverse it)) <a_side> no no that is a [Laughter] mechanical thing we try reverse but why didn't we give up at this stage we had a hope that because one circuit has been found out one possible circuit has been found out we know that there exists an indefinite number of circuit and therefore there is not reason why partial fraction work with a slight twist why continued fraction would not work with another slightly twist okay

<a_side> ((excuse me sir)) <a_side> yes <a_side> (()) (00:27:44) <a_side> correct what is the other part what do you mean other part

<a_side> ((initially we are started with removing the lowest one sir)) <a_side> correct

<a_side> ((then we get negative term from there means if too remove the highest power sir))

<a_side> ah we can start with either highest power or you can take the reciprocal

<a_side> ((sir if again we get a negative term can we shift to the lowest start moving the lowest one)) <a_side> yes why not anything is permissible so long as you don't change the dimensions so long as you don't change the numbers

<a_side> (()) (00:28:20) <a_side> you can change you can keep changing after every step it will be a mixture of cover one cover two and yes

<a_side> (()) (00:28:27) <a_side> right right so instead of the quotient being an impedance it will again been admittance okay [Noise] so two admittances will come in parallel or two impedances will come in come in series

<a_side> (()) (00:28:45) <a_side> oh it will not be a oh how do you break the circuit at that stage the two quotients if they have the same dimension then they will come either in parallel or in series they will not come as one in parallel and the next one is series that will not happen

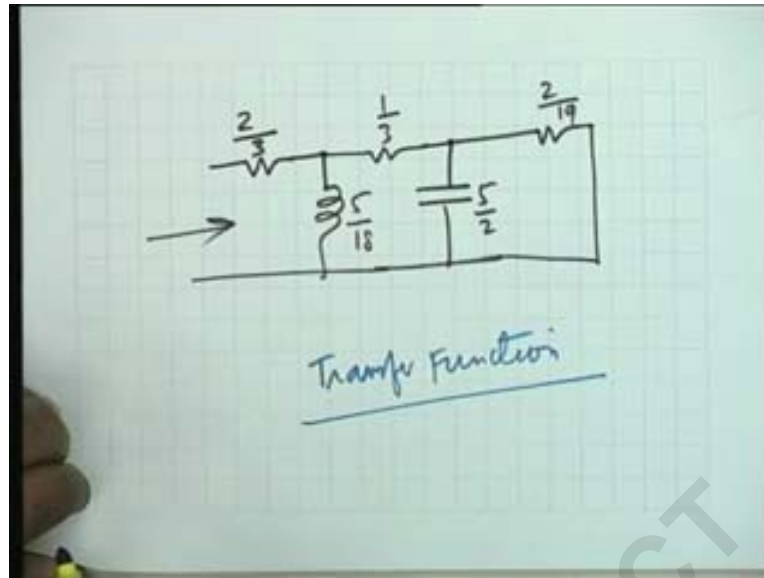
yes there is a question <a_side> (()) (00:29:02) <a_side> oh circuit realization okay let me complete it then or tell me what this what this two by nineteen so two by five s this is zero

now if we come back this is a z so this is y

<a_side> ((no sir one by z)) <a_side> what is a z <a_side> (()) (00:29:30) <a_side> oh this is not there this is a z no no no we took a z we started a z six plus five s plus s squared divides four plus five s plus s squared so it's a z z y this is z this is z [Noise]

so my circuit would be this

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two third then what would this be [Noise] inductors five by eighteen the next one
 <a_side> ((twenty-five by twenty-seven)) <a_side> no no no before that there is resistance
 of one third [Noise] next one is next one is capacitor five by two that was the effect of
 reversing the order

the next one is a resistance two by nineteen and then the remainder is zero impedance zero
 means a short circuit so this is the total circuit [Noise] okay

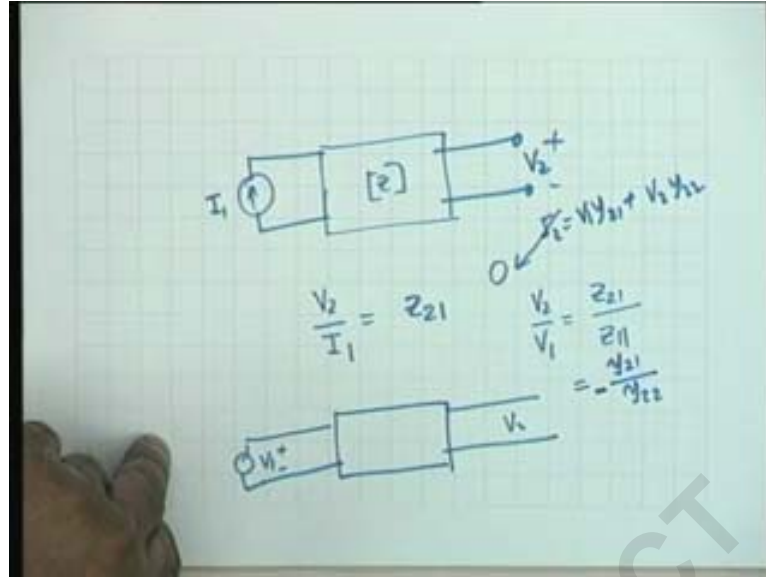
since ah a serious consideration of one port synthesis RLC is beyond the scope of this ah
 particular course we now turn attention to a glimpse into a transfer function synthesis and
 we recall a few facts

as you shall see by definition a transfer function is a network function that relates the output
 at one port to the input at some other port that is definition of transfer function right

otherwise if the port is the same because an effect at the same at the same port then you call
 it a rhyming point function

a transfer function therefore relates the effect or the response at one port to the cause or the
 excitation at some other port and you have already seen example [Noise]

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a current generator I_1 and you measure the voltage here V_2 then what is V_2 by I_1 ? it's a transfer impedance okay and how is it related to the parameters of the circuit and which parameter

z_{21} or z_{12} okay [Laughter]

z_{21} okay this is described by the z matrix this is z_{21} all right [Noise]

of course I know that if this is a reciprocal network which we have been discussing then this will also be equal to z_{12}

now suppose we take ah we take this network with a source impedance ah with a voltage source and you keep this open and measure the voltage

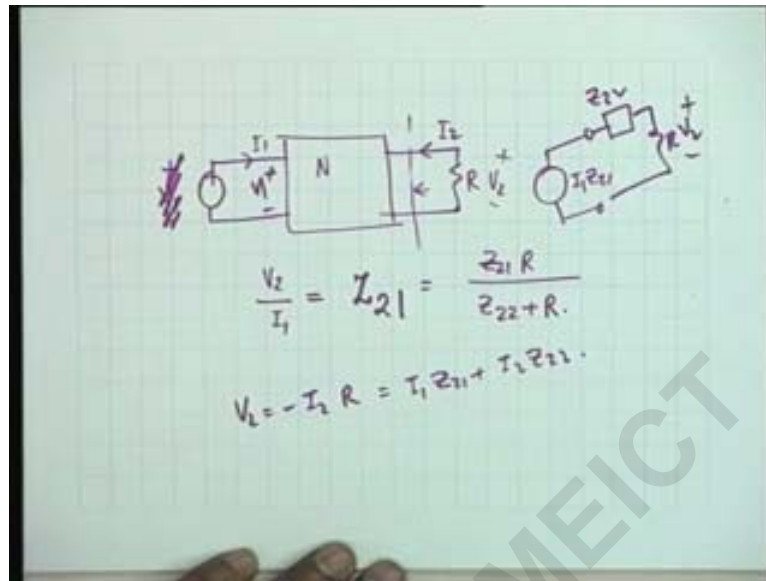
then V_2 by V_1 is the open circuit voltage transfer function and this is expressed in terms of z_{21} and z_{11} z_{21} by z_{11} its very simple write V_1 equal to $I_1 z_{11} + I_2 z_{12}$ and V_2 equal to $I_1 z_{21} + I_2 z_{22}$ but I_2 is zero therefore the ratio simply becomes z_{21} by z_{11}

I can also express this in terms of y parameters and if you don't recall this is y_{21} by y_{22} with a negative sign and it comes because I_2 equal to $V_1 y_{21} + V_2 y_{22}$ and I_2 is zero that's how it come [Noise] y_{22} and I_2 is zero that's how it come

these derivations I mean how you get this you should be able to remember okay ah simple things like this

now you complicate matters a little bit

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suppose we have a network N passive reciprocal network N driven by a voltage source V one and terminated in a resistance R the current is I two and what we are interested in is V two by [Noise] let's say I one this may be a current source now let's put is as a source it may be a current source it may be a voltage source

suppose we we want V two by I one you recall we had used a symbol for this this is a transfer function but no ah but not the z parameter of the network N because N is being augmented by capital R we call this a transfer impedance and we used a capital Z for this

Z two one that is the response point to the excitation point and this is simply ah equal to Z two one R divided by Z two two plus R

do you recall how this is done how this is derived how

<a_side> (()) (00:35:07) <a_side> z two one no this is capital Z two one no capital; Z two one is the transfer impedance of the terminated network there is a small z two one here which relates to N

<a_side> (()) (00:35:25) <a_side> [Noise] no no no no this is the actual condition terminated under terminated condition okay

now how did we obtain this give me a simple method

$V_2 = -I_2 R$ and $V_2 = -I_2 Z_{21} + I_1 Z_{22}$ that's wonderful absolutely wonderful

if you go back to the basics there is no ah you cannot make a mistake

well in a ah there is a simpler method if you don't want to write the equation you look at the Thevenin equivalent and derived by a current source what will be the open circuit voltage

$I_1 Z_{21}$ and the output impedance would be Z_{22} because current source and there will be a division between Z_{22} and R that's it

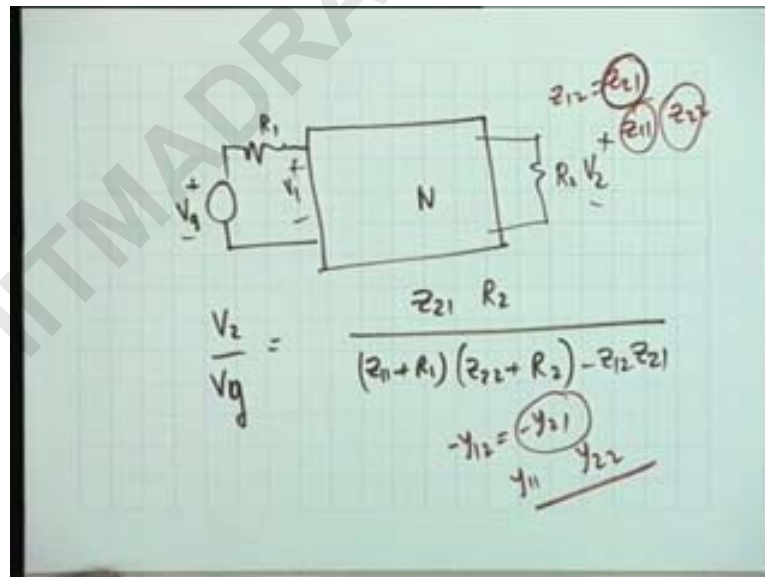
is the point clear not clear [Laughter]

this would be $I_1 Z_{21}$ this will be the open circuit voltage then opens the Thevenin impedance would be Z_{22} and you have an R this is V_2 okay

so there are simple ways i have done this in detail i am simply reviewing them but ah it is necessary [Noise] that you recall atleast a {simp} (00:36:57) simplest of that

we also treated the most general case

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that is we have a V_g generator in series with a resistance R and network N and a termination of R_2 two V_2

we have found out that V_2 by V_1 is equal to $Z_{21} R_2$ divided by $[Noise] Z_{11} + R_1$ this is the effective Z_{11} of this network $[Noise]$ multiplied by $Z_{21} + R_2$ effective $Z_{22} - Z_{12} Z_{21} [Noise]$

of course if you remember this then you can derive every other case as a special case of this this is the most general case terminated in both the ports and at one port is the excitation at the other port is a $[Noise]$ the the this point

the point to notice from all the simple examples is that the transfer function yes

$\langle a_{side} \rangle ((V_2 \text{ by } V_1)) \langle a_{side} \rangle V_2 \text{ by } V_1$ is here thank you for the correction that's wondering way people have not notice this is there any other mistake all right

Silent says that you have may be able to detect ah which means in all probability there is no other mistake $[Laughter]$ okay the thing that i want you to notice is that in in computing this transfer function simple transfer functions we have always struck to Z and Y parameters and this is one of the philosophies of synthesis

Z and Y parameter came fast in to the picture scattering came much later and uh the H parameters came the H parameters gained there prominence when transistor was brought in into the field of electrical engineering

so ah lets say around nineteen forty-nine or nineteen fifty the H parameters struck gained their prominence

the transmission parameters gained prominence when people wanted to build systems as a cascade okay but basic transfer functions we work only in terms of Z and Y parameters due to historical reasons

the other reason is that it is more convenient to use because impedance and admittance we know the properties $[Noise]$ you see all these transfer functions involve Z_{11} which is equal to Z_{22} and then these driving point functions Z_{11} Z_{22} we already know the properties of this okay

at least two element kind we know completely one element kind we know that they have to positive there

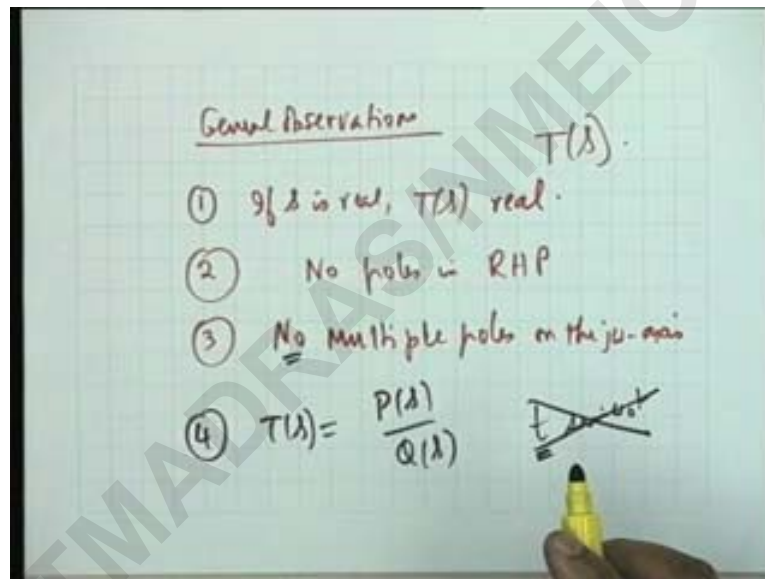
either this or you can express in terms of Y parameters that is minus Y_{12} and minus Y_{21} then Y_{11} and Y_{22}

we also know [Noise] the properties of driving point admittances they must also be positive real

so if we now know if we know investigate the properties of this transfer impedance and this transfer admittance that is Y_{21} and Z_{21} if we know the properties of this then we should be able to know the properties [Noise] of a transfer function properties of a two port all right

so um we shall now [Noise] spend sometime on the properties of this transfer impedance and this transfer admittance okay but before that ah some general observations you see whatever the transfer function is some general observations

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let us call [Noise] the transfer function as capital T of s the transfer function could be could have be dimension of an impedance could have be dimension of admittance or could be a dimensionless transfer function

if it is dimensionless then it could be a voltage ratio or a current ratio but whatever it is ah if s is real what can you say about T of s it must also be real why because T of s contains Z_{12} Z_{21} Z_{11} and Z_{22}

Z_{11} and Z_{22} we know they are real for s real what about Z_{12} Z_{21} or ah Y_{21} Y_{12} they must also be real they are [Noise] rational functions and there is no reason why they should not be real for s real

so if s is real T of s is real this should be [Noise] if you get a transfer function having a complex coefficient you say sorry we cannot realize this

second property irrespective of what the transfer function is can you have a pole in the right half plane no no poles in right half plane because we are talking of passive networks passive networks are always stable all right and therefore there cannot be any poles in right half plane

if there are poles on the $j\omega$ axis can we have multiple poles on the $j\omega$ axis no no multiple poles on the $j\omega$ axis and what is the logic now because this leads to instability that is even if the input is bounded the output shall not be bounded [Noise]

give an example suppose we have a we have two pairs of poles at plus minus $j\omega$ naught then the time domain response shall be of the form t times sign ω naught t or cosine ω naught t [Noise] and obviously as t goes to infinity these oscillation amplitudes grow indefinitely

so no multiple poles on the $j\omega$ axis this must be shared by all transfer functions if there is a multiple pole then you give up you you don't you don't proceed further

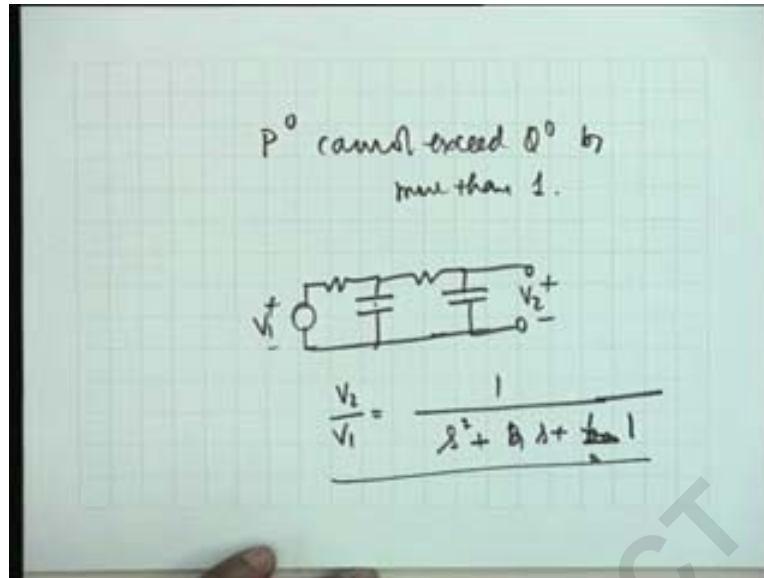
next one is a tricky question suppose T of s is written as P of s by Q of s what can be the maximum difference in degree between the two

well [Noise] i said it's a tricky question so [Noise] maximum difference maximum difference obviously can be only one isn't that right because if the difference is two we shall have a multiple pole at infinity which is a point on the $j\omega$ axis

if the if Q of s ah well ah i think i have said what i wanted to say [Laughter] there is the maximum difference cannot exceed one but can they be equal <a_side> ((yes sir)) <a_side> yes it can be equal [Noise] can no i made a wrong statement i take that [Noise]

maximum difference in degree can be more than what [Laughter] let me let me tell you

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i should what i should have said is that degree of P the numerator degree cannot exceed denominator degree by more than one this is the correct statement because Q of s degree can be three and P of s degree can be zero

for example you take this network you know that is can you write down its transfer function by inspection by just looking at it that is i have a voltage source and this is the voltage that i that is my [Noise] concern V_2 by V_1 what would be the form of this

<a_side> (()) (00:45:25) <a_side> one by s square plus let's say b one s plus [Noise] yes what should be this {coeff} (00:45:37) this coefficient

constant b zero is there a restriction on b zero or it must be positive but [Laughter] don't we know its value [Noise]

i appeal to your commonsense what is the DC transfer function if you apply DC here

<a_side> (()) (00:46:01) <a_side> of course it is a constant but what is the value of the constant

<a_side> ((constant)) <a_side> one it has to be one because a DC this is open this is open and therefore whatever voltage you apply here will come no current is flowing therefore b zero has to be one and you see that the numerator degree is zero the denominator degree is two they are differing by two they can differ by any number in fact but the restriction is that the numerator degree cannot exceed the denominator degree by more than one and the reason is that if it exceeds by more than one we shall have a multiple pole at infinity

here what is ah what is wrong what is right here and what is wrong here [Laughter]

there is a multiple zero at infinity that is permitted for a transfer function it is not permitted for a driving point function

why not why is such a thing not permitted in a driving point function because the reciprocal of a driving point function is also a driving point function and if there is a multiple zero at infinity there shall be a multiple pole at infinity of another driving point function which therefore ceases to qualify as a driving point function [Noise] the reciprocal of a transfer function is not necessarily a transfer function

for example the reciprocal of this cannot be realized why not because it has a multiple pole at infinity is the point clear so this is the first difference between a driving point function and a transfer function

that if a transfer function is written as P by Q the degree of P can be at the most one greater than the degree of Q but the degree of Q can be any number greater than the degree of P any number depends on the complexity of the transfer function [Noise]

just one more ah [Noise] one more observation obvious property then [Noise] then we will see we will call it a class

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$$T(s) = \frac{m_1 + n_1}{m_2 + n_2}$$

$$|T(j\omega)|^2 = \frac{m_1^2(j\omega) \cdot n_1^2(j\omega)}{m_2^2(j\omega) \cdot n_2^2(j\omega)}$$

n_1 s
 $n(j\omega)$ $j\omega$

suppose T of s is written as m one plus n one divided by m two by n two we have always been doing this with driving point function

now [Noise] i appeal to your commonsense and ask you what is $T j \omega$ squared in terms of m one of $j \omega$ and n one of $j \omega$

m one no m one of $j \omega$ square agreed then plus n one of $j \omega$ square divided by n two i set you a trap and you have fallen into it

<a_side> (()) (00:49:05) <a_side> no i am trying finding a magnitude squared magnitude square should be real part squared plus imaginary [Noise]

<a_side> (()) (00:49:15) <a_side> any other uh brilliant suggestion ah <a_side> (()) (00:49:24) <a_side> say it again

<a_side> (()) (00:49:30) <a_side> it wont be into $j \omega$ <a_side> ((this omega sir it's not a function of $j \omega$)) <a_side> all right we put s squared equal to minus omega square okay that is not the [Laughter] inaccuracy there is something else something else grossly wrong and i have cautioned against it several times

you see n one is the odd part suppose n one is equal to s then what is n one of $j \omega$ it is $j \omega$ omega would you bring here n one squared $j \omega$

<a_side> (()) (00:50:05) <a_side> that's it that correction is there you see the imaginary part is n one $j \omega$ divided by j

<a_side> (()) (00:50:19) <a_side> just a minute just a minute

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$$s^2 + 1 + 1$$

$$1 - \omega^2 + (j\omega)$$

$$(1 - \omega^2)^2 + \omega^2$$

$$- (j\omega)^2$$

we take a function lets say s squared plus s plus one

then one minus omega squared is the real part and plus j omega so the magnitude squared would be one minus omega squared whole square plus omega square

if i insist on taking this i must add a negative sign minus j omega whole square isn't that right please be careful [Laughter] about it

i have cautioned you again and again that [Noise] when you take the odd part you and put s equal to j omega then the imaginary part is that quantity divided by j because j comes into the picture

<a_side> (()) (00:51:18) <a_side> you can do that you can take the mod okay

now the question that i was asking is

<a_side> (()) (00:51:28) <a_side> yeah that is correct well it should have it should have been clear also from the consideration very simple consideration i was i was hood winking i was trying to take to detour and and confuse you and you have been confused please [Laughter] please come out of it

you see what you have to do is to find out the magnitude square

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$$T(s) = \frac{[m_1(s) + n_1(s)]}{[m_1(s) - n_1(s)]}$$

$$|T(j\omega)|^2 = \frac{|P(j\omega)|^2}{|Q(j\omega)|^2}$$

take m one plus n one and then you take m one minus n one j omega

this is this is the numerator P j omega and this is P minus j omega which will make the numerator magnitude squared

so $T(j\omega)$ magnitude squared is equal to the numerator [Noise] squared divided by the denominator squared

<a_side> (()) (00:52:30) <a_side> now but if multiply the two isn't it $m^2 - n^2$

<a_side> (()) (00:52:38) <a_side> oh let it do that also yeah you let it do that also [Noise]

the question that i am asking now is i have i have i have discussed this earlier what can you say about this magnitude squared function it must be purely even isn't it right

<a_side> ((yes sir)) <a_side> magnitude squared function cannot be negative it only contains even powers of ω in the numerator as well as the denominator

what about the magnitude that is the square root of this magnitude that must that also be an even function and what about the argument angle of $T(j\omega)$ must be an odd function okay

this we have said again and again and you will see the deflection of this evenness and oddness in this synthesis procedure they play a dominant part

we will continue this in next class [Noise]

thank you