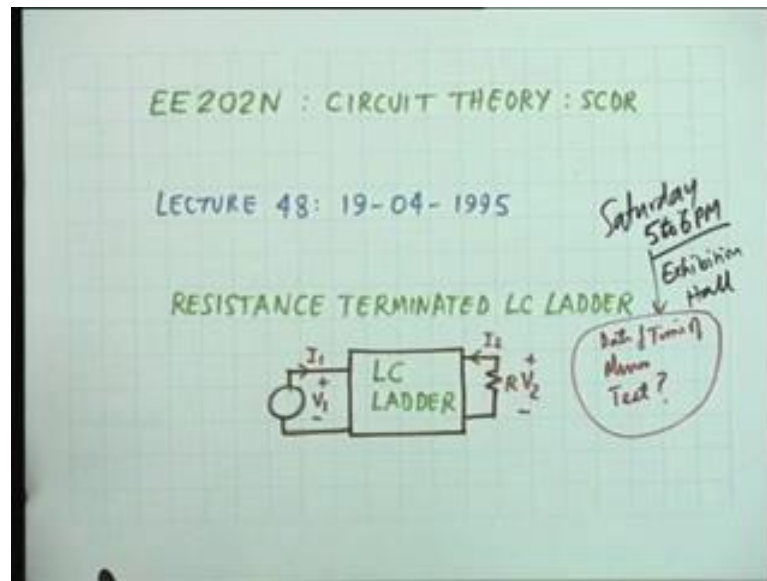


**Circuit Theory**  
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**Lecture - 48**  
**Resistance Terminated LC Ladder**

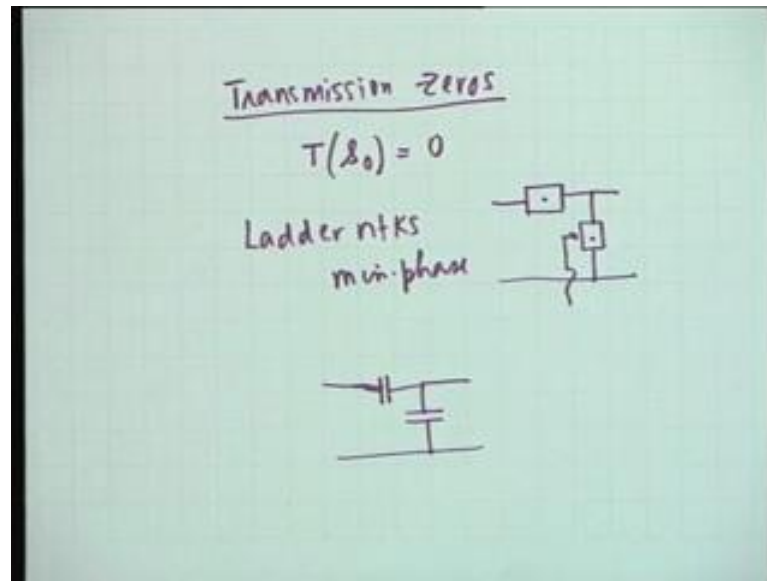
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This is the forty eighth lecture and we are going to discuss the synthesis of this particular configuration. That is LC ladder terminated at 1 end with a resistance, at the other end it is driven by either a voltage generator or a current generator. That's why I am not indicated, whether it is a voltage generator or current generator, it could be a voltage generator it could be a current generator.

Accordingly, the transfer function could be any of the 4 types. For example,  $V_2$  by  $I_1$  would be a transfer impedance,  $I_2$  by  $V_1$  would be a transfer admittance,  $V_2$  by  $V_1$  would be a transfer would be a voltage transfer function and  $I_2$  by  $I_1$  would be a current transfer function.

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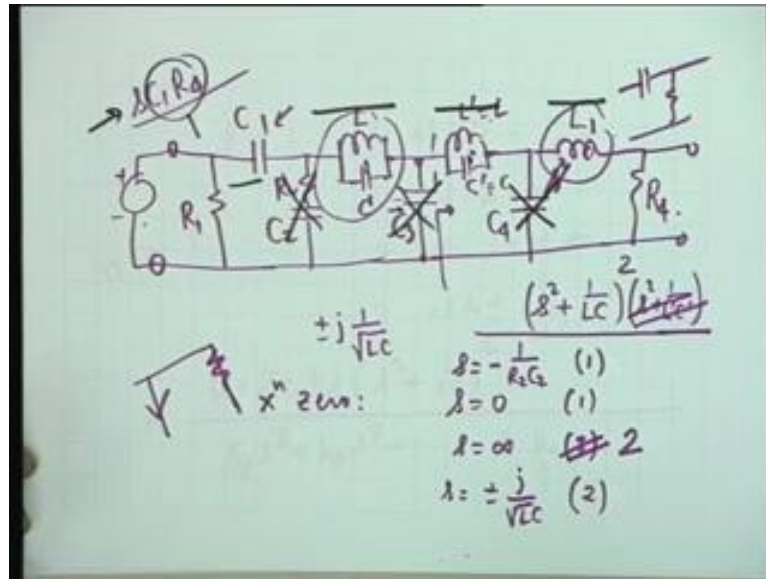


This is the specific thing that we shall we are going to discuss today, but a couple of points, about transmission 0s before we go on to this. You recall the transmission 0s are values of  $s$  at which the transfer function vanishes  $T$  of  $s_0$  equal to 0 if  $T$  is the transfer function. It could be impedance, admittance or a ratio of voltages or ratio of currents if there exist a frequency at which the value is 0, then that is called a transmission 0. We also saw that for a ladder network transmission 0s are either the series impedance poles or shunt impedance 0s.

For a ladder network, the transmission 0s are either series impedance poles or shunt impedance 0s. And since, these are driving point impedances they must be a positive real functions and therefore, ladder networks are necessarily, necessarily minimum phase networks; A minimum phase network is 1 in which the 0 of transmission is in the left half plane, ladder networks are minimum phase.

You would also recall, that a series impedance pole is not necessarily a transmission 0. If the input impedance looking from that impedance, if the impedance looking here also has a pole at the same frequency, then that is not a transmission 0. For example, as I said if you have a network like this well this does not have a transmission 0 at  $s$  equal to 0. Because, at  $s$  equal to 0 this gives rise to a pole, but this is also a pole and therefore, the voltage is distributed between the 2.

(Refer Slide Time: 03:03)



Let us, take a specific example to make some other points. We have let us, say a network like this suppose, we have in the ladder somewhere we have a an LC circuit like this. Then, perhaps, then we have let us say a capacitor another capacitor and so on. If we have a circuit like this obviously, plus minus  $j$  1 by square root  $LC$  shall be a pair of transmission 0 provided the input impedance here also does not have a pole there.

Now, what kind of a factor will this give rise to in the numerator of the transfer function. Whatever, the transfer function is as soon as you discover, that we have a pair of 0s at plus minus  $j$  by root  $LC$  what kind of a factor shall it give rise to in the numerator. Obviously,  $s$  squared.

Student: (Refer Time: 04:12)

1 by  $LC$  suppose, I have another such another such  $LC$  parallel circuit let us say,  $L$  prime  $C$  prime then obviously, I shall have another factor  $s$  squared plus 1 over  $L$  prime  $C$  prime.

Student: (Refer Time: 04:32)

No it is a transmission 0. The transfer function vanishes at that frequency and therefore, it has to be in the numerator. What was the question?

Student: (Refer Time: 04:47)

Pole of this is a 0 of transmission whenever this becomes open, there will be no transmission to the right it is a 0 of transmission of a ladder is either a series impedance pole or a shunt impedance 0. So, we shall have factors like this in the numerator suppose, now  $L'$  and  $C'$  are identical to  $L$  and  $C$  then; obviously, we shall have a double order 0 of transmission at  $\pm j \sqrt{LC}$  at  $\pm j \sqrt{LC}$  all right.

The point that I want to make is that there may be multiple 0s of transmission on the  $j\omega$  axis, which is not permitted in a driving point function. In a driving point function on the  $j\omega$  axis 0s and poles must be simple this is not, so far a transfer function a transfer function can have factor like this. Suppose, we continue  $L'$  equal to  $LC'$  prime equal to  $C$ . Suppose, we continue we have another capacitance here and let us, say an inductance here and suppose my ladder terminates here.

Let us, name them let us, call them  $R_1$  I have another resistance here  $R_2$ . Let us, call this as  $C_2$ , this is  $C_1$ , call this  $C_3$ ,  $C_4$ , then perhaps we have not used any other  $L$ . So, let us say  $L_1$  and let us, call this is  $R_4$ . By looking at this, looking at the ladder circuit, ladder network 1 can be find out 1 can conclude about the transmission 0s. For example, this capacitor does  $R_1$  give rise to a transmission 0 no it is a constant.

So, in any case if this is a voltage generator, then anything connected in parallel to voltage generator is in effective all right. So, that should not concern us anyway resistance is a constant it cannot give rise to a transmission 0. This  $C_1$  obviously, gives rise to a transmission 0 provided looking to the right is also not  $C_1$ , looking to the right at  $s$  equal to 0  $s$  equal to 0 what is the situation.

This is open, this is short, this is open, this is short, this is open, this is short, and then you have a resistance. So, there is a path from  $C_1$  to  $R_4$ , so  $C_1$  indeed blocks transmission. And therefore, there is let us, make a catalog of transmission 0 1 is at  $s$  equal to 0 due to  $C_1$  is there any other capacitance or inductance here which gives rise to a transmission 0 at  $s$  equal to 0 any other?

Student: (Refer Time: 08:03)

No. At infinity how many transmission 0s are there are there any first at infinity.

Student: (Refer Time: 08:10)

Obviously, this gives rise to a transmission 0, this gives rise to a transmission 0, at infinity and also this inductor at infinity. And therefore, at infinity there are 3 transmissions 0 all right. There are 3 transmissions agree 1 due to L1, 1 due to C4, 1 due to C3. There are 2 due to LC and LC, so  $s$  equal to plus minus  $j$  by square root LC there are 2 of them. Is there any other transmission 0? We have taken care of what about this 1? There is a transmission 0 at  $s$  equal to.

Student: (Refer Time: 08:58)

Minus 1 over  $R_2 C_2$  that is 1 transmission 0.

Student: (Refer Time: 09:03)

Yes.

Student: (Refer Time: 09:07)

This will also be a short at infinity yes so.

Student: (Refer Time: 09:23)

Let there be a current division, but this is open.

Student: (Refer Time: 09:29)

Due to C3 and C4 there are not 2 0s is that what.

Student: (Refer Time: 09:38)

Because, when C3 is short, so it is C4.

Student: (Refer Time: 09:42)

But, then so is open L1; L1 is also open at that point. There is in there is a 0 due to L1.

Student: (Refer Time: 09:48)

Suppose, there was a resistance instead of L1 yes.

Student: (Refer Time: 10:04)

What is the comment? There is an objection to making both C3 and C4 responsible for a transmission 0 at infinity. There is an objection to that you agree or you do not? You see, there is a that is what he is saying that if this is short obviously, there is no transmission. But looking here is the impedance equal to 0? Looking here is the impedance equal to 0? At infinite frequency?

Student: (Refer Time: 10:49)

Yes, it is 0, and therefore 1 of C3 and C4 creates a transmission the other does not. This is extremely interesting point and.

Student: (Refer Time: 11:00)

Only C4 creates, that is correct.

Student: (Refer Time: 11:06)

Where? No, we do not want to physical shift C3 I mean if it is in its place let it rest in place, but it does not create a transmission 0. Why not? This is precisely the point, this is precisely, why I use this particular circuit?

Student: (Refer Time: 11:26)

Does not matter because, of L1 it is open, but suppose L1 was not there that is why how multiple. For example, here if this is open, this is also open right, but the input impedance here is not a pole it is that it is 1 by s C3. And therefore, this creates a transmission 0, this also creates a transmission 0, that this is open does not hamper this also being open.

On the other hand, this short faces another short and therefore, C3 does not create a transmission 0. In other words, at s equal to infinity only 2 and that is due to not C3, but C4 and L1 now.

Student: (Refer Time: 12:20)

So, it is not a transmission 0. In other words, if there is a short circuit in parallel 2 short circuit the current there is a current division. But suppose, this was a finite impedance, then the current flows through this only, so it is a perfect transmission 0.

Student: (Refer Time: 12:44)

Instead of C4.

Student: (Refer Time: 12:59)

Instead of this.

Student: (Refer Time: 13:01)

Instead of inductor if there was a resistance.

Student: (Refer Time: 13:06)

Then, there would have been a transmission 0 due to L1 fine. There would not have been a.

Student: (Refer Time: 13:17)

Correct, but due to C4 there would have been 1 transmission.

Student: (Refer Time: 13:22)

No, no when C4 is a short nothing goes further. But when C3 is a short there something goes first. Now, on the basis of this data

Student: (Refer Time: 13:48)

Student: (Refer Time: 13:51)

L and L dash create a 0 at  $s$  equal to infinity no. Because, it is shorted C is shorted, so L and L prime at infinity are ineffective they are shorted up. Now, let me take this data and do some intelligent conclusions.

(Refer Slide Time: 14:12)

$$X^2 \quad \begin{array}{l} 1 \text{ at } s = -\frac{1}{R_2 C_2} \\ 1 \text{ at } s = 0 \\ 2 \text{ at } s = \infty \leftarrow \\ 2 \text{ at } s = \pm \frac{j}{\sqrt{LC}} \end{array}$$
$$K \frac{(s + \frac{1}{R_2 C_2}) s (s^2 + \frac{1}{LC})^2}{K_p s^4 + b_2 s^2 + \dots + b_0}$$

I have transmission 0s 1 at s equal to minus 1 by R 2 C 2 1 at s equal to 0. 2 at s equal to infinity and 2 at plus minus s equal to plus minus j by square root LC can I write from this the transfer function can I write. You see in the numerator will be s plus 1 over R2 C2 there is a 0 at the origin. So, the s must be a factor. Then, 2 at infinity what does this indicate?

Student: (Refer Time: 14:58)

Not at all 1 by s square will.

Student: (Refer Time: 15:05)

No.

Student: (Refer Time: 15:08)

Make another intelligent guess.

Student: (Refer Time: 15:10)

That is right it only means, that the degree of the denominator shall be 2 greater than the degree of the numerator right. So, I have to add this 2 s squared plus 1 over LC whole squared therefore, what will be the degree of the denominator now? 4 here 5 6 and



therefore, it would be 8. In other words, I shall have the denominator of the form  $b_8 s^8 + b_7 s^7 + \dots + b_0$  is that ok? In general right.

Can you also find, is this point clear? That the form of the transfer function, I have been able to write. In the numerator I wrote with leading coefficient unity, that is why in the denominator I did not make it unity. I could have made this unity by adding a constant  $K$  I could have done that. Now, is there any other information that I can get from here? Can I find out for example  $b_0$ ?

Student: (Refer Time: 16:23)

No, there are 2 factors because, I do not know these constants. But can I determine some of these constants  $b_0$  for example. Can I?

Student: (Refer Time: 16:35)

No at  $b_0$  we have to concentrate at  $s$  equal to 0. So, at  $s$  equal to 0 look at the network and see.

Student: (Refer Time: 16:50)

Correct, but which means, that the transfer function at  $s$  equal to 0 would be would tend to sum  $K$  prime  $s$  right. It would tend to.

Student: (Refer Time: 17:03)

If it is a transmission 0 at  $s$  equal to 0, then as  $s$  tends to 0 the transfer function must tend to  $s$  multiplied by a constant. Yes. Similarly, if it is a transmission 0 at infinity, then as  $s$  tends to infinity it must be constant divided by  $s$  or if there are 2 transmission 0s at infinity constant divided by  $s^2$ . From this information and from the network you can read, you can simplify the network at  $s$  equal to 0 you retain  $C1$ .

Because, this creates a transmission 0 you forget about this branch, this is infinite, this is a short, this is infinite, this is a short, this is infinite, this is a short, so all you have is  $C1$   $R4$ . Isn't that right? So, the transfer function would be  $R4$  divided by  $R4 + 1/sC1$ . So, the numerator would be of the form  $sC1 R4$  and the denominator would be of the form 1 as  $s$  tends to 0. So, you can find out this product  $C1 R4$ .

Student: (Refer Time: 18:17)

Student: (Refer Time: 18:20)

Let me, use a different color.

Student: (Refer Time: 18:22)

Correct you see what I was trying to do is can I find  $K$  by  $b_0$  in terms of the circuit elements. Well we can because at  $s$  as  $s$  tends to  $0$  well this is ineffective this remains this becomes open, this becomes short at  $s$  equal to  $0$  mind you this becomes a short, this becomes open, this becomes open, this is a short. So, all I have is  $C_1$  in series with  $R_4$  in shunt with  $R_4$  is that clear? At  $s$  equal to  $0$  the whole network reduces to a capacitance and a resistance.

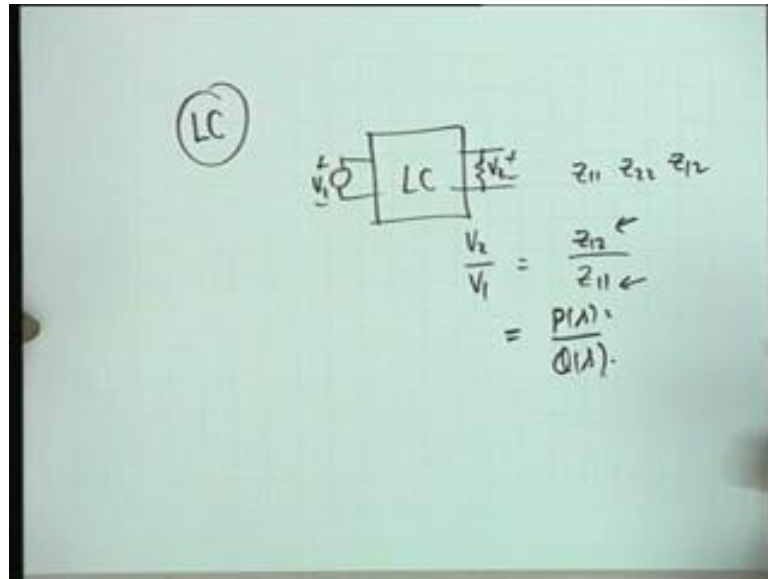
Student: (Refer Time: 19:07)

We are assuming a voltage source here.

Student: (Refer Time: 19:11)

What about  $R_1$ ?  $R_1$  is in parallel to voltage source, so it does not matter. If it is a current source yes it should there would have been a split. So, at  $s$  equal to  $0$  the transfer function tends to  $C_1 R_4$  and therefore,  $C_1 R_4$  must be equal to  $k$  divided by  $b_0$  point made. These are some of the intelligent guesses that I can make from this transfer functions. Let me ask you another question.

(Refer Slide Time: 19:47)



If you have an LC network if you have an LC some of the finer points of 2 poles synthesis and these are commonsense questions 1 has to think about this. You have a pure LC 2 port now you know that  $z_{11}$   $z_{22}$   $z_{12}$  all the 3 parameters are odd rational functions this we have proved. They are odd rational functions, if you want the voltage transfer function or the current transfer function for example, if you want the open circuit voltage transfer function  $V_1$   $V_2$  what is this ratio? In terms of these parameters it is  $z_{12}$  divided by.

Student: (Refer Time: 20:33)

No,  $z_{11}$ .

Now this is odd, this is odd and therefore, the transfer function would be a purely even rational function. That is if you write this as P of s divided by Q of s, then both P and Q shall be even polynomials. Is this point clear? But suppose there was a resistance somewhere in the circuit, maybe as a termination, then this properly is destroyed. Is that clear?

Student: (Refer Time: 21:10)

Yes.

Student: (Refer Time: 21:13)

It will modify the parameters and therefore, this property that will be purely even.

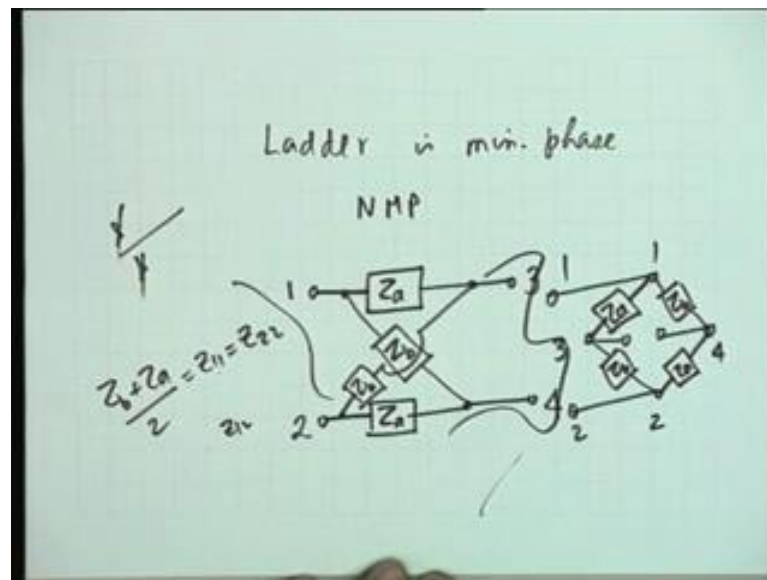
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It cannot be otherwise, you see if you have a even rational function, even rational function must be the ratio of a even polynomial to an even polynomial.

Student: (Refer Time: 21:43)

That cancels out we are taking P and Q to be primes with respect to each other, that is they have no common factor right another question.

(Refer Slide Time: 21:58)



You said a ladder is minimum phase.

Student: (Refer Time: 22:00)

Have to be even.

Student: (Refer Time: 22:09)

They are both odd s factor will cancel out. So, they will become even by even right. Odd polynomial is s times an even polynomial.

Student: (Refer Time: 22:21)

That is also even by even.

Student: (Refer Time: 22:27)

That means  $s$  is a factor in both numerator and denominator and that cancels out, so that is a trivial case. A ladder is a minimum phase network and it says that there exists non-minimum phase networks which are 1 of the common examples in all past networks. We have also d1 architecture, a structure of a non-minimum phase network and that is a lattice. Do you recall a lattice? We did this when being 2 ports.

A lattice we just recall what we did. Because, we will have occasion to talk about lattice. A lattice is a network like this, did we do lattice when, doing 2 ports?

Student: (Refer Time: 23:13)

Are you sure?

Student: (Refer Time: 23:17)

Only a problem let us, spend some time on this. A lattice usually, the form in which it is used is a symmetrical lattice. That is it goes like this there is an impedance  $Z_a$  here and there is an impedance  $Z_b$  here, it is a crisscross this is called a symmetrical lattice symmetrical lattice, whether, you a view whether you take this as 1 port or this as the first port it does not matter.

Because, it is symmetric the network is symmetric all right 1 should also realize that this the lattice is simply a redrawn configuration of a bridge network that it is a bridge can be very easily shown. Suppose, we take 1 2 3 and 4 between 1 and 3 there is a  $Z_a$ , between 3 and 2 there is a  $Z_b$ ; and between 3 and 2 there is a  $Z_b$ ; 3 and 2 between 2 and 4 there is a  $Z_a$  and between 4 and 1 there is a  $Z_b$  don't you see that this is 1 of the ports and the port is this agreed?

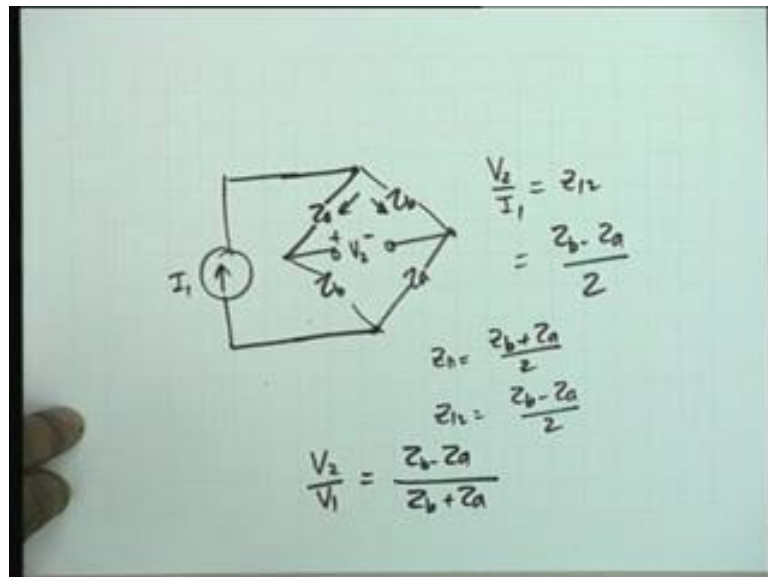
So, lattice is a redrawn form of a bridge: this bridge is not a balanced bridge. No.  $Z_a$  by  $Z_b$  is not necessarily equal to  $Z_b$  by  $Z_a$  unless they are equal all right which is a trivial case. So, it is not a balance network you had it been a balance bridge, then it would not have served as a 2 port why?

Student: (Refer Time: 25:20)

Because, then there is a transmission 0 at all frequencies isn't that right? So, it does not serve all purpose but it is important to identify, important to recognize that this is a bridge. And if you recognize this is a bridge, then you can determine, the Z parameters very easily. We did that I think this is the example that we did.

Z parameters input impedance with this open. So, it is simply Zb plus Za divided by 2 agreed? This is z11 is the point clear? Input impedance with the output port open. So, z11 is this and z12 since, it is symmetrical this is also equal to z22. And z12 is V2 by I1 let us do this would be interesting we recall because, we did this earlier.

(Refer Slide Time: 26:26)



What we have is  $Z_a$   $Z_b$  this is  $Z_a$  and this is  $Z_b$  we apply a current a generator  $I_1$  here and we measure the voltage  $V_2$  here plus minus. And  $V_2$  by  $I_1$ , shall be equal to  $Z_{12}$  you see  $V_2$  by  $V_2$  is this voltage minus this voltage which simply means, that it is  $Z_b$  minus  $Z_a$  divided by 2 why because, the current divides into 2 equal parts  $I_1$  by 2 and  $I_1$  by 2 we did this earlier.

So, our expressions are the  $z_{11}$  equal to  $Z_b$  plus  $Z_a$  by 2 and  $Z_{12}$  equal to  $Z_b$  minus  $Z_a$  by 2. And the transfer function if you want to find out  $V_2$  by  $V_1$  it is simply the ratio of this 2. And therefore, this is equal to  $Z_b$  minus  $Z_a$  divided by  $Z_b$  plus  $Z_a$  all right which can be non-minimum phase. Because, the 0s of transmission and the 0s of  $Z_b$  minus  $Z_a$  difference between 2 PR functions the 0s can occur anywhere in the s plane. For example, let us take a simple example.

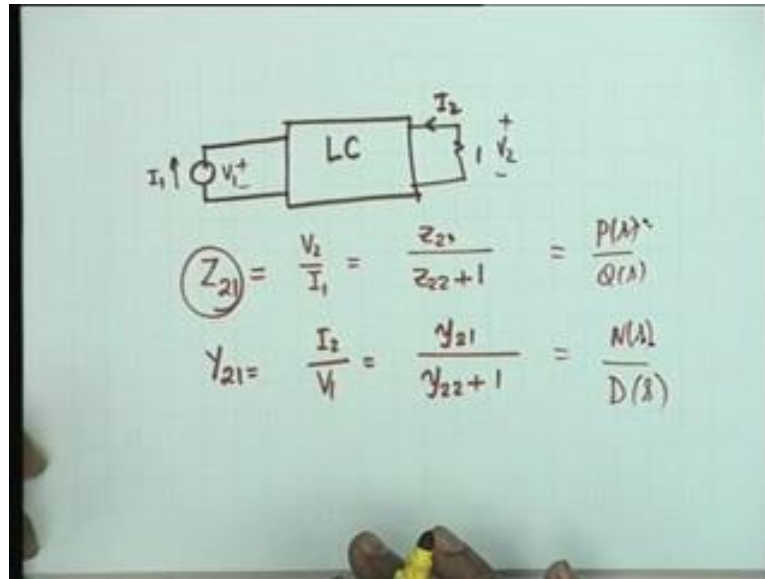
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The image shows a whiteboard with handwritten mathematical equations and a circuit diagram. On the left, the equation  $\frac{s-1}{s+1}$  is written, with a downward arrow pointing to  $\frac{1-\frac{1}{s}}{1+\frac{1}{s}}$ . To the right, the equation  $= \frac{Z_b - Z_a}{Z_b + Z_a}$  is written. Below this, a circuit diagram is drawn, consisting of two parallel branches. The top branch contains a resistor labeled  $Z_b$  in series with an inductor labeled  $L$ . The bottom branch contains a resistor labeled  $Z_a$  in series with a capacitor labeled  $C$ .

Suppose, I have the first order transfer function  $s$  minus first order all pass  $s$  minus 1 by  $s$  plus 1. Obviously, if we equate this  $2 Z_b$  minus  $Z_a$  divided by  $Z_b$  plus  $Z_a$  your  $Z_b$  would be a 1 Henry inductor and  $Z_a$  would be a 1 Ohm resistance. A simple lattice containing 2 inductors and 2 resistors that is serves the purpose. There is an alternative I could write this as  $1$  minus  $1$  over  $s$  divided by  $1$  plus  $1$  over  $s$ .

In other words,  $Z_b$  could be a 1 Ohm resistance and  $Z_a$  could be 1 Farad capacitor agreed. This is a simple synthesis of non-minimum phase transfer functions now, you go to that structure.

(Refer Slide Time: 28:55)



That is an LC network, LC network terminated in a resistance this is the most useful form of network synthesis, in practice most useful form. The it can be either a voltage generator or a current generator and the response could be either the voltage. We assume the termination to be 1 Ohm we normal the termination to 1 Ohm and this current is I2. So, we can have a transfer function like let us, say V2 by I1 what would you call this transfer impedance.

Student: (Refer Time: 29:35)

Trans impedance z21. And you can expression this in terms of the Z parameters that is z21 divided by anybody recall what this is? z22 plus the terminating resistance 1 z21 by z22 plus 1 you could also have a Y21 that is I2 by V1. And this simply in terms of the Y parameters of LC network it is Y21 divided by Y22 plus 1. There is a similarity between this that is why we take them, we take them together.

Now, in either case if this is let us say, P of s by Q of s and this is let us say N of s by D of s is there a conclusion that you can make about the numerator polynomials in the 2 cases. You see z21 that is right the numerator polynomials either P or N it must be either purely odd or purely even. Can you tell me why?

Student: (Refer Time: 30:57)



They cannot be even why not? You see, what is required is that  $z_{21}$  should be an odd rational function is either odd by even or even by odd right. It is the numerator that shall remain here  $P$  of  $s$  shall be the numerator of  $Z_{21}$ ,  $N$  of  $s$  shall be the numerator of  $Y_{21}$  the numerator can be either purely even or purely odd. Therefore,  $P$  or  $N$  of necessity have to be either purely even or purely odd?

Student: (Refer Time: 31:33)

No because, there is a resistance here.

Student: (Refer Time: 31:39)

$z_{21}$  is an odd rational function, so is  $z_{22}$ .

Student: (Refer Time: 31:46)

Total  $z_{21}$ .

Student: (Refer Time: 31:49)

Student: (Refer Time: 31:51)

That's right. This is neither purely even, not purely odd because this is not a pure LC network anymore it is a resistance, is a resistance.

Student: (Refer Time: 32:00)

Which part?

Student: (Refer Time: 32:02)

$z_{21}$  belongs to LC network  $z_{21}$  it is a  $Z$  parameter of the LC network, it is a  $V_2$  by  $I_1$ . And therefore, must be purely odd if it is purely odd it is either an odd by even or even by odd either case it is an odd rational function. And therefore, the  $P$  the numerator polynomial of  $Z_{21}$  shall be the same as the numerator polynomial of  $z_{21}$  and therefore, it should be either purely even or purely odd it cannot be the otherwise,

Student: (Refer Time: 32:38)

No I must have  $z_{21}$  is a rational function. P of s could be fourth degree polynomial you cannot have a network function which has 4 poles at infinity no.

Student: (Refer Time: 32:55)

(Refer Slide Time: 33:07)

$$z_{21} = \frac{P(s)}{P_1(s) + Q_1(s)}$$

$$z_{11} = \frac{P_1(s)}{Q_1(s) + Q_2(s)}$$

$$z_{22} = \frac{P_2(s)}{Q_1(s) + Q_2(s)}$$

$$z_{12} = \frac{P(s) Q_1(s)}{Q_1(s) Q_2(s)}$$

$$\frac{z_{21}}{z_{22} + 1}$$

No you see you remember,  $z_{11}$   $z_{22}$   $z_{12}$  that the same denominator in general, in general there is a same denominator polynomial. And therefore, if this is let us say this is P of s by Q of s let me, use some other  $Q_1$  of s. Then, the denominator or each of them shall be  $Q_1$  of s and therefore,  $z_{21}$  shall be P of s divided by P of s plus.

Student: (Refer Time: 33:41)

No some  $P_1$  of s plus Q if this is  $P_1$  of s.

Student: (Refer Time: 33:46)

Why into something.

Student: (Refer Time: 33:50)

Its 1 you see  $z_{21}$  is  $z_{21}$  divided by  $z_{22}$  plus 1 substitute this here.

Student: (Refer Time: 33:57)

If they are not the same how can it differ? Let, there be another factor  $Q_2$  s then this will be replaced  $Q_2$  s by  $Q_2$  s that is it. You see any pole of  $z_{12}$  must belong to these 2 whereas;  $z_{11}$  and  $z_{22}$  can have personal poles. If there are personal poles, there is another factor here see we had that here also in the numerator as well as denominator you can always make the denominator identical. The lesson from this exercise.

Student: (Refer Time: 34:38)

In the nodal analysis it was delta the node determinant node, admittance determinant.

Student: (Refer Time: 34:46)

All 3 this is the same.

(Refer Slide Time: 34:54)

$$Z_{21} = \frac{P(s)}{Q(s)} = \frac{z_{21}}{z_{21} + 1}$$

$$Q(s) = m + n \text{ is Hurwitz}$$

$$\left( \frac{m}{n} \right) Z_{LC}$$

The lesson from this exercise is that, if we consider let us say  $Z_{21}$  which is  $P$  of  $s$  by  $Q$  of  $s$  which is  $z_{21}$  by  $z_{22}$  plus 1. All we have commented is that, these numerator polynomial must be either purely even or purely odd. You can say this about  $Q$  of  $s$   $Q$  of  $s$  because,

Student: (Refer Time: 35:16)

That's it will be the sum of even and odd there is no other way. Because,  $z_{22}$  is odd and if you clear them of the denominator polynomial  $Q$  of  $s$  will surely be the sum of an even and odd polynomial. So, if  $Q$  of  $s$  is let us say  $m$  plus  $n$  if you break this up into even and

odd parts 1 thing that you can say about  $Q$  of  $s$  is that it must be Hurwitz. Isn't that right? Any transfer function to be realizable has to have a Hurwitz denominator. So, this is Hurwitz. If it is Hurwitz what is the test for Hurwitz? The ratio of the even to odd part.

Student: (Refer Time: 35:59)

Continued fraction expansion of  $n$  shall lead to.

Student: (Refer Time: 36:03)

Positive coefficient quotients  $s$  agreed? Continued fraction expansion of this shall lead to quotients which are positive coefficients. Now, continued fraction expansion is also cover 1 or cover 2 isn't that right? And therefore, do we agree that  $m$  by  $n$  will be an LC impedance do we agree that? It will be an LC impedance isn't it? If you make a continued fraction expansion with all quotients having positive coefficient; that means, you are determining inductors and capacitors agreed. So,  $m$  by  $n$  shall be LC it is either an impedance or admittance it does not matter now therefore, the synthesis.

Student: (Refer Time: 36:53)

This is the test Hurwitz? How do you test a Hurwitz polynomial? It is a there are 2 things you see LC impedances 1 of the properties is that poles and 0s interrelates. It is also true that any LC impedance can be expanded in continued fraction with positive coefficient quotients. Now, it is the other round here any Hurwitz polynomial the even and odd part. The ratio even and odd part can be expanded in continued fraction expansion with positive quotient, positive coefficient quotients. Therefore,  $m$  by  $m$  must be LC and of necessity it is poles and 0s must interrelate there is no other way.

Student: (Refer Time: 37:36)

Denominator because, the numerator is purely even or purely odd.

Student: (Refer Time: 37:44)

Yes what is but.

Student: (Refer Time: 37:48)

We require the whole we are coming to this.

(Refer Slide Time: 37:59)

$$Z_{21} = \frac{P(s)}{Q(s)} = \frac{z_{21}}{z_{21}+1} \quad \text{LC}$$

$$= \frac{P(s)}{m+n}$$

$P(s)$  even

$$\frac{P(s)/n}{\frac{m}{n} + 1}$$

Arrows:  $z_{21}$  points to  $P(s)/n$ ,  $z_{22}$  points to  $\frac{m}{n} + 1$

Now, the synthesis procedure should be quite simple I will illustrate this with  $Z_{21}$  that it is given as  $P$  of  $s$  by  $Q$  of  $s$ , where  $P$  of  $s$  is either purely even or purely odd. Because, we want to realize this is an LC network terminated in 1 Ohm resistance. So,  $P$  of  $s$  is purely even or purely odd and  $Q$  of  $s$  must be Hurwitz. So, what we do is we write the, we separate the even or odd parts.

Then, if  $P$  is even, if  $P$  of  $s$  is even, then we divide not the step we divide both the numerator and denominator by the odd part of the denominator agreed. If I do that then obviously, we shall get  $P$  of  $s$  by  $n$  divided by  $m$  by  $n$  plus 1 is the point clear? Now, do not you see that this is an odd rational function and therefore, you can identify this as  $z_{21}$ .

So, this as we have all ready argued is an LC driving point function therefore, this can be equated to  $z_{22}$  is that right? They have the same poles, they have the same poles  $z_{21}$  is odd,  $z_{22}$  is odd,  $z_{22}$  is LC driving point function  $z_{21}$  is not necessarily a driving point it is a transfer function.

Student: (Refer Time: 39:35)

That is right. It is  $z_{21}$  divided by  $z_{22}$  plus 1. So, what we have done is we have identified 2 parameters of this LC network and that is enough for synthesis,  $z_{21}$  by  $z_{22}$  plus 1 in order to, in order that this idea soaks in let us an examples.

(Refer Slide Time: 40:09)

$$Z_{21}(s) = \frac{2}{s^3 + 3s^2 + 4s + 2} = \frac{z_{21}}{z_{22} + 1}$$

$$= \frac{2/(s^3 + 4s)}{\frac{3s^2 + 2}{s^2 + 4s} + 1}$$

Then, we will see suppose, our function is 2 by  $s$  cube plus 3 $s$  squared plus 4 $s$  plus 2 suppose this is my  $Z_{21}$ . The numerator has to be either purely even or purely odd incidentally what kind of a function is this? What kind of filtering does this perform? Where are the 0s?

Student: (Refer Time: 40:34)

This is low pass obviously. What if the DC value?

Student: (Refer Time: 40:37)

1 infinity frequency value is 0 there are 3 transmission 0s at infinity.

Student: (Refer Time: 40:45)

That's right. Now, this say a lot if you make this commonsense conclusions the network is obvious. We will argue it out in a minute, but let us proceed systemically. We have this as  $z_{21}$   $z_{22}$  plus 1 since the numerator is even we divide both numerator and denominator by the odd part of the denominator that is  $s$  cube plus 4 $s$ . And I write this as

$3s^2 + 2$  divided by  $s^3 + 4s + 1$  all right. Therefore, this is my  $z_{21}$  and this is my  $z_{22}$ .

The problem now is that, it is not realization of just 1 parameter I can realize  $z_{22}$  by first 1, first 2, cover 1, cover 2. I can do any of this, but I have to simultaneously realize  $z_{21}$ . whatever, network I realize must satisfy the prescriptions on  $z_{22}$  and  $z_{21}$ . Now, we do not have to worry about the poles because, if you realize  $z_{22}$  by any manner you like the poles of  $z_{21}$  shall be the same. So, all we have to worry is about the transmission 0s. In other words, the problem of synthesis mark by words now is to synthesize  $z_{22}$  in such a manner that transmission 0s are realized.

In other words, the problem of synthesis now reduces to that of synthesizing a driving point function  $z_{22}$  is a driving point function. We have to realize  $z_{22}$  in such a manner that 3 transmission 0s at infinity are realized. Now,  $z_{22}$  means what we start from port number 2 with port 1 open. Now, how do we realize 3 transmission 0s at infinity obviously, we must have capacitors in parallel in shunt.

Student: (Refer Time: 43:08)

Not or we have to have. You see it is a third order function we should not require more than 3 reactive elements, more than 3 and there are exactly 3 transmission 0s. Therefore, my network must be of this form: 2 capacitors and 1 inductor incidentally, you see that this is a low pass filter.

Student: (Refer Time: 43:24)

There is no transmission 0 here either at  $s$  equal to 0. You see, my what is my network now? I can draw the total network now. What should I have here voltage generator or current generator?

Student: (Refer Time: 43:49)

No this is  $z_{21}$  current generator. So, I shall have a  $I_1$  and this is connected to 1 ohm resistance this is  $V_2$  agreed? So, all you have to do is to.

Student: (Refer Time: 44:05)

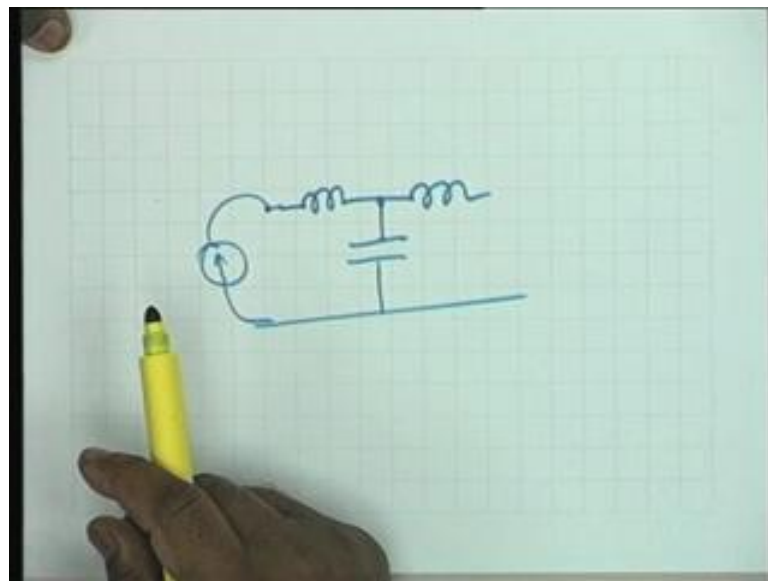
Can I please repeat yes of course, you see the problem has reduced that of synthesizing  $z_{22}$ . In such a manner that the transmission 0s are realized, if the transmission 0s are realized, then  $z_{21}$  shall realized. Because, they have the same poles whichever way you realize  $z_{22}$  the poles of  $z_{21}$  are being realized all you have to worry about is the 0s of  $z_{21}$  and the 0s of  $z_{21}$  are the transmission 0s.

Now, when we see that there are 3 transmission 0s at infinity and  $Z_{22}$  has to be realized such that, there are 3 transmission 0s at infinity. 2 of them are being taken care of by these 2 capacitors and the third 1 is being taken care.

Student: (Refer Time: 44:53)

How do we get that there are that is a good question. Why not 2 inductors and 1 capacitor that is a good question. I am glad you asked that question.

(Refer Slide Time: 45:07)



Suppose, we do that what is the problem? The problem is that if you connect a current generator here this pure inductor is in series with a current generator and is absolutely ineffective.

Student: (Refer Time: 45:23)

There will be 2 transmissions there cannot be 3 and therefore, I cannot choose this structure I must go back to 2 capacitors.



Student: (Refer Time: 45:34)

Depending upon the input suppose the input was a voltage generator, then this would not have been correct it would have been the 2 inductors and 1 capacitor is that ok? Now, what remains for the synthesis, what remains for the synthesis is to find these 3 elements. And all that you have to do now is to expand  $z_{22}$  in continued fraction right starting with which powers highest or lowest?

Student: (Refer Time: 46:10)

Highest powers. Now, can you start with  $z_{22}$ ? Can you start with  $z_{22}$ ? Or you have to take the reciprocal  $z_{22}$  does not have a pole at infinity.

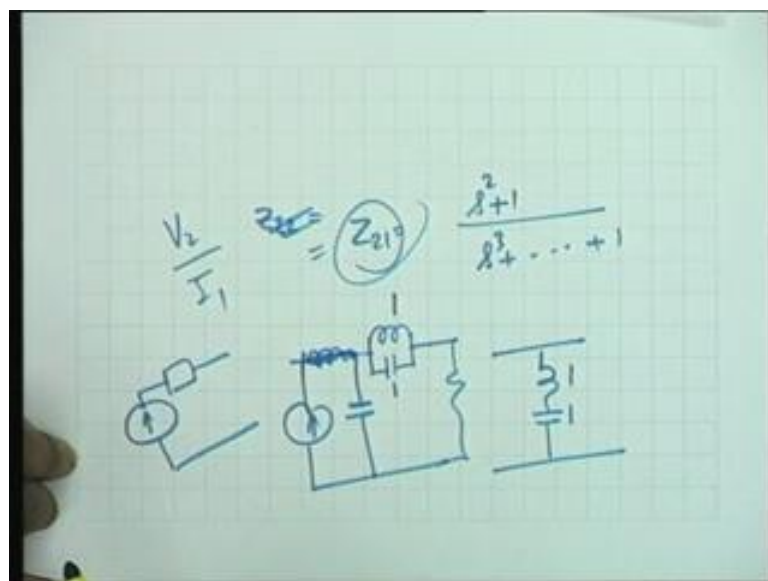
Student: (Refer Time: 46:23)

You take the reciprocal and make continued fraction expansion, then you will get  $C_1$ , you will get  $L$  you will get  $C_2$ . Finally, you shall get an admittance 0 which means that impedance open. Is this clear? The procedure.

Student: (Refer Time: 46:45)

Last 1 what we have let us, carry this out.

(Refer Slide Time: 46:52)



I have to realize  $z_{22}$ .

Student: (Refer Time: 46:53)

Intuitively, yes. We draw it intuitively absolutely.

Student: (Refer Time: 47:01)

Of course, you can if you know the transmission 0s. Let me, tell you let me how we think.

Student: (Refer Time: 47:10)

That's right suppose, I have a  $z_{21}$  let us take something else. Suppose, I have  $z_{21}$  which is  $s^2 + 1$  divided by let us, say  $s^3 + 1$  there are  $s^2$  and  $s$  term. Suppose, I have a transmission, a transfer function like this, then I know that either I shall have a parallel resonance circuit like this with 1 and 1 or I shall have a series resonance circuit like this in shunt. I know this because, I have to create a transmission 0.

In addition, if the degree of the denominator is 3, then I have 1 transmission 0 at infinity. Now, neither of this create a transmission 0 at infinity. So, I must have a capacitor either here or here.

Student: (Refer Time: 48:12)

No I cannot have another inductor.

Student: (Refer Time: 48:14)

Inductor in series here, I would not be able to realize  $z_{21}$  because, the whole thing now shall come in series with the current generator no way. The structure itself will show the impossibility you see this is not this current generator  $z_{21}$  is  $V_2$  by  $I_1$  and whole thing is in series, so it does not effect.

Student: (Refer Time: 48:41)

Because, it is a  $z_{21}$ ,  $z_{21}$  is  $V_2$  by  $I_1$  if we had  $Y_{21}$  that is the  $I_2$  by  $V_1$  yes this would have been in order, but not otherwise.

Student: (Refer Time: 48:55)

We will have we will continue.

Student: (Refer Time: 49:01)

Yes. The structure shall be obvious true common sense all you have to do is to argue, argue it out.

Student: (Refer Time: 48:10)

We have to have a capacitor in parallel, we have to here there is no other way, you cannot have the capacitor here yes.

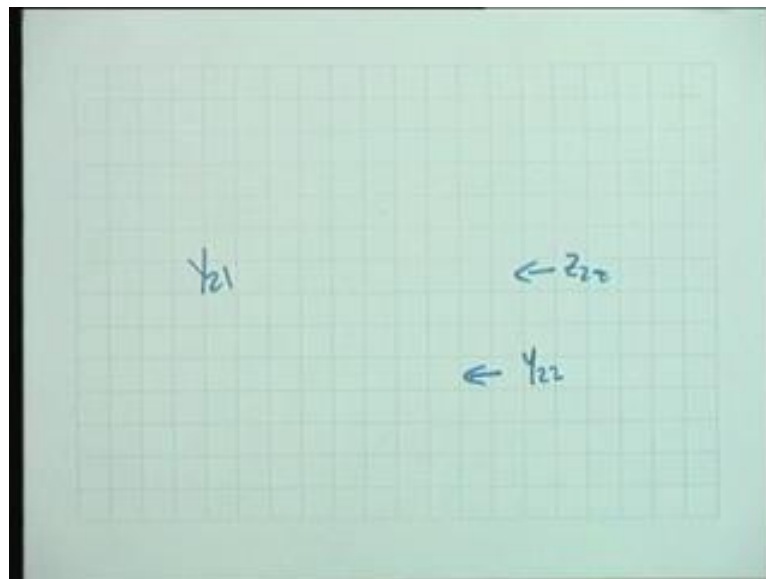
Student: (Refer Time: 49:25)

Anything in series with the a current source any impedance is in effective this is equivalent to the current source itself. Because, by definition a current source delivers, the same current irrespective of the load right. Therefore, anything in series with the current generator is in effective anything in parallel to.

Student: (Refer Time: 49:49)

Voltage generated is in effective.

(Refer Slide Time: 49:56)



So, when you develop  $z_{22}$  the last element must come in shunt with a current generator. On the other hand, when you develop  $Y_{22}$  if the transfer function is  $Y_{21}$  when, you develop  $Y_{22}$  the last element must be in series with voltage generator. These are the checks and balances if it comes otherwise then obviously, it does not make sense. We will start from here from tomorrow.

Student: (Refer Time: 50:29)

We will repeat yes.

Thank you.