Circuit Theory
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Lecture No. 06
Second Order Circuits: Magnetically Coupled Circuits.

This is the 6th lecture and our topic would be, second order circuits, which we had started last time. We will complete it this time, a discussion on second order circuits and we will also talk about magnetically coupled circuits. It is 1:02 now. As you recall, our second order circuit that we had started discussing, consisted of a battery of voltage V, a switch which is switched on at t equal to 0, a resistance R and inductance L and a capacitance C and we had, our aim was to calculate the current i of t in this circuit. We had taken I L of 0 minus equal to 0 and V c of 0 minus equal to 0.

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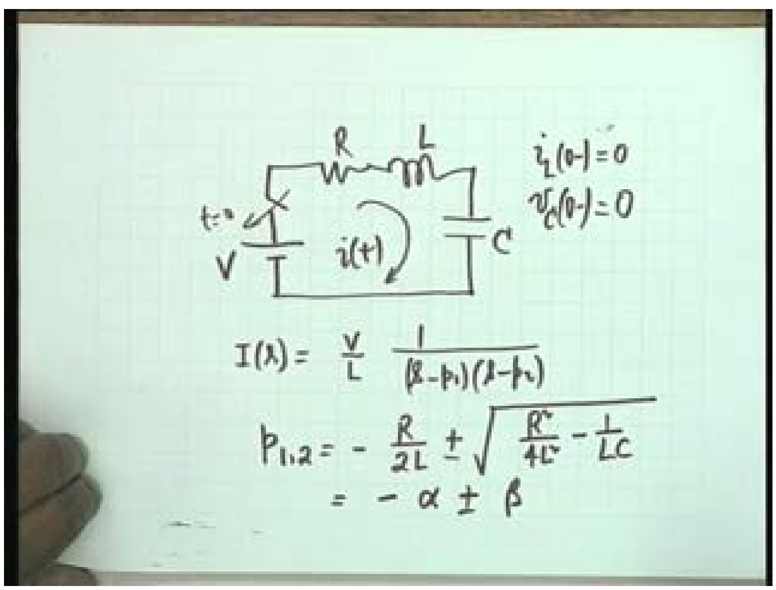

That is, the circuit is initially relaxed and by taking Laplace transform, we had arrived at the solution I of s equal to V by L divided by s minus p 1 times s minus p 2 where p and p 2 are the poles of the circuit and are given by p(1,2) is equal to minus R by 2 L plus minus square root of R squared by 4 L squared minus 1 over L C and we had decided to call this as minus alpha plus minus beta, and then, we are good that 3 cases may arise: One is that beta is real. Beta is real, that is, R squared by 4 L squared is greater than 1 by L C and the second case that we considered was when beta is equal to 0. That is, R squared by 4 L squared is equal to 1 by L C.

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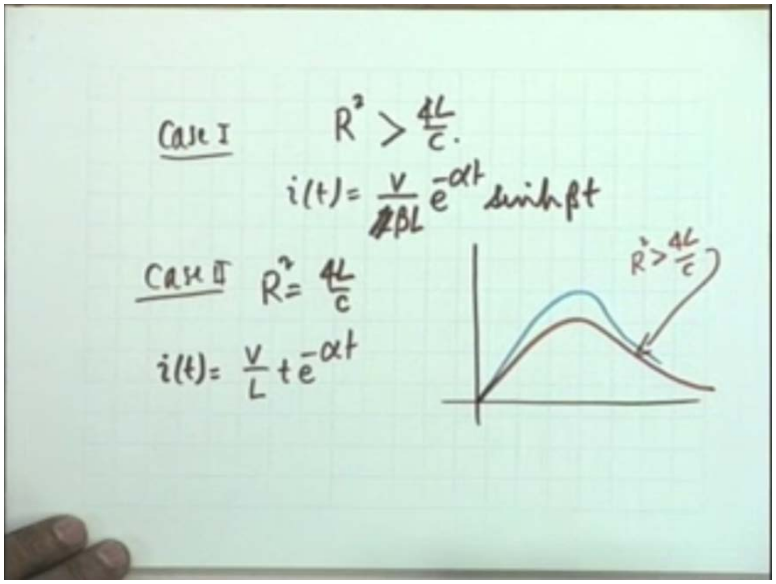

In the first case, in the first case, case 1, when R squared was greater than 4 L by C, our solution, our solution was i of t is equal to V by 2 beta L, 2 beta L e to the minus alpha t times. Well, I could call this as, cancel these 2. Then, this is sine hyperbolic beta t. Is it correct?

Does it check with what we had obtained earlier? These 2 factors, I am taking into account and I am writing it in this form and as I said, this represents a current which rises, which rises like this, has a maximum and then falls to 0 at infinity. This is the case, R squared greater than 4 L by C. On the other hand, in case 2, we had considered, R squared is equal to 4 L by C and under that condition, we had shown that the current is given by i of t equal to V by L t e to the minus alpha t. Because under this condition beta equals to 0 and therefore, we have 2 repeated roots and that is why this product by t comes in and for this, for this case, the plot would be something like this.

Yes, of course. We must multiply by u of t, both the solutions. That is correct. This case is for R squared equal to 4 L by C and we said that, this case R squared, that is case 2, corresponds to what is known as critical damping, critical damping and the case corresponding to red line here is to be called the over damped case.

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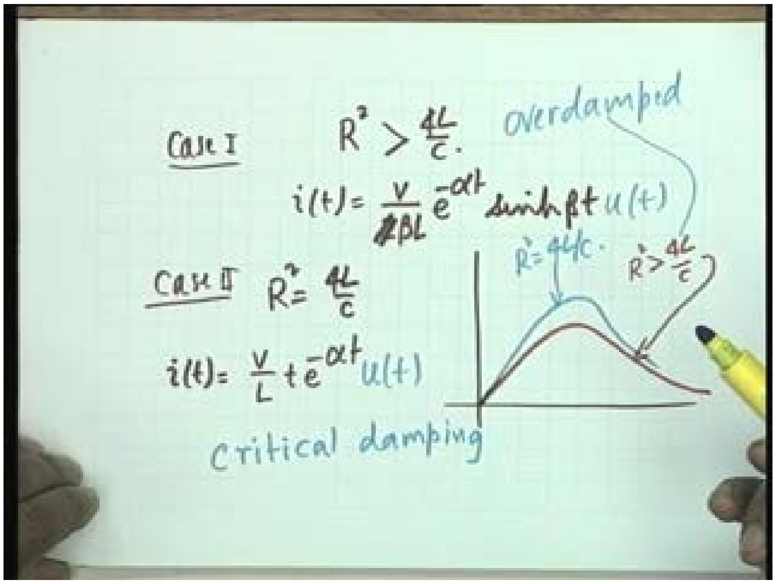

These two cases, we consider in details last time. It remains to consider case 2, when R squared becomes less than 4 L by C. That is, the resistance decreases, resistance decreases and as I, as I had commented last time, if the resistance is such that R squared is less than 4 L by C then the circuit breaks into oscillations and energy keeps on, keeps on exchanging between the inductor and the capacitor.

That is, the electrostatic form to electro dynamic form or electrical energy to magnetic energy and this keeps on oscillating till the presence of the resistance. After all, the resistance is a dissipating factor and that accounts for gradual damping, gradual decay of the oscillations. Well, to, to treat this case mathematically, let us look at our solution,

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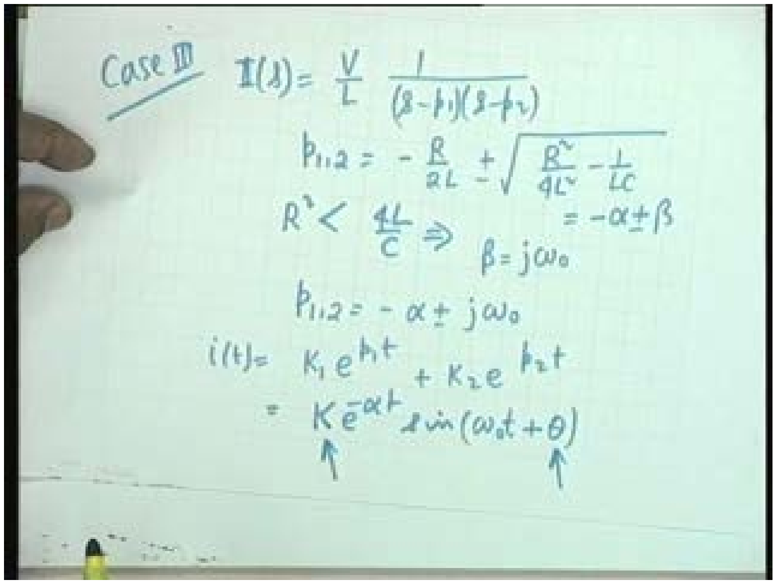

That is, i of, I am sorry, capital I of s is equal to V by L 1 by s minus p 1 s minus p 2 and our p 1, 2. I am considering case 3. p 1,2 is minus R by 2 L plus minus square root of R squared by 4 L squared minus 1 over L C and our case is that R squared is less than 4 L by C and therefore, this is also equal to minus alpha plus minus beta. Therefore, beta is purely imaginary. Therefore, we put beta equal to let us say j omega 0.

Then, my poles are minus alpha plus minus j omega 0 and it is very easy to show by writing i of t as some constant K 1 e to the p 1 t plus another constant K 2 e to the p 2 t and by combining the complex, the imaginary terms, it is very easy to show that this can be written in the form: K e to the minus alpha t, e to the minus alpha t comes from e to the p 1 t and e to the p 2 t, both have minus alpha as a common factor and therefore, e to the minus alpha t, the damping factor, goes out and then you have sine of omega 0 t plus some constant theta and, we are now required, we are now required to find out k and theta, the 2 constants.

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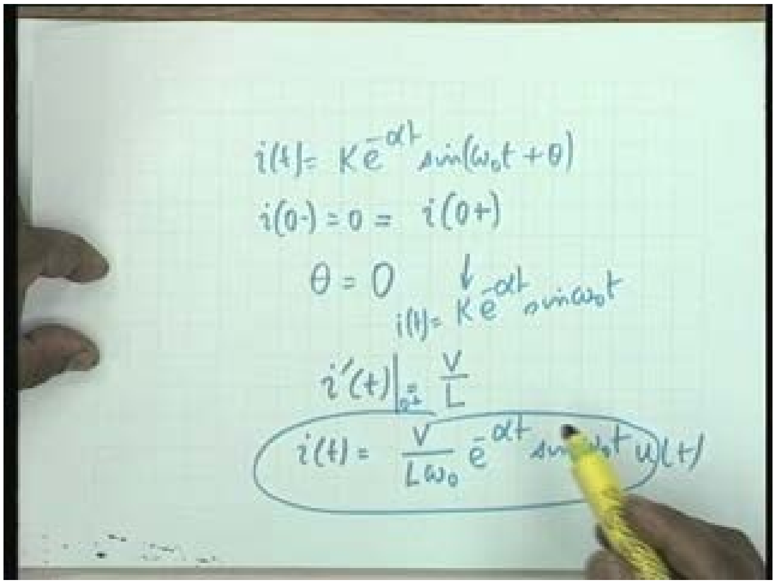

To find theta, first i of t equal to k e to the minus alpha t sine of omega 0 t plus theta. To find theta, you notice that i of 0 minus was equal to 0 and since the current in the inductor has to be continuous, this is also equal to i of 0 plus and therefore, if I put this value here, I get theta equal to, pardon me, how much would be theta? It would be 0 t equal to 0. So sine theta, sine theta is 0 and therefore theta is 0. Therefore, my equation becomes i of t equal to k e to the minus alpha t sine of omega 0 t. Then, how do I find out k?

Obviously, I require an i prime (..) and I have already shown that irrespective of the case that is considered, it is simply equal to plus V by L, not minus. It is equal to plus V by L. The current increases from 0 and therefore, the slope is positive. If you substitute this and find out k then the ultimate solution, I can write as V by L omega 0 e into the minus alpha t sine of omega 0 t. This is my total solution and you, and you indeed see that this is an oscillation - sine omega 0 t. The current starts from 0. That is why theta is 0. The current starts from 0 value here.

Student: (..) 0 plus sir, i dash 0 plus.

Sir: I dash 0 plus, yes. I dash 0 plus equal to V by L and of course, there shall be the ever present u of t here and you indeed see that if e to the minus alpha t was not there, then it is a pure sinusoidal oscillation; sine oscillation. Because of the factor e to the minus alpha t, the amplitude gradually decays.

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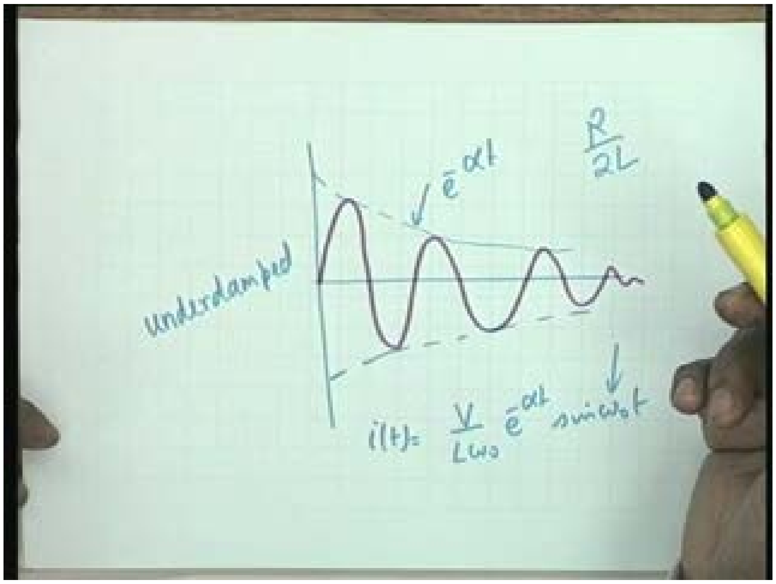

That is our solution. Our solution shall look like this. It will gradually decay and this envelope goes according to e to the minus alpha t. This envelope is e to the minus alpha t. And this is called, under damped case, and after you have identified under damp, the meaning of the word critical damping becomes more clear. Critical damping is the border line between oscillations and no oscillations. If the damping is slightly less than critical damping, there will be oscillations. If the damping is slightly more than critical damping, there would be no oscillation and that is why critical damping is called critical damping. Yes?

Student: Frequency should not (..)

Sir: Frequency, well, the expression is i of t equal to V by L omega 0 e to the minus alpha t sine of omega 0 t. So the frequency is omega 0. It neither decreases nor increases.

Student: The way we have drawn it, the figure?

Sir: No, forgive my bad drawing. The period is the same. I have always been poor at drawing, so you have to take this as sine of omega 0 t. The frequency in a linear system linear system can neither decrease nor increase. Frequency has to remain the same. It is only in the case of a non linear system that frequency can change. It also shows that if alpha was equal to 0, now alpha by definition is R by 2 L, if alpha is equal to 0, then there is no damping. Damping is 0 and the oscillation would have continued at infinitum without any reduction in the amplitude and this case, we have already treated. One special case, we have already treated: a pure inductor and a pure capacitor connected in series or in parallel, in a loop.

Two elements in the series or in parallel mean the same thing. Connected in parallel and they create sustained sinusoidal oscillations. This cannot be achieved in practice because of the inherent losses in the inductor and capacitor and that is where we require an active device. To make an oscillator, we require an active device which shall supply the energy necessary to counter the dissipation in the inevitable resistance and in the inductance and capacitance, that is, this decay factor. In order to counter this, we require an active device like a transistor or an op-amp.

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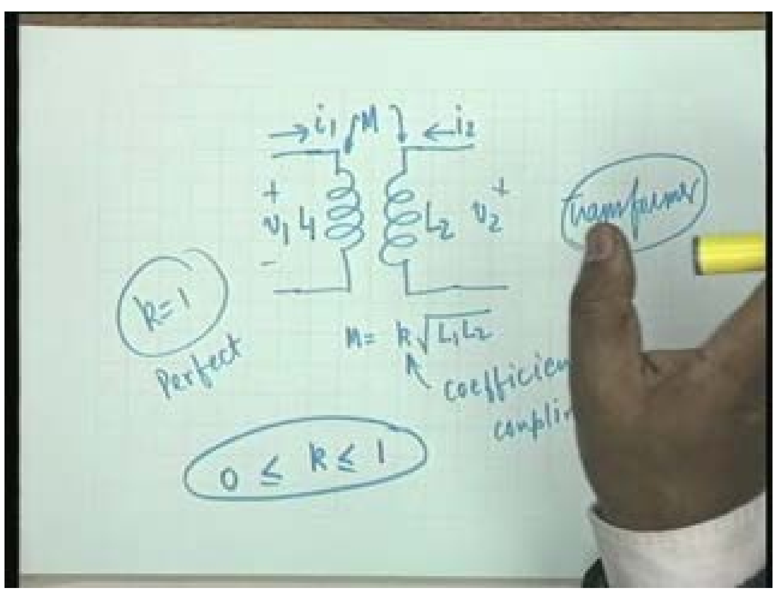

Now, we consider a very interesting an important class of circuits known as magnetically coupled circuits. Faradays laws of induction states that if you have 2 coils near each other and if in both of the coils current flows, let us say i 2 and i 1, the voltages are V 1 and V 2, then the 2 coils effect each other. In other words, the current in coil number 1, which conventionally is called the primary, the coil to left is called the primary coil. The coil to the right is called the secondary coil but there is no sacred rule regarding this convention. Whichever coil is exited, is known as primary. Now if both the coils are exited then obviously there is no distinction between primary and secondary, but nevertheless, the terms are often used.

Faradays laws of induction says that whenever there is a coil or a closed loop in a magnetic field and the magnetic field is varying, then a voltage would be induced in the coil. Now, naturally when 2 coils are brought close to each other, if current flows in both, then the flux of one coil links with the flux of the other coil. The flux of one coil links with another and this is mutual. That is, they share the magnetic flux and if in addition, the flux is changing with time, then there are induced voltages in both and to describe this phenomenon the corresponding element, corresponding circuit element is a transformer, known by various other names - coupled coils or magnetically coupled circuit, all kinds of names but basically, it is a transformer.

The name arises because it can transform one voltage into another, it can transform one current into another higher or lower. It can also transform a given impedance to some other impedance and that is why the word transform is there. This is a physical transformer; it is not a magnetic, it is not a mathematical transformation like Laplace or Fourier. This is a physical transformation: changes one level of voltage to another, either higher or lower, similarly, one level of current to another either higher or lower or a level of impedance. You can get a higher impedance or a lower impedance depending on the design of the transformer.

But basically, what is happening is that if the current, if there are currents in both and these currents are changing then they induce a voltage in each other and this phenomenon is described by means of a parameter known as mutual inductance M, between 2 coils. The mutual inductance M is related to the self inductance of the 2 coils L 1 and L 2 by the relationship: M equal to K square root of L 1 L 2, where K is called the coefficient of coupling and has the highest value highest possible value is 1. K must be must lie between 0 and 1 and it can be less than equal to less than equal to 1 and K equal to 0 indicates that the coils are not coupled at all. That is flux in one, does not affect the other.

If one of the coils is here and the other coil is in the bio chemical engineering block, in all probability, the magnetic field produced here will be so weak that it will not, it would not affect the other and therefore they will decouple. On the other hand k equal to 1 means, there is perfect coupling that is whatever flux is produced in the coil L 1 couples with L 2 also. In practice, this never happens, because some flux always leaks out to nearby magnetic substances. Some flux always leaks out and therefore small k usually is less than 1. K, small k equal to 1 is a very ideal situation and such, a transformer in which k equal to 1 goes by the name of perfect transformer.

Perfect, the adjective perfect is used to denote the case in which the flux linkage is perfect. That means, there is no leakage flux, no flux can leak out. All flux generated by L 1 also links with L 2. However, as far as the induced voltage is concerned, there will be no induced voltage if the flux is steady. In other words, if this, if there is a direct steady current, then direct steady current flowing in L 1 and nothing flows in L 2, the induced voltage would be equal to 0 because L d i d t, there the current must be changing with time in order to induce a voltage.

So a transformer does not work at dc. A transformer does not work at dc. It cannot produce a voltage in the secondary if the primary current is not fluctuating. Fluctuation is an essential condition for a transformer to work. Transformer works on ac or any other varying current but it does not work on dc. This is one of the most fundamental things that you should remember.

While we are discussing about, there are several things which we which we must clarify at this point, while we are discussing about perfect transformer, let me also include, let me also explain, the term ideal transformer.

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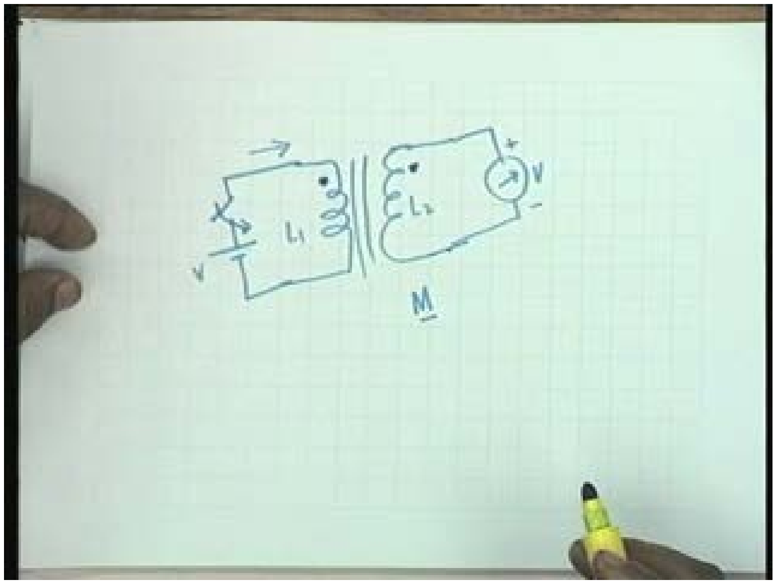

Perfect and ideal are not the same thing. One can be a perfect group. Obviously, he is not an ideal human being. An ideal human being has other characteristics associated. One can be perfectly but even then he may not be ideal. There is difference between perfect and ideal. In the case of a transformer, all that is needed for perfect transformer is k equal to 1. That is, coefficient of coupling is 1. Flux generated in 1 coil always links with the other. In the ideal case in addition to k equal to 1, in addition to k equal to 1, there are 2 other conditions attached. That is, the primary inductance tends to infinity, the secondary inductance tends to infinity. That is, very large inductances. What does this mean?

That means, very negligible amount of current is required to create a flux. After all, the flux in a coil free is the product of inductance and the current carried. So, if inductance tends to infinity, 0 current should be sufficient to produce a finite flux, finite, non-zero flux. In practice, capital L, well in practice, we cannot have an ideal. There is no ideal transformer. Ideal transformer is a concept which is very useful in circuit theory.

Student: Sir, does an ideal transformer implies perfect transformer?

Sir: An ideal transformer is perfect but all perfect transformers are not ideal. An ideal transformer must be perfect because k equal to 1 is an essential condition, is the essential condition. The second condition is that both L 1 and L 2, both of these inductances are very large so that the magnetizing current required, that the current required to produce a flux is negligibly small or infinitesimally small. But both the inductances go to infinity, but their ratio is finite, the ratio cannot be infinity. The ratio is finite. For example, 10 thousand may be infinity to you, 9 thousand, 9 thousand 9 hundred may also be infinity. The ratio is finite. So this is the condition of ideal transformer.

An ideal transformer is a perfect transformer plus requiring no magnetizing current, either in the primary or in the secondary. The ratio of the two inductances is however is finite. This is the condition for an ideal transformer and we shall have, we shall have occasions to refer to an ideal transformer very often. It is a very useful concept, cannot be realized in the laboratory. Not even a perfect transformer can be realized. Coefficient of coupling like point 9 8 is very difficult to realize nevertheless. It can be realized with lot of shielding and all that, but an ideal transformer cannot be realized.

Student: Excuse me sir, transformation may be a kind of compromise because if you have a very small current exciting, then you have also then the inductance you are taking is very large. Then the voltage is also to be very large, if you assume that it is something like a sinusoidal. The d i of d t is not much, which means that if there is any small resistance the current would be again very large.

Sir: Oh, ideal transformer, current would be very large, yes. Ideal transformer in theory can supply infinite amount of energy. That is why it is ideal. It does not exist and ideal. In both perfect and ideal transformers one of the conditions it is not essential but one of the conditions is that they are loss, less that means there is no resistance, there is no dissipated. As you will, see these are not practical elements. We cannot make them in the laboratory. We cannot use them as circuit elements. However, there are many non ideal elements, which can be represented in terms of an ideal transformer, plus lumped elements, which account for non ideal. This where the use of ideal transformer comes.

Now in the case of an induced voltage, there is a question of polarity. As you know, you can wind a coil clockwise or anti clock wise. If the current flows clock wise, there is a certain direction of flux. If the current flows anti clock wise, these are the right hand or left hand?

Student: Right hand.

Sir: Right hand rule

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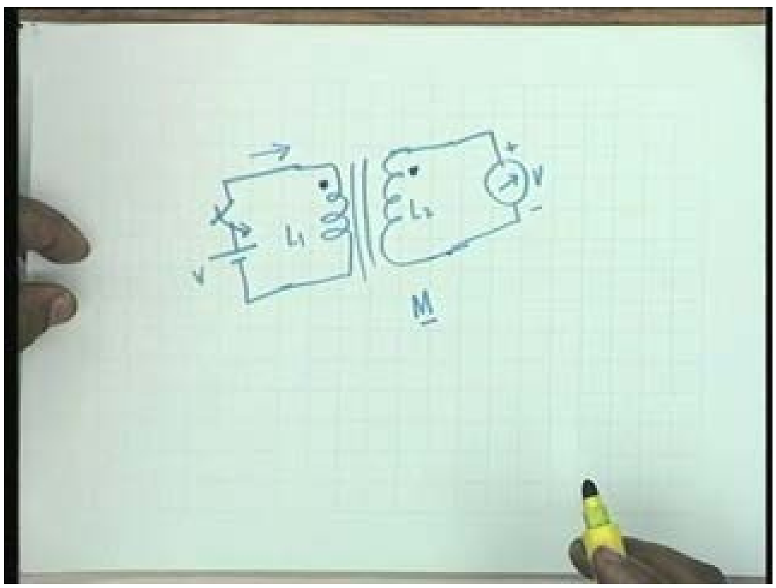

We do not have to go into the controversy regarding right hand rule: which finger to point and which is the direction and so on. But we must be careful about the polarity of the mutual inductance. I am showing a core. It is not necessary. There are L 1 and L 2 and as you will see there are dots marked on a transformer dots and these dots have the following interpretation. Once and for all, you should learn it and you should never make a mistake in future. Let me explain to you the meaning of a dot by taking a very simple example.

Suppose you take a battery V and a switch. Now if you put the switch on, the current naturally will pass like this, correct? This is battery. The current will pass like this. So, this terminal would be positive with respect to the lower terminal. Now what you do is, you include a volt meter here in the secondary with this polarity volt meter is to be current flows like this that is. This is positive and this is negative.

Now as soon as you put the switch on the volt meter will give a deflection, as soon as you put the switch on. You are creating a current from 0 current and therefore there is a change and this change will reflect in an induced voltage here. If the volt meter reads correctly, if the volt meter deflection is in the correct direction, this means that this point is also positive, with respect to the lower point, and that is the meaning of dot. If there are dots here and here it means that the potentials of these 2 points fall or rise together. Is that clear? And under this condition, the mutual inductance M is considered to be positive. On the other hand, if the dots are one is here and one is here, then the mutual inductance is considered to be negative. So capital M can be positive or negative.

Student: Sir, both are on the lower side?

Sir: If both are on the lower side, capital M is positive and you should be able to do that. Our convention would be the following, convention would be the following.

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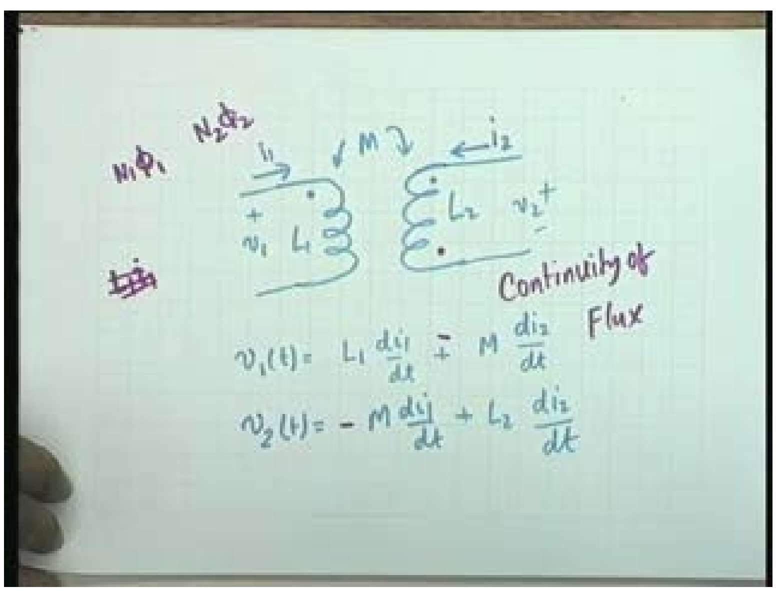

If I have a transformer like this, L 2 L 1 and the dots are like this, M is this and let us say V 1 is the voltage here, V 2 is the voltage here, the currents, if the dots are like this, currents should both be going in because if this polarity is positive then the current will go like this i 1 and i 2. Then the equations, the differential equations are that V 1 t is equal to L 1 d i 1 d t plus M d i 2 d t, this is the coupling term, this is the term that couples the second coil to be first. That is the voltage of the first coil is not determined by its own current only, but the other current also has an effect.

Similarly V 2 t is equal to M d i 1 d t, both are positive plus L 2 d i 2 d t. This is the convention, with reference to this figure this is the convention. Capital M is considered positive. On the other hand, if the dot is changed let us say from here to here. Everything else remains the same. Then this should come with a negative sign and this should with a negative sign. In other words, we consider capital M to be negative under that condition. Is the point clear?

Student: Sir, the voltage?

Sir: Which voltage?

Student: You see if I change. Now let me show it in a color. If I change the dots to this, nothing else changes, nothing else changes. Voltage, current, conventions remain the same. Then we will write this with a negative sign and this will be negative.

Student: Sir, when we write a minus sign, we consider M as positive?

Sir: When you add a minus sign, no. When this is positive, we will consider M is positive.

Student: Sir, when we write a minus sign?

Sir: When you get a minus sign, M is negative, no. M is negative means that the mutual inductance is minus M.

Student: Sir, the value we put in will be the absolute value or?

Sir: Absolute value here, if you take the sign, if you take account of the sign of course it is positive, absolute value. It is like borrowing and depositing. When you are borrowing, the amount is not negative amount is five rupees 10 rupees, whatever it is, you put a minus sign there. That is what we are doing here. So this is the convention. Now, if the mutual inductance is positive, then the total flux in the circuit, total flux in the circuit would be L 1 i 1. No, let us put it this way.

Suppose the number of turns in in L 1 is N 1 and the flux per unit turn is phi 1. Let the number of turns in the second coil the N 2 and the flux per unit turn be phi 2, then when M is positive, the total flux will be the sum of the 2. When M is negative which means that the directions are winding at different or the directions of current are different, either of the 2, then the total flux would be the difference between the 2 N 1 phi minus N 2 phi 2. And in a transformer the principle of continuity of flux shall be valid, that is, flux can neither be created nor destroyed.

If currents i 1 and i 2 change, the flux cannot change instantaneously, that is, the total flux shall remain the same and therefore, exactly like a capacitor, in a capacitor, there is continuity of charge, here there shall be continuity of flux this shall be valid.

Student: Sir does that imply that the total flux associated with L 1 will be N 1 phi 1 plus N 2 phi 2.

Sir: N 1 phi 1 plus N 2 phi 2. Yes.

Student: And vice versa.

Sir: That is correct. The total flux in the system shall be N 1 phi if the 2 coils are coupled not otherwise if they are coupled.

Student: Excuse me sir.

Sir: Yes.

Student: Sir, you said that M can be negative if the current is opposite or the currents are opposite.

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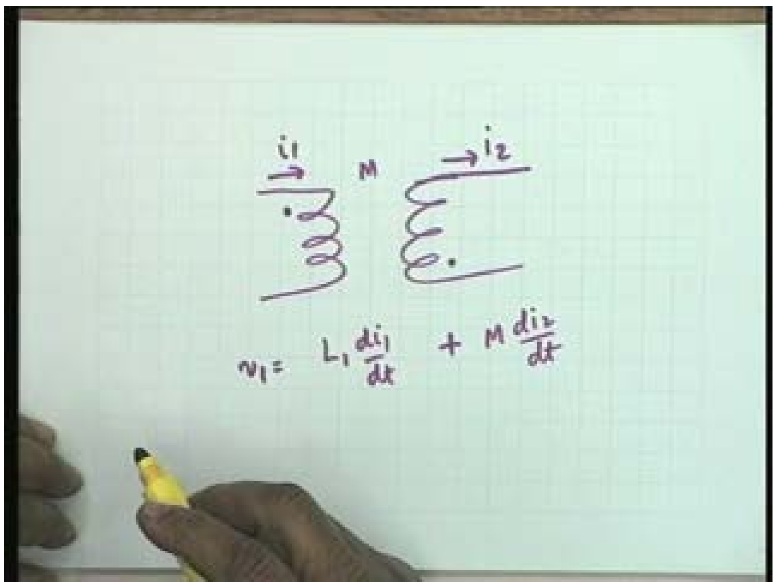

Let me take an example. I know what is bothering you. Suppose my dots are like this and the currents are like this i 1 and i 2. Suppose it is so. Capital M, the value of mutual inductance is the absolute value given. What would be your equations? V 1 equal to L 1 di 1 dt. Now u v. I know this the value would be M d i 2 d t. But shall we take a plus sign or minus sign?

Student: plus

Sir: Plus because there are 2 minuses, i 2 is in the negative direction and the dot is in the other direction. Therefore, it will be plus. Is that clear? This is the convention. This is because of the convention.

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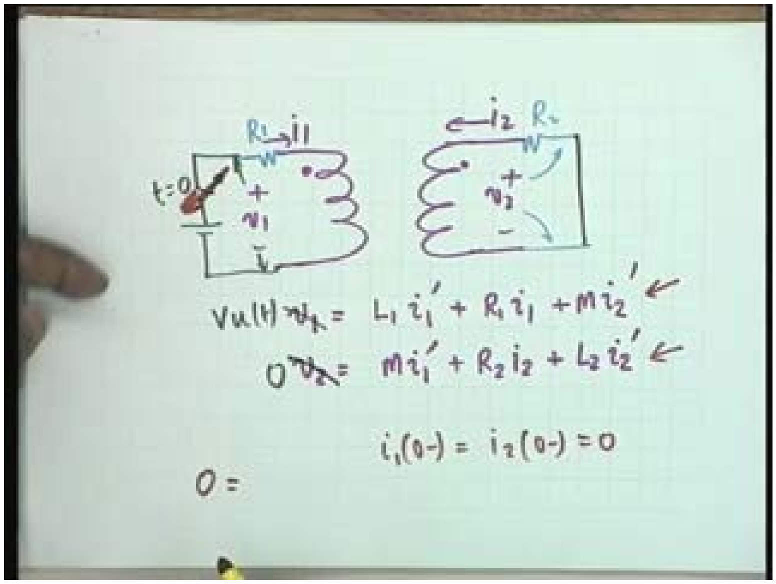

Let us go back to our original transformer and do a number of interesting things. We have V 1 i 1. This is the convention. We always write it like this and we consider dots like this V 2 plus. Whenever the dots are different and the current directions are different or voltages, polarity is different you can take care of it but this is what you must remember. Under this condition V 1 equal to, in addition let us say, the resistances of the 2 coils, these are also taken into account. Let us say we lump them into 2 resistances, R 2 and R 1. Let us consider that the coils are not perfect. We are taking account of the resistances. Resistance actually is distributed throughout the coil but we are drawing the equivalent circuit. We are drawing the magnetically coupled part here and the dissipated part separately.

Then, my equation shall be V 1 equal to L 1 i 1 prime, d d t, I put as prime plus R 1 i 1 plus M i 2 prime. I do not want to write d d t again and again, and the other equation is that V 2 is equal to M i 1 prime plus R 2 i 2 plus L 2 i 2 prime. These will be my equations. Now let us take a special case. Let us consider, let us consider V 2 as a short circuit. Take a special case. Let us consider this as a short circuit, so that a current i 2 flows like this. Then obviously V 2 shall be equal to 0, and let us say that V 1 arises because of a switch and a battery which is switched at t equal to 0. Then V 1 would be equal to V u t.

Student: sir V is the across the terminals of the (…)

Sir: No, now V 1 is here now V 1 is here because i have taken R 1 i 1 also into account. It is this job plus this voltage and that voltage contains L 1 i 1 prime and M i 2 prime, agreed? Now, what I do is I integrate both of these equations. Now you must be with me. You have not done this earlier and you must follow carefully. I am working in the time domain. What I do is I integrate, I have taken a specific example.

Student: Sir, the switch is closed at time t is equal to 0?

Sir: That is right.

Student: (..)

Sir: Oh, I am sorry, I beg your pardon. It closes at t equal to 0 with initial conditions. Let us do that also, i 1 0 minus equal to i 2 0 minus equal to 0. The initial condition, initially the circuit is completely relaxed, nothing to look into the past. It has no history of any current and the battery is closed. Now what I do is, I am interested in finding i 1 0 plus and i 2 0 plus. I want to find out the initial conditions of the circuit. It is given that before this switch is closed the coils are absolutely relaxed there is no flux, no energy.

So what I do is I integrate this equation, integrate this equation from 0 minus to 0 plus. If I integrate the left hand side, as unit step function from 0 minus to 0 plus what is the integral?

Student: 0

Sir: 0. Differential coefficient is delta, not integral. So what I get is 0 equal to the first equation.

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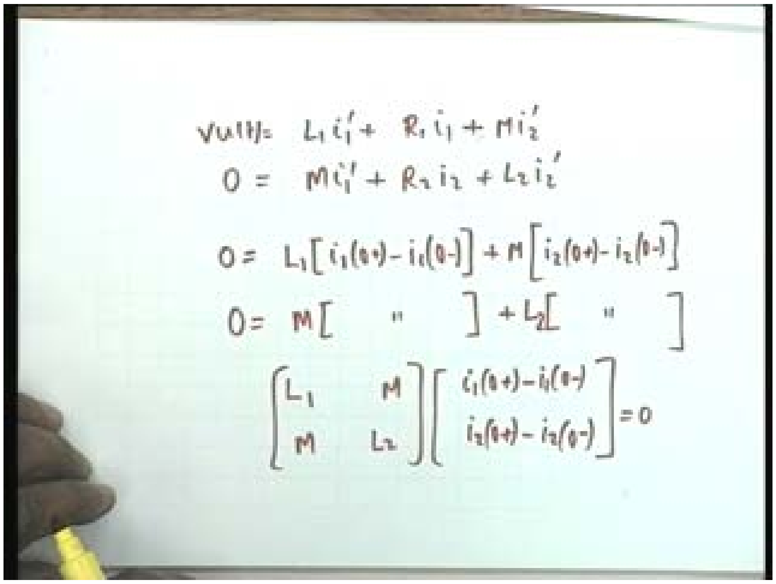

Let me write this again. V u t equal to L 1 i 1 prime plus R 1 i 1 plus M i 2 prime and 0 equal to M i 1 prime plus R 2 i 2 plus L 2 i 2 prime. I am integrating from 0 minus to 0 plus. So, the first equation gets, 0 equal to L 1 i get i 1 of 0 plus minus i 1 of 0 minus. the first term. Second term, i 1 of integrated between 0 minus and 0 plus is 0. Third term is, M i 2 prime so i 2 0 plus minus i 2 0 minus. In a similar manner, the second equation shall give me M times this plus L times L 2 times, the second equation.

Now if you have an equation set like this L 1 M M L 2, then you have i 1 0 plus minus i 1 0 minus i 2 0 plus minus i 2 0 minus, this is equal to 0. It means, well if I expand this, I will get the same equation, but that is not what I want to do. What I want to do is the following. Suppose I have, please try to understand this, suppose i have 2 equations like this, a 1 x plus b 1 y equal to 0, a 2 x plus b 2 y equal to 0.

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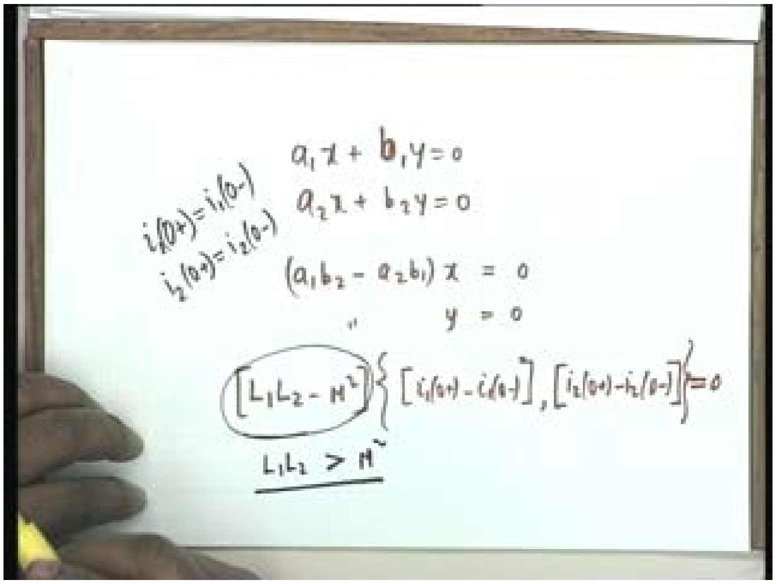

Suppose I have a set of equations like this, then it is very easy to show that the determinant of the coefficients which a 1 b 2 minus a 2 b 1 multiplied by x or y, both shall be equal to 0. Is that clear? This can be very easily shown. So in my case, L 1 L 2 minus M squared, if I multiply by I 1 0 plus minus i 1 0 minus, this shall be equal to 0. If I multiply instead by i 2 0 plus minus i 2 0 minus, this shall also be equal to 0. You understand the meaning of this equation? That is, this multiplies either this or that, in both cases it will be equal to 0. Now that is very interesting. How can this be 0? What are the conditions?

Suppose I had a non perfect transformer. That is, L 1 L 2 is greater than M squared. Non perfect means the coefficient of coupling is less than unity. If this is so, then this quantity is positive, is greater than 0 which means that if the product of this and this has to be 0, then this must be 0 by a similar argument this must also be 0. In other words, what I get is that in a non perfect transformer i 1 0 plus would be equal to i 1 0 minus and i 2 0 plus would be equal to i 2 0 minus.

That is, the current shall be continuous. Not only the flux is continuous, the currents are continuous is that. In sharp contrast will be the case when the transformer is ideal. That is, if L 1

L 2, we shall show this but before that we will take an example. We shall show that if L 1 L 2 is equal to M squared, then these do not have to be 0, because there is a 0 already here. If L 1 L 2 is equal to M square, we shall indeed show that, not only they need not be 0. They cannot be 0. That is, we will show that in an ideal transformer the currents of necessity have to be discontinuous. We will show this, but before showing this let us let us take an example.

Student: What was the need to short circuit the second one?

Sir: Otherwise, the current cannot flow. Just a second, let us go back here. If it is not circuited, the current i 2 cannot flow and there would be a V 2, I would not been able to put this equal to 0. that was the need. Let us consider an example. We consider an example, which the switch, let me see, I must be careful about closing and opening. This closes at t equal to 0 and the voltage is, let us, say 6 volt, the primary resistance is 3 and the inductance is 1 Henry. With this dot, the inductance is 2 Henry with the upper terminal doted. Then there is a resistance is of 8 ohms and the whole thing is short circuited. This current is i 1 and this current is i 2. The mutual inductance, is it positive or negative?

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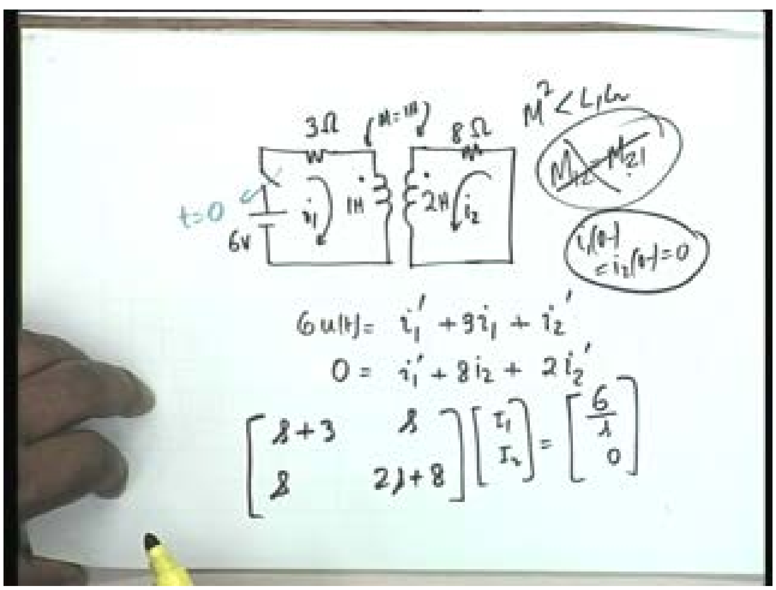

Student: Positive.

Sir: Positive, it is 1 Henry. Obviously M squared, yes.

Student: Sir, in case (...)

Sir: They cannot be different. You must have learnt in your high school that M 1 2 is equal to M 2 1. It is always true.

Student: Sir, if the direction of current are reversed and we do not say that M is negative, we say that M is positive?

Sir: You say whatever you like. You write the equations correctly.

Student: But M is positive because.

Sir: If you take the sign correctly, I am willing to allow you any language that you use. The differential equations must be written correctly. My definition was that, if the currents are like this and the dots are in opposite direction, then we say M is negative. That means, in the equation M d i d t terms will come with a negative sign. That is what I mean anyway. Now this is my circuit and you see, the equations I can write down the equations by inspection. 6 u t shall be equal to L 1 i 1 prime. L 1 is 1 Henry. So, i 1 prime plus 3 i 1 plus M is 1 Henry, so, i 2 prime. Agreed? And the other equation would be 0 equal to i 1 prime 1 Henry i 1 prime plus 8 i 2 plus 2 i 2 prime. This is the equation.

If I take the Laplace transform, let us say if I take the, if I want to solve this solve for i 1 and i 2. Oh, it is given that i 1 0 minus equal to i 2 0 minus.

Student: Excuse me sir

Sir: Pardon me

Student: M is

Sir: That is, that will be true if k is equal to 1.

Student: M is given.

Sir: M is given so K is less than 1. M squared is less than L 1 L 2. Therefore, the coefficient of coupling must be less than 1. I want to solve now it is given the initial the conditions before the switch was put on was there both the currents were equal to 0 if this is given then I am to solve the equations. Then, if I follow Laplace transform method, I do not need to bother about what is i 1 0 plus and what is i 2 0 plus. I do not have to bother about this and very simple thing would be just take Laplace transforms you get s plus 3, s plus 3 then s 2 s plus 8, multiplied by i 1 i 2 would be equal to 6 by s and 0. The characteristic equation or the poles can be found out from the determinant, from the determinant of the left hand side, the coefficient matrix. And obviously, the determinant is s plus 3 multiplied by 2 s plus 8 minus s squared. Do you know this characteristic equation? This is equal to 0. This is the characteristic equation.

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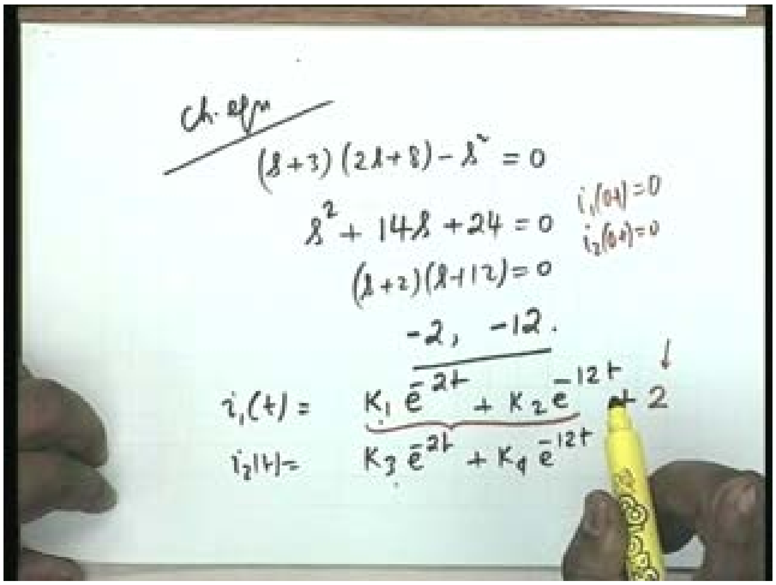

Student: Sir, instead of taking Laplace transform we have taken that i 0 plus i 1 0 plus and i 2 0 plus is 0.

Sir: That is correct. We have implied this that was a catch in the whole thing. is the point clear? This equation

Student: Sir, that will cause problem in that equation. L1 L2 is not equal to M squared.

Sir: Here L 1 L 2 is greater than M squared and therefore there shall be continuity of current and therefore, what I wrote is correct here in this particular case, not in all cases. Is this point clear? I have already shown that if the coefficient of coupling is less than unity, then the currents in the primary and secondary, both the coils have to be continuous at t equal to 0 that is 0 minus should be equal to 0 plus. So when I take the Laplace of i 1 prime, I simply write small s multiplied by capital I 1. I do not have to take care of i 1 0 plus because that is identically equal to 0.

So the characteristic equation of the system becomes this and you can see that this s squared plus 6 plus 8 14 s plus 24 equal to 0, which means that s plus 2 times s plus 12 would be equal to 0. So, the natural frequencies of the system or the poles of the system would be at minus 2 and minus 12. Now, what I can do now, once I have identified this, I can write the two currents as some K 1 e to the minus 2 t plus K 2 e to the minus 12 t and i 2 t is equal to K 3 e to the minus 2 t plus k four e to the minus 12 t. The roots of the system, that is, the natural frequencies are real and negative and therefore, I can write in terms of exponentials.

However, there is a one mistake namely, that in i 1 of t i 1 of t at t equal to infinity at t equal to infinity. What is the current in this? It would be simply 6 by 3, 2 amperes. It is not 0 and therefore, I must add a term of 2 amperes here, which signifies the particular solution or the particular integral and this is the solution to the homogeneous equation, this is the complementary function. Now in i 2 of t at t equal to infinity, obviously, this current shall be 0 when flux is self stabilized, there is no induced voltage and therefore, I do not need to add a constant here. So these are the 2 equations. Now I can find out K 1, K 2, K 3, K 4 from the initial conditions.

Now, as I said, if I had taken the Laplace transform, if I have taken this and simply found capital I 1 capital I 2, took the Laplace inverse, the solution comes out very easily. But suppose I do not want to work in the frequency domain, I want to work in the time domain only. Then how do I evaluate? You understand my proposition? You can work in the frequency domain from this. You can find out capital I 1 capital I 2. Take the Laplace inverse and be done with it. But suppose I do not want to do that. I want to solve it completely in the time domain, that is, I want to find out K 1 K 2 K 3 K 4. Then one of the conditions, two conditions are that i 1 0 minus equal to 0 therefore, i 1 0 plus is also equal to 0 which means that K 1 plus K 2 plus 2 would be equal to 0, the 1 equation. The other is i 2 0 plus equal to 0 which means that K 3 plus K 4 would be equal to 0. Now I have four constants to determine, I have two equations so I require two more equations. Now how do I get those equations? I go back to the original.

Student: original equation

Sir: That is right. I go back to the original.

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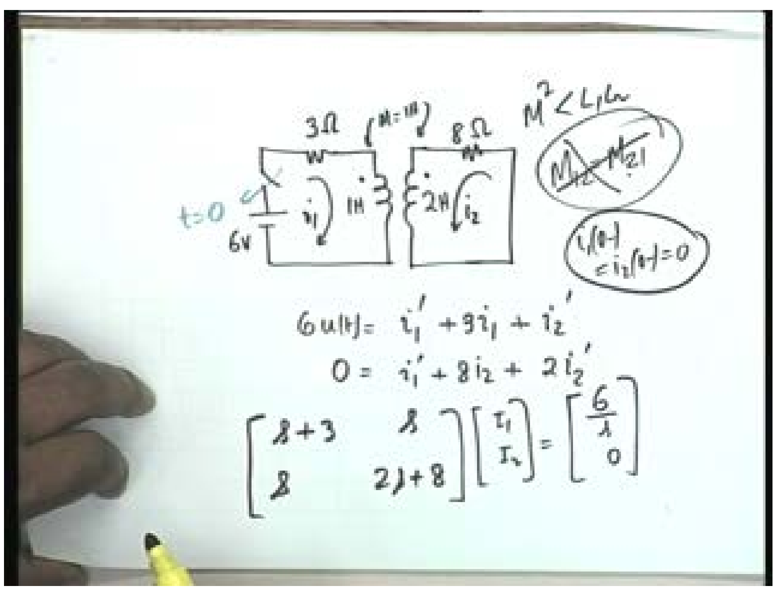

I put t equal to 0 plus, then the left hand side would be 6, would be equal to i 1 prime 0 plus

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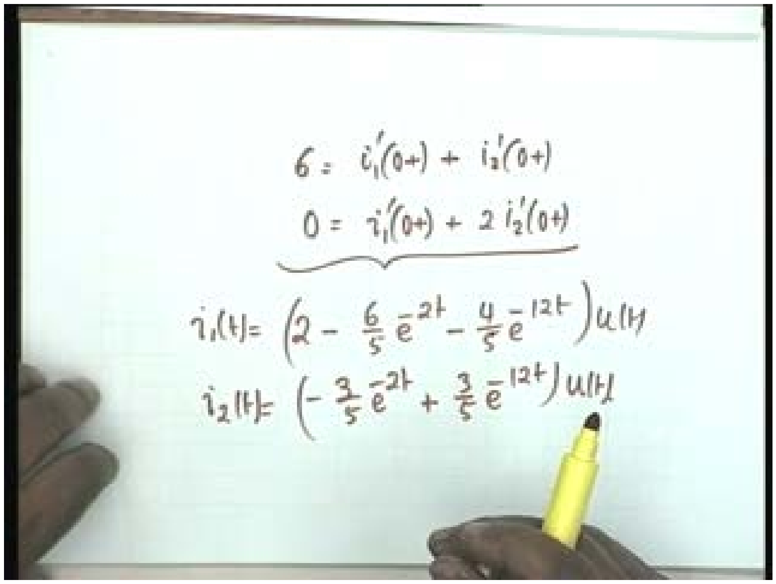

Student: plus 0

Sir: plus 0 because i 1 is 0 and i 2 prime 0 plus and then 0 equal to i 1 prime 0 plus plus 2 i 2 prime 0 plus from which, by solving these 2 simultaneous equations, I can find out i 1 prime 0 plus, i 2 prime 0 plus and then going back to original equation, that is in terms of K 1 and K 2, I can solve for K 1 and K 2. What I will do is, I will simply give you the final solution. Therefore, you can verify this final solution. Is the procedure clear to all of you?

Student: Yes sir.

Sir: Therefore i 1 is 2 minus 6 by 5 e to the minus 2 t minus 4 by 5 e to the minus 12 t. This whole thing multiplied by u of t and i 2 t is minus 3 5th e to the minus 2 t plus 3 5th e to the minus 12 t times u t. Does this surprise you that these two coefficients are equal and opposite? No, because i 2 0 is equal to 0 and therefore these two. Also, it should not surprise you that the sum of these two coefficients is equal to 2.

Student: No.

Sir: It has to be because i i 1 0 is also equal to 0. These are the various checks in the system and in circuit theory, if you cannot check your results against common sense, you must have made a mistake. As I said, you did not have to go through the time domain complication. We could have, we could have used Laplace transform domain only. But this is very instructive. It is very instructive to do it. On the other hand, when you go to the case of a perfect transformer, that is, L 1 L 2 is equal to M squared, that is, the coefficient of coupling equal to 1. Things will become a little more complicated and if you go to the time domain, if you go to the time domain, there is more illumination. There is more light. There is more knowledge, more interpretation and more sources of joy. As you shall see in a few moments, I have a few moments to spare.

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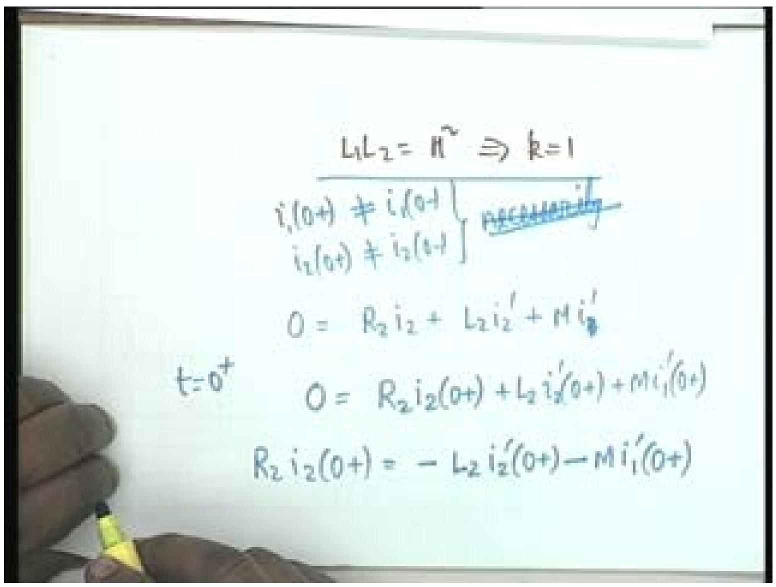

Now if I take, suppose this is the condition, suppose L 1 L 2 equal to M squared, then as you will see i 1 of 0 plus is not necessarily equal to i 1 of 0 minus and i 2 0 plus is not necessarily equal to i 2 0 minus. I must say necessarily. No, I think I have put 2 negatives is not it? No, let me put it this way. My language has become slightly complicated. i 1 of 0 plus is not equal to i 1 of 0 minus. It may also be equal, to start with. No, we will show that they have to be different. So I will cut necessarily, we will show that they have to be different. This is what we are going to show.

The first thing we do is we write the equation to the secondary that is 0. As you recall, is equal to R 2 i 2 plus L 2 i 2 prime plus M i 1 prime right? This is the secondary equation. Suppose I put here t equal to 0 plus I put here t equal to 0 plus. Then I get 0 equal to R 2 i 2 0 plus, plus L 2 i 2 prime 0 plus, plus M i 1 prime 0 plus. Is that okay? Absolutely put t equal to 0 plus nothing else and in this equation, I can simplify this to R 2 i 2 0 plus is equal to minus L 2 i 2 prime 0 plus minus M i 1 prime 0 plus. This is trivial manipulation.

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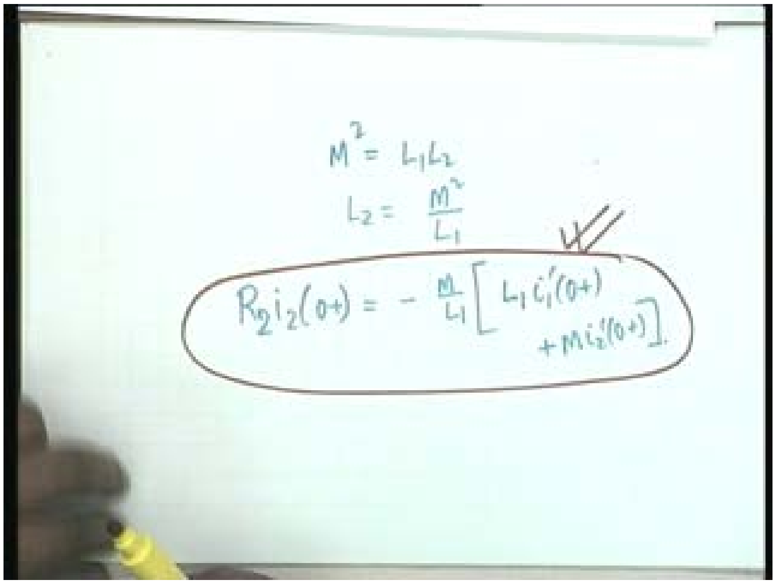

But because M squared is equal to L 1 L 2, I get L 2 equals to, excuse me, M squared by L 1. The fun starts here. Therefore, I can write R 2 i 2 0 plus equal to, I substitute for L 2 and take minus M by L 1 common. Then, what do I get? You see, here I had minus M i 1 prime 0 plus and therefore, if I take M by L 1 common, I shall get L 1 i 1 prime 0 plus and the second one, what do I get plus.

Students: M i 2 prime 0 plus

Sir: M i 2 prime 0 plus. This equation will prove to be the most interesting equation that we have derived in this 6th lecture, as we shall show in the 7th lecture. We will continue this discussion.

Student: (..)

Sir: This is correct. I certify that this is correct. Then we shall go back to the, this is the term that occurs in the first equation and that’s why fun starts. We will show, we will continue this discussion next class.