Nonlinear and Adaptive Control
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Lecture – 06
Adaptive Command Tracking

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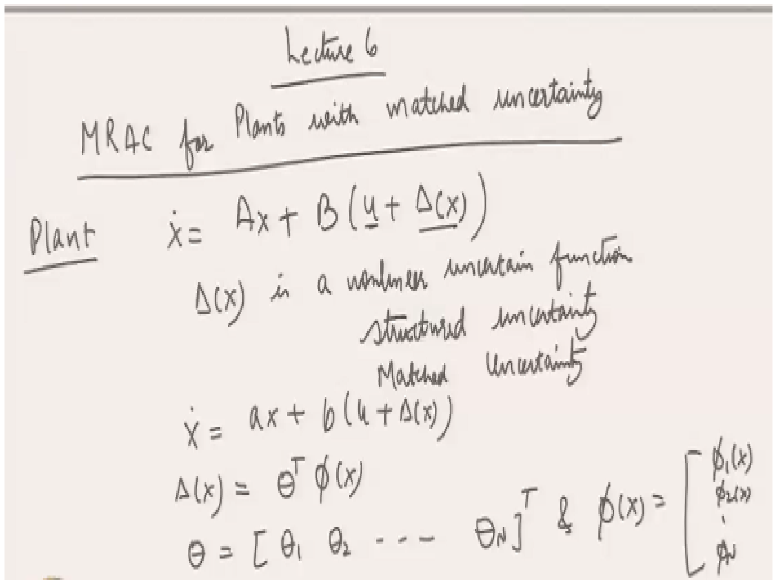

Welcome everyone to lecture 6 and in lecture 5 we had finished the analysis for the indirect and direct MRAC cases. In this lecture we will talk about the case where the plant is not entirely linear which is the case for real systems where you may have some nonlinear terms in your system dynamics. So, let us assume that the plant in this case is given as Ax+ Bu+ delta x so instead of the lti plant Ax+ Bu here we have a nonlinear term delta of x.

So, it is a non-linear function uncertain so delta of x is not known. So, how do we design the controller u in this case so typically when we have a certain sum uncertain terms and uncertain non- linear terms in the system dynamics. We try to cancel those nonlinearities using the control input. In many cases the nonlinear terms main fact will be helpful for the stability but then we need to know exactly know what those nonlinear terms are.

In case those terms are uncertain we are not sure whether those are in fact going to be useful for the stability and it is in fact better to somehow account for these terms using a robust controller.

Certain non – linear terms can also be dealt with in a different way using adaptive control. As we see this in this lecture so we assume that these nonlinear term and delta of x is in fact we structured uncertainty.

So, there are two types of uncertainties one is structured uncertainty and the other is an unstructured uncertainty. Structured uncertainty is one where all over the entire term is not known but we know the structure of the uncertain term and unstructured uncertainty is where we do not know the structure of the uncertain term. So here we assume that this delta of x is structured uncertainty.

And in this case it is also a matched uncertainty so why it is matched because delta of x occurs in the same channel as the controller so using our control input in u. You can directly account for the effect of this nonlinear term that is delta of x if say we had some term like+c times delta of x. Then we could say that term is unmatched uncertainty that would be more difficult to deal with the control input u.

In this case we assume that the uncertainty is matched structured non-linear uncertainty so again in this case I consider a scalar case so let us reduce these matrices to scalar terms just make the analysis much easier and then you can later work on the vector case. Okay as I mentioned we assume that delta of x has a structure and it also has a very special structure which is called as linear parameterization.

So delta of x is given as theta transpose phi of x so it is a linear combination of basis functions these basis functions and phi of x are known the ways that this basis functions are combined are unknown and given by theta. So, theta is the vector of unknown parameters and theta is defined as theta 1 theta 2 till theta n and phi of x denotes the basis function and it is a vector of known basis functions and 1 of phi x phi 2 of x so notice that theta transpose phi of x is theta scalar.

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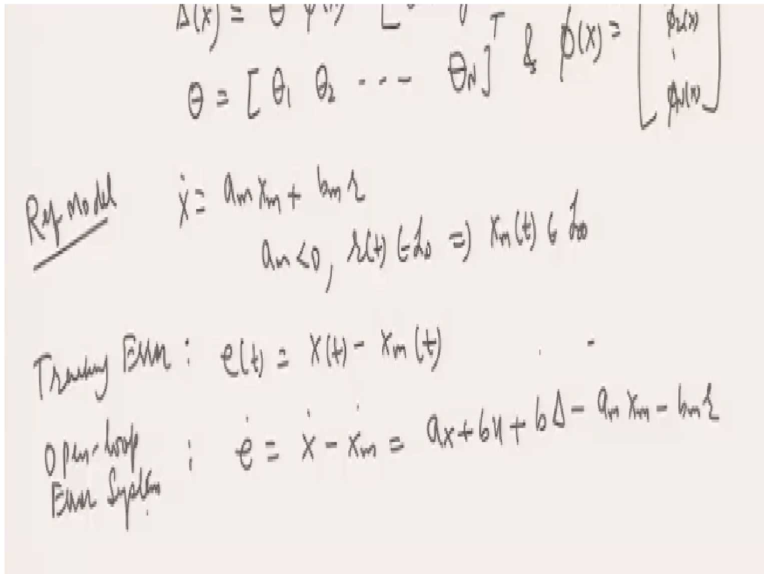

Okay so the objective is again to follow the reference mod and so we want you to be a model reference adaptive controller. Which follows the reference model given by am xm+ bmr and again we say that am <0 r is bounded which means that xm is also bounded. Okay so I should write that this uncertainty is linearly parameterizable and that is why it can be handled using an adaptive controller.

Otherwise you have to use some other kind of controller so we will see how this. The fact that this uncertainty is linearly parameterizable helps us in adaptive control design. Okay so again we get the mechanics remains the same and by now I hope that you are familiar with the mechanics. We start with the tracking error which is the difference between the plant state and the model state and then we construct the open loop fun system given by e.

In this case we get the b delta term because of this nonlinear term which we did not get before but in this case we do consider the system. Okay so for the controller how do we decide challenge here is how to design the controller u to stabilize the system so if the delta term was not there we would go ahead and design the direct MRAC and indirect MRAC case because there is a delta term.

We would have to modify this controller may be add an extra term to account for this non-linearity. So, the control intuitively would be a combination of two terms one would be a nominal controller and the other one would be a term which is used to account for this non-linear term delta.

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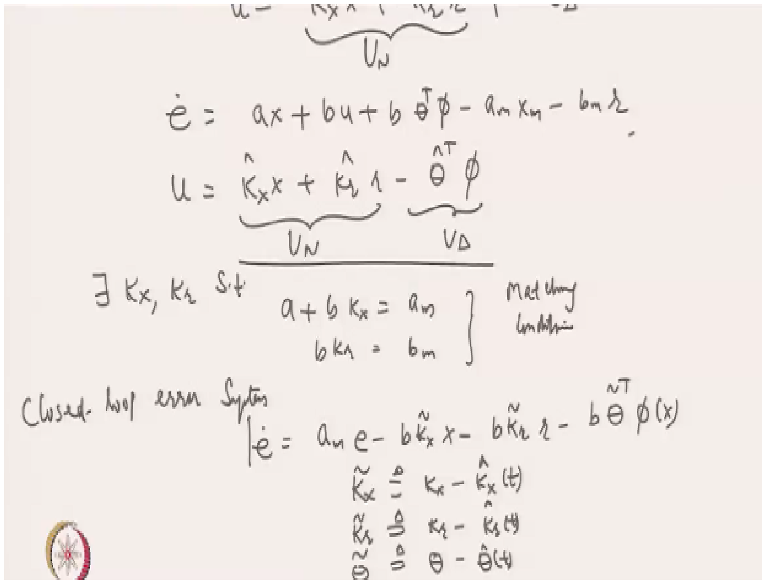

So u would be a combination of the nominal MRAC. So u would be kx hat+ k r hat r+ u delta term and let us call it as nominal term. This is the term u delta is the term which use to account for this term b delta so let us just substitute for the term delta. The e. is given by ax+ not substitute for u right now since theta is unknown we can choose u to be a certainty equivalence controller.

Certainty equivalence controller is one where you write the controller first and there is no uncertainty then whatever terms are uncertain you just consider their estimates. So, in this case suppose kx and kr were known we would choose the control input to be kxx+ krr+ for this case we would chose theta transpose phi–theta transpose phi. And cancel and trying to cancel this nonlinear term but since we do not know this theta we chose an estimate.

So, u is given by kx hat x+ kr hat r and let just say that this is - theta hat transpose phi. Okay so this is the nominal controller and this is the part which is used to account for the nonlinear term. Okay so we need to design the update laws for kx hat kr hat and theta hat. The fact that this term is parameterizable as right said write it simply as linear combination of the function x r and Phi. So, again we for nominal case it is a simple case of MRAC.

So, we consider that there exists ideal kx and kr a+ b kx=am and bkr=bm and these are the matching conditions. They are trivially satisfied in either case. For the vector case they may not be satisfied so you have to be careful when you do the vector case. They may not exist enough structural flexibility in the plant to satisfy these magic conditions in the vector case. So, the sedentary equivalents controller is given by this combination of two terms.

When we substitute that in the error system we get the closed loop error system which is given by. So, here like before kx tilde is the parameter error estimation error for kx and is defined as kx – k x hat kr tilde is defined as kr - kr hat and theta tilde is the parameter estimation error for the parameter theta. It is given by theta – theta hat. So, these are hats are time varying and kx kr theta are constants.

So, this is the closed loop error system which will be useful for us in when we do the Lyapunov analysis.

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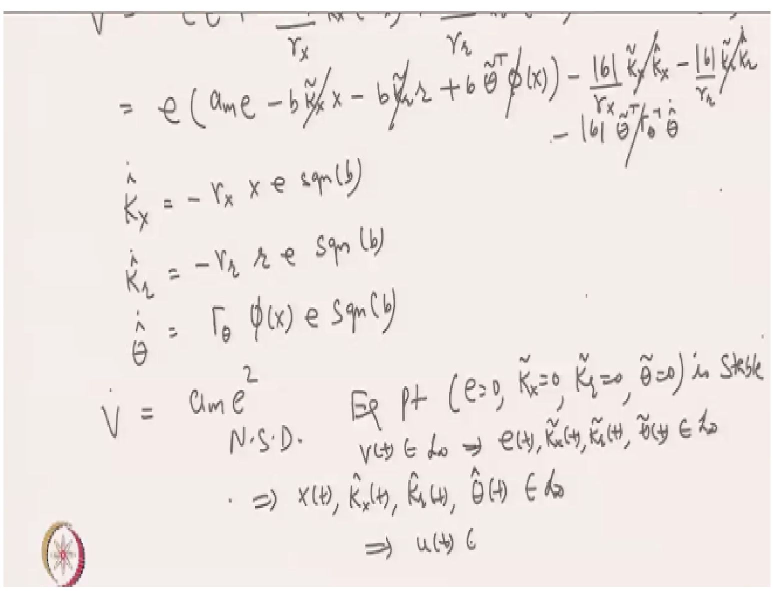

So, let us go head and chose Lyapunov analysis function candidate which is always the next step. Let us chose Lyapunov function candidate so V in this case is the function of e kx tilde and kr tilde and it may also be function of time. But what we chose is in fact trying in variant function so as I said there can be other choices of Lyapunov functions from what I showed here. This is just one way to choose Lyapunov function candidate.

But they may exist many ways to choose Lyapunov function You can try some other options and see if they work out. So, if you remember for the case where both A and B are unknown. So, this is the scaler case we consider for the direct MRAC we found that a good Lyapunov function would involve the absolute value of the parameter b so this is the positive number.

And by 2 gamma x and this is very similar to the Lyapunov function candidate that it shows for the direct case without the non-linearity. Since we have a non-linear term here which is linearly parametrizable with the unknown parameter theta. We need to include that as well in the Lyapunov expression since theta is the vector so the way to include that would be through this expression.

Okay so here we consider that this gamma theta is adaption again and positive definite so this is very similar to the gamma x and gamma r that we have chosen which are the nominator gamma theta inverse. So the next step is so this is Lyapunov function candidate that we have chosen. Let us look at the time derivative of v v. is given by e e.+ kx tilde – kx .+ b gamma r kr tilde – kr .+ 2 and so 2 will disappear tilde transpose gamma theta inverse - theta hat .

So, let us substitute for the closed loop error system from above that we get ame. I think there is sin mistake here we get the positive term - bkx tilde x – bkr tilde r+ b theta tilde transpose phi of x then we have – b/gamma r K r tilde Kr hat. So, now we have set the the problem for designing the update laws for kx hat and kr hat and theta hat. So we have to generate the update laws for K x hat. kr hat. theta kr hat.

Okay for kx hat and kr hat we can simply use what we have obtained earlier is in fact going to cancel these 2 terms. So, those would be – gamma x x e sin of b. Okay if you chose it then this term will cancel with this term. Let us chose kr hat –gamma r r e sin of b then this term will cancel this term. Okay for this case we have to choose the update law as gamma theta Phi of x so you want to cancel this term.

So, the update law for theta hat would be like this. So, here we assume that all the b is unknown but the sin of b is known. If you extend this to vector case the sin of the matrix b it is very hard to think about. You could probably think about knowing the sins of the different entries of the b matrix but that would be bit unreasonable to assume. So, the case where both the A and B metrics are unknown. It is little hard but you can try that on your own.

There are some assumptions you have to make and the result may not be global and although you may end up with global result. If you come up with the result like that you would probably publish the paper in good journal. Okay so with this we are able to cancel all this terms and or we end up with v.=am e square so which means that the equilibrium point so this is the negative semi definite.

The equilibrium point given by e= 0 kx tilde= 0 kr tilde=0. The origin is Lyapunov stable since v=0 and v.>0 we can similarly say that v is bounded which means that v of tk. So, the signal changing is something that you all should do although it looks very similar to what we had done before it is important that you go through with it in the indirect case we found that when we go through with it the control input in fact and not bounded in certain case.

So, we have to modify the update law so it is important that you prove that all signals are bounded. So using this we can further show that x of t kx hat of t kr hat of t and theta hat t are bounded. Okay so what is left is to prove that control u is bounded so if you go back to the controller we see that we are prove that K x hat is bounded. Theta hat we just proved to be bounded from the Lyapunov analysis and phi.

So, we have to assume here that Phi is bounded whenever x is bounded so that is an extra assumption that we make on basis function and the basis functions here such that so assumption here for the basis function is that. That Phi of x is bounded if x is bounded so we have already proved using this analysis that x is bounded. So we can say that the basis function Phi is also bounded.

So, we can say that the control input u of t is bounded. Then similar to the previous case we can also go ahead and use the barbalats lemma and you can easily show previously that e of t and e . of t are bounded and the fact that e of t is L 2 and 2 barbalats lemma can be used to show that e of t goes to 0 as t goes to infinity and so again in this case we have been able to prove that the tracking error converges to 0.

Although the same cannot be said about the parameter estimation errors all we can say about the parameter estimation errors are that they are bounded in many cases this is all we need our most important objective is for the tracking goes to 0. Okay so that is all about the case where you have structured a matched structured uncertainty like this we have done the scalar case. So, you can also attempt the vector case for this it is very similar to this analysis.

And this is in fact a realistic scenario because real systems are not entirely linear and there is always some non-linearity or other and if we can say that the non-linearity is linearly parameterizable then we can use this kind of model reference Lyapunov controller to stabilize the overall system. So, now I move on to slightly different problem which is the command tracking problem.

So, far what we have discussed is a model reference adaptive control approach where there is a plant which is which is any system that you are going considering for example an aircraft or a chemical system or a biological system which is linear time in variant system. Or the case that we just did with structure and certainty and then there is a reference model which we want the plant to follow.

So, the objective is given in terms of reference model which in capsulate the desired behavior that we require from the plant. And sometimes the objective can simply be a tracking objective that is we want the plant state to track some desired time wearing signal.

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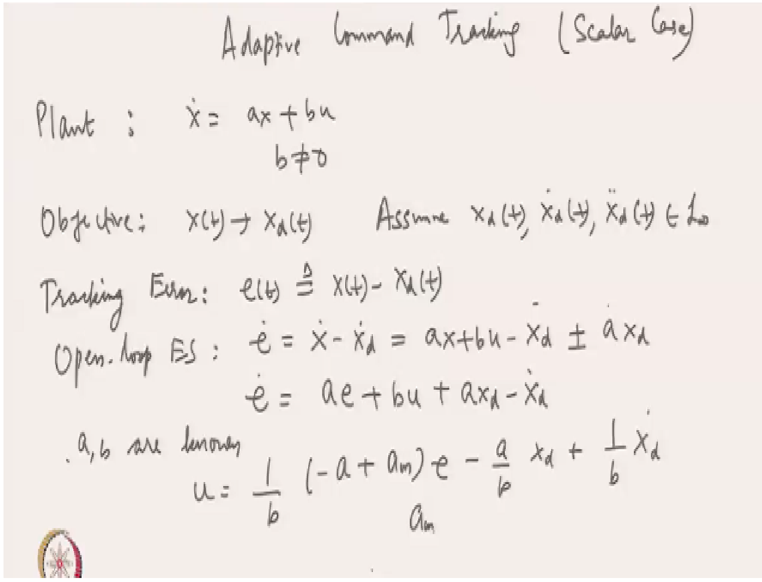

So, that is the problem of adaptive command tracking which we will do now adaptive command tracking. So, again I consider the scaler case but you could again go ahead and do the vector case to be slightly more involved in pulling traces in the Lyapunov function but the approach is similar and and by using this they scale it because it it is easier to follow and for students who are interested in learning more about it.

They can go ahead and do the more complicated vector case okay so again the plant is a linear time invariant plant given by the ax+bu we say that b is non-zero for controllability and the object is so there is no reference model here. So, we do not want x of t to follow some state of reference model instead the objective is given in terms of a trajectory which is x of t so we want x of t to follow x t of t so x d of t could be any time varying signal of your liking.

That you want the plant to follow and what we assume here is that this desired trajectory is smooth it can be differentiated and these derivatives are bounded. So, we say that x t of t so until the second derivative we say that it is bounded. So, we will see where this assumption is used okay so whenever we are given a problem like this we always start with the with the tracking error e of t is defined as x of t-xd of t and then we try to find the error dynamics.

So, the open loop error system is given by taking the time derivative of the tracking error ax+bu-xt. Okay okay so just for just for my ease of analysis what i do here is i add and subtract that a x d okay. So, what it does is it converts the error system into a form which is similar to what we had obtained before. So, it just helps to make the analysis easier so but does not change anything mathematically.

So, e. is ae+bu+axd-xd. okay now assuming a and b are exactly known how would you design the controller u. Suppose a so I make the assumption so that a and b are known so then we could design the controller u as 1/b so we want to cancel terms that are unwanted here. So, we since a all the ways is known and it can be either positive or negative. So, we want to cancel this term and we want to place our own term am here.

And we also want to cancel these two terms because they are not really helpful in proving that e goes to 0 . So, we want to cancel these terms involving in desired projectory so we would use -a/p xd+1/b xd. So if we use this and substitute this in our open loop error system expression then we would get e.=ame where am we have chosen to be <0 so am it could be any desired value. But you could also think of it as a pole as a desired pole location.

That you want the system to obtain so the closed loop error system in this case would come out to be e.=ame and am would then represent the the desired goal location so this could also be thought of a pole placement controller adaptive pole placement controller. Here the idea is that instead of instead of the the open loop pole location we want to place the pole at some desired location given by am which is on the left hand left hand plane.

And this controller would do the job but here we assume that both a and b are exactly known so what if when we are not exactly known. So, like in the previous case we simply use certainty equivalent controller. So, for the case where a and b are unknown which is the which is the adaptive case the controller u is simply given by taking the estimates of the unknown terms. So, a-ahat+am so wherever you have these terms we replace them by their estimates okay.

And we can call these coefficients as k1, k2 and k3 k1 hat k2 hat and k3 hat and these could be k1 k2 and k3 okay since we do not exactly know a and b in we we also have k1 hat k2 hat and k3 hat.

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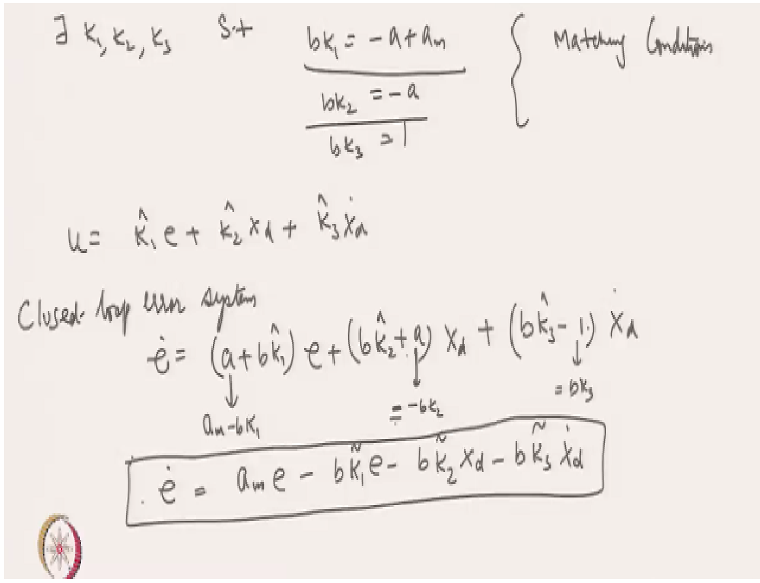

Or what we do assume is that there exist k1 k2 and k3 such that the matching conditions again they come into place again directly found from the algebraic equation expressions that we had formed. So, bk1=-a+am bk2let us look at that look at what he so b this entire expression is =k1 so bk1-a+am -a=k2so bk2=-a amd 1/b=k3 so bk3=1 so these are our matching conditions for this case.

Okay u is given y k1 hat e +k2 hat xd+k3 hat xt. here k1 hat k2 hat and k3 hat are the estimates of k1 k2 and k3 okay let us now substitute this into the open loop error system and get the closed loop error system. So, the closed loop error o system is if given by e.ae+bu a+b k1 hat e+p k2 hat+a xd+b k3 hat -1 xd. okay so here we use the matching conditions and replace the terms which are unknown that is a and b.

But we should be a little smart about this substitution so here of course we want to combine bk1 hat but this term we choose the matching condition given by this to replace by a/am-bk1 so this is am -bk1 this is obtained from the from the matching condition from the matching condition 1 the second term in the bracket bk2 +a we we can manipulate this by by replacing a in this case by-bk2 which is given by second matching condition.

So, here we replace by -bk2 and because we want to combine bk3 hat with a matching term that we get an error we replace 1 here by bk3 okay. So, after making these manipulations what we get is ame-dk1 tilde e -bk2 tilde xd-bk3 tilde xd. alright so this is our closed loop error system for the command tracking case. Okay so now the next step would be to again consider Lyapunov function candidate and try and prove stability.

So, Lyapunov analysis will also help us in designing the update laws for k1 hat k2 hat and k3 hat.

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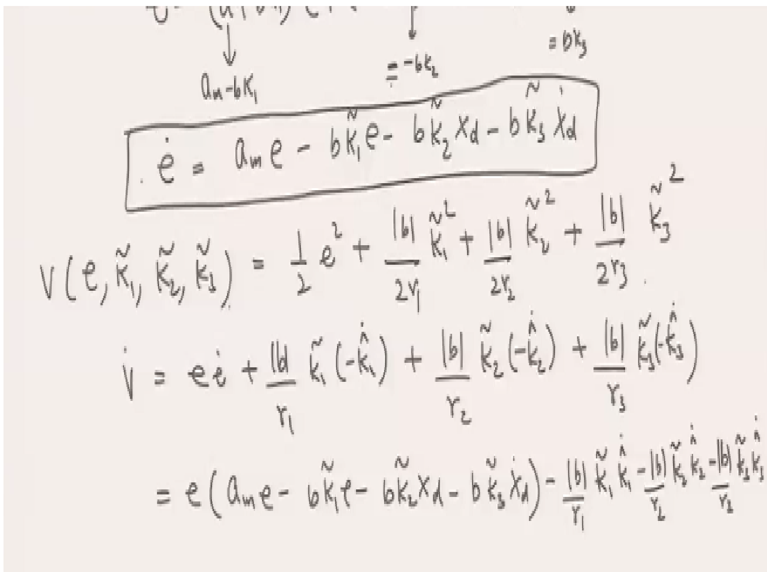

Okay let us choose the Lyapunov function we want to choose the Lyapunov function which involves these error states the tracking error and all the parameter estimation errors k1 tilde k2 tilde and k3 tilde and we want that the entire function should be a scalar should be positive definite. So, these are some of the things that we have to keep in mind the first thing that you always try is the quadratic function it just makes it simpler.

But there could be many choices available which could yield the yield similar result okay let us choose the Lyapunov function as because in the scaler case it becomes easier for me to choose it very similar to the direct MRAC case so gamma 1 and gamma 2 are positivize quantities which we see would be the adaptation gains. Okay, okay so let us now take the time derivative and substitute for the error dynamics.

So, v. s given by e e.+ okay now I have set up the problem for the development of the update laws. Okay, so let us substitute for the closed loop error system -b/gamma 2 k2 tilde k2 hat.-b/omega 3 k3 tilde k3 hat.

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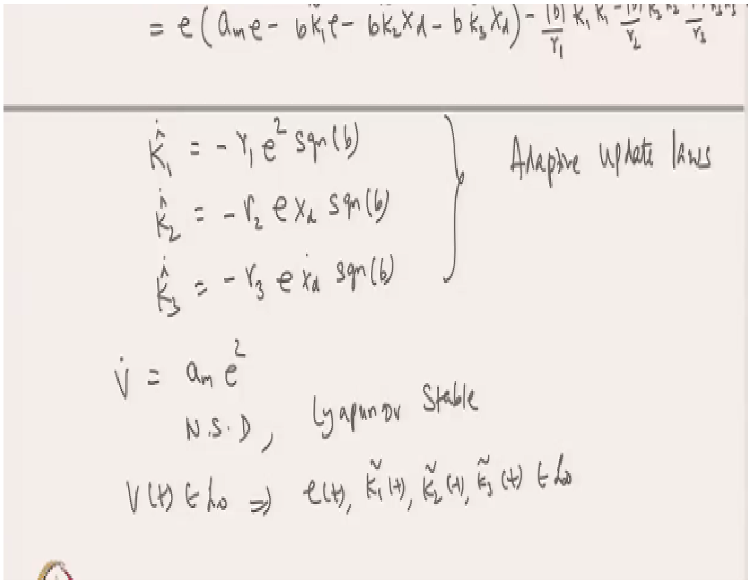

Okay, okay now so let us choose the update laws for k1 hat k2 hat and k3 hat okay so k1 hat we want to choose so as to cancel this term involving bk1 tilde e so the way to choose this would be to use a -gamma 1 e square and sin of b then we want to choose k2 hat. in such a way that we could cancel this term again while we are cancelling this term because the they are not really important not really needed in the Lyapunov derivative.

We want to have terms which are sin determinant and preferably negative so that we can take care of them and these terms which we want to cancel are the sins are not really known because they are dependent on these errors state and the designed trajectories which vary with time and their sins also may vary. So, k2 hat. is designed like this and so ext sin um of b okay and k3 hat. we will design similarly to cancel last term in the in the brackets.

So, which will be e x d. signum of b okay so if we choose these update laws so these are the the adaptive update laws. One more thing to notice here is that so since we have used k1 hat k2 hat and k3 hat and we have gone ahead with the analysis with these estimates. This is a direct approach we can also do an indirect approach where we simply considered a hat b hat and go ahead with the analysis.

And then try and design the update laws for a hat. and b hat.so that is something that you could try on your own. This is the direct adaptive command tracking controller okay so substituting these in v. what do we get is v.is am time v square okay so again this is negative semi definite and you could say that the it will be on point is Lyapunov stable important to go through with the analysis.

And we can show that v is bounded which implies that e of t k1 tilde of t k2 tilde of t k3 tilde of t are all bounded.

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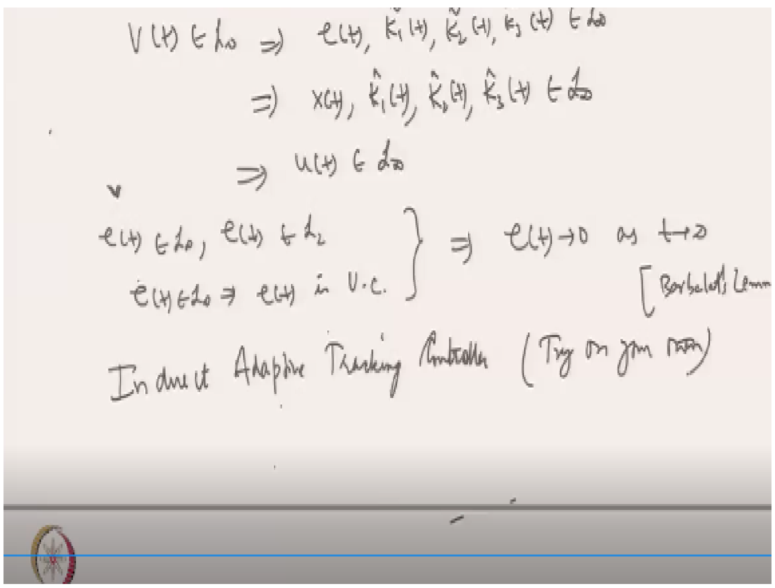

Which further implies that x of t is bounded because we have assumed that xd of t is bounded and since k1 is simply a constant k1 hat of t is also bounded k2 hat of t and k3 hat of t are all bounded okay and then we can go back and try and prove that the controller is bounded okay so here we see that k1 hat is bounded e is bounded k2 hat is bounded xt is bounded. K3 hat is bounded xt. is bounded because that is something that we had assumed.

So, since all these terms are bounded what we can say is that in the controller u of t is also bounded. Okay so what were not shown here so far is that the tracking error e of t goes to 0 so let us go ahead and prove that we have already proved that e of t is bounded and using previous similar to previous analysis we could also prove that e of t is L2 that is apparent. The thing we have to be careful about is proving that e. e is uniformly continuous.

So, for that we need to look at e. and prove that e. is bounded. So, let us look at e. so e is bounded k1 tilde is bounded e is bounded tilde is bounded xd is bounded k3. tilde is bounded xt is bounded so all these is bounded so what we can say is e. of t which implies e of t is uniformly continuous okay I am using these 3 facts we can claim using Barbalats lemma. Then e of t goes to 0 as t goes to infinity.

So, okay okay so in in this lecture what we have so you can in fact go ahead and do the indirect adaptive controllers. Because in this lecture we have studied two different controllers from what we had done before. The first one is a case where we have matched structure and certainty the fact that the uncertainty is linearly Para materializable allows us to use adaptive control approach. And and design the entire controller as an adaptive controller.

So, there we have a nominal controller which is the MRAC controller and controller which is used to account for the nonlinear term which is again an adaptive controller for cases where we have unstructured uncertainties or uncertainty which are not linearly para materializable it is not straightforward to use adaptive controllers. They are popularly used for cases where the uncertainties are linearly para materializable.

The second case that we discussed today was a related case where instead of the plant following a reference model the plant state tracks a desired time varying signal given by xt which which we assume to be smooth with bounded derivatives. So, using direct adaptive control approach we designed a stabilized controller for this case. And you can go ahead and do the indirect command tracking adaptive controller.

Okay in the next lectures in fact in the next few lectures what we will discuss is robust adaptive control where instead of the uncertainty being structured uncertainty. The uncertainty is unstructured and how do we tackle that case so normally we use some kind of a robustifying elements in our control laws. Either the controller or the or the adaptive laws which make sure that the stability is preserved.

So, in the next couple of lectures we will study robust adaptive controllers thank you.