INDIAN INSTITUTE OF TECHNOLOGY DELHI

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NPTEL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING

Video Course on
Electric Vehicles Part 1

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Lecture # 12
Vehicle_Dynamics_Simulation_And_Dynamic_Equation

Hello everyone, welcome to NPTEL Online Course on Electrical Vehicles. In our previous interaction we have started a topic vehicle dynamics
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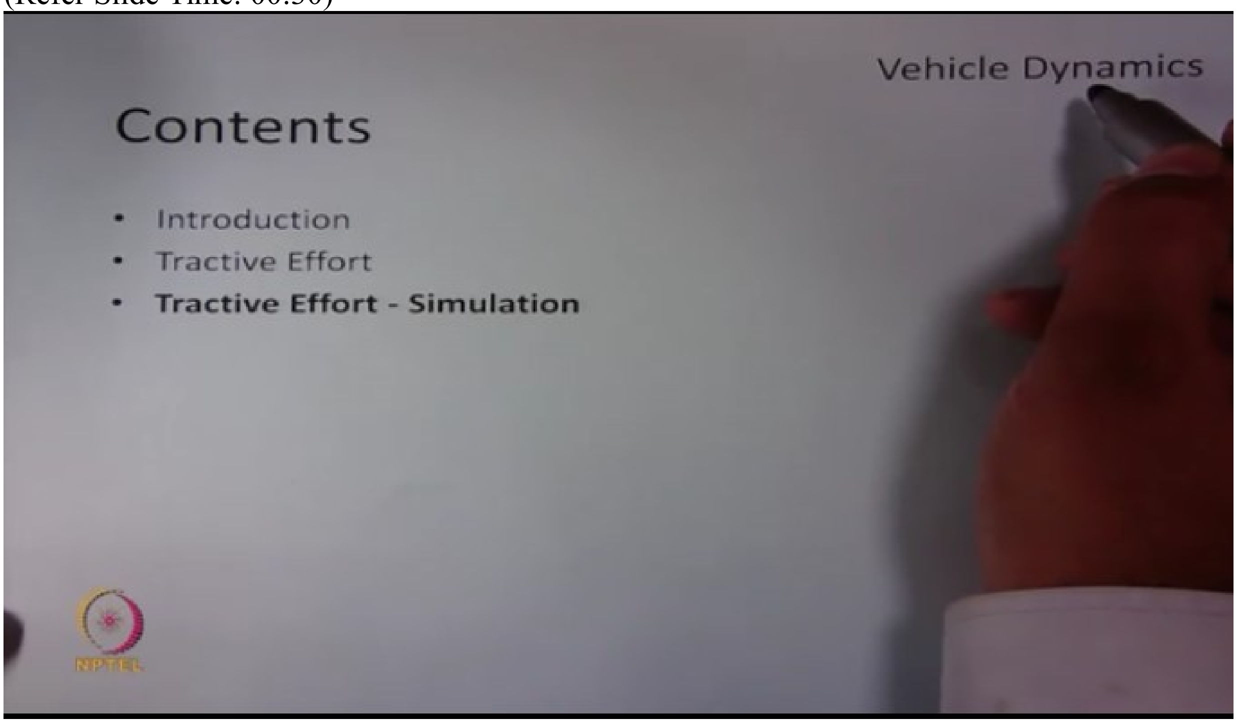

and we have covered full topics under the vehicle dynamics such as introduction and tractive effort.

Let us see some simulations with respect to the topic of tractive effort in today's interaction.

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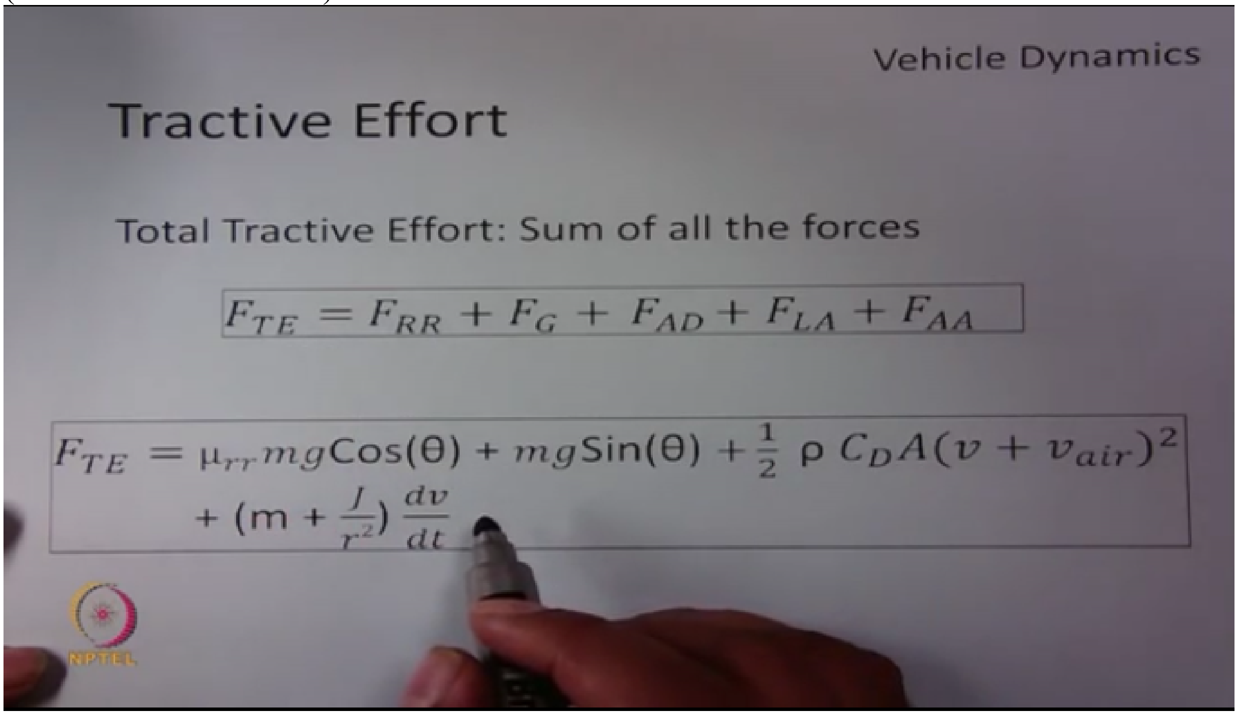

So we have seen that the total effort that is required to move the vehicles you say sum of various forces such as force due to rolling resistance, force due to gradient or slope, force due to aerodynamic drag, and force that is required such as linear acceleration and angular acceleration, if you want to provide change in a speed when the vehicle is moving.

So if we substitute the formulas of each of these forces we get this equation, so force due to tractive effort is FRR which is Mu RR MG cos theta, forced due to gradient is MG sin theta, forced due to a dynamic drag is 1/2 rho CDA and the square of the addition of vehicle velocity and air velocity. And also we have acceleration forces due to linear moment and angular moment.

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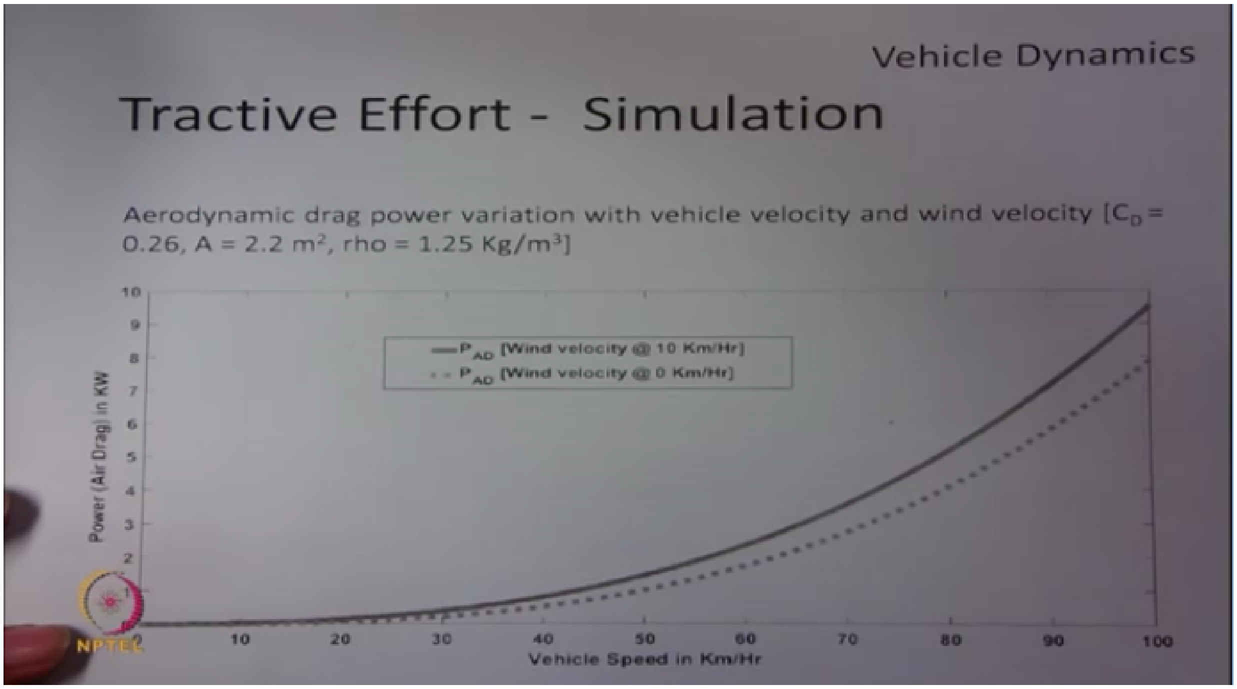

So we know that the dynamic aero dynamic drag force is a function of, square of vehicle velocity, so if you want to see the variation in aerodynamic drag power with respect to vehicle velocity and also with respect to wind velocity, it will be good idea to plot the aerodynamic power with respect to different vehicle speeds for non-value of drag coefficient, frontal area and air density, so let us assume that we have a drag efficient of 0.26, frontal area of 2.2 meter square and air density of 1.2 KG per meter cube, so if you see the power due to aerodynamic drag at different vehicle speeds, let's say we'll see the variations from 0 kilometer per hour to 100 kilometer per hour, we'll see a graph which is this graph.

So we can see that the power is a cube function of speed, so therefore the power required at very high speed is significantly high, so let's say at 50 kilometer per hour, the power required is around 1 kilowatt, while the when the vehicle is operating at 100 kilometer per hour the requirement of power is 8 kilowatt, so just by doubling the speed from 50 kilometer per hour to 100 kilometer power the power required changes from 1 kilowatt almost 8 kilowatt, so this is a huge variation.

Secondly let us see that in addition to vehicle velocity if there is a strong wind of let's say 10 kilometer per hour opposing the vehicle movement then what will happen? So this second curve shows that effect, so if the wind velocity of 10 kilometer per hour is opposing the vehicle movement you need extra power of around 2 kilowatt at 100 kilometer per hour.

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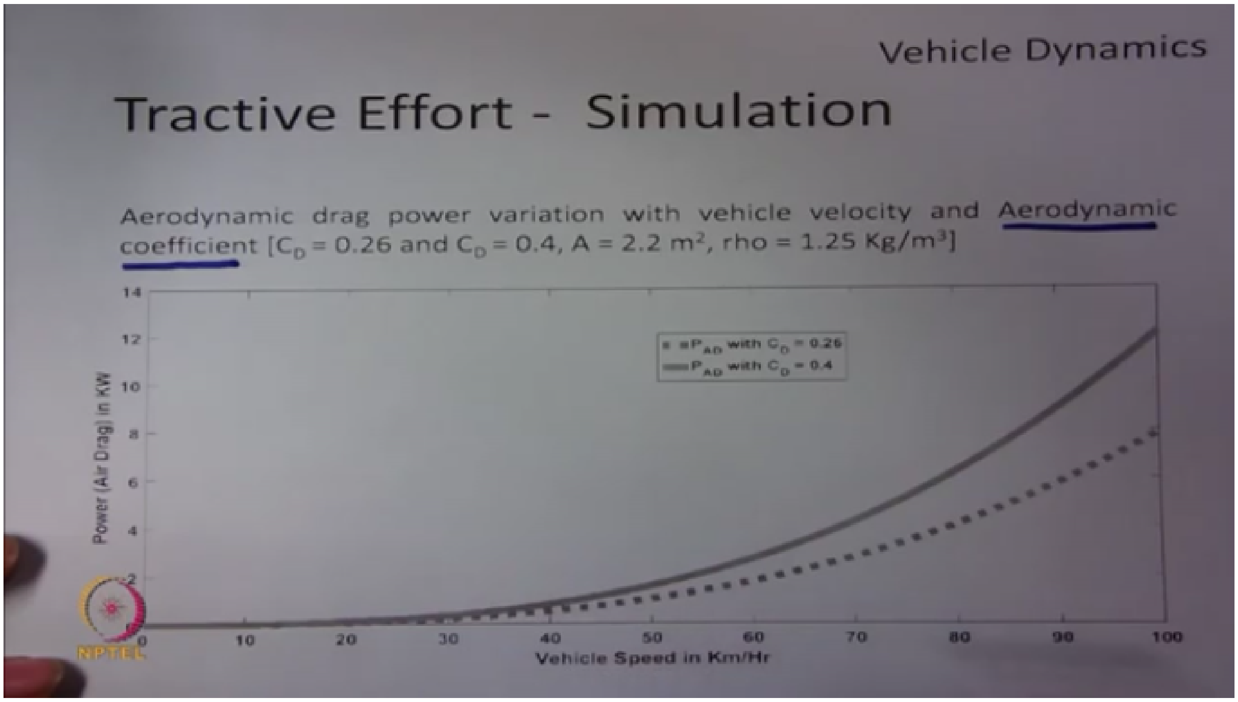

Now let us see the effect of variation of aerodynamic drag efficient which is CD on the aerodynamic drag power, so let us see that the previous case example, but with two values of CD so we have CD of 0.26 and CD of 0.04, area is similar 2.2 metric square and rho 1.2, 5 kilometer per metric cube, so if you take this example we can see that if we increase the CD from 0.26 to 0.4 the amount of power needed increases from 8 kilowatt which we need when the CD is 0.26 to around 12 kilowatt when we are operating with a CD of 0.4, so we need to support extra 4 kilowatt power if you are running at 100 kilometer per hour.

So you can see that the effect of this coefficient is quite high and there should be every effort to reduce the drag coefficient in electric vehicles, because this amount of power requirement can put significant stress on battery energy capacity.

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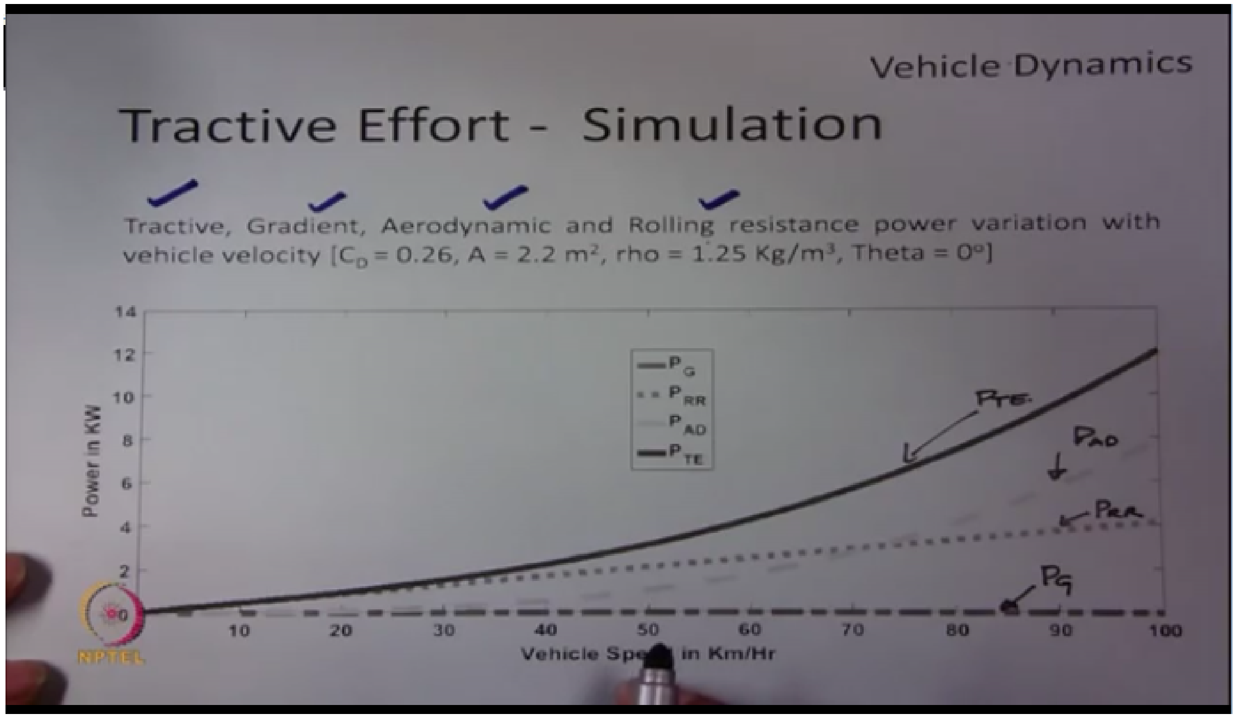

Now let us see all the forces and their variation together, so let us see the gradient force, the aerodynamic force and rolling resistance force and addition of this force is which is attractive effort together in a same graph, so we want to see the effect of this vehicle velocity on this forces and the power variation.

Again we will take the example which you have taken earlier that is CD of 0.26, air of 2.2, rho of 1.25 and let assume a flat road where the theta is 0. So when the theta is 0 we know that there is no power required in terms of gradient force, so it is 0, first MG sin theta and sin theta is 0.

And these is the variation of force and the corresponding power required to support rolling resistance force, so you say commutely straight line so this is a weak function of speed as we have seen, and when we talk about aerodynamic force which we have already seen so it requires around 8 kilowatt at 100 kilometer per hour so this is continuing the same graph as we have seen earlier.

So the total tractive effort is somewhere around 10 kilowatt, so you see addition of so PTE is addition of PAD + P rolling resistance + P gradient, so gradient force is 0 here.
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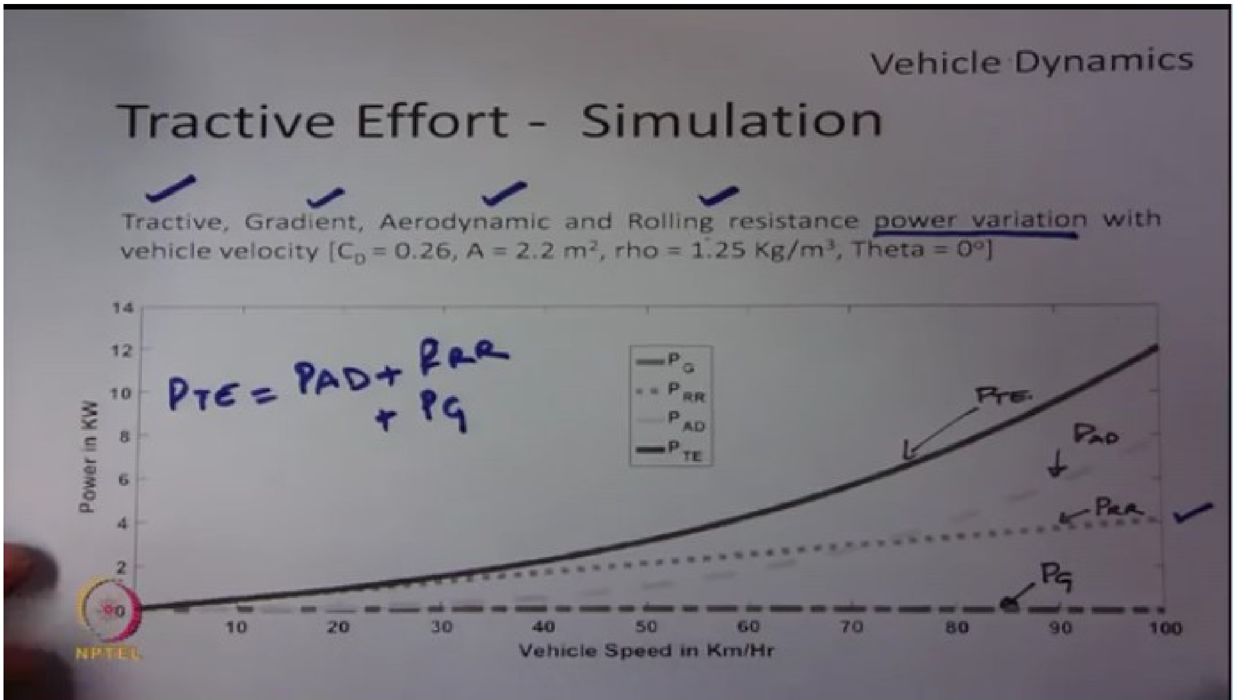

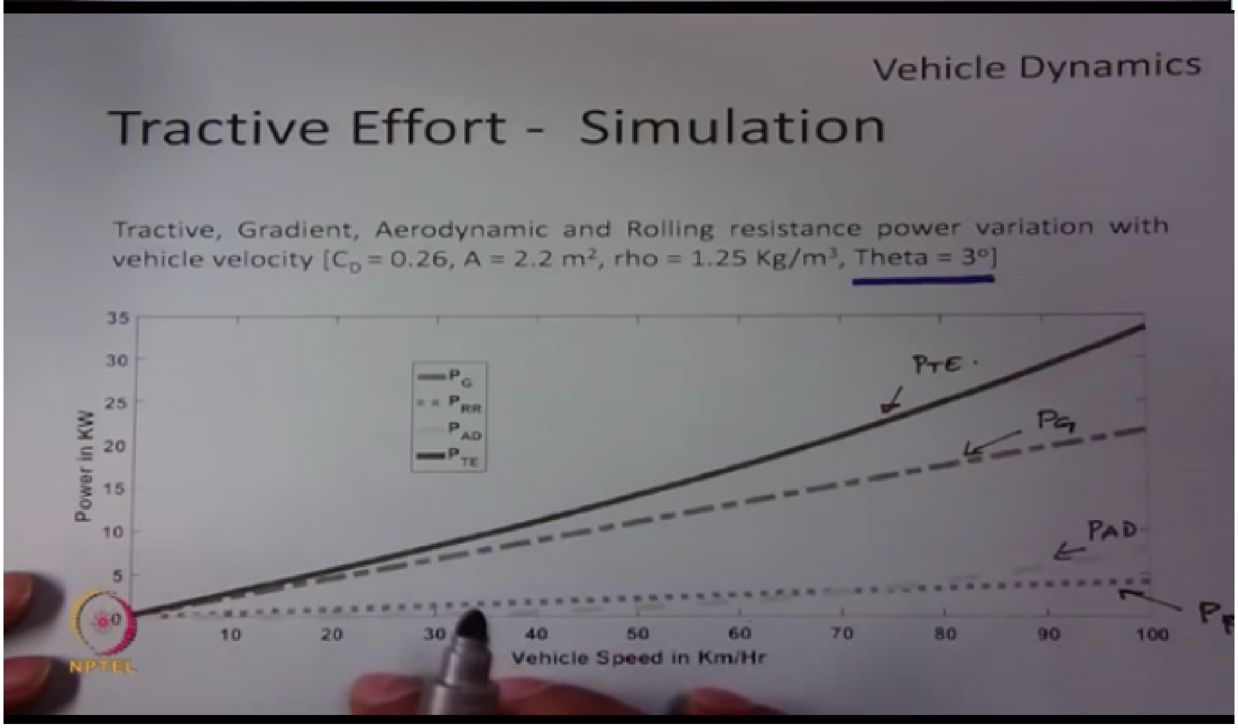

So let us see the similar result when we have our operating at the theta of 3 degrees, so let say the vehicle is moving on a slope of 3 degrees and when we are operating the vehicle at different speeds, so in this case the only variation will be in terms of gradient power, so gradient power now extremely high so at 100 kilometer per hour it requires almost 20 kilowatt power, so if we are driving the vehicle on a slope of 3 degrees the gradient power itself is so huge, it is around 20 kilowatt, the total tractive effort will be somewhere close to 35 kilowatt, so you can see the effect of slope, so moving the vehicle on a slope versus a flat road is inorganic, so this effect will be very high if you are running at high speed on a slope, so effect increases linearly, so it also tells us that you know if you are operating a electric vehicle on a hill, or in a slope terrain it push the heavy burden on the vehicle battery, so it is always advice to operate the electric vehicle on a flat road terrains for having a good range.

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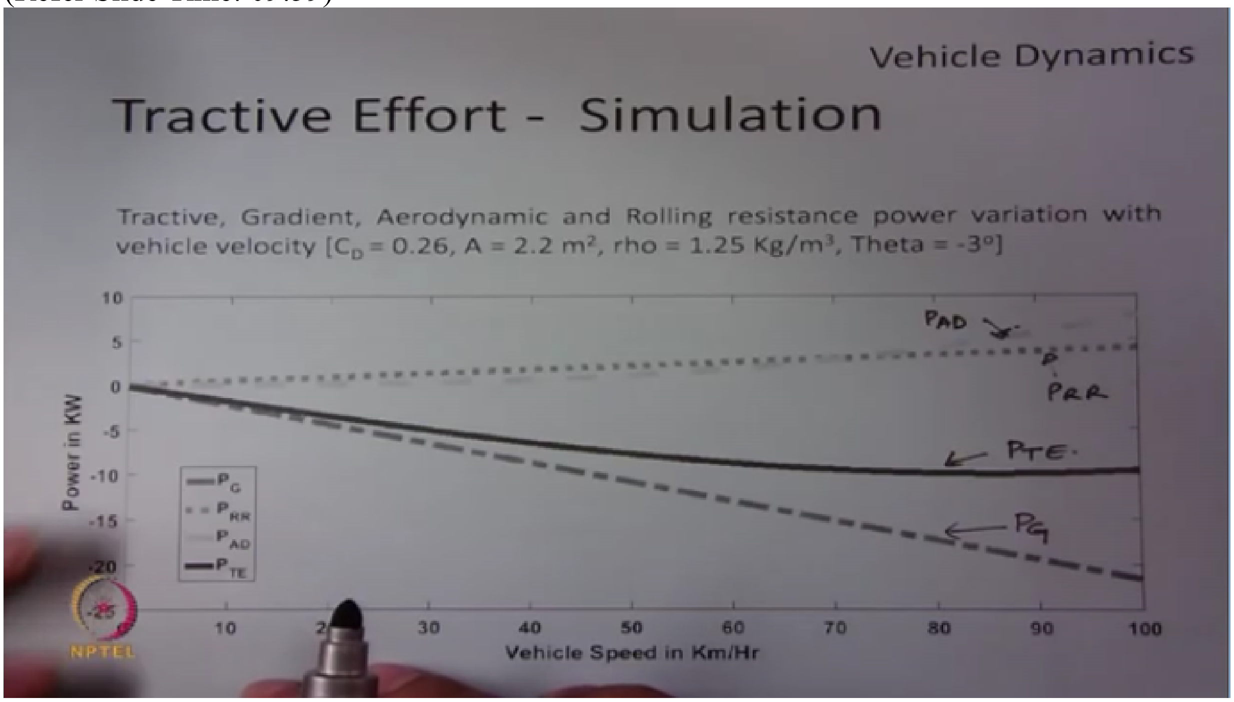

Similarly let us see the effect of all this forces when you are operating at a theta of -3 degrees, it means that we are driving the vehicle down the slope, so in this case again similar to the previous case that gradient power required is around 22 kilowatt at 100 kilometer per hour speed.

So when we see the total tractive effort now it is negative, so it increases with a tractive effort up to 70 kilometer per hour and after that it becomes flat and maybe started decreasing, so what happens at that higher speed? At higher speeds the power taken away by aerodynamic drags becomes very significant and the power what we can regenerate becomes lesser, lesser, because the aerodynamic power becomes significant at higher speeds.

So this graph can be plotted using simple simulation files, so in this course we are using MATLAB as the simulation tool,

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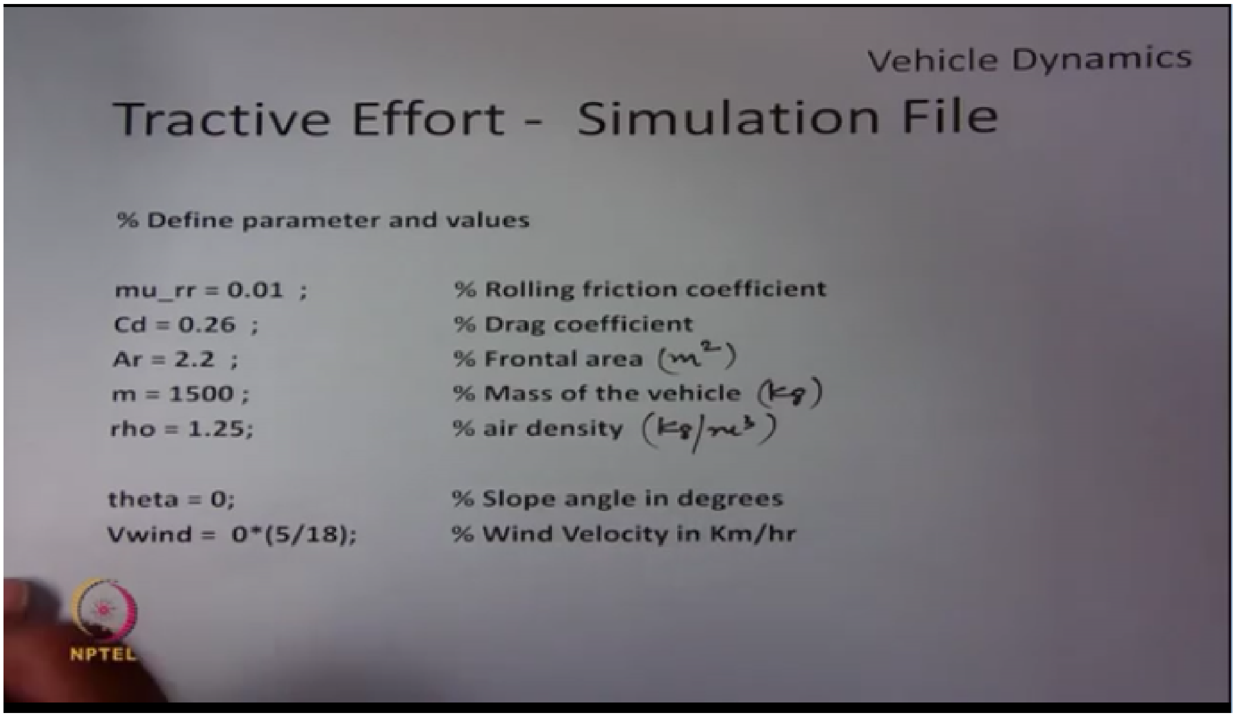

so the simulation file use to plot this graphs is elaborated here, so first general in the summation file we have to define the parameters and their values, so in our case we need to define the rolling friction coefficient which let's say Mu RR, it's a 0.01.

Drag coefficient of 0.26 area of 2.2 meter square, mass of 1500 KGS and density of 1.25 KG per meter cube, theta we can define in degrees, vehicle of the wind we can give in kilometer per hour but if we multiplied it by 5/18, we convert it to meter per second.

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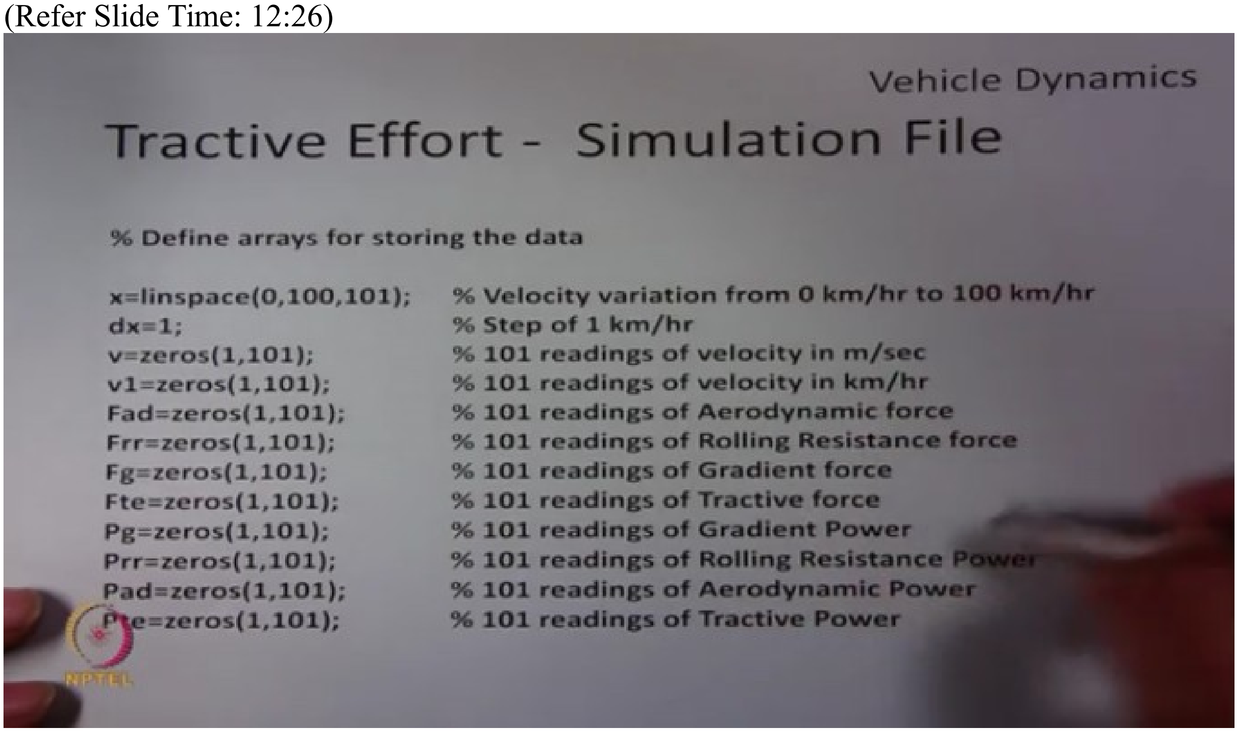

Then since we are plotting all the powers and the forces, so these are forces due to all aerodynamic force, rolling resistance, gradient and total tractive power and power, so we have to define all these variables and also, we have to define arrays because we need to store lot of data, so against each speed we have to calculate the power and store as a data, so we have to create arrays and where the values of this different forces and power will be stored, so we have to define velocity as well, velocity, forces and powers.

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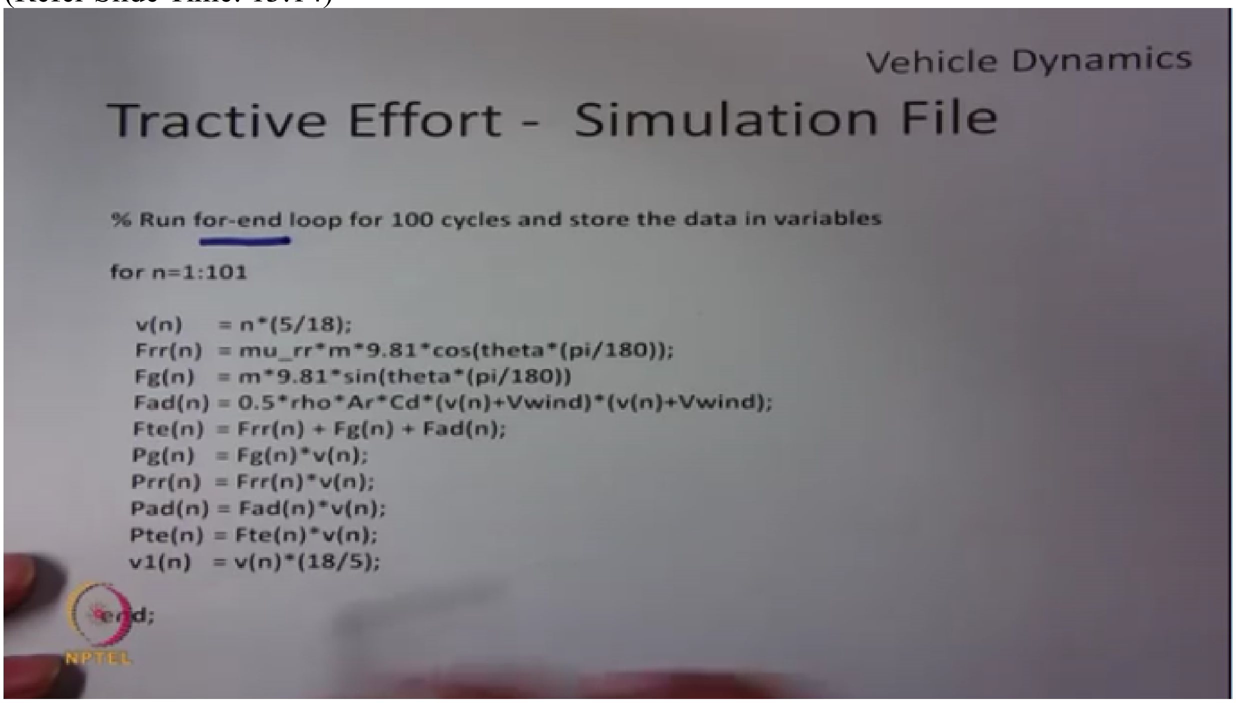

Then we can run a for loop and the loop has to run in number of steps, so let us say that we are operating at the step of 1 kilometer per hour, so we are varying this speed from 0 kilometer per hour to 100 kilometer per hour at a step of 1 kilometer per hour, so for each iteration we will calculate all the forces that total tractive effort, all the power and the total tractive power for each velocity and we'll store the results in the array of 100 values.

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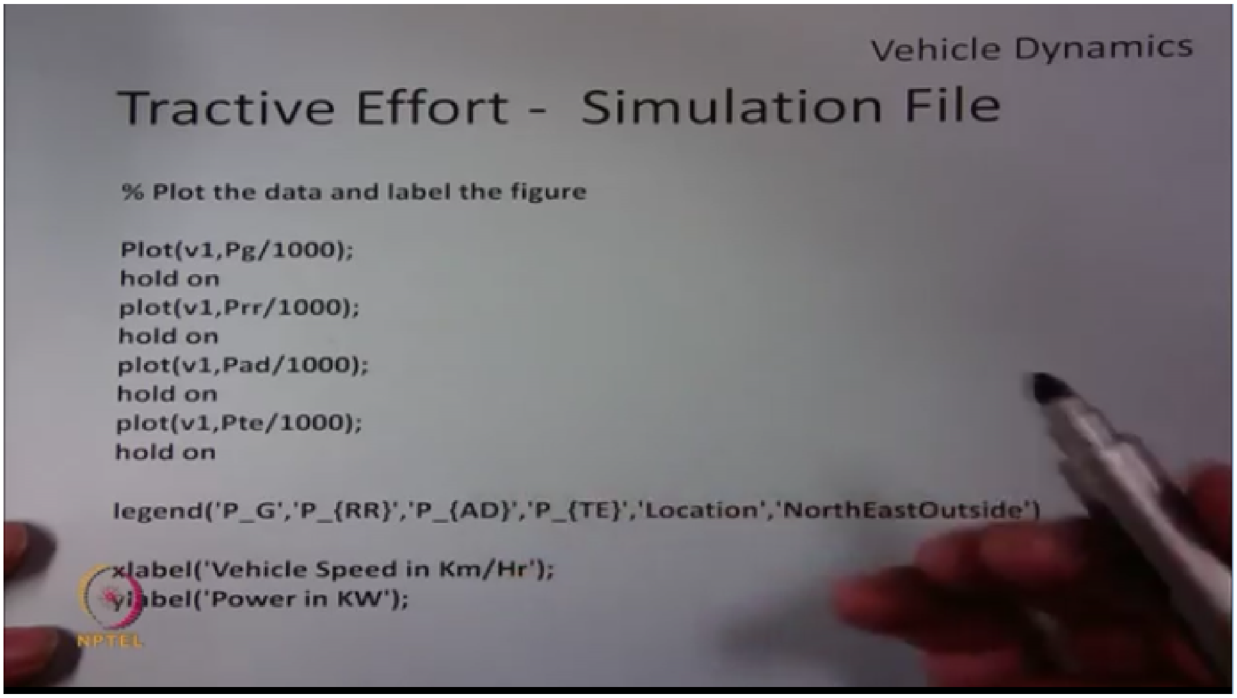

So having got the values we can plot this power versus velocity at the meter per second or kilometer per hour, so whichever is visible, and since the power is quite huge we can plot in terms of kilowatt, so if we divide by 1000 it means the power will be now plotted in kilowatt and speed in kilometer per hour, so it is simple exercise it can be also done in C language, which is easily available to the students.

Now let us go to our next topic which is dynamic equations,
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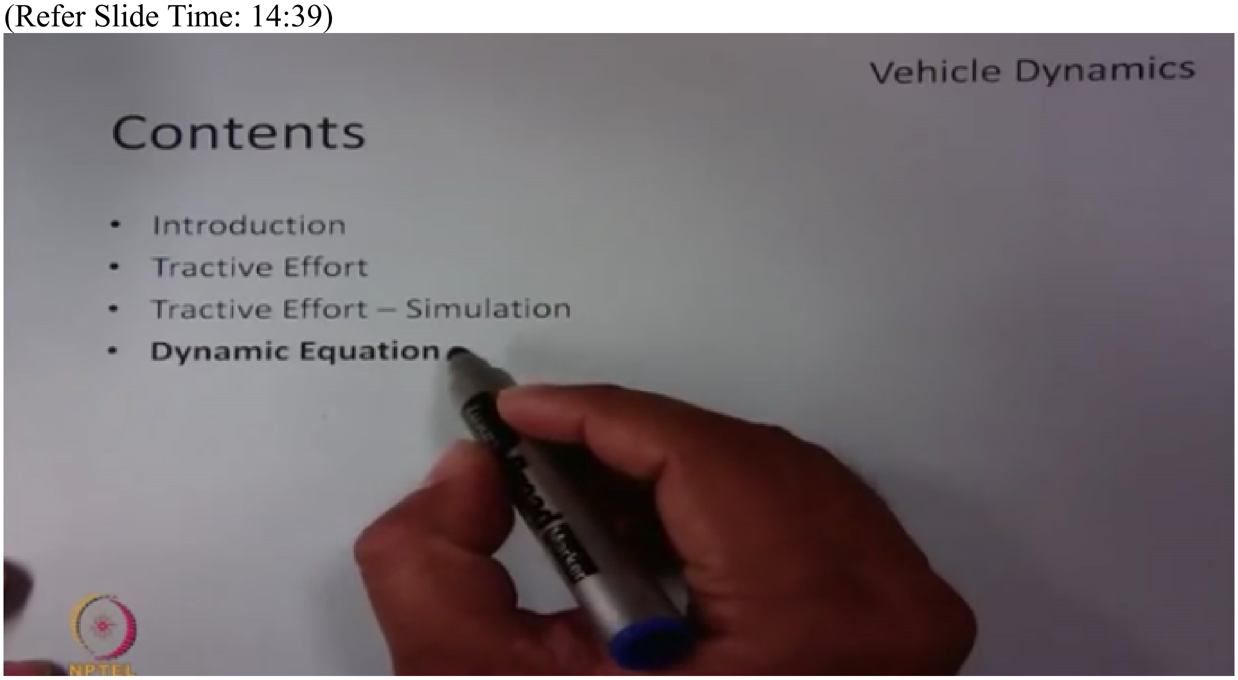

so we will try to drive dynamic equations which can be used for understanding the dynamics of vehicle for different kind of force input, so the input is the force what we get from the position or attraction unit, so how the system will behave for different kind of distraction input force is important analysis parameter,

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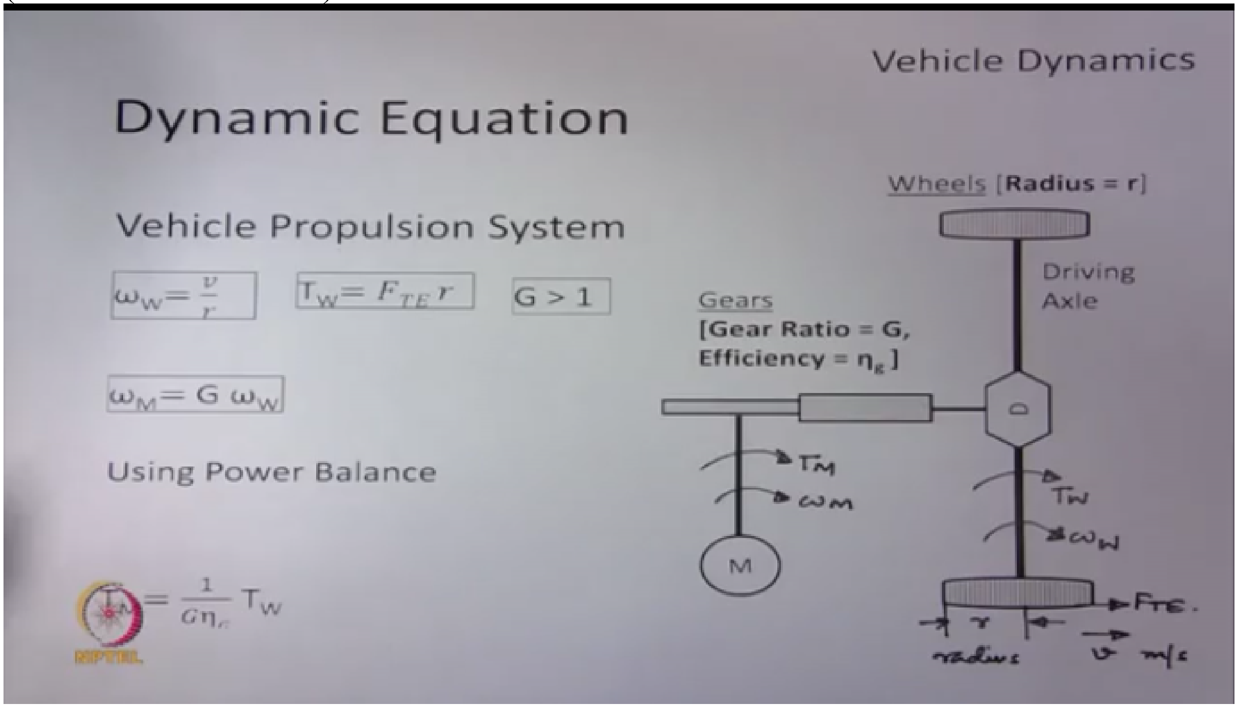

so this is the typical drivetrain of a electric vehicle, where a electric motor is connected via gears and differential to the wheels where driving axle, so we use fixed gear generally in a electric vehicle, so let us assume the gear ratio of the gear is G, and its efficiency is eta G, so let us try to find some basic relationship between different forces and speeds, so if you assume that the vehicle is moving at a velocity of V meter per second under a tractive effort of FTE, then we can say that the torque on the driving excel which is T wheel is equal to FT into R, so this is the simple relationship between force and torque.

Similarly the angular velocity of the driving axle which you can denote as omega wheel can also be found out using this relationship, so omega V is velocity by R, so in a typical electric vehicle the gear ratio is normally taken as greater than 1, it will be in the range of 8 to 10 in most of the cases, we can also find the angular velocity of the shaft of the electrical motor using the gear ratio, so we can say that omega M is G into omega wheel.

In a electric vehicle drivetrain generally the speed of the motor is higher while the speed of the wheel is lower, so speed is reduced from motoring side to wheel side, we can also find the relationship of torque of the motor shaft with respect to torque on the driving axle, this can be done using power balance equation which we can write as follows, so we can say that the power on the electrical machine shaft can be equal to power delivered on the driving axle, but since we are losing some energy in the gears, because of this efficiency so we can also accommodate the losses in the gears by substituting eta G as a dividing factor to the power delivered on the driving axle, so generally this losses has to be supported by the electrical machine.

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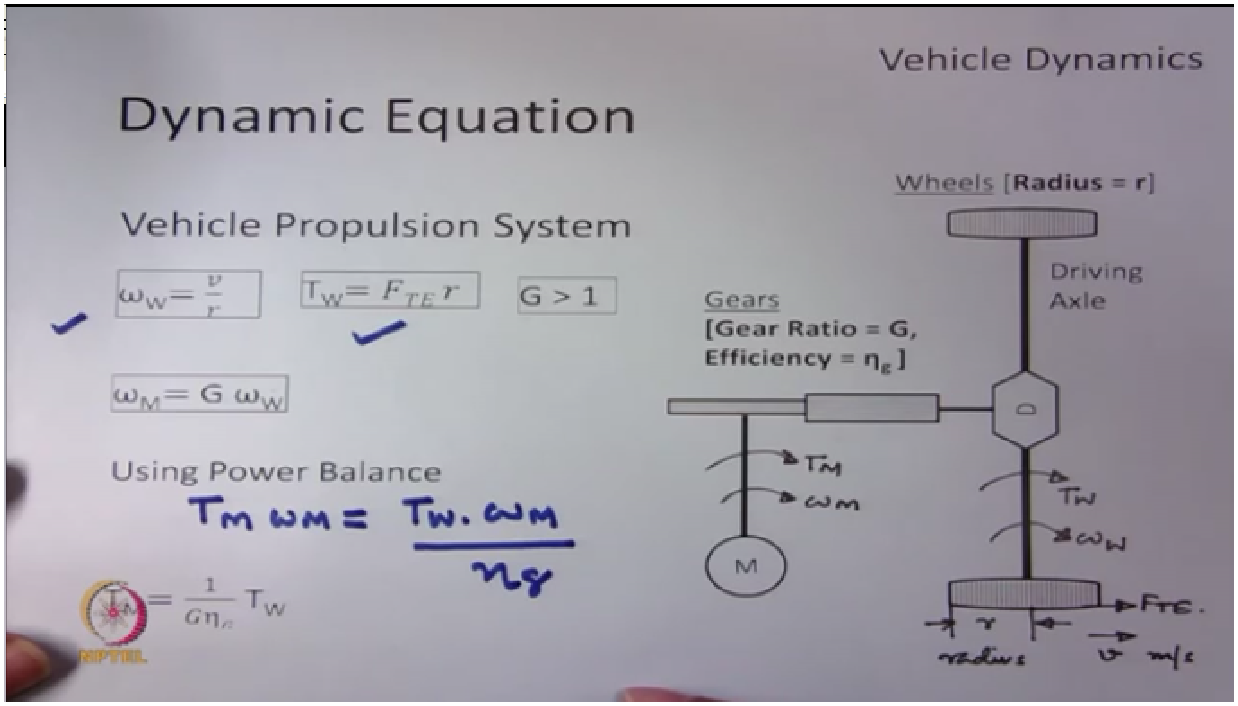

So since we know the relation between omega M and omega wheel we can find the relation between TM and T wheels as this, so TM will be 1/G eta G into T omega, so here we can see that the motor torque is lower compared to wheel torque since it is divided by G and G is greater than 1.

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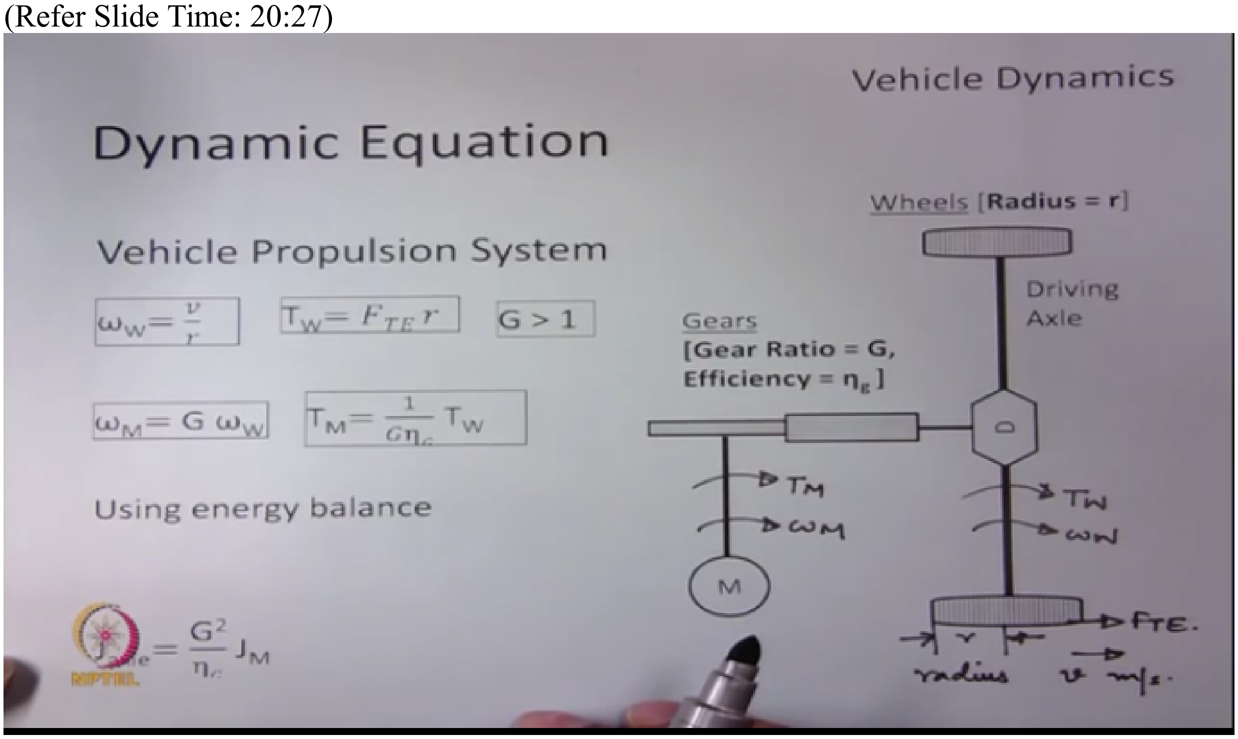

So in our previous interaction we have discussed about J axle which is moment of inertia of the driving axle, so this is the moment of inertia of all the components as seen by the driving axle. So if we want to accommodate and understand the relation of the moment of inertia of motor shaft in force equation we may have to find the relation between JM and J axle, so how we can do that? We can do that by using energy balance equation, so we know that the energy stored in the mechanical shaft of electrical motor is 1/2 JM omega M square, and this has to be equalized to the energy stored in that driving axle which is 1/2 J axle into omega wheels square,

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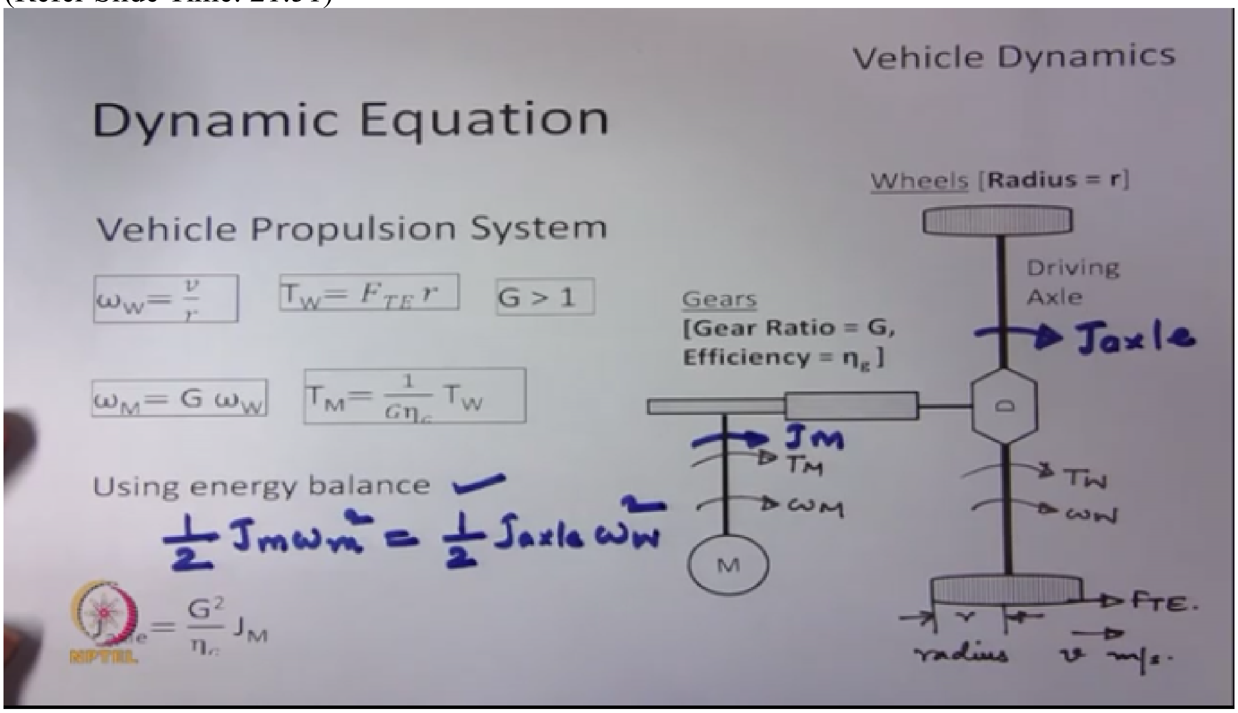

but since we have to accommodate the losses in the gears we have to divide it by eta G.

So here also we know the relation between M and omega wheel and which can be used to find the relation between J axle and JM as G square/eta G, so let us substitute this relationship on the force equations,

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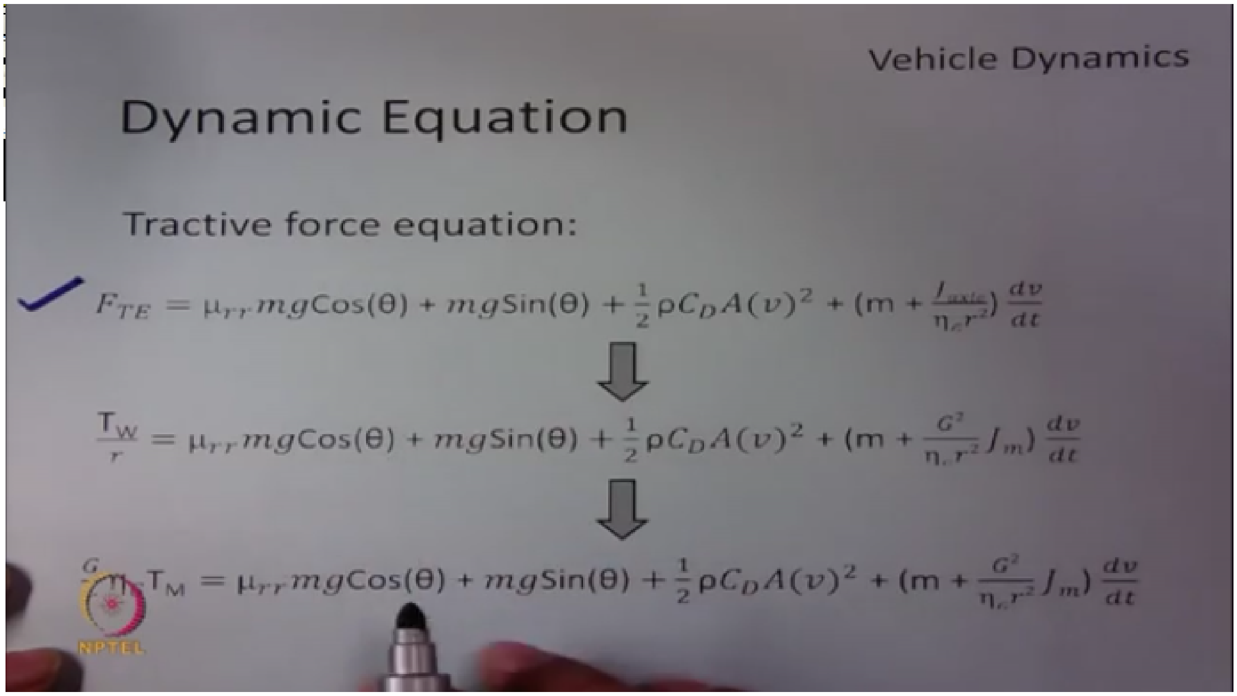

so we know the force equation as addition of different forces. And let us now substitute you know torque of the vehicle axle on FTE, so we know FTE is torque of the wheel divided by R and also the relation of J axle with respect to inertia of the motor.

We can also substitute the motor torque in place of axle torque, by means of relation we have just seen as this. So now this equation is in terms of motor torque and inertia of the motor, and the velocity we are still keeping the vehicle velocity, so using this equation we can find the amount of torque that is required to be delivered from the motor, if you want to operate the vehicle at a velocity which is V with all this resistive forces in place.

Let us try to redefine this equation in terms of dynamic equation in terms of velocity,
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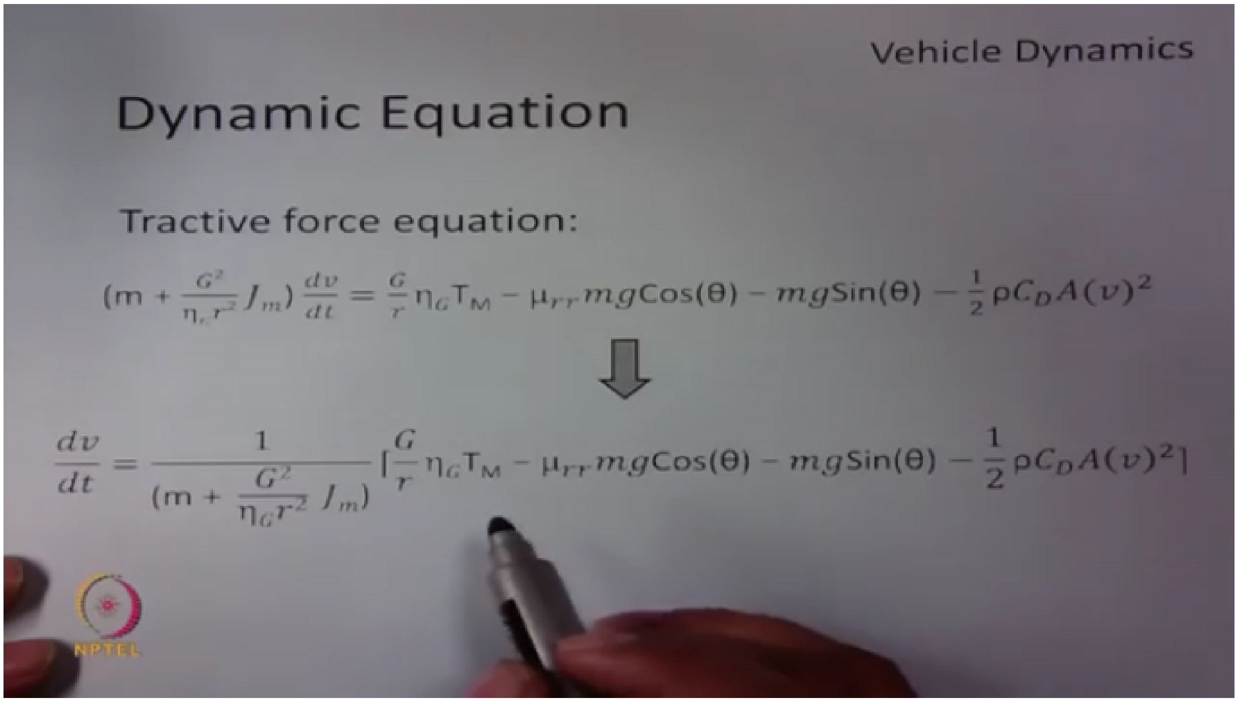

so if we re-substitute the thing and we bring this term which is proportional to DV/DT on one side, and the tractive effort and the subtraction of all the resistive forces on other side we can get this equation D/DT will be G/R eta G TM – all the resistive forces and this will be divided by the equivalent mass on the vehicle axle.

Now when the vehicle is in motion there can be different types of tractive effort that can be applied from the motor, so let us start with a scenario where we are applying a constant tractive effort, means the FTE is constant,

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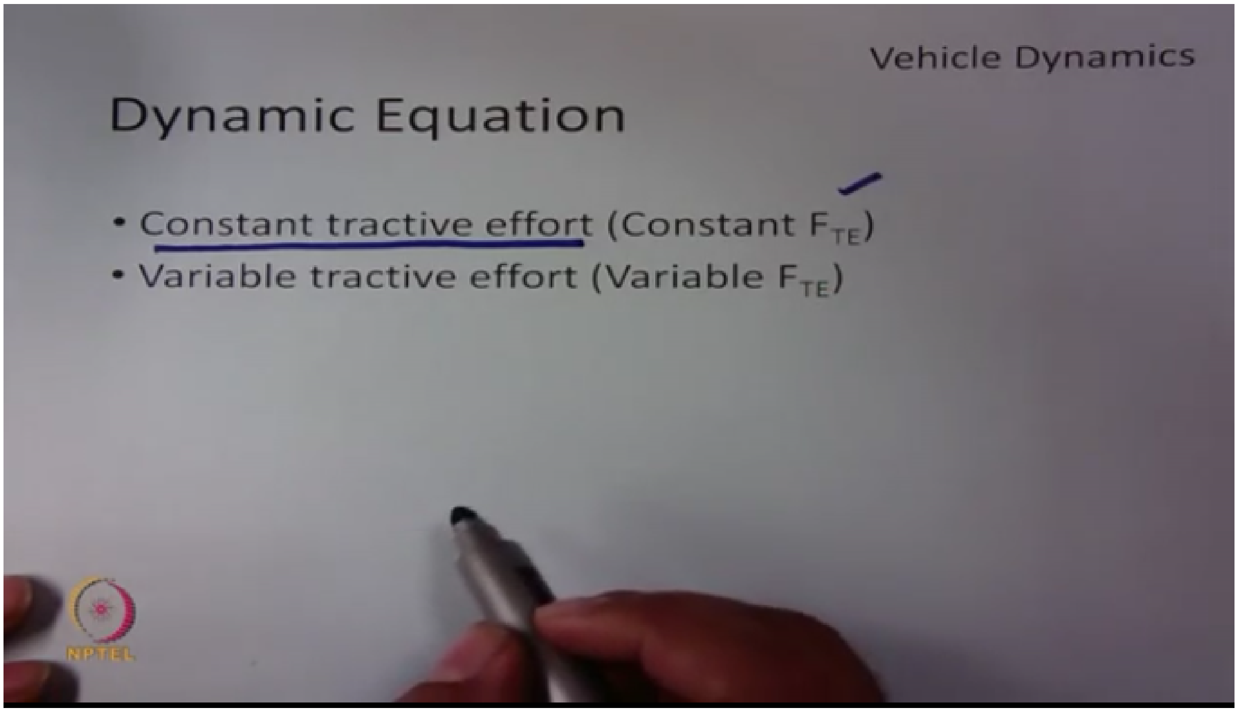

but in a practical scenario this FTE is not constant and it will be applied in a certain manner depending on the driving cycle of the vehicle, so this also will be discussed later.

So let us start with the derivation of dynamic equations when we are operating with a constant FTE,

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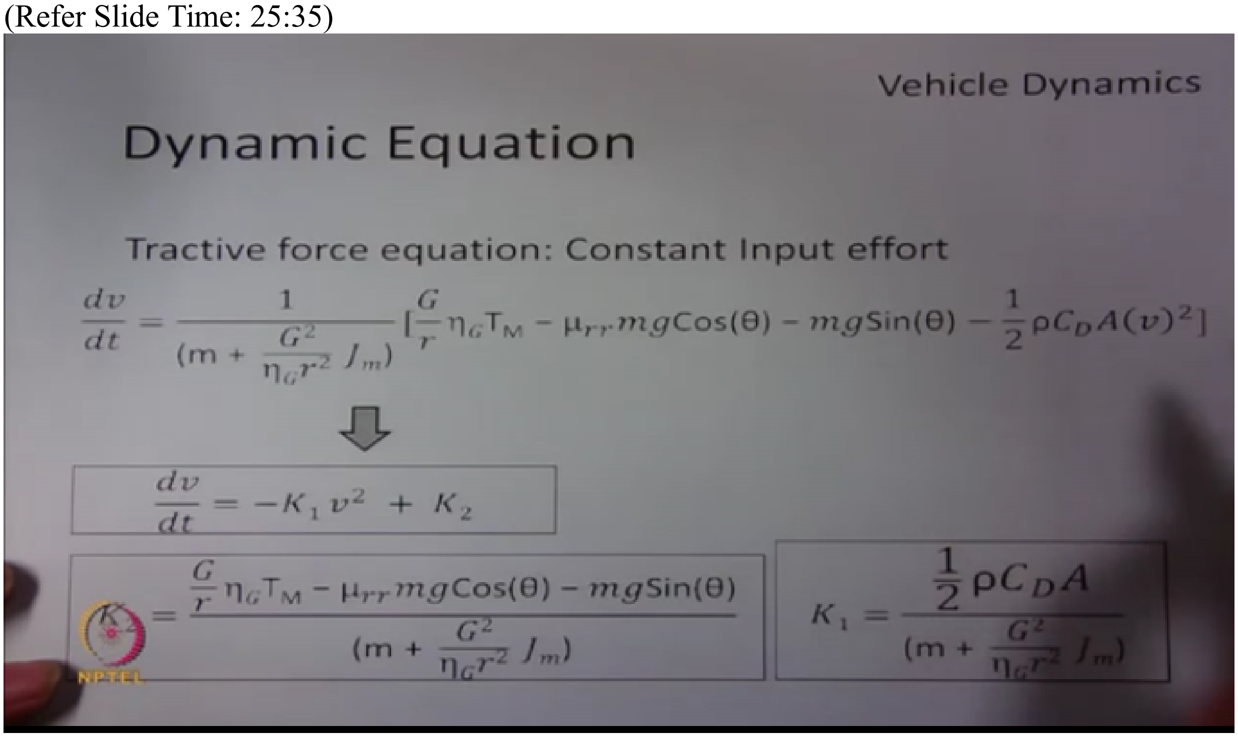

so when we are operating at constant FTE that TM will be also constant and this whole equation can be written in terms of some simple constants, so DV/DT can be written as –K1 V square + K2, so K2 is all constant terms + this K1 is a function of this aerodynamic force equivalents, so this is the equation, so we can see the values of K1 and K2, so K2 is basically the constant parameters and K1 is aerodynamic forces equivalent.

So if you want to understand the variation of velocity for this constant force input we need to solve this equation as a function of time.
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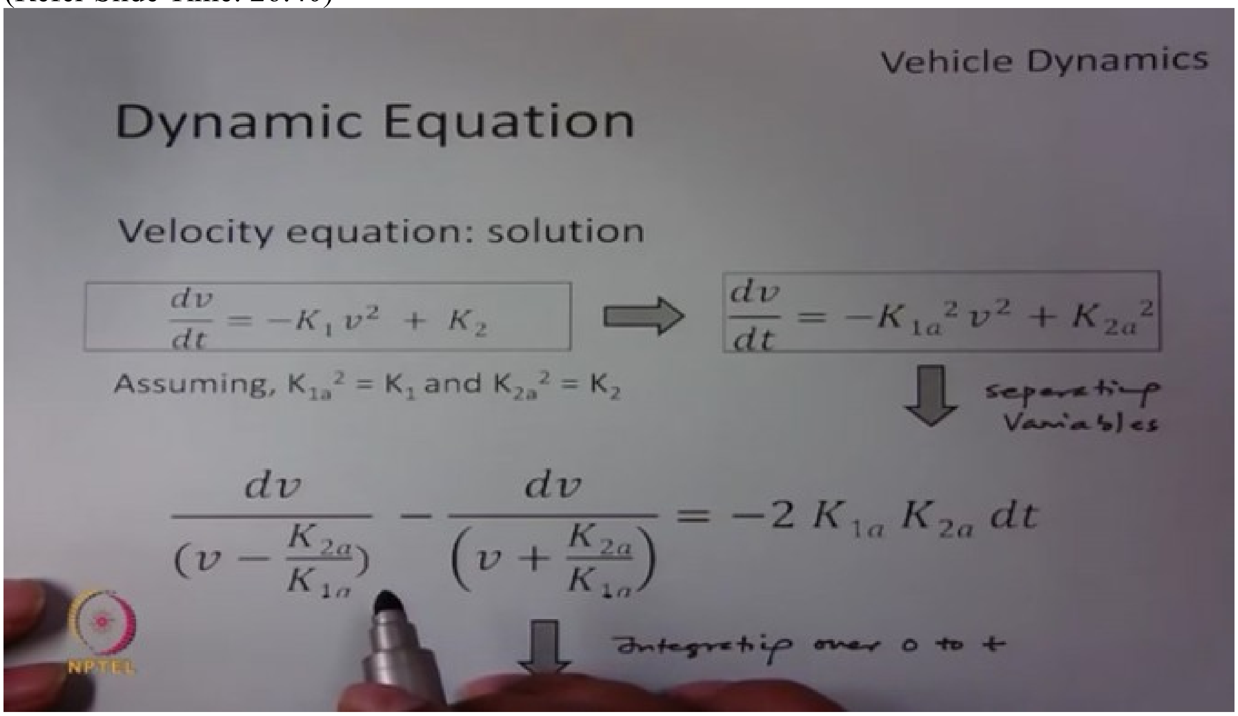

So first solving this equation let us assume few things let say assume that K1 = K1 A square and K2 is K2 A square, if you do this substitution the solution can be easily derived, so this equation will become this equation, so K1 and K2 will be substituted by K1 A and K2 A, so once this equation is there we can rewrite this equation in terms of this way, so this you can rewrite in terms of this way, and this is separating variables method and we want to understand the variation of velocity with respect to time, let us integrate this over 0 to time T,
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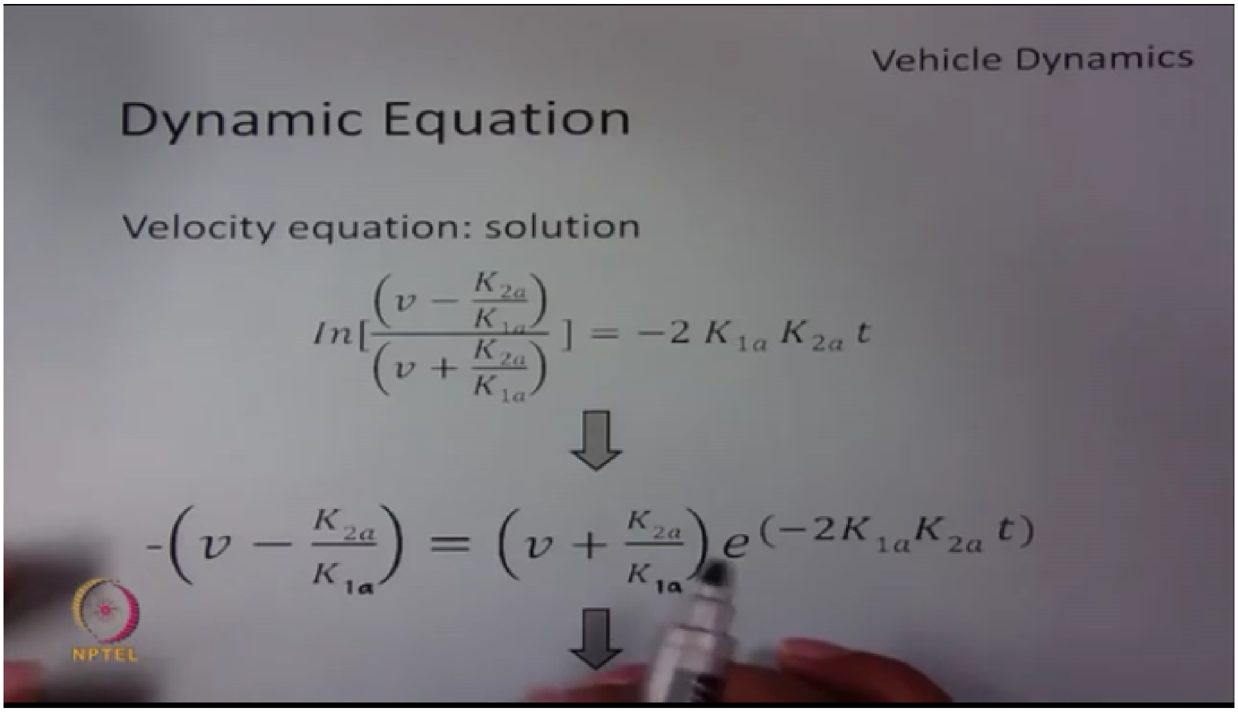

if we do that we'll get a logarithmic equation which is like this, and this can be first solved in terms of exponential terms, this we can remove this logarithmic term like this.

So now it can be further simplified as this equation and this equation is a,
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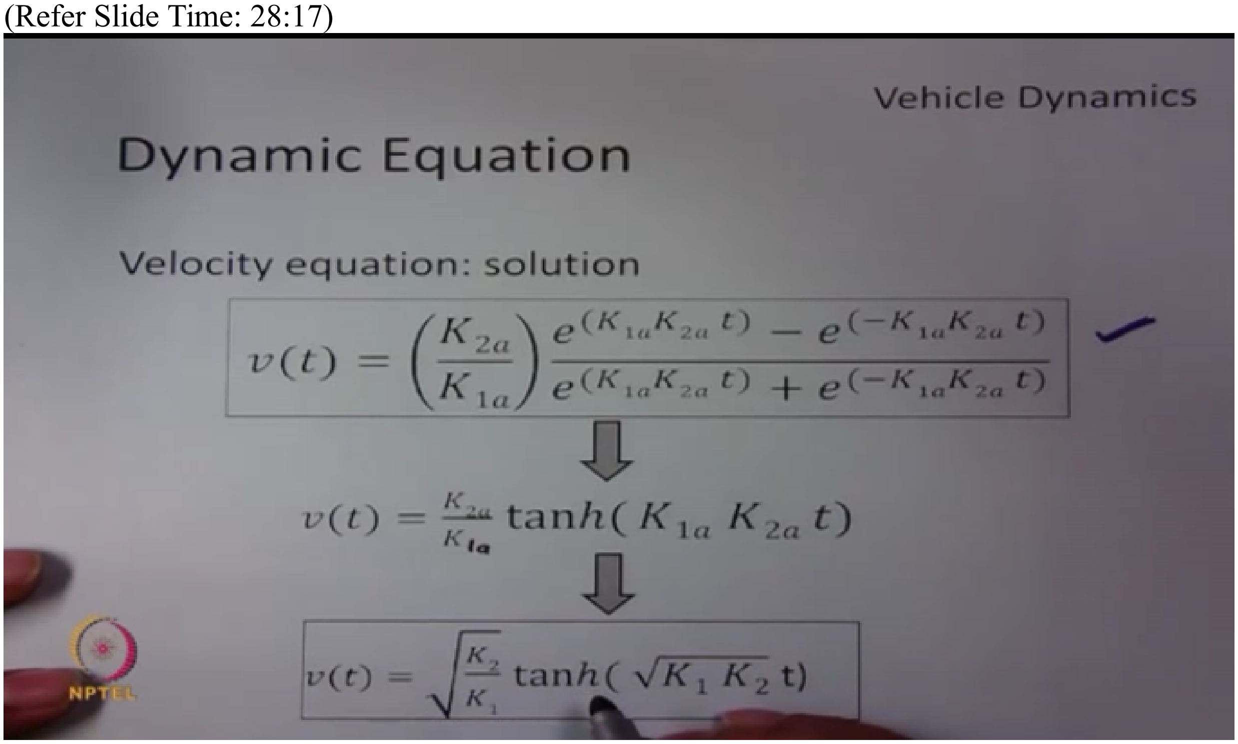

can be easily rewritten as V(T) is K2A/K1A Tan H (K1A, K2A into T) so this is the equation we were looking at, so this can be further simplified and we will again re-substitute the value of

K1A and K2A in terms of K1 K2, so K1A is route of K1 and K2A is route of K2, so if we substitute this we'll get this equation.

So this is the equation of velocity variation with respect to time in terms of variable K1 and K2 which are you know the parameters of the system considering all kind of opposing forces and accelerating forces, so what this equation known for it?
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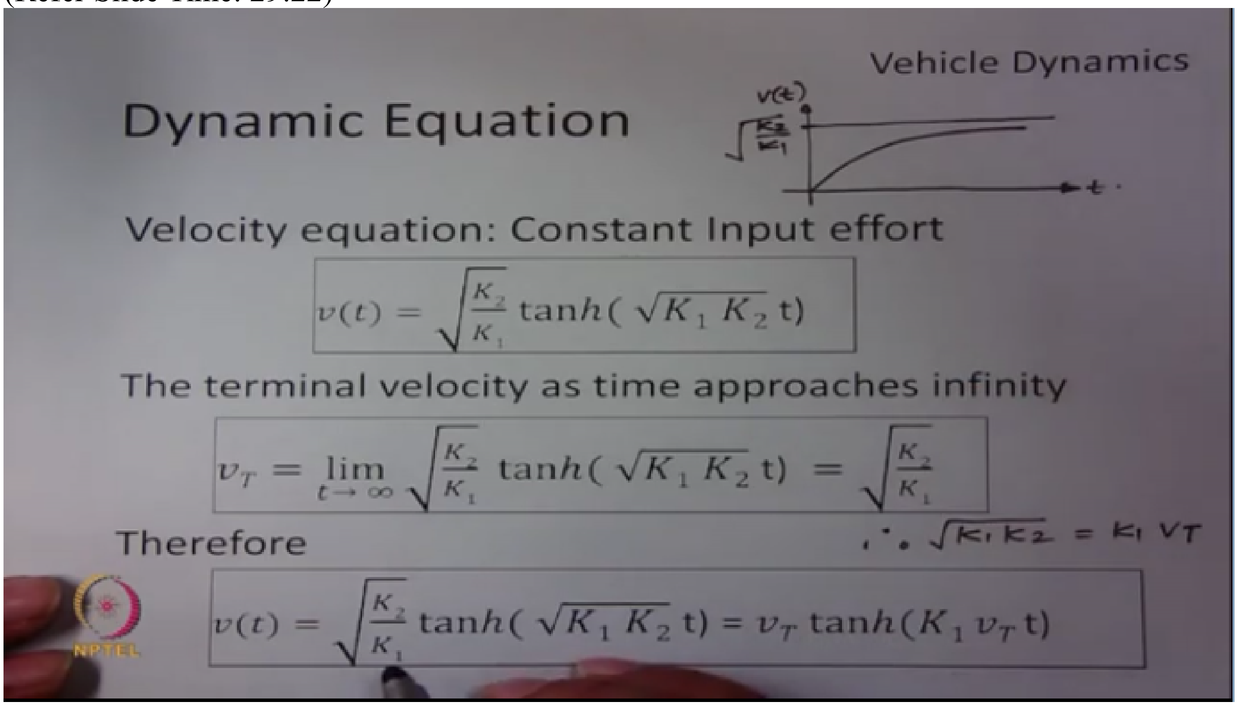

So this infers at this the velocity will start from let's say 0 meter per second and it will move towards statistics value as that time elapses, so what is the statistics value of this velocity?

The statistic value of the terminal velocity can be find if we find the limit value as time approaches infinity, if it substitute this limit as time tends to infinity, we will get a very simplified expression which is root over K2/K1, so terminal velocity is this so we can say that this V terminal value is equal to root of K2/K1, so which is a very simple expression, so once this value is known we can do this simplified substitution which is root over K1 into K2 is now K1 into VT, because if you re-substitute this in a proper way you can able to find this expression, and this can be substitute it in this expression of VT as this, so this is VT and this is K1 VT, so this is the simplified expression of VT in terms of terminal velocity which a system will finally end too.

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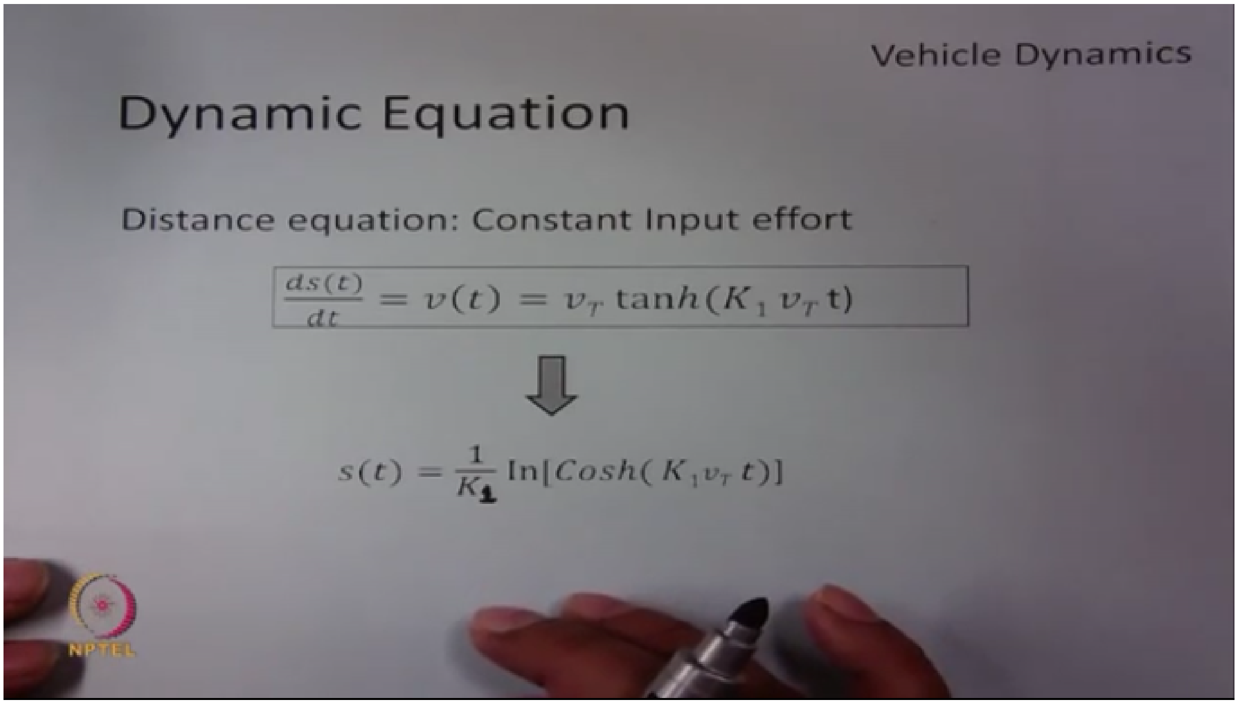

We can also use this equation to find the distance the system travels in a finite amount of time, so VT is nothing but D/DT of ST, so S is basically distance, so if you integrate this we can get the expression of S(T), so S(T) is 1/K1 logarithmic of cos H K1 VT, so this is exponential graph and let's say we want to understand you know the starting acceleration,

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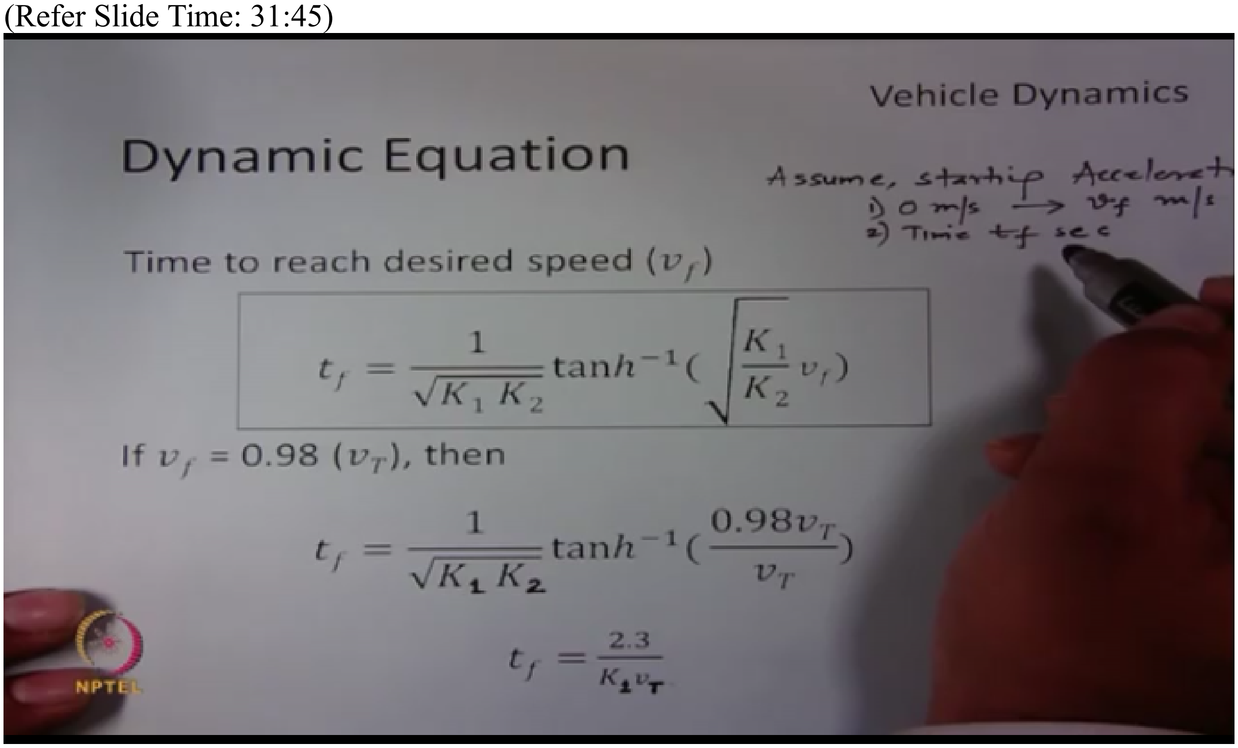

let's say we want to understand that if you are operating the vehicle and increasing the speed by cos and torque input and we are increasing the speed from 0 meter per second to let's say VF meter per second, and typically it takes a time equal to TF, so when we are operating and going from 0 to VF speed the time required is TF, so what is this TF? So we know the starting value of speed and the let's say final value of VF which is lesser than VT, then what is the time required for achieving that speed, so that can be simply evaluated by using the expression of V, so in spite of V(T) we will substitute V(F) and T will take it on the other side and it can be just 1/K1 K2 tan H inverse and this expression, so this is just a reserve expression of the velocity expression.

Let's say roughly we want to find the time, when the velocity reaches almost 98% of the VT, so if we want to understand TF or this speed range then we can simply substitute VF as 0.98 and we all know that root of K2/K1 is VT, so if you substitute this we'll get a very simplified expression for TF, so these are rough value of time we need to achieve, 98% of the terminal velocity which is 2.3/K1 into VT.

So these are some simple formulas which can be used to some performance analysis,
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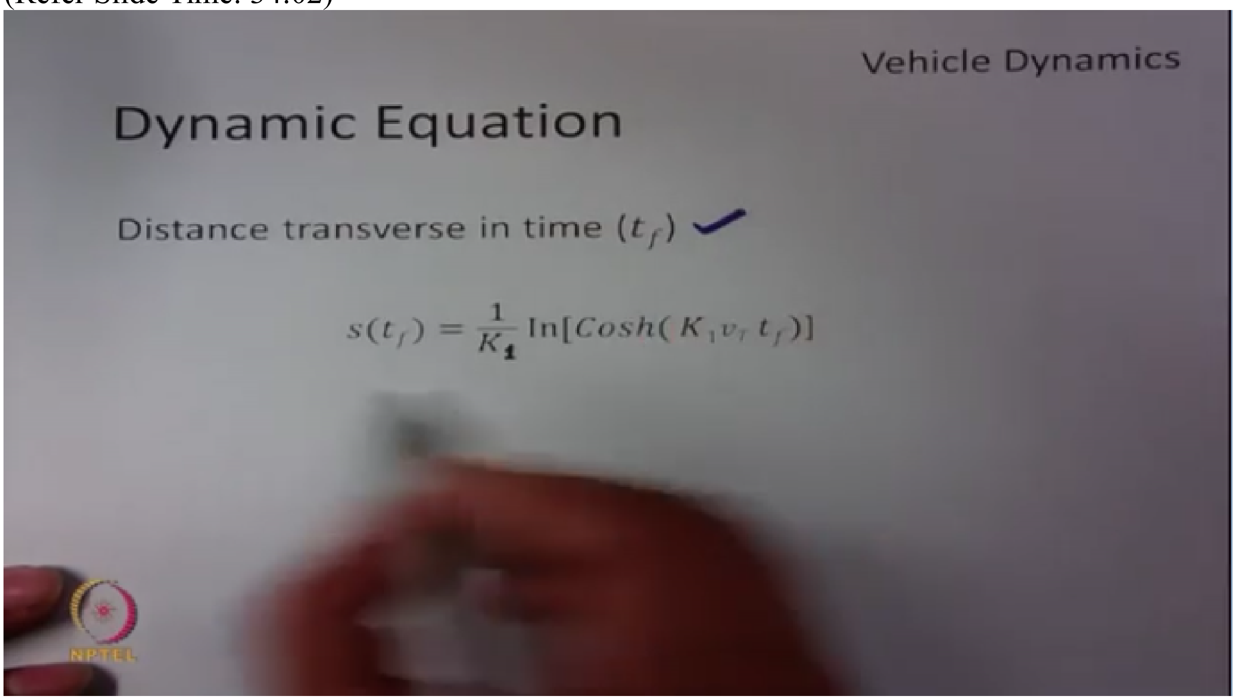

so once the TF is known we can also find that distance which is transferred or achieved when we are reaching the velocity of VF, so we just substitute that TF in the expression of distance.

We would also like to calculate the instantaneous power that is required to achieve this kind of operation,
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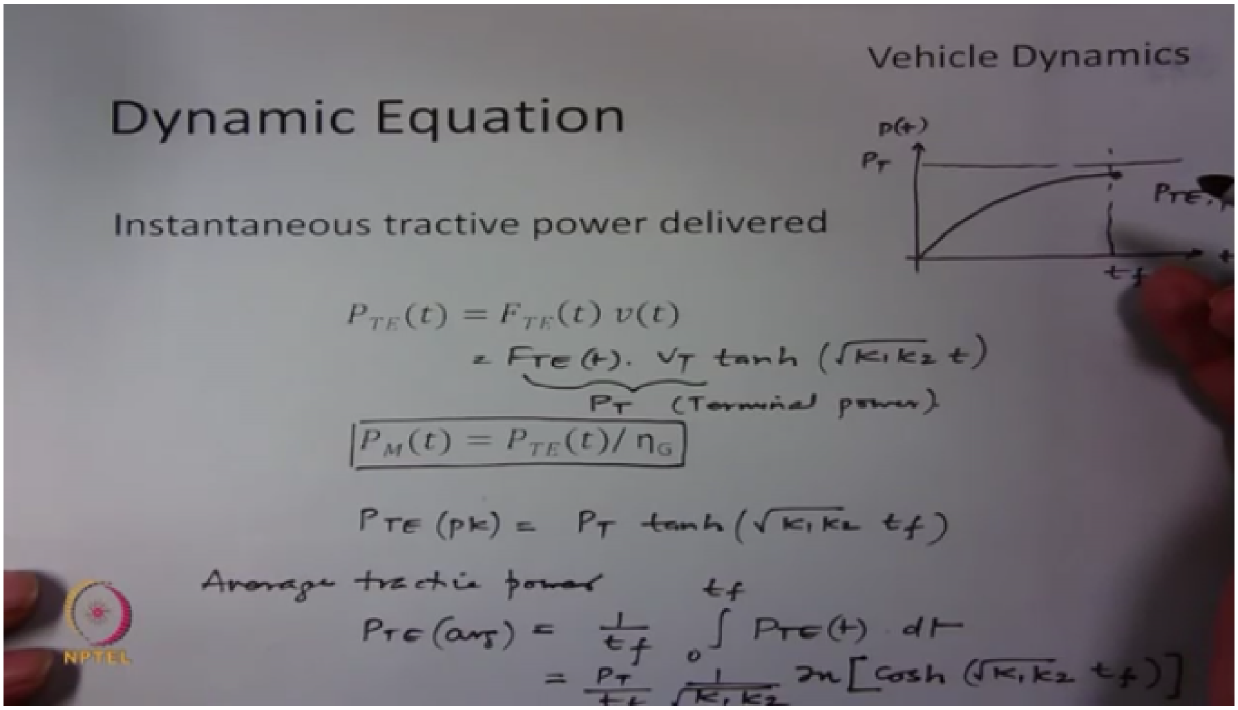

so since we are operating at constant input force of FTE the plot of power will be similar to velocity plot, so it will be also similar to this, so we are achieving you know the velocity VF at time TF, so let us see the expression of power, so P tractive effort is a product of F tractive effort into velocity, so this can be just written, so this is a constant number and VT is also constant number, so this constant numbers can be clubbed and we can say that this is nothing but terminal power, so when the VT is reached the power required is P(T), and the power required by the shaft of the motor is you know this power divided by efficiency of the gear.

So we can also find the power that is required when we are operating at velocity VF during time TF by just substituting TS, TF here, so the instantaneous power equation is very important but for energy calculations we need to find the average power that is required by the vehicle when it's operating from 0 RPM to the required RPM, so it's always a good idea to calculate the average tractive power that is required, so average tractive power will be somewhere here, so it can be evaluated by doing the integration 1/TF, 0/TF and the expression of T respect to time and we'll get this expression.

This is very important expression in terms of energy requirement that we'll see, so in electric vehicle all the power is coming from battery and the integration of the power with respect to time is the energy, so PTE is nothing but D/DT of energy, or energy is a integration of the power required,
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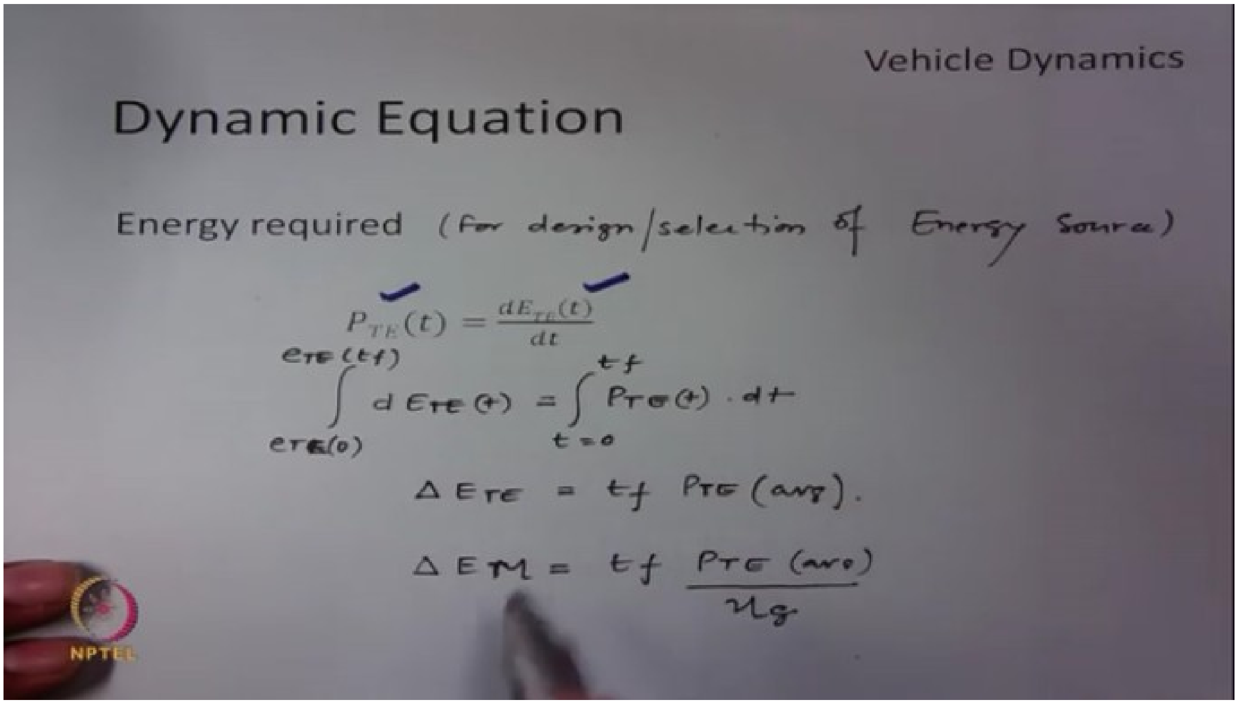

so if we can understand the PTE average we can just multiply by TF to calculate the delta energy required from the batteries, so that's why this expression is important and when we want to understand the energy required to be delivered from the motor shaft we have to just divided by the efficiency of the gas.

So that is all under you know dynamic equations, so here we have covered the derivation of dynamic equations in terms of force, and velocity, expression of distance, power, energy all this for a constant input effort, so we have discussed constant input effort,
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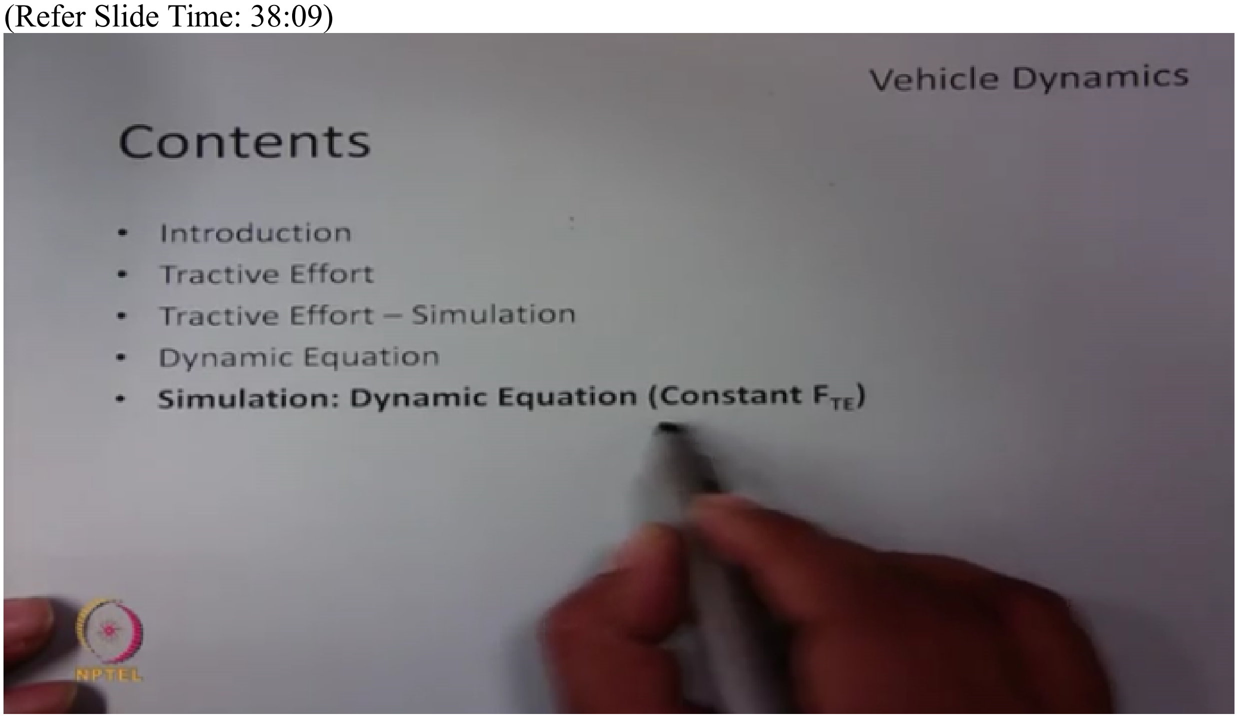

so we are yet to discuss the dynamic equations when the FTE is not constant.

So in our next interaction we will try to simulate this expressions of velocity, distance, power, energy, all this equations for a known system and try to see its effect for different types of vehicle parameters etcetera, so we will do that in our next interaction, so thank you for listening the lecture.

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Produced by
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Services Centre
IIT Delhi