Electrical Machines
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Lecture no 15: Electromechanical Energy Conversion-II

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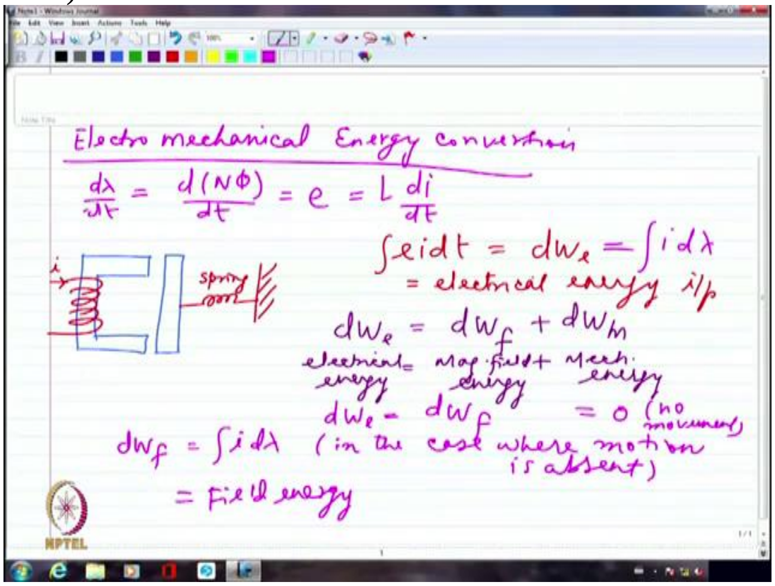

So we had looked that mainly couple of things, one is we said basically $\frac{d\lambda}{dt}$, which is also indirectly $\frac{d\lambda}{dt} = \frac{d(N\phi)}{dt}$. Where ϕ is the flux linkage, is actually e. If we are looking at only the induced EMF, which is there in the inductance. So, I can write the same thing as,

$$\frac{d\lambda}{dt} = \frac{d(N\phi)}{dt} = e = L\frac{di}{dt}$$

If I am assuming that the inductance is a constant. If there is no movement in the electromechanical system, what we considered earlier. So, we considered basically a system somewhat like this.

We had looked at a core which is actually C-shaped. And we also took one more maybe rod kind of structure which will be attracted by this electromagnet, if sufficient current flows through this. So, let us say I have a coil and if I have a sufficient current flowing through this. Then I am going to have the electromagnetic force that is created is going to be quite strong enough. And it will go against whatever is the spring action, I may have here. So, I am showing a spring which is attached to your frame. So, this is spring.

So we said that when there is no movement, the entire electrical energy that I am feeding in which I may call actually as $\int eidt$. I may call this as, *ei* is the power, so if I integrate it over dt or whatever is the time interval that I have. Then I may call this as the electrical energy input.

$$\int eidt = dW_e = \int id\lambda = \text{electrical energy input}$$

So, this is electrical energy input. So, this will essentially go towards increasing the magnetic field energy, if there is no mechanical movement imparted. So, I am going to say normally under general circumstances $dW_e = dW_f + dW_m$. Where this is the mechanical energy, this is the magnetic field energy, and this is the electrical energy.

So, we are looking at electromechanical energy conversion system where electrical to mechanical energy conversion is taking place which is like a motor or an actuator in this particular case. So, if I am going to have no movement this will be equal to zero, if there is no movement. So, I will have $dW_e = dW_f$. And that is what we wrote as rather than writing $\int eidt$, I can write the same thing as $\int id\lambda$. Because dλ/dt, I can call that as e, because of which I can say *edt* is equal to *dλ*. So, I am writing this like this. So, we said

$$dW_f = \int id\lambda \text{ (in the case where motion is absent)}$$
$$= \text{Field energy}$$

So, we call this as the field energy. I am just recalling what we did in the last class.

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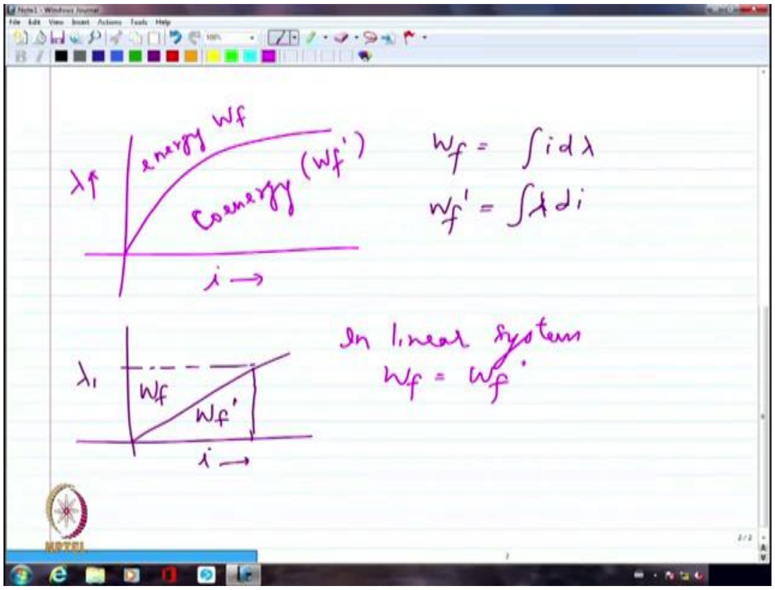

And we said if we are actually plotting a graph between i and λ, we are probably going to plot it like this with non-linear characteristics. If I say that there is an iron core which is contributing to non-linearity between the relationship between i and λ or Nϕ.

So, I am going to call this portion as energy or field energy which is W_f and I am going to call this portion as co-energy or W_f'. So, I may say that $W_f = \int i d\lambda$, for a given position of the moving part. If I do not have any movement it is at a given position. Then I may say at a particular x value $W_f = \int i d\lambda$ and $W_f' = \int \lambda di$. And energy has at least some kind of physical connotation whereas co-energy does not have any clear physical connotation.

So, if I may say that it is the linear system which has linear characteristics between i and λ, I am going to have essentially whatever is this rectangle that represents directly λi. So, in this case this particular line divides this area under the rectangle exactly into two halves. So, I will have energy and co-energy to be equal to each other. So, whenever I consider a linear magnetization characteristics, I am going to have $W_f = W_f'$. So only in linear system this is going to be true. So I can say in linear system I am going to have $W_f = W_f'$.

After this we started considering a small movement that is where we left off, actually in the last class. So, let us again look at system what we had considered. So, this is actually the iron core and I am going to have a current pass through this. So, the current is i.

Student: What do you mean by linear system here?

Professor: The linear system is where there is no saturation that has happened. It is still in the linear portion of the magnetization characteristic.

So, if I look at i versus ϕ or λ versus i. I am going to have only a linear relationship between them. That will be true for lower values of i provided, neglect hysteresis. If you consider hysteresis it is not linear anymore. I hope you understand because hysteresis will always talk about some amount of remanent flux density. So, if there is a remanent flux, the linearity is lost. So, I will not have, when i is zero, the flux will not be zero. If there is a remanent flux. So, I cannot say it is exactly a linear relationship. After that also once the saturation creeps in, as you say if the i value is very large. At that point again it is nonlinear. So we are neglecting of course hysteresis property when we are talking about this.

So, if I am looking at a system somewhat like this. And I have let us say a moving portion like this which is attached to a frame through a spring. And let us say I have a distance of maybe x_1 at this point in time, rather I am calling the air gap length to be x_1. And let us say I am passing sufficiently large enough current through this because of which the magnetic field created is strong enough to pull that moving part towards itself. Because of which may be the distance has reduced now from original value of x_1 to a smaller value which is x_2. So, the movement has taken in such a way taken place in such a way that from x_1 the distance or the air gap has decreased to x_2.

So, if I tried to draw the two magnetization curves corresponding to these two points. This is i and this is let us say λ. So for x_1 if I draw a magnetization characteristic like this, x_2 considering the air gap is lower I should probably create a little bit higher flux or flux density hopefully. Because I am essentially looking at the distance decreasing, the air gap decreasing, the reluctance decreasing because of which I should have the flux increasing slightly.

So, this corresponds to $x = x_1$ whereas this corresponds to $x=x_2$. And let me assume probably that the movement is taking place pretty slowly. How slowly is a matter of detail very clearly. But let us say I have an electrical time constant for this circuit which is L/ R, some L/R. So, if I am allowing the movement to take place slowly the current can reach steady state at every point in time. And considering that resistance of the circuit is a constant, the current can remain as a constant. What I mean is I am giving sufficient time, so that the current almost persists at a constant value, no matter what.

That means the time constant of the circuit is going to be much smaller than the time taken for the movement. That is what I am assuming. So, let me take the case where the movement is slow. Which means the time taken for movement is actually greater than or much higher than the electrical time constant. So that I would not see really much of variation in the current at all. So, I am going to look at one particular current value coming up like this. Maybe I am just taking i equal to constant in this particular case. If it is an alternating current circuit, I have to look at i_{rms} to be a constant ultimately. Obviously in an alternating current circuit you cannot expect the current will be constant. From instant to instant. So, I am looking at maybe the RMS value of the current is a constant.

So, under this situation I will have actually two different values of λ maybe this is of one λ value which is the final λ. Which I may call as λ_2. And let me talk about another lambda value which is actually somewhere here which is actually λ_1. Let me write these points or name these points as O, this is P, this is Q, this is R and this is S. So I am just naming the points as this.

Now what actually I have input to the entire system is the electrical energy input what I have given as electrical energy input is the input to the entire system. So that will be from what we said earlier $\int eidt$. This is the electrical energy input. And after all edt we said is $d\lambda$. So, I have to integrate this as an electrical energy input going into system as λ_1 to λ_2 $id\lambda$. That is all. Please remember the same $id\lambda$ happened to be the field energy provided there was no movement, now there is movement.

So, I am writing the same thing now as the electrical energy input. Which actually I can specify as the area under this particular rectangle. Because I have to write this as actually $\lambda_2 - \lambda_1$ multiplied by whatever is the current or I may say area of the rectangle which is actually PQRS or PQSR whatever. That is the area which is representing the electrical energy input during this particular duration of the movement complete movement that has taken place.

We are essentially writing only the boundary curve. We are not writing in between curves. The curve which is corresponding to OP that represents the initial condition. And whatever is OQ, that represents the final condition. So, the flux linkage has changed from λ_1 to λ_2 while the movement has happened from x_1 to x_2. So, we are looking at the two end points and then we are trying to see how much of mechanical work has been done. So, we are taking stock of the situation after looking at the initial condition and the final condition. Last class we wrote $dW_e = dW_f + dW_m$. We are going to look at how much is the change in field energy, how much is the change in the electrical energy or how much is the electrical energy input, and how much is the mechanical energy contribution. So, we are looking at every single thing in this equation.

So, the electrical energy input we have accounted for as the area across that shaded portion. Whatever is the area of the shaded portion which is corresponding to $\int eidt$. That is electrical energy input. Let us try to look at whether there is any change in the field energy. We said field energy under a given situation is $\int id\lambda$. That is what we said. So, if I try to look at actually the field energy, in the first case I should have actually this as idλ, this entire area. That is what is idλ corresponding to the first condition where I have had the moving part a distance of x_1.

So I should say corresponding to or I can say initial field energy is actually OPR, this is the area. Please note that is OPR, that entire area enclosed by the magnetization characteristics corresponding to x_1 and the lambda axis itself. Whereas if I look at what is the final field energy, that is going to be actually OQS. This is the area which is representing the final field energy.

So, if I want to do get the difference in field energy, in that case I have to minus whatever is the final field energy. So, I can say dW_f will be whatever is the area which is OQS minus OPR. So, these two areas whatever is the difference that is going to be my difference in the field energy. So, I can say from here this is actually area of PQRS, equal to this is area of OQS minus OPR plus dWm which is the mechanical energy.

So, I should be able to write the mechanical energy finally as that is. So, from this equation I should be able to write $dW_m = \text{area PQRS} + \text{area OPR}$. Because this minus when I get it to the other side it will becomes plus. So that is this entire area. Are you getting my point? This entire area is actually area OPQRS + area OPR - area OQS. That is what it means.

So, if I look at the entire area, I am getting this entire area as the total area which is enclosed by the summation of these two, minus whatever is the area that is actually taken up only by this portion. Anyway, I can say basically this area I have to minus until this portion. Which means I am going to get only area OPQ.   So, this is area OPQ.

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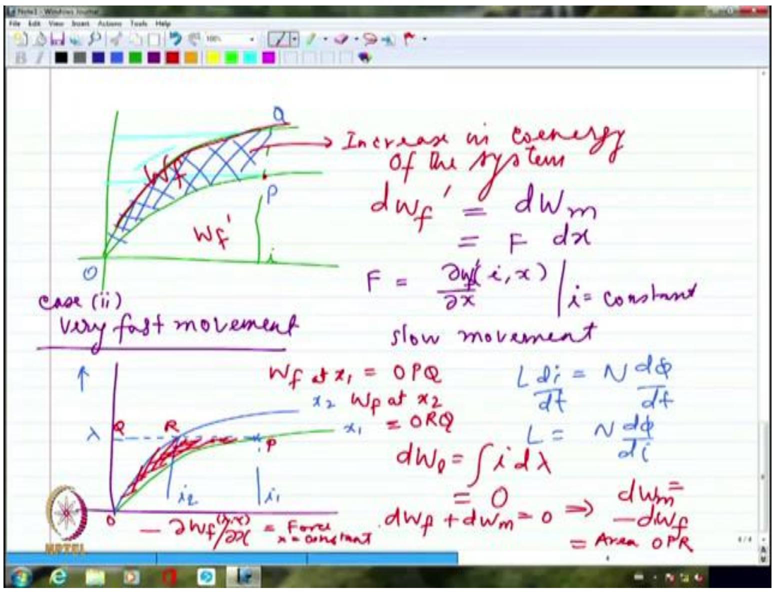

Let me try to probably redraw the figure in the next thing. What we have done is first we showed one curve like this, we showed one more curve like this. And we showed this i to be the same i. And we showed that this particular area that is this rectangular area gives me whatever is the electrical energy and this particular area whatever we have got this area is going to give me whatever is the field energy originally that was present.

So, I have added those two. But the final field energy present is only this area. If I am talking about this as the area. So this particular area that is which is enclosed by the two curves. So, if I may call this as OPQ. This is essentially representing whatever is my actual value of mechanical energy that have been outputted by the system. So, the mechanical energy output that has come out of the system. Then the movement has taken place from x_1 to x_2 corresponds to this area.

Now we will have to check what is really this area. We said basically this portion is W_f, this entire portion is W_f. whereas this is W_f'. That is what we said earlier. So, if I look at what was W_f originally when the curve was in P or the operating point was in P. This was all W_f and this is all

W_f'. Whereas now if I am looking at only this the portion above this curve W_f and below that is W_f'. So, this essentially gives in the increase in the co-energy.

Although co-energy does not have any physical significance, then the system moved from x_1 to x_2. Whatever was the increase that happened in the co energy which is W_f' that is represented by the shaded area. So, the shaded area actually represents increase in co-energy of the system.

So, I can say basically this is dW_f'. The two together actually I am getting the electrical energy input. That is all I am saying. So, my electrical energy input it is going towards two tasks. Rather I should say three tasks. One is resistance loss, definitely there will be resistance loss, whether I like it or not. The second task it is doing is to increase the field energy. Apart from that whatever is left over it is going towards mechanical energy conversion.

Student: For case(a) and case(b), is the electrical energy input same?

Professor: We do not know whether it is the same input. That is a big question mark. Because we considered actually, the lambda is changing from zero to particular lambda value and we integrated the whole thing and we said area under that curve. Between the lambda axis and this was you know your field energy. But now we do not know how much is the electrical input. We are saying ok that is some electrical energy input. We have not quantified anything. We have roughly said from λ_1 to λ_2 the flux linkages have changed.

We do not know whether the quantity is the same in the previous case and this case. In the previous case, we considered there was no movement. We considered only one particular magnetization characteristics. Where as in this particular case we have considered two magnetization characteristics. One at length x_1. And the other one at air gap length of x_2. And we considered x_2 is smaller. That is the reason why we considered that curve above the previous curve.

Assuming that the reluctance is smaller because of which the flux should be hopefully larger. So this essentially tells me, I hope your question is somewhat clarified, if not completely. We are not violating energy conservation principle, very clearly. In the last case we did not consider any movement. So, the mechanical energy was zero. In this case the mechanical energy is a non-zero quantity and we are trying to quantify it. In the process of quantification, we looked at different areas as per our definition. And then we ultimately arrived at the fact that we are getting the difference in the co-energy or increasing in the co-energy is equal to the mechanical work done.

If we are doing it slowly, automatically the current will remain as a constant RMS value. Current is not really controlled by you completely unless you are varying some resistance or something. We are telling right now that the resistance is a constant. The inductance is changing clearly for every change in inductance there will be a transient, no doubt. But we are doing it so slowly that the transients are not so visible. Because the time constant happens to be really really small as compared to whatever is the movement the rate movement.

You are keeping the voltage constant your current remains as a constant because you are allowing it to reach steady state at every point. The reluctance is changing, the inductance is changing. But transient wise it will change, you are allowing at every point the current to reach steady state as simple as that. So that is why we said right in the beginning. If you are considering a slower movement the mechanical time constant generally is much higher than the electrical time constant. You can consider either way. Here even if we consider AC current, we are looking at the RMS value. If you actually considered DC current, you have to again consider the time constant there too.

So I am passing a current let us say it reaches the value of say V/R. At the point you are actually trying to move when you are moving you are going to see a variation in the current instantaneously. But you allow it to settle again and again. I would say it is better to look at an AC current in this case because we are looking at the induced EMF, $d\lambda/dt$ and so on and so forth. AC can reach a steady state. You call AC steady state as the steady state when you consider $R+j\omega L$ as the impedance.

You cannot talk about otherwise ωL. Please understand the phasor diagrams that you draw or for steady state. They are steady state. If you still have doubt, please put this into your head that what we are talking about as impedance as $j\omega L$ or $1/j\omega C$. All these things are in steady state. When you talk about transient that is $Ri+Ldi/dt$. When you write that, that is transient. At steady state the current should not change is your perception. That is not steady state. If the initial value at the beginning of the cycle and the final value at the end of the cycle are the same. You call that as the steady state.

Steady state does not mean it is not a constant I mean it should be a constant. No, you have simple harmonic compilations. Do not you say it is steady state? It is oscillation, of course it is oscillation. But it is steady state please understand. So if I have an LC oscillation I would say reach steady state very clearly, as long as in the beginning of the cycle and end of cycle the values are the same. Is this clear. So sinusoidal excitation if I have there is nothing like a constant value, never ever. RMS values are constant, yes fine. Once it reaches steady state. We are having basically the flux is changing reluctance is also changing. So MMF can remain as a constant. You will look at it that way. Flux is changing it has gone from λ_1 to λ_2, please note that.

So, I hope so that it will clarify matters if you please read through P C Sen or any other machines book again from electromechanical energy conversion principle. So, I would say $dW_f' = dW_m$ in this particular case. Now this I can write as $dW_f' = dW_m = Fdx$. Where $dx = x_2 - x_1$ or $x_1 - x_2$, whatever is the difference. So, from this I should be able to write $F = \frac{\partial W_f(i,x)}{\partial x}|_{i\ =\ \text{constant}}$, I am writing a partial derivative because W_f is actually W_f' rather is a function of both i as well as x. Very clearly I can write W_f' as a function of i as well as x. But I am going to keep during this condition i as a constant. Which means it is slow movement. There is another case very clearly when I say there is slow movement there should be some other case with faster movement.

So maybe I would consider that case as well, very fast movement which is case 2. Again, I will compare this in terms of time constant. The electrical time constant happens to be maybe comparable or even greater than the way the movement is taking place which is kind of very unrealistic. Because mechanical systems are normally very slow whereas the electrical systems time constants are generally much smaller generally. But let us probably take a system where it is a miniaturized machine. Very small miniaturized machine. Maybe the inner sphere and other things will be really small because of which the movement is taking place really fast.

If the movement is taking place really fast, I cannot assume now that I am going to have i as a constant. Now the transient will definitely get into picture. So, I am looking at again let us say two graphs, one is may be like this and another one probably like this. I am just taking two graphs. Again let me say me one of them is at x_1 the other one is at x_2 and let us say I am initially at an operating point somewhere here. And i is not a constant.

So, I am actually going to have rather flux as the constant. Please understand that generally the cause is i and the effect is λ, cause is i and effect is flux. And it takes a little while for the flux to show up that change in the current. If there is a change in the current, you are going to have a little bit of time lag before it shows up as a change in the flux. That is why we call it as hysteresis. Hysteresis means lagging behind the flux change always lags behind the current. So, it is going to take a little while. But I am doing the movement so fast, that the flux linkage is not even given a chance to change. The flux linkage is remaining as a constant, almost. I am not going to have any change in the flux linkage at all.

Let us say I am just arbitrarily taking some operating point. And I am showing that the flux is basically remaining, or flux linkage is remaining as a constant. Whereas the current is very clearly changing from maybe originally it was i_1 and here it is i_2. There are two different points in the current. So, the current has changed. That means the difference in the inductance in one sense is manifested in this particular case because of which in the two cases, the currents happen to be different, they are not the same. You are not allowing the current to reach the steady state. You are doing it so fast that the flux linkage is really not able to follow the you know change in the position of anything it is not able to do anything basically.

Student: When the flux linkage is not changing quickly, is it related to electrical time constant of the circuit?

Professor: The time constant and the lag between the current and the flux, roughly you can say that because the flux is manifestation of magnetism. The inductance what you are actually putting it fictitious quantity that is also manifestation of magnetism. That is why we write generally Ldi/dt as the voltage drop which is also equal to $Nd\phi/dt$. That is why we write

$$L = N\frac{d\phi}{di} = \frac{N\phi}{i}$$

So, inductance actually is a construct of human beings or electrical engineers who want to depict the relationship between magnetism and electricity in the form of a circuit.

We wanted to do KVL, KCL and KVL, KCL you cannot do, unless you represent the magnetic field also as a circuit quantity. So the circuit quantity that you are manifesting by means of actually equating $L = N\frac{d\phi}{di}$.

So, Faraday's law came first probably and then the inductance was introduced that is how it was. So the inductance is basically the representation of the magnetism in circuit parameters. So, we have here this as λ. We already said that $dW_e = \int i d\lambda$. There is no change in λ. So, the electrical energy input is literally zero. Are you getting my point? We are really not getting any electrical energy input. So, I have now two quantities only. One is dW_f the other one is the dW_m that equal to zero. Which means I am going to have $dW_m = -dW_f$. Are we having a reduction in the field energy? Yes, very clearly.

Originally our field energy was OPQ. Now I am going to have maybe ORQ. So, I should say W_f at x_1 is OPQ. Whereas W_f at x_2 is ORQ. So, I am having a reduction in the field energy and the reduction is the field energy is this shaded area. This is what is the reduction in the field energy. Which is OPR. So, this should be actually equal to area OPR in the graph.

So, I can say you know in a nutshell, if the movement is taking place very slowly the increase in the co-energy represents the work done. And if the movement is taking place fast then the reduction taking place in the field energy that represents the work done. Mechanical energy or work done. So, I should be able to write in this case also $\frac{\partial W_f}{\partial x}$, where I am going to have $-\frac{\partial W_f}{\partial x}$ of course. This is the function of λ and x of course. This will be actually the force with lambda as a constant, lambda as a constant. $-\frac{\partial W_f}{\partial x} = \text{Force } (\lambda \text{ as a constant})$

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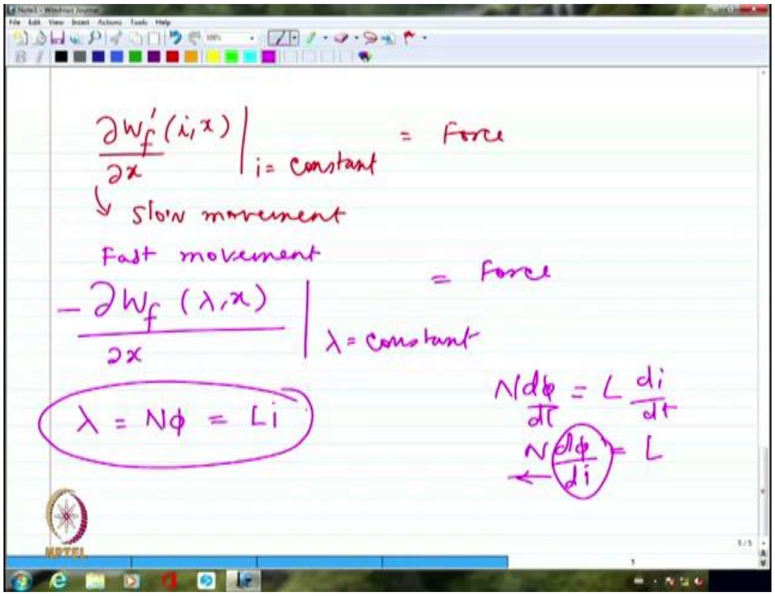

So, let me write these two expressions once again. Let me write that $\frac{\partial W_f'(i,x)}{\partial x} |_{i\ =\ \text{constant}} = \text{Force}$.

This is for slow movement. If I consider fast movement under that condition, I am going to have $-\frac{\partial W_f(\lambda,x)}{\partial x} |_{\lambda\ =\ \text{constant}} = \text{Force}$.

Student: How can the current decrease when lambda is remaining as constant in fast movement case?

Professor: So you are saying basically that in the constant case here lambda is remaining as a constant agreed. Reluctance essentially decreasing. So reluctance multiplied by the flux which is actually also decreasing. So, the current is decreasing as well, is not it? The current is decreasing originally i_1 is the higher value i_2 is the smaller value. It is very much fine. Why should it increase? Lambda multiplied by the reluctance. Reluctance has decreased lambda has remained the same. So, lambda multiplied by reluctance is MMF. MMF has decreased. MMF has decreased. In the first case MMF has remained probably the same whatever. But here MMF very clearly have decreased.

So, we have basically seen that we are having an expression for the force that is developed in a translational system. What we have to study is about rotational system. Because all our machines are rotating machines. We look at it eventually. But before that whatever we have specified as λ, $\lambda = N\phi = Li$, provided we consider L to be a constant.

If we consider L to be a constant probably. It is not going to have variable permeability even at a particular point. Then I can directly equate $\lambda = N\phi = Li$. Because we wrote like this, $N\frac{d\phi}{dt} = L\frac{di}{dt}$. From which we wrote $N\frac{d\phi}{di} = L$. This $\frac{d\phi}{di}$ we are considering this way only because we are assuming that, that ϕ and i do not have a linear variation with each other. If I assume, they are having an initial I mean they are having continuously linear variation. Rather than writing $\frac{d\phi}{di}$, I should be able to directly write that as Nϕ/i, directly.

So, if I am considering a linear system I should probably be holding onto this expression. Rather than writing this in terms of calculus. So now we will look at the linear system, where we are going to look at all the quantities in terms of linearized magnetization characteristics where we are going to assume $N\phi = Li$ or $\lambda = Li$. Thank you.