Electrical Machines

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Lecture-16

Electromechanical Energy Conservation -III

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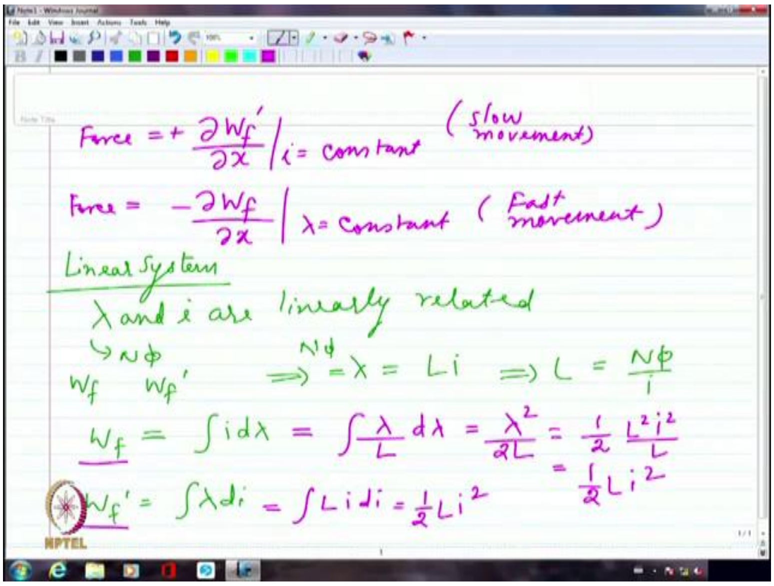

We were talking about electro mechanical energy conversion under two different situations when the movement is really fast and when the movement is really slow. So, in the two cases we derived the expression for force as in one case it was $\frac{\partial W_f}{\partial x}$, actually reduction in the field energy with I would say in this particular case the movement was actually slow. So, because of which we said i is constant.

So, we call this as the case corresponding to slow movement. This was the fast. In the first case we were talking about the field energy in the slow case it was the co-energy. So, in the other case which was corresponding to the reduction in the field energy because of which we said $-W_f$ and of course we said λ is constant. And we said this is fast movement. And then we just started talking about what is a linear system. And that is where we had stopped the class in the last session.

So, if you are looking at actually a linear system what actually we are trying to imply by saying it is a linear system is, λ and i are linearly related after all λ we said as $\lambda = N\phi$ and which means number of turns anyway is not changing. So, the flux and current are linearly related. So, if I tried to plot magnetization characteristics, I am trying to say that they are mainly operating in the linear region of operation.

So, we are going to have lambda and i linearly related. So when they are linearly related we said already that W_f and W_f' are one and the same. Because, we said that essentially the total area will correspond to λi which will be actually the rectangle and then you are going to have the diagonal exactly intersecting the area into exact two halves because of which W_f and W_f' will turn out to be one and the same in terms of the magnitude.

So, we are actually looking to reproduce a linear system. We are essentially saying that $\lambda = Li$ already we said $\lambda = N\phi$. There is no change in that but we are also writing $N\phi = Li$ so because of which we are going to take actually $L = N\phi / i$ rather than taking $Nd\phi / i$. We are taking this as $N\phi / i$. That is what we are taking as the inductance. So, if I am talking about a W_f we said W_f under static condition when there is no movement is $W_f = \int i d\lambda$, $W_f' = \int \lambda di$.

$$W_f = \int i d\lambda = \int \frac{\lambda}{L} d\lambda = \frac{\lambda^2}{2L} = \frac{1}{2}\frac{L^2 i^2}{L} = \frac{1}{2} L i^2$$

$$W_f' = \int \lambda di = \int L i di == \frac{1}{2} L i^2$$

We are just actually iterating them that is all we are trying to do so whether I Look at W_f or whether I look at W_f ' both of them happen to be the same.

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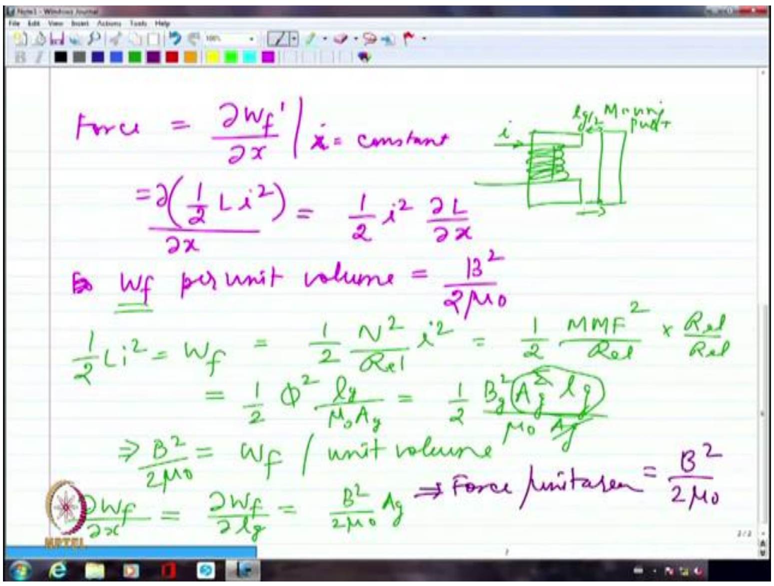

So, if I try to actually look at what is the force, force we said

$$\text{Force} = \frac{\partial W_f'}{\partial x}\Big|_{i\ =\ \text{constant}} = \frac{\partial(\frac{1}{2}Li^2)}{\partial x} = \frac{1}{2}i^2\frac{\partial L}{\partial x}$$

So, we are taking a partial derivative basically of the inductance in that inductance might vary with respect to current as well, because if I reach saturation the inductance value is going to decrease because the permeability is not a constant but given *i* as a constant we are trying to look at how the inductance varies one with respect to the position alone and this gives me what is the force so if you recall what we had done earlier.

As for W_f per unit volume,

$$W_f \text{ per unit volume } = \frac{B^2}{2\mu_0}$$

We are basically talking about a similar system what we had conceived earlier that is we are actually having basically a system maybe a translational motion system we are still not we have not migrated to rotational motion this is translational movement system and we are talking about this as the moving part. And this as the stationary part from where I am getting the excitation. This is the current that I am forcing. At this point we are basically talking about if I am allowing the movement, at that point. And that to allowing a slow motion in that case we are basically talking about how the force is evaluating the force involved how it is evaluated? For a given value of current.

Student: Is the inductance function continuous or discontinuous?

Professor: See, actually if you look at the evaluation of L it will be a continuous function because you are not seeing an abrupt movement. It is a slow motion. It is probably starting off from x_1 and then it is moving to abortion until x_2 so you are looking at air gap gradually decreasing. So, you will have a continuous variation in the inductive because you are going to see a continuous variation in the reluctance. That is what we are looking.

So, when we are actually looking at the linear motion for that we are saying that how the inductances varying with respect to the position. That is what we are looking at, right? so, if we actually looked at the original value of W_f whatever we derived before some time to we can drive it again $\frac{1}{2}Li^2 = W_f$, that is what we said as the W_f value. This I should be able to write this as

$$\frac{1}{2}Li^2 = W_f$$
$$= \frac{1}{2}\frac{N^2}{\mathrm{Re}l}i^2 = \frac{1}{2}\frac{MMF^2}{\mathrm{Re}l}\times\frac{\mathrm{Re}l}{\mathrm{Re}l}$$
$$= \frac{1}{2}\phi^2\frac{\mathrm{l_g}}{\mu_0 A_g} = \frac{1}{2}\frac{B_g{}^2 A_g{}^2\,\mathrm{l_g}}{\mu_0 A_g} \Rightarrow \frac{B_g{}^2}{2\mu_0} = W_f \text{ per unit volume}$$

I am writing reluctance as Rel it so that I do not confuse it with the resistance. So, if I am going to talk about mainly the air gap stored energy because μ_0 is much much smaller than $\mu_0\mu_r$. I am basically considering this. So, this area is actually area of air gap and l_g is going to be the length of the air gap. $A_g l_g$, is actually the volume so this is what we said as $\frac{B_g{}^2}{2\mu_0}$, is the field energy per unit volume.

This is what we said earlier also just reiterating that, that is all. So, if this is the field energy per unit volume as we just express it. If I want the full field energy I have to multiply that by $A_g l_g$, that's all.

Student: Why is the stored energy in the core being neglected?

Professor: Because the air gap stored energy happens to be much much higher as compared to what is being stored in the core, because the core permeability is quite high as compared to what is going to happen in terms of the air gap. So, most of the energy that is going towards magnetising the entire structure goes into actually magnetizing the air gap because the air gap is difficult to magnetize you are going to have higher and higher reluctance. So, that is the reason why we are saying we are mainly looking at the air gap.

So, this actually will be the W_f value per unit volume. If I try to now look at what is

$\frac{\partial W_f}{\partial x} = \frac{\partial W_f'}{\partial \mathrm{l_g}} = \frac{B^2}{2\mu_0}A_g$, because W_f and W_f' are the same that is what we said. So, if I try to look at this, I should do this actually with respect to the distance. So, that is essentially the length of the air gap itself. So, I should be actually doing this with respect to $\partial \mathrm{l_g}$ because that is what is the length of the complete magnetic path. So, maybe if I say this is l_g or I should call this as $l_g/2$ and another $l_g/2$ here so that when they add up it becomes l_g that is all. Basically, I am looking at what is the length of the air gap. That is, it.

So, this is going to be again If I say $\frac{B^2}{2\mu_0} A_g l_g$, l_g will just be eliminating when it differentiated that is all. So, this is going to be the force that is produced in the air gap whose flux density is B. And whose area of cross section is A_g. So, in which case I can see a force per unit area will be $\frac{B^2}{2\mu_0}$. Because area is A_g. So, if I tried to calculate what is the force per unit area, we said first energy stored per unit volume in the form of field energy that is $\frac{B^2}{2\mu_0}$. And if I tried to calculate what is the force per unit area as visualized by the areas which are exposed to the air gap. I will again get that to be actually $\frac{B^2}{2\mu_0}$.

So we are just making some jugglery so that we are able to arrive at some expressions which are more comprehensible in terms of flux density because finally you may be given a material whose maximum flux density that you can arrive at is so and so in which case you would be able to calculate what is probably the maximum force it can develop. Provided I pass sufficient current to drive it to maximum flux density criteria. That is, it. So, as long as I actually operate with the limitation on the flux density, I also have a limitation in terms of what is the maximum force I will be able to develop for a given material. Per unit area of course if I increase the area I am going to get definitely more force.

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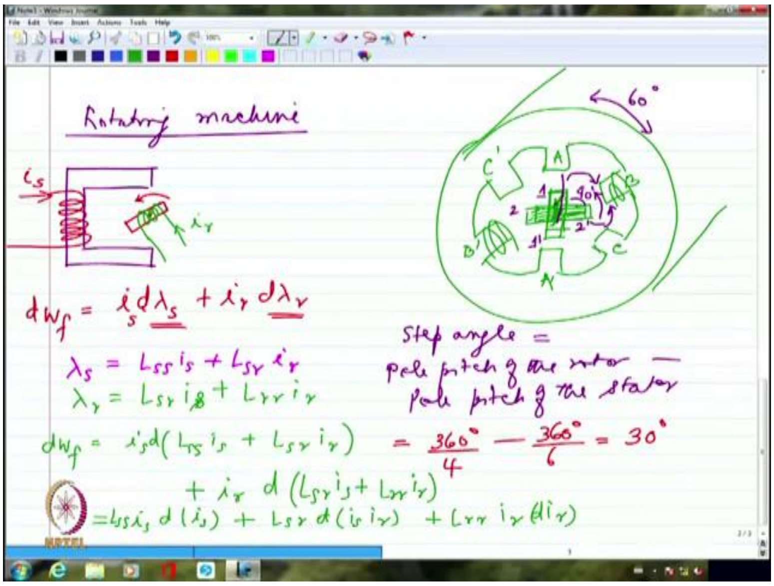

So, let us now having derived the force expression, all these whatever we talked about was with reference to only a machine which is translational in nature which is like an actuator, That is it. Nothing better than that, maybe a relay maybe an actuator, so we would like to derive a similar thing for a rotating machine because we are going to deal with rotating machines basically. So, let us say I am looking at a rotating machine.

So, in a rotating machine as I told you we will have very clearly two members. One will be a stator and the other one will be a rotor. So, let me probably take this as the stator again I am just taking a C shaped magnet. So, let us say this is my stator and I am going to have a coil wound around this data structure which will try to magnetize it. So, this is the current. Which I may call that as i_s because it is the stator of current. Now I may have a rotor probably which I mount it here in such a way that it will rotate probably around this. So, I am just having the rotor structure it is going to rotate continuously maybe in anticlockwise direction or clockwise direction. It is riveted in the centre and it is free to rotate probably around that centre.

So, I am looking at the rotor and the rotor might also carry a current. It is not singly excited. It is doubly excited. It is excited from both stator as well as rotor. So, maybe I am going to have one more current passed here which is actually the rotor current and both of them are definitely going to interact with each other ultimately to produce the rotational motion. So that means torque will be in all probability a function of both stator current and rotor current.

The question was, why do you say they are might be current in the rotor? There is always a current in the rotor it is not true. For example, I will give you an example of a stepper motor. In a stepper motor I will just show you crude structure of a stepper motor. You may have something like eight poles or six poles sitting in a stepper motor. I am just showing these are the poles. So, this gives you at least the practical example of a machine where I have only one side excitation, so this is the stator. Imagine it is cylindrical machine so I am showing essentially this expanding into the paper. Now I may call this as A A’ B B’ and C C’ so I have three pairs of poles. Now let us say I have the rotor in the form of a + actually it will not have so much of air gap.

I am essentially accentuating the air gap so you can imagine it as though it is a little longer than what I have shown. So, this is the entire rotor. The rotor does not have any winding, in many of the stepper motors you will not have a winding as a rule in the rotor of a stepper motor. It may be made up of a permanent magnet sometimes but generally it is need not have any winding or even a magnet.

So, what I may do is let us say I start off with exciting B and B’, I will have windings around each of the stator poles. So if I excite B and B’ the natural tendency of any magnetic system will be to minimize the reluctance. So, either this has to come here or this has to go here. So, if I may name this as 1 1’ and 2 2’ either 2 2’ have to align themselves with V or 1 1’ have to align themselves all the way it has to come towards B and align itself. I will not excite all the three pairs of poles simultaneously. Never ever I will only excite B B’ then I may excite C C’ then I may excite A A’.

I will do it sequentially so any system would like to do minimum work that is true for every mechanism so obviously it is going to make sure that this to move here will be much simpler rather than this all the way to move towards the B B’ because this will be only about 30 degrees or something whereas that will be 60 degrees movement. Please realize that this is right now in a vertical position and it has to move by 60 degrees whereas this has to move only by 30 degrees so in all probability 2 2’ will move by 30 degrees in anticlockwise direction and align themselves with whatever is the pole that is being excited which is B B’ now. No winding is there in the rotor, but this will work only if I have the rotor poles to be attracted because of them being made up of Ferro magnetic material.

The rotor has to be made up of a Ferro magnetic material I cannot have it out any other thing, it has to be attracted because of the electromagnet that has been created in the stator because of the excitation that we have given the excitation given is now to B and B’ after giving the excitation to B and B’ and once the poles are aligned I can remove the excitation from B and B’ and give it to C and C’ then again whatever is the closest to pole that will try to align itself with C and C’.

So, for every pulse what I am giving, I am calling this as a pulse A and A’. I am giving one pulse then B and B’ I am giving another pulse and C and C’ I am giving the next pulse all of them are definitely not excited simultaneously. So, as I give the pulses one after another, I will see the alignment of rotor poles whichever is the closest to the excited phase or excited poles in the stator. So, generally is this particular case we call the moment as the step angle because for every excitation it will move by a particular angle called step angle. So, in this case we saw it as about 30 degrees because 90 degrees actually is the angle between this pole and this pole, rotor pole is displaced from each other by 90 degrees whereas the stator pole are displaced from each other by 60 degrees I have not drawn very well otherwise you should see the 60 degree clear.

So, normally the movement is whatever is the pole pitch of the rotor minus the pole pitch of the stator. Pole pitch of the rotor is 360 degrees divided by four because there are four poles and this is 360 degree divided by six. So, I am going to get 90-60=30 degrees. So, for every step I will get basically 30-degree motion. So, if I am going with A B C and then A B C I will get anti clockwise movement if I am giving rather C B A, I will get clockwise moment please analyse it and check it out you will get it that B if I tried to do it at C B A I will get clockwise movement.

And if I get A give ABC for naming what I have done for ABC I will get anti clockwise movement so there are very much machines available which have only Ferro magnetic materials in their rotor no winding what so ever and they are really rugged you cannot really spoil them very easily because there is no question of a winding being thrown out there is no question of any winding actually shaking and coming out or whether it is getting short-circuited all those things are completely eliminated because of which the structure is pretty much rugged.

They will not get spoiled so easy these kinds of machines but stepper motors are available only for smaller ratings not for very large rating maybe until one kilo watt that is the normal rating we see where as you look at synchronous machines, induction machines can easily go up to a couple of megawatts, synchronous machines can go until thousands of megawatts so you cannot compare the capacities in terms of what stepper motor can offer vis-a-vis what synchronous machine can offer. So, we do have let us say we are going to talk mainly about doubly excited machine in this course mostly that is the reason why I took an example with two excitations from stator site as well as rotor site and under a static condition I should be able to say $\int i d\lambda$ that is what we call as the field energy.

So, because of which now I have to consider the field energy corresponding to the stator as well as rotor, so I am going to say $i_s d\lambda + i_r d\lambda = \text{Total amount of field energy}$. That is the total amount of field energy that I will get or I should say this is

$$dW_f = i_s d\lambda_s + i_r d\lambda_r = \text{Total amount of field energy}$$

In which case I have to know first of all what is λ_s and what is λ_r and λs is essentially telling me that $d\lambda_s/dt$ will be induced EMF in the Stator. Similarly, $d\lambda_r/dt$ will be induced EMF in the rotor. So, will there be an induction in the stator if I have a current flowing in the rotor. If the two coils are mutually coupled. If one flux is going to influence the other one definitely, they will be influencing each other in terms of what is the amount of induced EMF.

We cannot help but have the influence because you are having a ferromagnetic core and rotor winding is very much in the vicinity of ferromagnetic core on which the stator is wound and vice versa. So, I can write basically that

$$\lambda_s = L_{ss} i_s + L_{sr} i_r$$

This is what is my stator side flux linkage. Where L_{ss} is its own self-inductance, L_{sr} is the mutual inductance between the stator and the rotor. As far as the rotor side flux linkage is concerned I am going to have

$$\lambda_r = L_{sr} i_s + L_{rr} i_r$$

So, both these things are going to be actually corresponding to the stator and the rotor flux linkages respectively and we are assuming a linear system that is why we are able to put it in a way that it is L times i. That is what we are writing all along. So I should be able to write now

$$\begin{aligned} dW_f &= i_s d(L_{ss} i_s + L_{sr} i_r) + i_r d(L_{sr} i_s + L_{rr} i_r) \\ &= L_{ss} i_s d(i_s) + L_{sr} d(i_s i_r) + L_{rr} i_r (di_r) \end{aligned}$$

So, I am having three terms here one completely corresponding to the self-inductance of the stator, one completely corresponding to the self-inductance of the rotor which involves only the rotor current and the third one which is corresponding to the stator and rotor currents with mutual inductance between them. So, let us try to look at again W_f what we wrote was dW_f now what I want is W_f. So, this entire thing integrated,

$$W_f = \frac{1}{2} L_{ss} i_s^2 + \frac{1}{2} L_{rr} i_r^2 + L_{sr} i_s i_r$$

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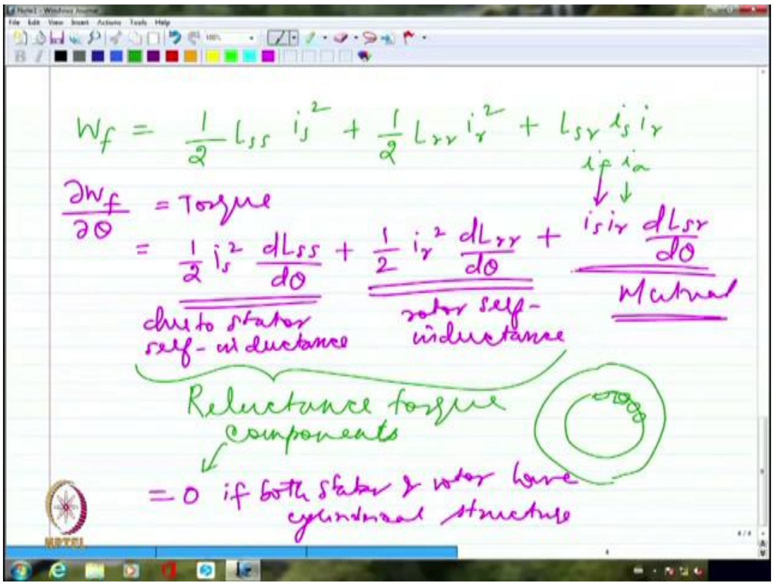

So, I should be able to say clearly the first term will be $\frac{1}{2}L_{ss}i_s^2$. Which is the stored energy because of the stator winding carrying a current of i_s the second one will be corresponding to the rotor which looks similar. So, this is the rotor self-inductance multiplied by the rotor current itself. And the third one which does not have a half there. We are going to have essentially whatever is the mutual inductance between the stator and rotor multiplied by is and i_r.

L_{sr} at this instant we are looking at first of all static condition we are looking at static condition currently what we are looking at this otherwise I could not have equated $\int id\lambda$ directly to the field energy how can I, $\int id\lambda$ is actually the electrical energy input all along we said we can equate that to the field energy only if the entire system is static. That is why I can take L_{sr} as a constant. Otherwise it is definitely a function of feature very clearly when they are in alignment I will have the maximum inductance when then they are not in alignment I am going to have at 90 degrees I am going to have minimum inductance.

So, we will have the definitely variations no doubt. So, this is going to be my W_f now if I try to look at $\frac{\partial W_f}{\partial \theta}$ because I have to look at the whole thing with respect to theta now. This will give me rather than force I should call this as now torque. Because we are just migrating to the rotational system directly. So, in which case I should have this to be

$$\frac{\partial W_f}{\partial \theta} = \text{Torque}$$
$$= \underbrace{\underbrace{\frac{1}{2}i_s^2\frac{dL_{ss}}{d\theta}}_{\substack{\text{due to stator}\\\text{self-inductance}}} + \underbrace{\frac{1}{2}i_r^2\frac{dL_{rr}}{d\theta}}_{\text{rotor self-inductance}}}_{\text{Reluctance torque components}} + \underbrace{i_s i_r\frac{dL_{sr}}{d\theta}}_{\text{mutual}}$$

Because we are talking about this when the current is a constant because W_f and W_f' are the same. So, we are looking at when we are deriving torque we assuming current is a constant. So, these are essentially the three portions of an electrical machines torque and these two are due to the self-inductance variation of the stator and the rotor respectively. The self-inductance will vary or not. It depends upon the structure of the machine. For example if we have looked at this stepper motor very clearly if I am trying to look at the inductance of this coil.

Let us say the coil that is wound around B and B' or A and A' or C and C'. Then I am going to have a complete alignment in the inductance will be maximum even for the coils which are wound around the state poles when the rotor is aligned completely with the stator. I am going to have maximum inductance because minimum reluctance will be there. The air gap path basically will not be followed as much as possible if I try to look at the magnetic field lines under aligned condition it is going to go like this.

This is all the magnetic field lines will be. So please understand that the coil inductance depends upon how much is the air gap that is visualized for a magnetic circuits or magnetic field line. So, the magnetic field line actually will pass through really minimum amount of air gap when the stator and rotor poles are aligned so you are going to have a variation in the stator coil inductance even though the rotor is really not having any coil. In this case the self-inductance variation is the one which is actually creating the torque because the rotor current is not even there.

So, if I trying to look at the expression now what we got this is due to stator self-inductance variation and this is due to rotor self-inductance variation and this is due to mutual. How can you assume L_{sr} to be a constant. So, L_{sr} definitely is not a constant is very clear from this that $\frac{dL_{sr}}{d\theta}$ is the one that which is really contributing towards the part because of the interaction between the stator and rotor currents. If both of them are carrying current I will definitely have interaction between the stator and rotor currents and their mutual coupling also. That is what is producing interactive torque finally between stator and rotor and that happens here. So, this is due to the mutual inductance that is coming up between the stator and the rotor and the interaction between the stator and rotor currents. So, if I may say that these two are due to variation in the reluctance the self-inductance variation is essentially due to the variation in the reluctant path.

So, these two are generally known as reluctance torque components. So, these are reluctance torque components. So, the reluctance torque components will be visualized only if physically the reluctance itself is changing for some reason or the other. If the reluctance is changing for some reason or the other maybe because of the movement of the rotor which is having protruding structure, you guys understand it is not a smooth structure. It is having protrusions in the form of poles, but I may have machines there my stator is also cylindrical my rotor is another concentric circles filling up two cylinders inserted inside one another.

Both will have a pretty smooth structure. If I just look at them superficially so I may have definitely some slots and so on and so forth where the windings are inserted but by and large if I look at the air gap the air gap is completely uniform throughout the circumference. In which case there is not going to be much variation in the self-inductance at all. Whether I look at the stator or whether I look at the rotor for any position I am going to have the air gap to be uniform, so it is not going to differentiate between position A and position B.

There is no question of any variation in the air gap and hence in the reluctance. So, I will not get any reluctance torque. So, this particular component will be equal to zero give both stator and rotor have cylindrical structure. So, if both of them have cylindrical structure. The air gap is uniform. Because the air gap is uniform. I will not see normally any torque produce because of the reluctance variation at all. Because there is no reluctance variation absolutely.

Whereas this generally will always be there in Most of the machines provided I excite both stator and rotor of course stepper motor is an exception because I am not really exciting the rotor, so I have only the stator side excitation so obviously what I have there is only the stator side reluctance torque. I will not have the torque due to rotor side current because rotor side current zero and I will not have $i_s i_r \frac{dL_{sr}}{d\theta}$ because I am not having i_r at all. So, whenever a machine is doubly excited, I am going to have for sure the mutual inductance prone torque. Whenever I am going to have a cylindrical machine, I will definitely not have reluctance torque, and whenever I have a singly excited machine, I will have only reluctance torque due to the member which is being excited. For example, if I look at the DC machine generally it is a cylindrical structure the stator as well as the rotor will have fairly a cylindrical structure so in which case, I am not going to get these two. The reluctant torque whatever I am talking about they will not be present. So, only thing that may be present is this. So, we call one of the members which carries the current is as the field system. So, one of them happens to be the field current. The other one seems to be the armature current. So, I am going to have $\text{Torque} \propto i_f i_a$. Now I have to look at how exactly L_{sr} varies with respect to theta.

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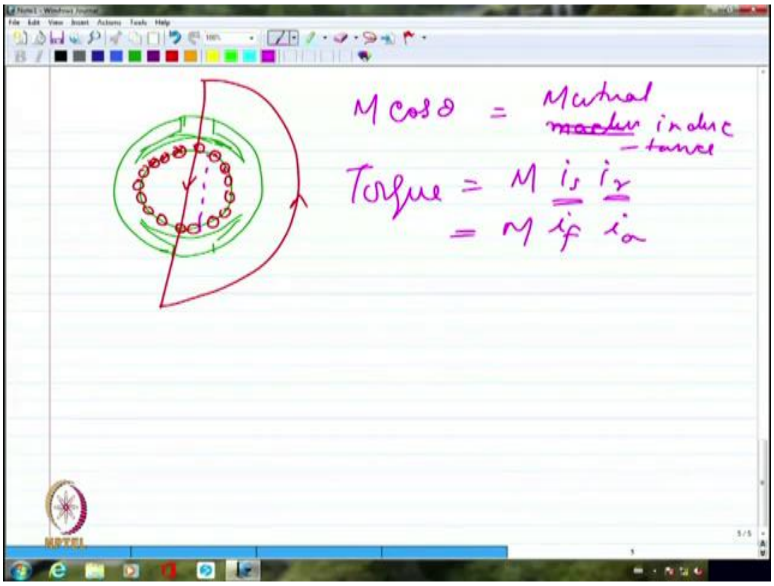

We are slowly migrating into the DC machine that is what we are doing. So, if i have let us say. This is going to be probably my stator poles of course there will be a small proportion. Normally I may have a north pole here. I may have a south pole here. This will try to encompass as much area as possible. That is how it could be. So, I am going to have stator and rotor probably. This is stator and I am going to have the rotor winding which is actually in the outer periphery of the rotor like this, of course they will be in the slot.

So, if I am saying that I am having basically the magnetic field line going like this and then completing itself through the Yoke Region. Yoke I have not shown, the poles cannot be attached to nothing. They have to be attached to the Yoke see here. So, there will be a Yoke Region. So, through the Yoke the magnetic field line return. So, if i try to look at really what is the kind of maybe all these things are carrying dots and all these things are carrying cross of current. Let us see, arbitrarily I am just taking, because of which I will have probably this magnetic field corresponding to the stator is exactly passing through the rotor and then completing itself through the yoke. If I look at this, I have to say normally I am going to have this as if it is dots and this is cross, I would have had the correct magnetic field line going up. Now maybe I will have the magnetic field line going down in the case of Rotor as well maybe. So, if I am having the magnetic field line aligned exactly with that of the stator winding. I am going to say theta probably is equal to zero but at any point in time theta is going to be continuously change.

So, depending upon whether the stator windings and the rotor windings are aligned with each other I am going to have either θ=0 or θ= 90 or they are aligned, misaligned exactly opposite I will say θ= 180 and so on and so forth. So, if I look at the magnetic rather mutual inductance it will be a function of cos theta when they are aligned, I will have maximum mutual inductance then they are completely unaligned then I am going to have minimal mutual inductance. So, I should have basically the mutual induction as a function of cosθ. This is how it will be in generally in any electrical machine. So, this is going to be the mutual inductance function.

So, when I say $dL_{sr}/d\theta$ this will become sinθ basically because cosine when I actually differentiate, I am going to get a sine forget about the sign -sinθ or something just this is going to be a sine function basically. So I know that $dL_{sr}/d\theta$ is going to be sine function ultimately what I get back if I ensure that between these two that is whatever is the actual displacement between the stator and rotor windings which is actually theta if I tried to make sure that they are always aligned somehow or the other, or they are always at some 90 degrees I can do this by modifying the structure in some way or the other. So, what we do in a DC machine is to make them exactly at 90 degrees to each other. So, $dL_{sr}/d\theta$ which is actually Msinθ. Theta is always made as 90 degrees by inserting the commutator. That is what we do in a machine. The commutator is essentially a DC to AC or AC to DC convertor.

So, what we do in a DC machine is to make sure that the maximum contribution comes from the mutual inductance between the stator and rotor and that essentially contribute to the torque being simply M times. You know i_s and i_r and what is i_s that is actually the field current, what is i_r that is armature current, so the torque come out to be

$$\begin{aligned}\text{Torque} &= M\, i_s\, i_r \\ &= M\, i_f\, i_a\end{aligned}$$

So, the simpler scalar expression i_f is also DC, i_a is also DC. So, I am able to really look at whole thing simple scaler equation so that is the major advantage of DC machines whereas in AC machines you are always going to have a phase angle for the current with respect to the voltage. And you do not even know whether the rotor current and stator current will have same phase angle. There is no guaranty. So, I just really do not know how the really fluxes are explained.