Electrical Machines
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Lecture 18
DC Machines – EMF and Torque Equations & Generator Operation

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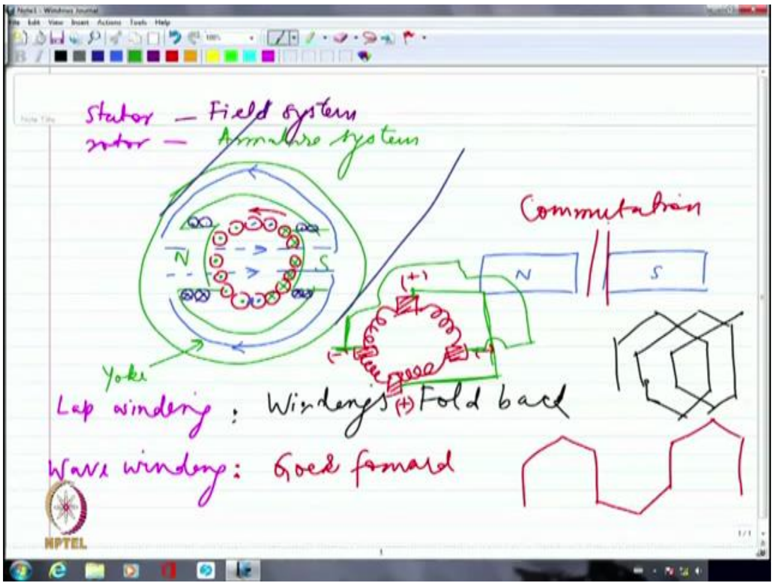

Okay, so we are back on DC machines once again, and we had looked at in the last class the basic constructional features of a DC machine, we started off in fact with Fleming's left hand and right hand rule we said right hand rule will be applicable for generator, left hand rule will be applicable for motor, and then we started off with the constructional features of a DC machines, we said that the stator in a DC machine accommodates field system and the rotor accommodates the armature system this is not really true for all the machines, this is a unique feature of a DC machine.

Because you are going to have basically if you look at the field system in a synchronous machine that is actually accommodated inside the rotor whereas the armature actually is accommodated inside the stator because of some various, I would say a potential constraints that are their normally in a synchronous machine which we will see when we are discussing synchronous machine that will be the last machine that we will be discussing at the end of almost towards the fag end of the course.

So, in DC machine we said that the stator basically is going to be having the field system and that is essentially shown in the form of its own cross section somewhat like this, so this is going to be north pole and this is going to be south pole this is the yoke region with the field lines going somewhat like this and then it is going to complete the path like this, this is how the magnetic field lines are, and we said that if it is actually an electromagnet we are going to have the conductors placed on the top of this field and imagine that this is actually going into the board like this.

So, we are having the conductors also going inside into the machine and wound around the pole. Now this has to be in all probability dot and this have to be cross so that I have the magnetic field lines as shown in the figure, only then it is going to make them as north pole and south pole we also looked at the winding arrangement initially I showed you only two actually two sides of one particular coil and I said that one of them will be dot, one of them will be cross depending upon whether the rotor is being rotated in clockwise direction or anticlockwise direction but, definitely both of them will be of opposite polarity.

If one is dot the other one will be cross and we said the winding could be arranged in two different fashion which we called as lap and wave so, I am showing multiple number of the conductors that are placed on the outer periphery of the rotor, so if I may say may be the direction of rotation is anticlockwise or something, then I am going to have may be some of them having dot I am just showing these things having dots in which case the other side will be having cross, and approximately both sides should have equal number of windings the one which are not really in the vicinity of any of the pole, like I would say this one and this one similarly, may be a couple of them, here in the inter polar region. Inter polar region meaning in between the two poles they will not have much of currents really because they are actually in the cusp of the two poles so I have the border line of north pole and the border line of south pole then I have the border line of north pole and border line of south pole, there is hardly going to be any current carried by them.

In fact, they are defecting from dot to cross or cross to dot, because of which you are going to have very very minimal current, some of you pointed out in the last class when we were drawing the winding diagram that if we had taken say north pole here and south pole here, the conductors that were somewhere in the cusp of south pole or north pole one of them was dot one of them was cross may be something will carry current in upward direction the other one will carry current in the downward direction, they are right adjacent into each other, how can it be? that question was arising for some of this students.

Please understand that in the cusp, they are hardly going to have much of current because you are looking at the current completely changing its direction from dot to cross and cross to dot. So even if they are of kind of opposite polarity it should not matter so much because they are anyway on the verge of changing this process of changing the current from positive to negative, or negative to positive we call that as commutation which we will discuss in detail at the later class not today.

Because, first of all we not to understand the EMF equations and torque equation for a DC machine when we will classify the DC machine after classification we will be looking at exactly how each of them work, and when we are looking at each of them working in detail we will also look at commutation so, just to reiterate what we said in the last class with respect to lap winding and wave winding, we said lap winding actually going to have the windings fold back, that is they are essentially looked upon as I am going to have a winding going around like this, and it folds back.

So it is going to be lapping or overlapping with each other so we call this as lap winding so, it is essentially folding back where as wave winding goes forward continuously, it does not fold back, it does not fold back at all, so I am going to have essentially the winding going on and on and on, until is covers the entire circumference of the cylinder and then it will fold back only after that.

So it is going to go further and further so because of which if I actually look at the structure, I am going to have the lap winding looking as though if I have three turns, three turns are going to come like this, one brush will come up here, three more turns I will have one more brush coming up here, three more turns and I will have one more brush coming up here, and three more turns between these two brushes. This is how it is going to be.

So, I am essentially looking at, if it is a four pole machine, for example this is going to be plus which is affiliated to north pole, this will also be plus, this will be minus, and this will be minus as well. And just to make sure that the two pluses are not at different voltages normally they will be short circuited externally with thick copper conductor, so we will have thick copper conductors short circuited these two similarly, another thick copper conductor short circuiting these two like this. So, this is how it is going to be.

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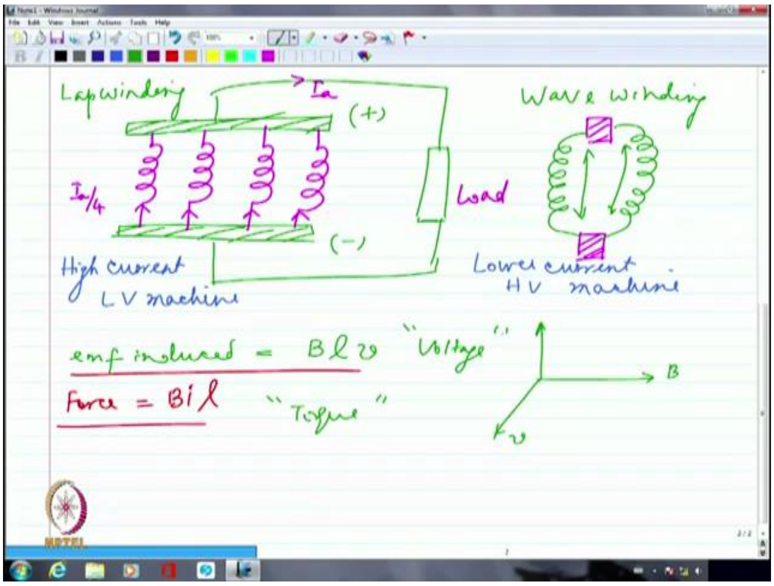

So, I can write the whole thing in the form of a diagram as though there is the positive bus that is located here, that is the negative bus that is located here, and between both of them so, this is positive, this is negative, I will actually connect my load probably between these two, may be the load is a resistor, may be load is a DC motor, whatever it is. I am connecting the load like this and in between these two I have actually the turns so three turns for this, three more turns for this, three more turns for this and three more turns for this. This is how it is.

So this is the arrangement of a lap winding please note that, there are as many parallel number of paths as the number of poles, so I am going to have the load carrying a current which is corresponding to, if I say, from plus the current is going if this is I_a, the armature current, each of them will be approximately $I_a/4$. each of them will be $I_a/4$, if there are 4 plus that are there in my machine arrangement rather if it is wave winding I am rather going to have probably one brush on one side another brush on another side, and if I assume that again there are 12 coils or 12 turns on the whole I am going to have actually on one side 6 turns.

And on the other side also 6 turns this is how it is going to be, 6+6 may totally it will make up 12 turns so, I have only two parallel paths. Please note that all these are connected in series. So, which means the voltage rating definitely is going to be higher because I am going to have totally 6 turns, which means 12 conductors are connected in series continuously so because of which I am going to have a higher voltage rating so this corresponds to lap winding whereas this corresponds to wave winding.

So, we just completed the last class saying that lap winding is meant for high current low voltage machine. Whereas this is lower current and high voltage machine so, normally this is the way we, design the stuff depending upon what kind of power rating we are designing it for that will be the first criteria, then the next criteria will be whether the voltage will be higher or current will be higher and then correspondingly we will choose the winding, that is how it is done normally.

With this structure in mind let, us try to look at the EMF equation and torque equation of the machine. The fundamental theorem that is the Biot-Savart's law as well as the Fleming's left hand and right hand rule give you basically a way to calculate the force and as well as whatever is the EMF induced.

So this you guys must have definitely studied that EMF induced when I have a conductor whose length is *l* and if it is moving in a magnetic field whose magnetic flux density is B and if I say that the velocity with which the conductor is moving is v, Blv is going to be actually the induced EMF for this particular conductor of length l. So, we normally, write as though if I am going to have v like this may be I am going to have my v exactly perpendicular to this, I am going to have the induced EMF which is perpendicular to both of them, basically. That is how it is going to be.

Similarly, I should be able to write the force that comes in to play when I am having B and i that is if a current carrying conductor of length l is placed in a magnetic field which is having a flux density of B, it will create a force which is actually Bil where i is the current carried by the conductor, B is the flux density inside which is placed and l is the length of the conductor so, that is also perpendicular to both, the actual current carrying conductors length in which plane it is placed and the flux density on which plane we are looking at the flux density it will be perpendicular to both of them.

So we were going to use these two expressions actually to get what is the value of torque in a DC machine and similarly what is the value of voltage induced in a DC machine, both we are going to look at by these two equations. So, let me actually look at, what is the flux per pole, in probably a DC machine.

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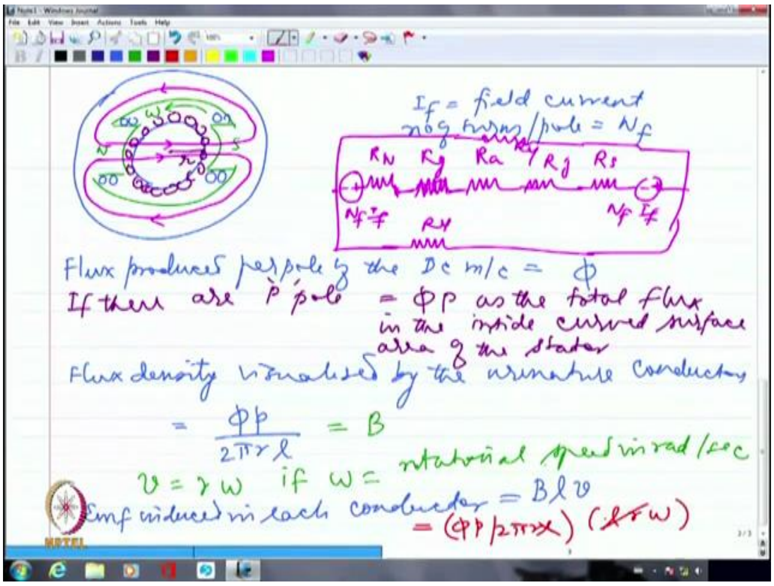

We drew this if you may recall when we were talking about magnetic circuit we said that if I have a magnetic pole here, I have the south pole here, and I am going to probably, push a current which is corresponding to some I_f, I_f is the field current I am talking about.

So what I am passing as the field current is I_f and the number of turns per pole I am calling that as N_f I am talking everything about field winding so, N_fI_f will be the MMF per pole what is available as MMF per pole will be N_fI_f and I should be able to calculate very clearly what is the magnetic circuit's reluctance I am going to have definitely the rotor sitting here, I am not showing the conductors right now, and the rotor will try to occupy as much ways as possible so that the air gap width is kind of limited.

If the air gap width is very large I will definitely require more and more amount of MMF to produce the particular value of field strength. So, I want to make sure that the air gap is very very small in the case of any of the electrical machines, but it should be large enough for the rotor to rotate without hitting the stator, it cannot hit the stator so it should be able to rotate without any impediment from the stator. So, I have to make an optimal design of air gap such that the rotor is able to rotate freely, but, at the same time the air gap is not so large to demand a very very large field current or magnetizing current.

So, it is a trade of basically we have to make sure that, that is done, now, if I look at the overall field line it is going to go like this and then complete its path like this. Similarly, I am going to have the field line going like this. So, I should be able to calculate clearly the reluctance corresponding to this magnetic circuit path that will correspond to whatever is the reluctance offered by the north pole for example then I am going to have a reluctance of the air gap, not inductance, reluctance of the air gap and I am going to have the reluctance of the armature. I am going to again have a reluctance of the air gap and I am going to have one more of the pole which is the south pole and then I will have reluctance of the yoke on this side and I will have definitely the reluctance of the yoke on the other side also I have to connect them together and then ultimately I am going to show as though there is one MMF here one more MMF here, there are two MMFs one corresponding to north pole, one corresponding to south pole.

This we drew it already when we were discussing with respect to the calculations in a magnetic circuit so I have to show it as though I am going to have plus minus here, plus minus here, so this is also N_fI_f this is also N_fI_f. This is how it is going to be. So this will tell me how much flux is produced per pole, I will be able to calculate because I have a clear magnetic circuit defined I should be able to calculate how much is the flux produced per pole, so let me say,

the flux produced per pole, of the DC machine=ϕ webers.

And if I look at the complete armature conductors they are all occupying this area, complete the surface area of this cylinder, so if I say that the radius of this cylinder is r, roughly, so $2\pi rl$ is going to be the area or the curve surface area of the cylinder so, if I have 4 poles or 8 poles whatever each of this pole is actually circulating some amount of magnetic field lines. So, the complete magnetic field lines or the flux what I can get for P number of poles, if there are P poles,

total flux in the inside area or inside curved surface area of the stator $= \phi P$

Which will be faced by all the conductors which are actually on the outer periphery of the rotor.

Flux density visualized by the armature conductors, $B = \frac{\phi P}{2\pi rl}$

because $2\pi rl$ is the overall surface area of the armature and total flux that is available in the air gap between the stator and rotor that is ϕP, so $\frac{\phi P}{2\pi rl}$ is approximately the flux density B.

So, I have B with me now, I have to look at the calculation of *Blv*. If I am rotating this armature with the velocity of ω rad/s. I should say basically every conductor will have a linear velocity of $v = r\omega$ if ω is the rotational speed in rad/s, and of course l is the length of the conductor. So, I can now say,

$$\text{EMF induced in each conductor} = Blv = \left(\frac{\phi P}{2\pi r l}\right) l(r\omega)$$

$$= \frac{\phi P}{2\pi}\omega$$

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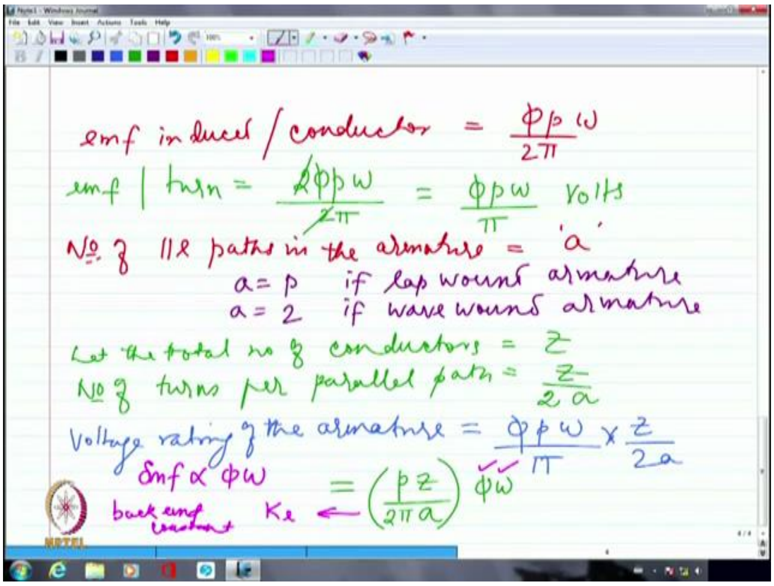

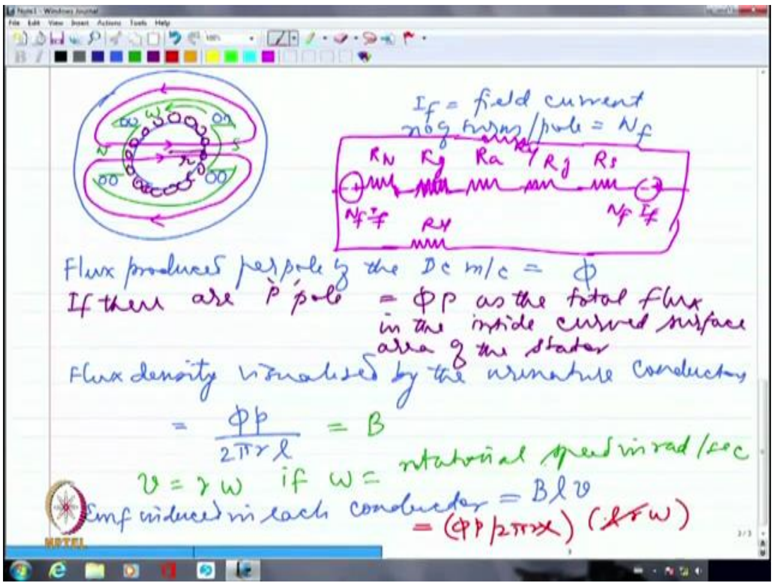

$\frac{\phi P}{2\pi}\omega$, this is the EMF induced per conductor then I have to get the EMF induced per turn because both of them two conductors which are connected together in series we call it as one turn. So, I can say

$$\text{emf/turn} = 2\left(\frac{\phi P\omega}{2\pi}\right) = \frac{\phi P\omega}{\pi} \text{ volts}$$

Now, I have to check whether all the turns are going to be connected in series or whether some of the turns are are going to be connected in series and then they will be in parallel, depending upon whether it is lap winding or wave winding. I am going to have basically, different number of turns being connected in series and different number of turns being connected in parallel. So, let us say in general I am going to say that,

$$\text{number of parallel paths in the armature} = a$$

So, if I am talking about wave winding $a = 2$, if I am talking about lap winding $a = P$. So, let me write that as well I should say

$$a = P \text{ if lap wound armature}$$
$$a = 2 \text{ if wave wound armature}$$

Let the total number of conductors = Z

$$\text{The number of turns per parallel path} = \frac{Z}{2a}$$

Now, all I need to do is, to multiply whatever is the EMF per turn by the number of turns per parallel path. That will give you the total voltage generated.

So, I can just say

$$\text{voltage rating of the armature} = \frac{\phi P\omega}{\pi}\frac{z}{2a}$$

where a =2 if it is wave wound armature, a =P if it is actually a lap wound armature.

So let me write this a little differently I am going to say P is actually the number of poles which is already decided for a given machine Z is the number of conductors which is also decided for a given machine a is also decided for a given machine 2π of course is a constant. So, I am going to write like this,

$$\text{voltage rating of the armature} = \frac{\phi P\omega}{\pi}\frac{z}{2a} = \left(\frac{PZ}{2\pi a}\right)\phi\omega$$

ϕ is actually the flux per pole, and that can be varied if I increased the field current or decrease the field current. Normally, we will try to operate it at rated flux, whatever is the rated flux of the machine.

But for some reason I want a lower voltage I may go with less amount of flux. Normally, I would like to operate it at rated flux. Similarly, normally I would like to rotate it in such a way that I will get rated voltage at that particular speed when I am injecting rated field current into the machine, so I am going to get basically rated voltage at rated speed at rated excitation. That is how the machine will be designed. So, the speed is definitely under my control, in one sense the excitation is also under my control. that is the reason I have taken them out, because they can be varied if I want to vary them.

So, this particular constant, $K_e = \left(\frac{PZ}{2\pi a}\right)$ of the machine generally is known as back EMF constant. We call this as EMF constant or back EMF constant. I am so used to saying back EMF constant because in the case of motor invariably we call this K_e as back EMF constant and I can call it as EMF constant that is fine. No problem. Because in a generator there is nothing like back EMF, so it is the EMF constant.

So, this gives me basically the equation for the EMF generated in the machine.

So, in general I should be able to say, EMF generated in the machine, $emf \propto \phi\omega$. ϕ is the flux ω is the speed at which I am rotating the armature. So, generally this is a very very important fact about DC machine, EMF generated is always proportional to flux multiplied by omega. Very clearly this also coincides with the $\frac{d\phi}{dt}$ because $d\phi$ is essentially how much is the change in the flux dt is at what rate the conductors are cutting the flux.

Obviously they are rotating faster I am definitely going to have, the rate of change flux being visualized by each of the conductors is going to be much higher. So, this is the EMF equation of the machine.

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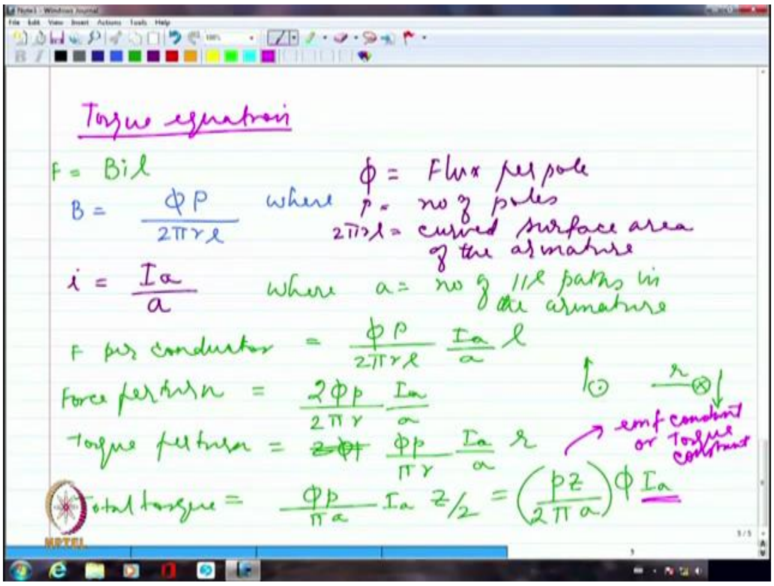

Next, let us try to see whether we can again derive the torque equation, we already talked about very briefly the torque equation when we were looking at electromechanically energy conversion process, but, we are going to now derive the same thing with a little different approach which is actually *Bil* equal to force (F).

$$F = Bil$$

Again we are going to take one conductor at a time and then we are going to look at what is the force developed by each of the conductors and then ultimately arrive at the total torque.

We already said,

$$B = \frac{\phi P}{2\pi rl}$$

I am just reiterating this, ϕ is the flux per pole P is the number of poles and $2\pi rl$ is the curved surface area of the armature. i is the current carried by each of the conductors if I have the number of parallel paths as a as I just showed couple of slides before we said that if I am going to have, I_a as the total current each of the conductors is going to carry only $I_a/4$, if there are 4 parallel paths.

So clearly if I have multiple number of parallel paths, I am going to have

$$i = \frac{I_a}{a}$$

where a is the number of parallel paths.

So,

$$\text{Force per conductor} = \frac{\phi P}{2\pi rl}\frac{I_a}{a}l$$

This is what is the force per conductor. If I want rather force for two conductors or one turn I have to multiply this by 2.

$$\text{Force per turn} = \frac{2\phi P}{2\pi rl}\frac{I_a}{a}l$$

Now, after all if I have 2 conductors which are making up for one turn I am going to have one conductor here, one conductor here, and may be one of them will carry dot and the other one will carry cross because of which I will have in one sense the force upward the other one will have force downward so it will act like a couple.

So, if I want the torque, I have to multiply this by the radius of rotation or radius of gyration and the radius is actually r that is what we assumed. That is why we took the curve surface area as $2\pi r$ and all these conductors are going to act in synchronism basically all of them join hands ultimately to provide the complete rotational motion. So, I have to say

$$\text{Torque/turn} = \frac{\phi P}{\pi r}\frac{I_a}{a}r$$

Now, I have to multiply this by the total number of turns which will be $Z/2$, because I have already taken care of the parallel paths by writing $\frac{I_a}{a}$. So all I need to do is, simply multiply this by the number of turns.

$$\text{Total torque} = \frac{\phi P}{\pi a}I_a\frac{Z}{2}$$

So, I would get again let me take ϕ and I_a out so I will have,

$$\text{Total torque} = \frac{\phi P}{\pi a}I_a\frac{Z}{2} = \left(\frac{PZ}{2\pi a}\right)\phi I_a$$

So, $\left(\frac{PZ}{2\pi a}\right)$ is the constant of the machine. I hope so, that we have arrive at the same thing whether we looked at $\left(\frac{PZ}{2\pi a}\right)\phi\omega$ was EMF similarly, $\left(\frac{PZ}{2\pi a}\right)\phi I_a$ is the torque. So, I can call this sometime this constant is specified as EMF constant or torque constant both are equally valid because as long as I tell the speed ω in terms of rad/s.

If I express the speed in rad/s, my torque constant as well as EMF constant are the one and the same. So I would say $\left(\frac{PZ}{2\pi a}\right)$ is the constant for a given machine and then I am going have essentially the same thing working as torque constant as well as EMF constant and if I vary the armature current, I will get varying torque keeping the flux as a constant, if the machine is operating at rated flux. If I actually pass rated armature current through the machine I am going to get rated torque.

Similarly, if I am having a generator which is rotating at rated speed and if it has rated flux, it is going to generate rated value of EMF. So, that is how basically the machine name plate details, we call that as name plate details because if you look at the any machine that will be one metal plate that will be stuck to the machine it will be riveted to the machine which will have what is the kilowatt rating of the machine.

What is the kind of speed it will run at normally what is the voltage and what is the current you can push the through the machine if it is a motor. If it is a generator it will indirectly tell you what is the maximum current you can draw from the machine, what is the voltage it can generate if it is rotated at rated speed with a given excitation. So, that is known as the name plate details, the name plate details will carry basically what is the rated voltage, rated current, rated speed, rated flux or rated field current, for example in a DC machine, the power rating. All these things will be written on the name plate details of the machine normally. The armature resistance or the field resistance if you want to get those things are generally measured, either we do a VI test that is voltage we apply and then we pass a current and then we say V/ I is R. So we tested that way or we use a multimeter depending upon how, accurate your multimeter is or what kind of accuracy you require in your test finally.

So, generally the name plate details will contain all these data at rated excitation at rated speed. The voltage and current will be specified for that, and the torque also. Now, that we have seen basically the EMF equation and torque equation which are required for a DC machine, let us first of all start with the generator operations.

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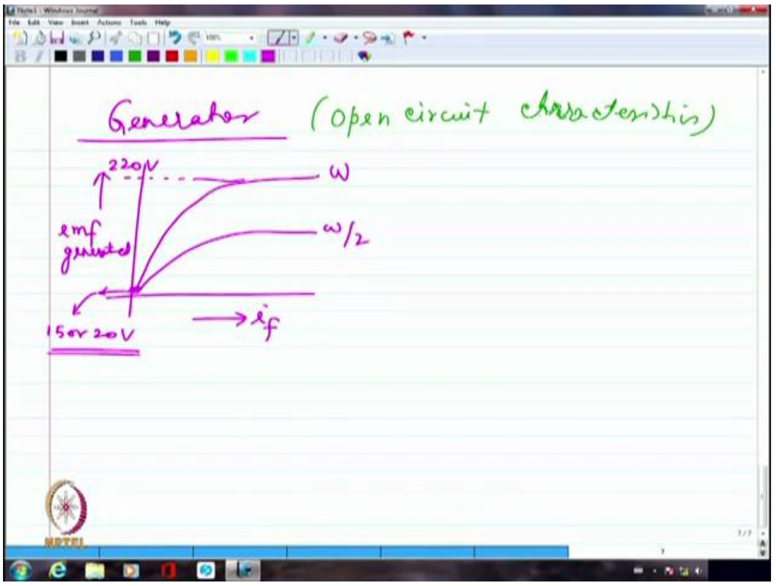

One of the first and foremost important characteristics of any generator is, if I am not loading it, how much it is able to generate as the voltage because we are looking at the open circuit voltage, how much it is able to generate as the voltage at the given speed and at a given excitation. We call this as when it is not supplying any current it is on open circuit so we call this as open circuit characteristics. So, open circuit characteristics of a generator essentially tells you, on the Y axis it will be EMF generated, overall EMF generated, by the machine and generally here it will be I_f what is the field current that you are passing through the machine at the given speed.

So, if I am running the machine, at let us say some omega (ω), I should have at ω this is the EMF generated, if I am talking about ω/2 it will be half that. So, I will have basically initially fairly a linear characteristics as long as may be I am not hitting saturation when my iron oriented flux is dominating over the air gap flux that is when the saturation will creep in. Definitely I have two components of flux one will be actually due to the reluctance in the air gap which will be actually very very small the flux will be small as compared to the flux due to iron.

So, when I am talking about the ferromagnetic portion flux dominating over whatever is the flux created due to the air reluctance path, I am going to have the saturation slowly creeping in. Initially, it may look linear eventually it will become completely saturated because the air gap flux is not making any dent on the overall flux that is the reason.

So, I am going to see this kind of characteristics if I keep on increasing the field current slowly. This I am assuming there is no remanent flux, if there is already remanent flux I should show actually some voltage even with zero field current. Imagine that there is no field current at all. If there is some remaining flux for remanent flux in the machine it will generate a voltage even when I rotate it at certain speed, without any field current.

So, this remnant flux will probably generate a voltage corresponding to 10 volts or 15 volts or whatever depending upon at what speed I am rotating it. The rated voltage, probably being approximately say 220 volts this may be only 15 or 20 volts nothing more than that. If I am applying constant field current I should have the constant voltage being generated, I am showing very clearly if I am putting variable field current because I want to know the capability of my DC machine, I am rather looking at what kind of back EMF constant it has, or EMF constant it has, that I will be able to get from this is not it.

Because I know at what speed I am rotating the machine. I know what is the field current I am applying in the machine. So, I should be able to get whatever is the EMF generated divided by the speed in rad/s divided by the field current will give me the back EMF constant. So, open circuit characteristics (OCC) will allow you to determine what is the back EMF constant or EMF constant of the machine which will be useful even in calculating the torque eventually. So, we will look at further classification of DC machines based on the field connection in the next class.