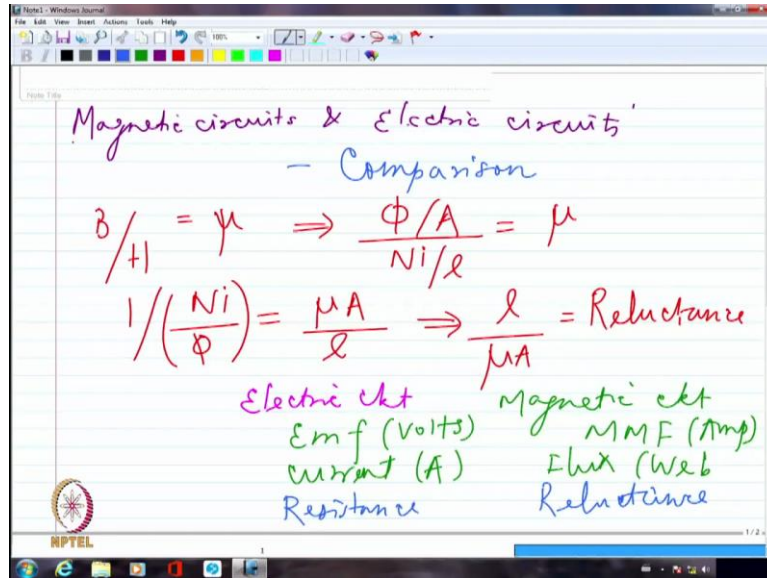


**Electrical Machines**  
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**Module 1**  
**Lecture 2**  
**Magnetic Circuit -II**

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So we had talked about the comparison between Electrical and magnetic circuit and we had started off with the relation, it was  $\mu = B/H$  from which we actually said  $\phi = BA = \frac{Ni}{l}$ . So I would say this is rather phi by A. let me probably write this as phi by A. So let me just write this as phi by A. So I can write this as mu from which I should be able to write this as  $\frac{Ni}{\phi}$ .

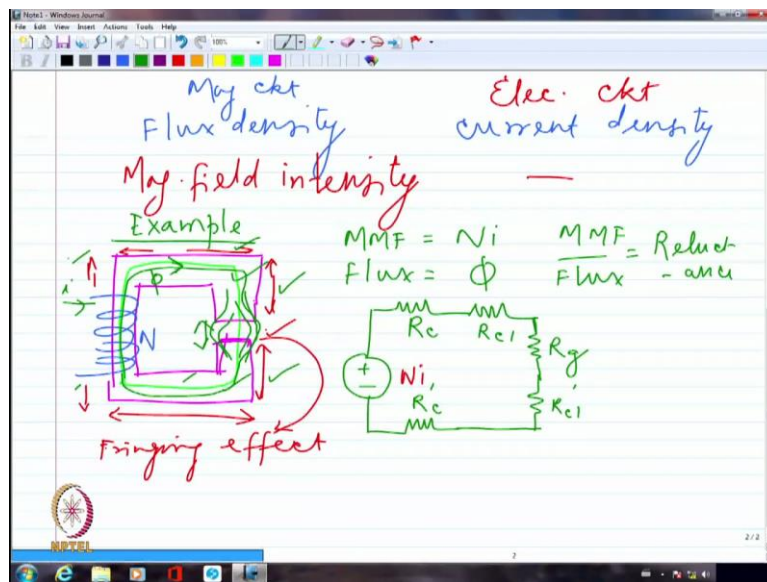
So this is essentially something called reluctance which is equivalent to the resistance in the case of an electric circuit. So if I have an electric circuit which is excited by a voltage source V. I am going to have a current which is flowing actually through the electric circuit which will be actually less if the resistance is more and it will be more if the resistance is less, I am going to have it that way something is wrong the way I have done  $\frac{Ni}{\mu}$ .

Student: phi by Ni.

Professor: So I can say  $\mathfrak{R} = \frac{l}{\mu A}$ . This is the reluctance  $\mathfrak{R}$ . So if I compare an electric circuit and magnetic circuit, I should say on the one side let me write electric circuit, on the other side, let me write magnetic circuit, so if I say that in this particular case, this is going to be EMF, in this particular case, it is going to be MMF.

Let me write the units also in the bracket. This will be volts. Whereas this will be in ampere turns but I am going to write that as amperes. Similarly, I can write here current whereas this will be in amperes and here it is going to be flux which will be in Wb. Now if I try to look at what is the impeding factor here that is resistance whereas here it is going to be reluctance  $\mathfrak{R}$ . So this will be weber, ampere per weber. Ampere turn per weber I should write as the reluctance whereas I am removing the turn because it is a dimensionless quantity.

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And we also said when we were talking about flux density in the case of a magnetic circuit that is corresponding to current density in the case of an electrical circuit. If I am looking at magnetic field intensity whereas here, there is no corresponding quantity, although one of your classmates mentioned that electric field is V/m, so we should be able to say that as an equivalent quantity, invariably, we talk about electric field only in electrostatics.

Hardly ever we talk about it when we talk about current electricity. So this is essentially the way we are going to analyze the effect of an electric current when we consider a magnetic circuit. So if for example, let me take the same diagram what we had taken last time, so let us say I have a core like this. With a small air gap, so I am going to have some air gap.

Let us say I am going to have some coil wound around here. So let us say it has  $N$  turns and let us say I am going to have an electric current of  $I$  flowing through this. So I would say that MMF in this particular case, so we are considering an example basically, so in this MMF will be  $N$  times  $I$ . If I try to look at what is the flux established, I am assuming that the flux is established, it is going to confine itself as much as possible to the core if it is a ferromagnetic core because if you look at air, it is not going to have as much of permeability as what ferromagnetic core has.

In fact, there will be a ratio of almost 4000 or 3500 compared to the permeability of the ferromagnetic core vis-a-vis the permeability of air. So because of which, I am going to assume that if I have some flux line flowing through this, it is going to completely confine itself to the ferromagnetic core alone. And I am going to call that as actually  $\phi$ , that is the flux. Now this flux MMF divided by flux is going to be the reluctance.

And this reluctance if I try to look at it here, I have some reluctance for the ferromagnetic core alone, from here to here I will have some reluctance, from here to here I will have some reluctance, I will also have some reluctance for this, I will have some reluctance for this, of course I am going to have some reluctance for this limb as well. But there is some more reluctance that I am going to have for the air gap alone and the air gap reluctance will be in all probability much higher as compared to what I have as the reluctance for rest of the stuff.

So if I try to draw this in the form of an electric circuit, I should show it as though there is an MMF source, just like how we show an EMF source, the same way I have to show an MMF source which will be  $Ni$ , and after that, I am going to have probably a resistance corresponding to the limbs corresponding to this portion as well as this portion, I am drawing one resistance. So let me write that as some  $RC$  which is the reluctance of the core.

Similarly, I am going to have the reluctance corresponding to this portion as well as this portion which I am going to write this as another  $RC$  maybe dash if the dimensions are slightly different.

So I am writing that as  $R'_c$ . Now I am going to have maybe one more  $R_{c1}$  corresponding to this portion and one more  $R'_{c1}$  corresponding to this portion for example. Now there is one more reluctance which is corresponding to the air gap which I may call as  $R_g$ .

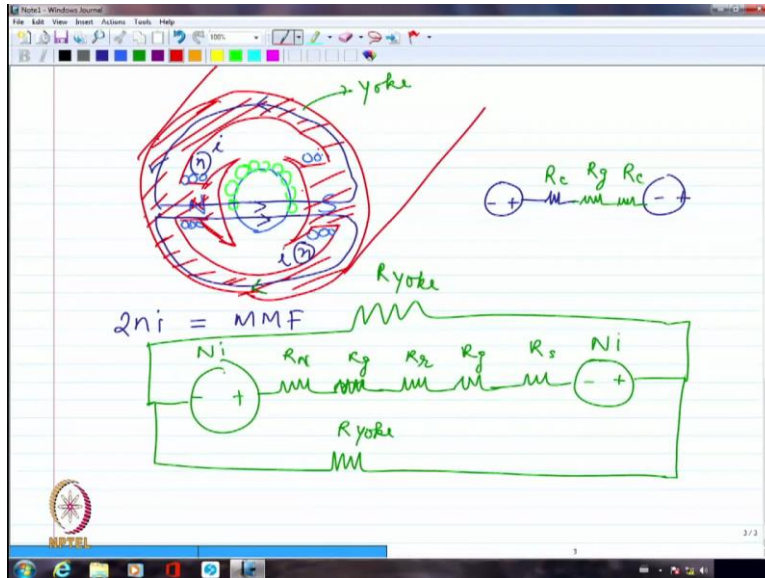
So I have all these reluctances connected in series. Actually speaking, at this point where the air gap is, because the reluctance is higher here, I should have expected the flux to diminish, but what is going to happen is, in all probability, these lines of flux will try to bulge here so that they cover larger cross-sectional area because the flux density for a given material is kind of fixed, I can't have, that is the maximum flux density I am talking about.

The maximum flux density for a given material is limited by certain value. So if I say that for iron it is something like 1 or 1.2, for air it will be much less than that, maybe 1.2 divided by 3500 or something like that that is how what is going to be. So I have to definitely allow more area, rather it is not my allowing, I would say the flux lines will automatically take up more area so that the flux density does not go too high and it is actually our assumption that the flux density at this point is same as that of the flux density at this point.

It need not be, especially because intervening air gap is there, some of them may get lost in the process. So I should say this particular effect what happens whenever there is an intervening air gap, this is known as fringing effect or fringing of magnetic field lines. This happens every time there is an intervening air gap, which is going to cause some amount of bulging of the flux lines.

So if I look at the magnetic field lines here, basically many of them will flow like this, but here they will bulge and then again they will complete themselves. So this is how the magnetic field lines are going to take shape whenever I have an intervening air gap. This happens all the time in rotating machines. So let us try to take a probably quick look at one of the machines maybe a DC machine.

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If I take a DC machine, there will always be a stator and there will be a rotor. The stator in the case of DC machine produces the magnetism that is required which is acting like a via media between electromechanical energy conversion process. So let us say this is going to be my stator's outer periphery. So I am going to have basically a magnet here, I am showing it like this and one more magnet here.

One will act like a north pole, the other one will act like a south pole. So this is the magnet. So this portion is all made up of iron basically. All the shaded regions are made up of iron. So I am going to have north pole here, this is north pole and this is going to be south pole. Obviously I have to have the bunch of conductors which is known as armature which will actually experience the force if it is a motor or which will actually have the generated voltage if it is a generator.

That should be sitting in between. So let us say this is my armature and I am going to have the conductors at the outer periphery, so I am going to have conductors all over in the outer periphery like this. So these are the conductors. Now when I am having current flowing through my field coil, so let me show the field coil here, this is probably going to be the field coils which are sitting here and the current is going to flow through these field coils.

Considering that this entire machine is extending into the board, it is a cylinder whose cross-section I am showing. So I am essentially going to have this producing the magnetism, the north pole and south pole will produce magnetism and if I look at the complete magnetic path, I am

probably going to have, so the MMF consists of how many ever number of turns I have here, this will also have certain number of turns  $M$  here and some current  $I$  is going to flow through this, same current  $I$  is probably going to flow through this as well.

So I will have in all probability,  $2Ni$  as the MMF in this case, I am showing two poles, each of them consisting of  $N$  number of turns and both of them carrying a common current  $i$  if I connect them in series, so this is going to be the MMF. Now when I look at the magnetic path, it is probably going to go like this and then it will complete its path like this, this is the way the magnetic field line is going to be.

Similarly, I can also think of the same thing happening from downside, so it is going to have like this and it is going to flow like this. This is how it is going to be. So I can show this magnetic circuit somewhat like this, I can show as though I have one source here, one more source here. So I can say about this as plus and minus, similarly I am going to have probably this as plus and minus.

Both of them together are going to contribute to the total magnetism or magnetic flux. They are not opposing each other, they are essentially aiding each other. So I have shown them as additive and in between them, I should probably show first of all the reluctance of the pole itself. Then I have to include the reluctance of the air gap, then the reluctance of the pole again. I should also include the reluctance of rotor. So multiple number of resistances will come if I assume that for this entire thing, that is for the North pole as well as the rotor half the rotor portion am calling as  $R_c$ , this is also  $R_c$ , this is  $R_g$ .

In fact, I should have had more number of resistances to indicate for example, the north pole reluctance, then the gap reluctance, then the rotor reluctance. So I can have plus and minus here, one resistance corresponding to our North pole, then I can have the gap reluctance, then I can have you know the rotor reluctance, then one more, gap reluctance, one more South pole reluctance and apart from that, I should have you know the other one which is the MMF corresponding to the South Pole. So this is one  $Ni$ .

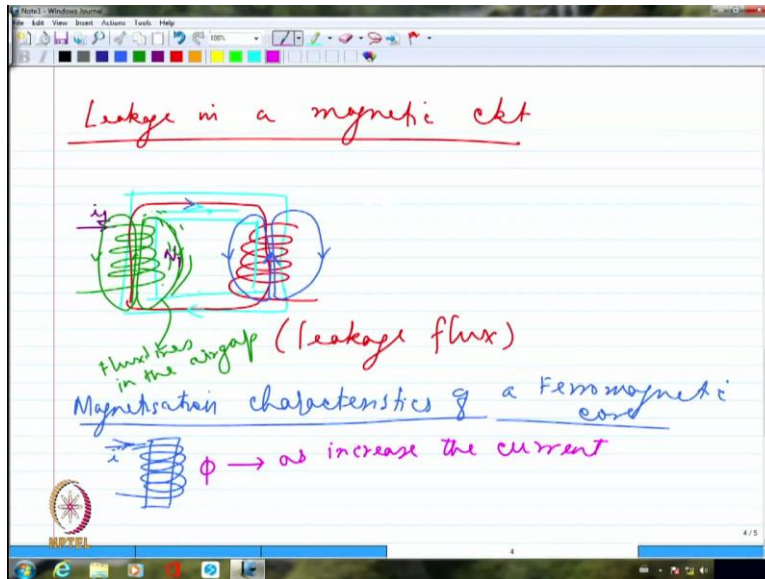
And ultimately, I am going to have one more reluctance which is corresponding to the yoke portion, which holds the poles properly in place, that is to provide mechanical strength and also

to provide a path for the magnetic field lines because the return path is provided by the yoke, here is where the return path is. So it is provided by the yoke.

Similarly, on the top also I have to include one more R yoke. So this shows me the representation of the magnetic circuit that is created in a DC machine. So you can say that if we have to make the calculation, this is also an approximation because you please understand that whatever we are having as the cross-sectional area here, will not be same as the cross-sectional area here but we are going to lump the whole thing and say that this is the reluctance of North pole.

Similarly, South pole, we are going to lump it together. So it is not an exact calculation but at least it is good enough as an engineering approximation. Whatever it would give as the flux value will almost be you know in-line with what actually we are calculating finally as the generated voltage and so on based on those flux values. So it is good enough. So this essentially tells you how to get an equivalent magnetic circuit.

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There is one more thing that comes into picture in electromagnetism which is known as leakage in a magnetic circuit. If I have let us say a transformer and let me represent the transformer core like this. Let me show the windings of the transformer somewhat like this, this is my primary and this is going to be the secondary. So let us say I am pumping in a current of  $i$  here. Let me call that as  $i_1$  because it is essentially the primary current.

I am going to call that as the primary winding, so I am calling that as  $i_1$ . Let us say the number of turns is  $N_1$ . On the secondary side, if I keep it like an open circuit, there is not going to be any current but if I connect a load, I will definitely have a current through that as well. Let me first of all discuss only about the primary, if this current is flowing, I am going to have probably whenever there is a positive half cycle, if it is an alternating during the positive half cycle, probably I am going to have a flux established like this.

So I am going to show the direction of the flux probably somewhat like this. I am assuming that all the flux that is produced because of the primary winding's excitation is confining itself to the core. That is my assumption but if let us say there are some 10,000 magnetic field lines that have been produced, although the ratio of the air gap reluctance to the reluctance of the iron core is about 4000, I will still have maybe 2-3 lines at least escaping and following the path of air gap.



I am essentially looking at the ratio of the number of lines confining themselves to the iron core and the number of lines that are coming through the air gap. That is supposed to be about 4000. So if I say totally 10,000 lines are there on the whole as flux lines, at least 2 or 3 can definitely be following the path through the air gap. So if I say that these are majority of the flux lines path, some of the flux lines can definitely go through this, like this.

So I am essentially showing as though it is coming like this, it can come out and then it can go like this. This is the flux line which is passing through the air gap. The same way, it can also go on the other side. So these are flux lines in the air gap, these are the flux lines in the air gap. Please note that these flux lines are not linking with the secondary winding at all.

These are only linking with its own self, the primary only. So it is not going to link with the secondary winding. Essentially, the induced voltage in the secondary winding is because of the linking of flux. The primary flux is linking with the secondary, only then I am going to have an induced EMF. Most of the flux actually links with the secondary but some of the flux lines, maybe very few, they will not link with the secondary, so those are useless.

They are not going to really contribute towards the voltage production on the secondary side. So we will call this as the leakage flux. So leakage flux is present whenever there is something which is not linking the other member, we generally expect that both members of an electrical machine should be linked with a common flux. If I have a stator and if I have a rotor, both of them have to be linked with each other.

Only then it is going to make better sense in terms of voltage production and so on. In this particular case, whatever does not link with the other member, we call that as the leakage flux. The same thing holds good in the case of secondary winding as well. I am probably going to have a flux like this and it can just link itself like this. So I am going to have essentially again leakage flux for the secondary winding as well.

Please note that I have shown the direction of flux lines this way, which means what is coming here will be upward. But what is coming from the primary is downward, in the same limb. I hope to understand because I was showing basically the primary flux in this direction. So it will go

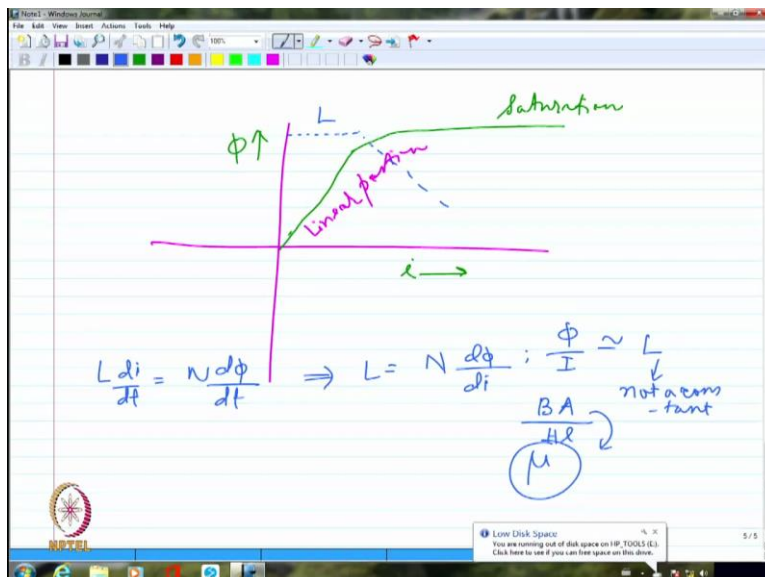
down. So primary and secondary flux will obviously oppose each other. They cannot be aiding each other, Lenz's law.

If they aid each other, the flux could have really gone to infinity. So they have to oppose each other. We will talk about this definitely when we talk about transformer in a greater detail. So we are going to have every time there is an intervening air gap anywhere, we will have huge amount of leakage, we will have definitely fringing, all these things have to be taken into consideration if I want an accurate calculation of the magnetic circuit parameters.

So, so much so for the case of magnetic circuit with ferromagnetic core and air gap. Now let us try to see what happens exactly to the behavior of a ferromagnetic core when I am actually pumping in current through that. So let us try to look at the magnetization characteristics of a ferromagnetic core. Let us say I am going to take a fresh sample of iron and let us say it is just a piece of iron like this and then I am going to wind some coil around here and I am going to pass a current of I through this.

And I am going to look at what is the value of flux I am getting as I increase the current. So this particular behavior whatever we get in the form of a plot between the flux produced versus current, that is known as the magnetization characteristics of any of the portions of an electrical machine.

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If you actually plot this, you are going to have it somewhat like this, let us say this is the x-axis and this is the y-axis. So I am going to have essentially on the x-axis, I am going to have  $I$  and on the y-axis, I am going to have the flux. So initially, I may say that as  $I$  increase the current, the flux is increasingly linearly, but beyond a particular point, the increase will become you know less and less and it will become almost flat.

Where it becomes flat, this portion is known as the saturated portion of the magnetization characteristics and initial portion what I have got is the linear portion, so I am taking a fresh sample of iron, it will start from 0. For 0 current, there will not be any flux and for increasing current, it is going to increase and once it reaches whatever is the value which is responding to the maximum flux that can be encountered by this iron, I am going to have that as the saturation point.

Now if I try to look at what is the corresponding inductance measure of the coil. I have wound a coil. So I want to get what is the inductance corresponding to these portions of the magnetization characteristics. Let us try to again take a look at what we wrote in the last class. We can write  $L \frac{di}{dt} = N \frac{d\phi}{dt}$ . So I can simply write  $L = N \frac{d\phi}{di}$ .

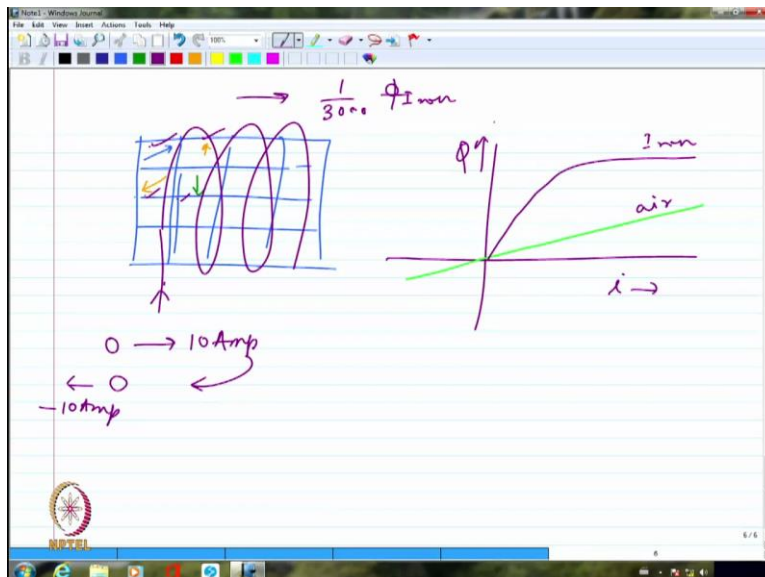
So the flux and current are directly proportional to each other, which is the linear portion representing, I would have almost the inductance to be a constant. Whereas, the moment saturation creeps in or even before the saturation creeps in, I am going to have the inductance definitely decreasing. So if I try to plot the inductance correspondingly, I should plot it as though it is almost a constant until here and then it is going to come down.

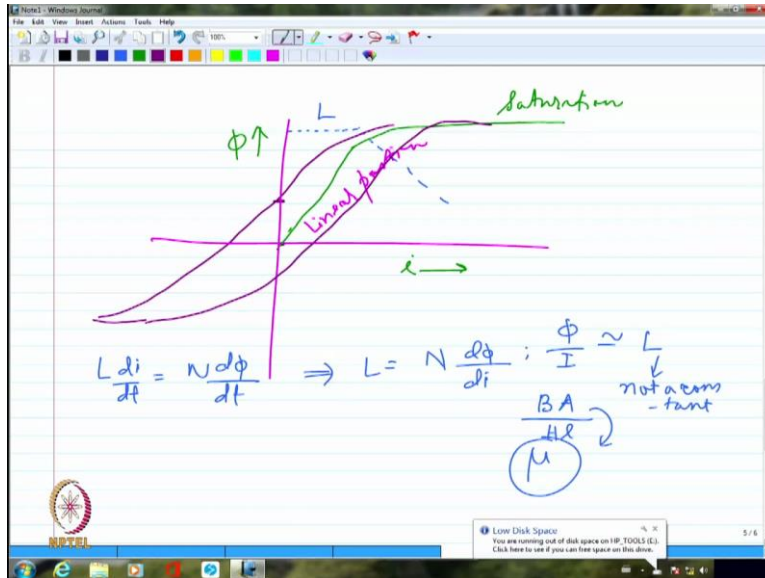
So this is how the inductance will vary. So if I say that  $\phi$  and  $I$  are not directly related to each other in some portions of the magnetization characteristics, I should say correspondingly  $\phi$  by  $I$  is roughly representing the inductance, of course I have removed the  $N$ , number of turns I have removed, that is approximately representing the inductance. So this  $\phi = BA$ , and I can write this as you know if I write  $N$  also here, then I should say  $\frac{Hl}{N}$ , I can write it like this.

So I have basically this inductance is not a constant means I am also not going to have here  $\mu$  comes in as the part of the relationship. So  $\mu$  is also not a constant in the case of a ferromagnetic core. The  $\mu$  value whatever I am talking about which is relating  $B$  and  $H$  together, that will not be a constant.

If I take that into consideration, I will not have very simple calculations in the magnetic circuit, but we are kind of you know constrained by the fact that iron is going to give me more magnetic flux for a given current, so we are bound to use that. So that's all the more reason, we should also take a look at what happens when this non-linear behavior creeps in into the magnetization characteristics.

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So we will just recall the current ring theory of magnetism. So if I am actually looking at a piece of iron, so if I take a piece of iron. I can divide that into some domains, each one will have roughly a width of about 0.1 millimeter. So I can just divide this into several domains of each about 0.1 millimeter width. This has definitely  $N$  number of atoms within that 0.1 millimeter width. And if you look at these iron atoms, they have 4 electrons in their outermost orbit and all the 4 electrons seem to spin along the same direction for whatever reasons.

So if I look at the iron atom, all the 4 electrons will spin in a particular direction and their axis are being parallel to each other. Whether I like it or not, this is how in a piece of iron, generally a particular domain or an atom behaves. And all that atoms in a particular domain seem to have the axis of the spinning of the electrons along the same direction, because of which if I look at any of these small domains, all of them are having some kind of current spinning in a particular direction.

So the electron spinning is equivalent to a current because of which magnetism is produced and the magnetic moment created by each of them probably or in a particular direction if it is here, this way, this may be this way, this may be this way, this may be at the bottom. So I am going to have multiple directions towards which all these magnetic moments are aligned. Now if I look at a new sample of iron, because all of them are randomly aligned in different directions, the net magnetic moment available is 0 in a new sample of iron.

It is not going to have any value at all but the moment I pass a current, maybe I just wind you know a coil around this and I pass a current in a particular direction, because of which an extrinsic magnetic field or external magnetic field is going to be created, maybe along this direction. So what is going to happen is, this magnetic field will try to align all these domains one by one along its own direction.

So I am going to see that slowly as the strength of the magnetic field increases due to higher and higher current I am going to have more and more domains aligned along the direction of the external magnetic field, but after all the domains are aligned, there is nothing that is left over to be aligned further. So it reaches a saturation. So I should say the external magnetic field maybe from 1 by 3000 or 1 by 4000 times the original magnetic field which is due to iron.

So if I try to look at the flux that is created it due to the intrinsic magnetic domain, that will be much higher as compared to what is happening in the surrounding air. Air is never going to go through saturation absolutely. Whereas, the magnetic field strength that is created or the magnetic field you know the overall magnetic flux that is created in the air is much less as compared to what is the flux that is created due to the domains that are intrinsically existing in iron.

So when I draw the magnetization characteristics, actually I should show two portions if I have to be really accurate, one should be corresponded to air, the other one should be corresponding to iron. Two things I have to show, so this is flux and this is current. So this is due to iron and this is due to air. But the air is almost 1 by 3000 or 4000 times.

So if I show both of them, this will almost coincide with the X axis itself, there will be hardly anything that will be left out. So this essentially causes saturation in an iron core, whenever I have an iron core. So that is the reason why I am not going to have the permeability of a ferromagnetic material to be a constant irrespective of the status of the domains which are residing within the iron core, very clear.

And once I reversed, I try to reverse the current, let us say the current has increased from 0 to 10 ampere. Now I am trying to reduce the current and I want to bring it back to 0 ampere. Many of the domains which are already aligned, they will not try to come back from their position

because already aligned you have done work on them. So they are not going to be disturbed any more because still the magnetic moment is in the same direction.

The extrinsic magnetic flux is in the same direction. Now, if I actually try to come to 0 and go towards minus 10 ampere. In an alternating current, this happens during every cycle. So when I have want to come to minus 10 ampere, now this current that I am pumping in, will create an extrinsic flux which is definitely in the opposite direction, but it has to do some work on these magnetic domains and realign them.

So when you do the work on the magnetic moment, you have to essentially move them against their will. So it takes a while for the current strength to build up and then only it can reverse or completely reverse the position of all the domains. That is the reason why what actually one of your classmate just asked when I was drawing the magnetization characteristics, why it is starting from 0.

If it is a new sample of iron none of the domains have been aligned or realigned, that is why it started from 0 and it reached a saturation value. After that when I am trying to reduce the current, they are going to slowly realign themselves. Even when the current is 0, I will still have some amount of magnetic flux left over, that is remaining. So that is known as remnant flux. That is what is remaining.

So it is known as remnant flux and then it will reach saturation on the other side, when I sufficiently increase the current then it is again going to go through the same kind of movement when I try to increase it from minus 10 ampere to plus 10 ampere. So the variation in the flux is always you know kind of left behind as compared to the variation in the current. The variation in the flux is left behind which is known as the hysteresis property of the iron.

Hysteresis means lagging behind, so the flux actually, the variation in the flux actually lags behind compared to the variation in the current. So this is essentially happening cycle after cycle when I am having a sinusoidal excitation or any kind of AC excitation that is given to the coil which is wound around an iron core. So this particular hysteresis property is a pain in the neck in many of the alternating current machines because it will first of all eliminate the linearity.

If I simply try to assume that flux and current are linear even in the beginning, that is not going to work. So the linearity property is lost which will give rise to several nonlinear behavior especially if I want to use this transformer for measurement. Let us say I have 1000 amperes of current, you have an ammeter only corresponding to 10 ampere. What I will do is to use a current transformer which will step down the current from 1000 ampere to 10 ampere.

I can't assume that when I am actually coming to 10 ampere in the main winding, it will not be really 10 divided by 100, I cannot really rely on that because it is not a linear behavior in all probability. So I am going to have a big problem because of the nonlinear behavior that is happening in my transformer or any other ferromagnetic core. So we will continue with this and then they will slowly move onto transformers from here.