Electrical Machines
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Lecture 08
Transformers – Voltage Regulation and Efficiency

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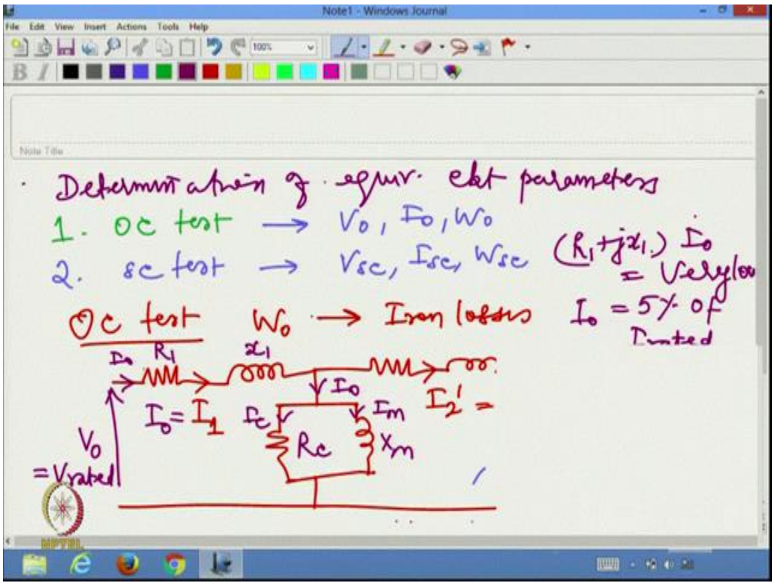

In the last class we have seen about determination of the parameter of a single phase transformer, determination of equivalent circuit parameters, So we actually looked at two of the tests, one is open circuit test, the other one is short-circuit test, these are the two tests we looked at and in open circuit test we said we will get three readings, V_0, I_0 and W_0 and short-circuit test we said we will get again three readings V_{SC}, I_{SC}, and W_{SC}.

We also said that in open circuit test the reading that we get as W_0 is corresponding to only the iron losses, there is no indication of any copper loss in this, that is because if you actually think about the equivalent circuit corresponding to open circuit test and short-circuit test together, I am going to have the equivalent circuit somewhat like this for the single phase transformer.

If I look at the open circuit test. I am going to have these on open circuit condition, which means the current that is flowing here which I make call as $I_2^{'}$ this is equal to 0 and if I am looking at actually the current that is flowing here which will be I_1 during the open circuit condition, what actually flows during the open circuit condition here is only I_C which is actually corresponding to core loss component of current and I_m which is corresponding to the magnetising component of current.

So I_1 will be equal to actually the summation of these two currents, I_C and I_m, which is actually I_0. So, I am going to have essentially only I_0, as the current that is flowing through the transformer under open circuit condition, when I apply rated voltage V_0, so V_0 is rated voltage what I am applying. So under this condition, I am going to have, we already talked about this that I_0 will be only about 5 percent of whatever is my I_{rated}, so if I look at the drop that takes place in R_1 and X_1, it is going to be the drop across R_1 and X_1, drop across R_1 that is $I_0(R_1 + jX_1)$ is going to be very low.

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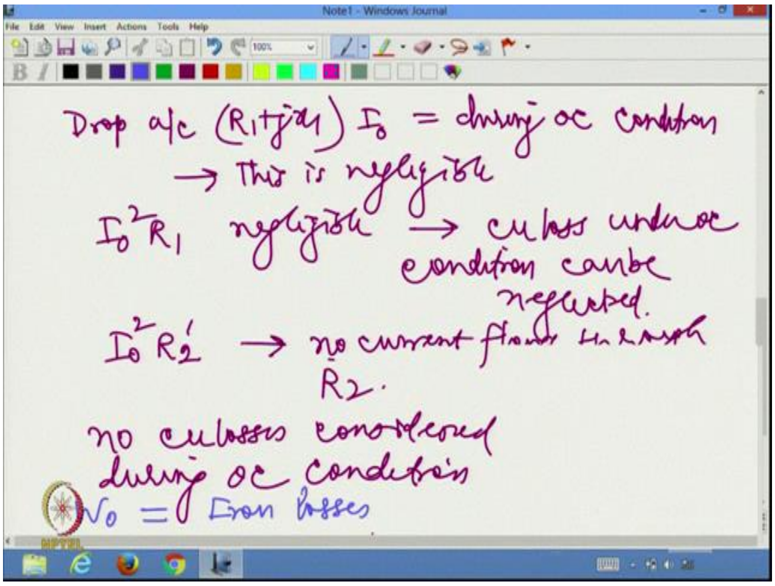

Because of which reason I am going to get actually these drop is really negligible, so I can say drop across $I_0(R_1 + jX_1)$ during open circuit condition, this is negligible and there is no current in the secondary side at all, so $I_0^2(R_1)$, so we would say copper loss under open circuit condition can be neglected. Whereas if I look at actually $I_0^2(R_2^{'})$, no current flows through R_2, Because the secondary is open circuited, so I am going to have no copper losses considered during open circuit condition. So, I can say that basically the W_0 value what I get during open circuit is equal to iron losses themselves, copper losses are not considered at all.

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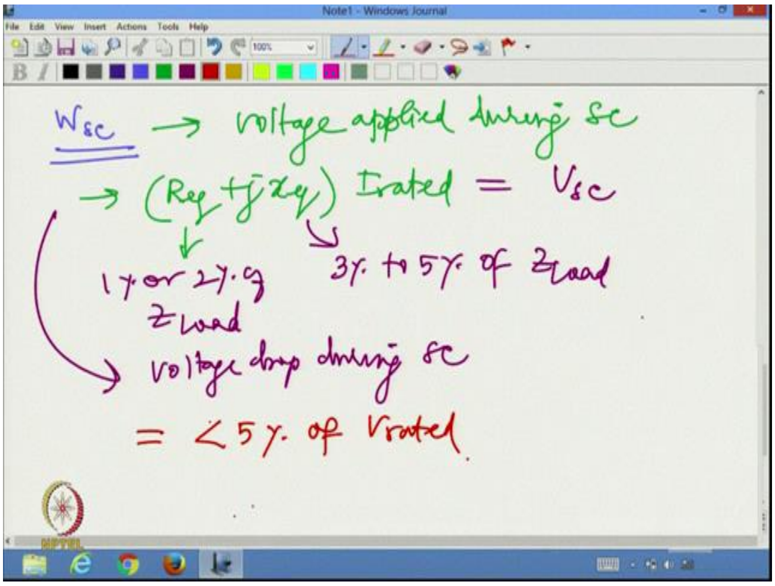

Whereas if I try to look at what is the value of the loss that I get during short-circuit condition, the voltage that I apply during short-circuit is going to be $V_{sc} = (R_{eq} + jX_{eq})I_{rated}$, this is only about 1 percent or 2 percent of Z normally or if I try to look at what is Z_{Load}. Similarly this will be only about 3 to 5 percent of Z_{Load}, so overall, I am going to look at this if I call this as V_{SC} that is the voltage drop during short-circuit, this is going to be only about 5percent or less than 5 percent of V_{rated}, if this is going to be less than 5 percent of V_{rated},

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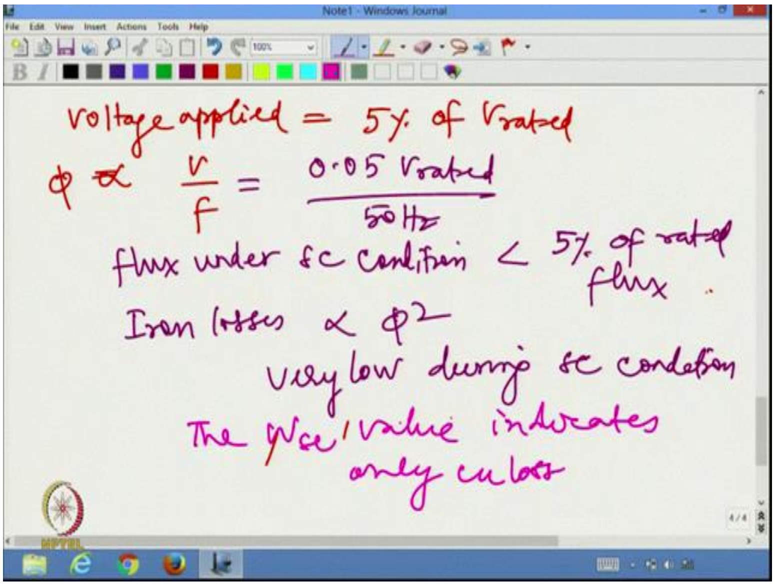

Then I can say the voltage applied is equal to less than 5 percent of V_{rated} in which case, so if it is less than 5 percent of V_{rated}, then I am going to have essentially $\frac{V}{f}$ that is what is flux?

So, flux is going to be proportional to $\frac{V}{f}$ due to which I can write this is $\frac{0.05V_{rated}}{50Hz}$ due to which I am going to have actually flux under short-circuit condition is less than 5 percent of rated flux. So, I am going to have essentially the iron losses which are proportional to flux square approximately will be very low during short-circuit condition, so the losses W_{SC} value indicates only copper losses.

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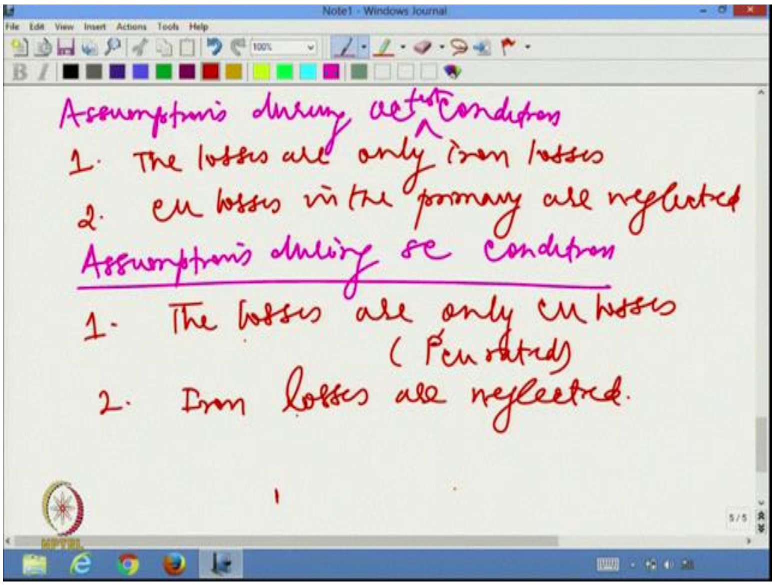

So, we make basically the assumptions during open circuit condition or open circuit test condition is first thing we are going to say is that the losses are only iron losses and copper losses in the primary are neglected, and of course there is no secondary copper loss at all because secondary is open circuited. Similarly, I can say assumptions during short-circuit condition is actually, we are going to apply or we are going to look at only the losses as only copper losses and that is actually PCU rated, that is rated copper losses, so the second thing is the iron losses are neglected, So these are the assumptions that we make during open circuit test and shortcut circuit test.

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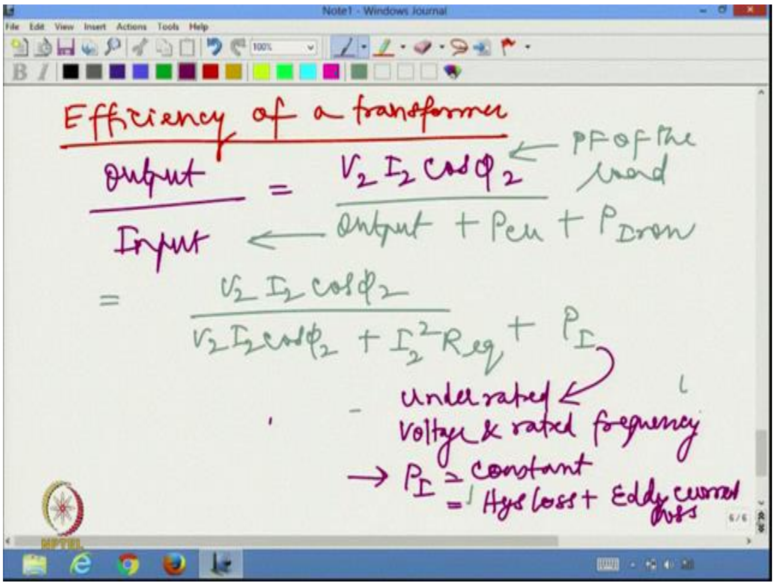

Now let us look at efficiency of a transformer. So the efficiency of a transformer is basically the $\frac{\text{output}}{\text{input}}$, so output, let me write this as $V_2 I_2 \cos\phi_2$, where $\cos\phi_2$ is actually the power factor of the load divided by again, I am going to have $\text{output} + P_{CU} + P_I$, so the iron losses and copper losses need to be added, so that makes up for the input,

So let me right, this as $\frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + I_2^2 R_{eq} + P_I}$, as long as I operate the transformer under rated voltage and rated frequencies, I will have basically the iron loss to be a constant, so P_I will be constant, so I can call this as either iron losses or constant losses or I can say this is the hysteresis loss plus eddy current loss.

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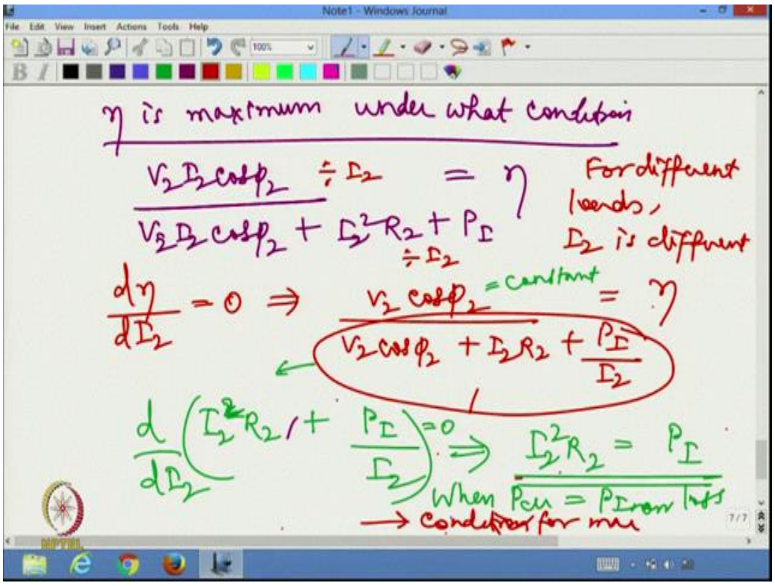

Now let me write, under what condition efficiency is maximum, let me try to derive this under what condition, So let me write the expression $\frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + I_2^2 R_2 + P_I}$, this is the efficiency, so let me write under different load condition, I am going to have I_2 to be different, so let me differentiate the efficiency with respect to I_2 and I make it as 0.

So, in which case I am going to have actually, let me right, let me properly divide the numerator as well as the divide the denominator I_2, so I can write $\frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + I_2 R_2 + \frac{P_I}{I_2}}$, this is what is equal to, the efficiency. So, I can write when I differentiate this, I will have to mainly differentiate only the denominator, nothing else, so if I differentiate this, that is sufficient because this looks like a constant, this is not going to matter.

So I can write basically when I differentiate this I would have $\frac{d}{dI_2}(I_2^2 R_2 + \frac{P_I}{I_2})$, so I am going to have basically this is $I_2^2 R_2 + \frac{P_I}{I_2}$ has to be differentiated with respect to I_2, so when I differentiate this I will have $I_2^2 R_2 = P_I$, this is what I am going to get as the expression I differentiated and then equated to 0, this is what I am going to get, which means when $P_{CU} = P_{Ironloss}$, that indicates the maximum efficiency condition, this is the condition for maximum efficiency,

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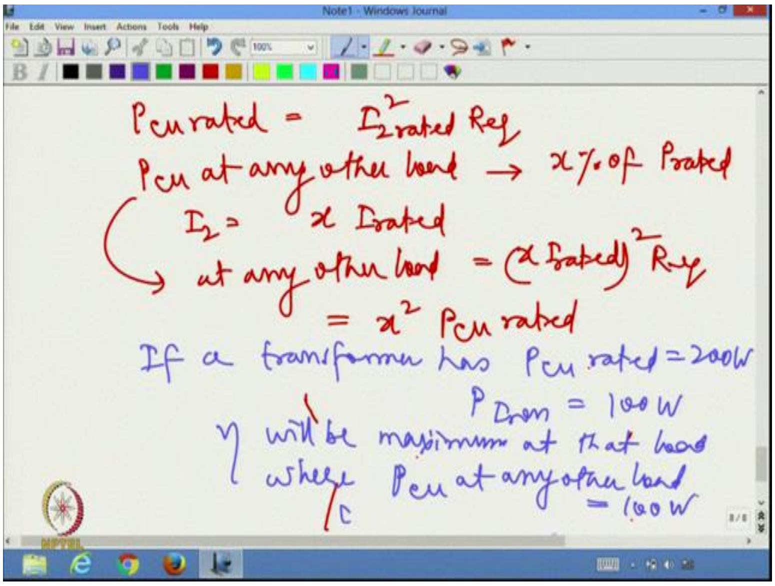

Let me understand this situation clearly, $P_{CUrated} = I^2_{2rated} R_{eq}$, P_{CU} at any other load which is may be x% of P_{rated}, then I should write current will be equal to $(x)I_{rated}$, so I can write P_{CU} at any other load will be $(x_{Irated})^2 R_{eq}$, which will be actually $(x)^2 P_{curated}$. So, I can say if a transformer has $P_{cu\ rated}$ to be 200 W and P_I to be 100 W, efficiency will be maximum at that load where P_{cu} at any other load equal to 100 W.

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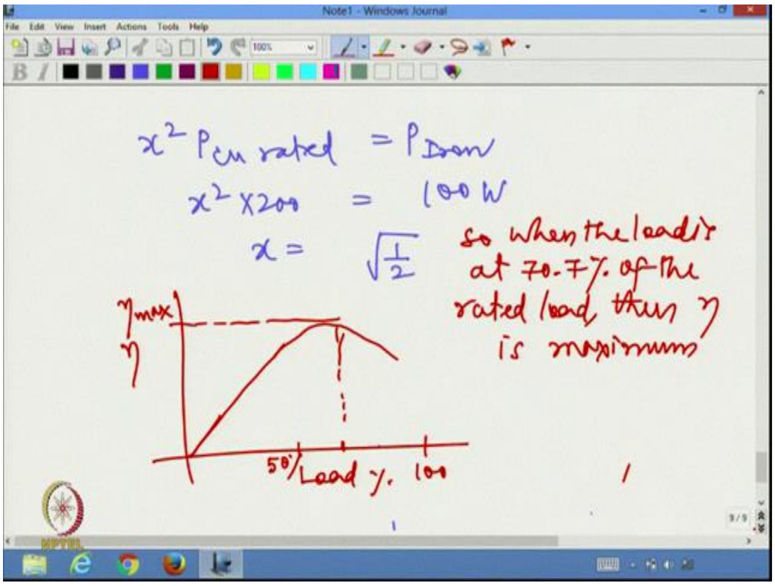

So, I can write this as $x^2 P_{curated} = P_{Iron}$, so this is $x^2 200 = 100$ W, so $x = \sqrt{\frac{1}{2}}$, so when the load is at 70.7 percent of the rated load, then efficiency, η is maximum for this particular transformer. So, if I try to plot actually efficiency versus load condition, so let us say this is

100 percent, this is 50 percent, so 70 lies somewhere here. So, I am going to have probably the efficiency will start with 0 and it will reach maximum around this condition and then it will start falling again, so this is what is the maximum efficiency. So, this is what we mean by saying, what is the maximum efficiency condition of a transformer.

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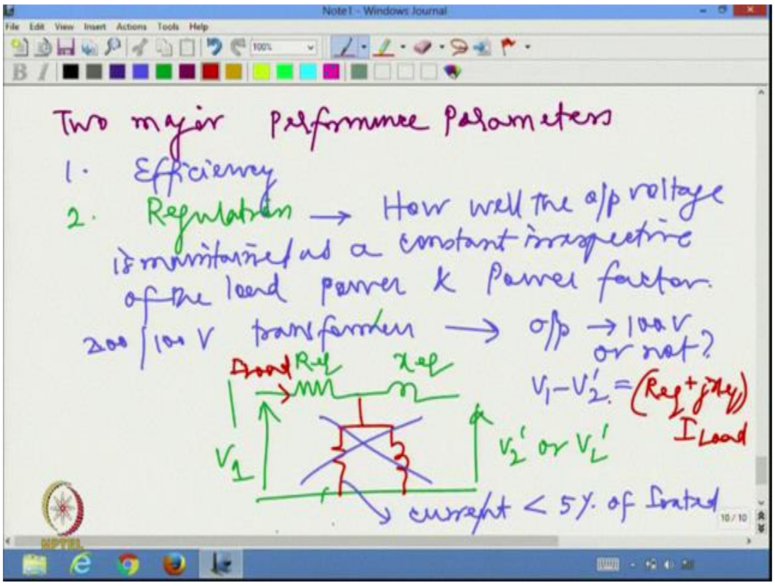

So, we told already that there are mainly two major performance parameters for a transformer, one is efficiency which we talked about already, the second one is going to be regulation, so regulation actually talks about how well the output voltage is maintained as a constant, irrespective of the load power, so this is what and power factor, of course, this is what we mean by voltage regulation, what we mean is, if I apply let us say it is are 200 by 100 V transformer, if I apply 200 V irrespective of the load whether the output voltage will be at 100 V or not, this is what we mean by talking about efficiency, regulation.

So when we actually look at the transformer equivalent circuit, we have R_{eq}, we have X_{eq}, we apply V_1 here and what we are getting here is $V_2^{'}$ or $V_L^{'}$, we are neglecting the parallel components, we do have some parallel components, somewhat like this, but we are planning to really neglect it because these are going to carry very small current, the current is really small current is less than 5 percent of I_{rated}, so I can afford to neglect it. So I can say that drop that is V_1- $V_2^{'}$ is going to be basically $\left(R_{eq}+jX_{eq}\right) I_{Load}$, this is what is going to be, so I_{Load} is actually flowing here, this is I_{Load}, so I can afford to neglect the parallel components, so I can say $^{'}V_L^{'} I_{Load}\left(R_{eq}+jX_{eq}\right)$ is going to be the voltage drop.

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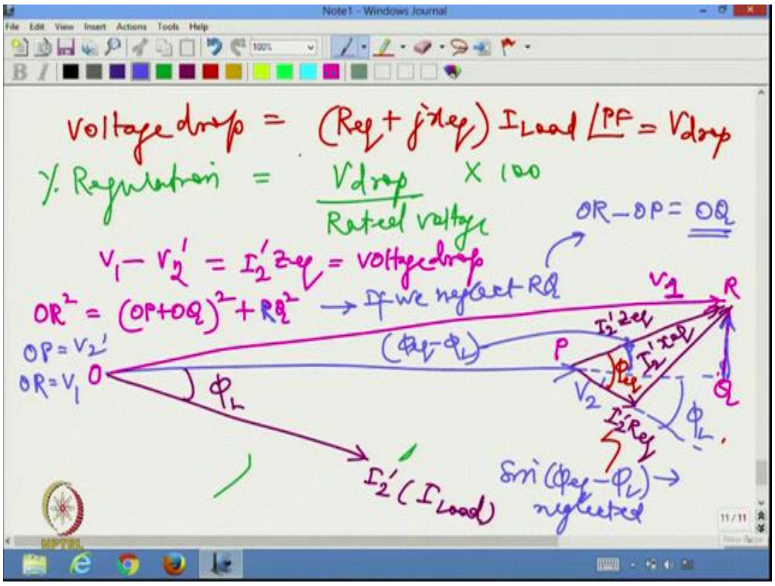

So, voltage drop $=\left(R_{eq}+jX_{eq}\right)\ I_{Load}$ and a have to look at I_{Load} also along with its power factor, otherwise it doesn't make sense. So now if I may call this is V_{drop} I can say regulation in percentage, if I try to look at will be $\dfrac{V_{drop}}{V_{rated}\times 100}$, this is what is percentage regulation, so if I actually draw the phasor diagram I should say this is what is my voltage which is actually $V_2^{'}$ or load voltage, now I would say maybe my current is lagging behind by some angle, let me call this as $I_2^{'}$ or $I_{Load}\angle\phi_L$, this is the load power factor angle.

Now if I try to look at what is $I_2^{'}R_{eq}$ it will be in parallel with this, so this is $I_2^{'}R_{eq}$, perpendicular to that will be $I_2^{'}X_{eq}$ and what I look at here as the summation of these two, this will be $I_2^{'}Z_{eq}$, if I actually extend this, please note that this angle, if I extend this here, this angle is same as ϕ_L, whereas this entire angle I am going to call as the inherent impedance angle of the transformer which is ϕ_{eq}.

Now when I add this $I_2^{'}Z_{eq}+V_2$, I will get the overall voltage which is actually V_1, of course I have drawn the whole thing in the exaggerated fashion, now I can say V_1- $V_2^{'}$ = $I_2^{'}Z_{eq}$. So this is what is the voltage is drop, I can write rather this as the summation of let me call this point as O, this point as P, this point as Q, this point as R, so I can say that $(OR)^2=(OP+OQ)^2+(PQ)^2$, And OP is after all, if I write what is OP? OP= $V_2^{'}$, OR=V1.

So, if I neglect this $(RQ)^2$, not PQ am sorry, $(RQ)^2$, so if we neglect RQ, then I can write this as OR-OP=OQ, so OQ becomes my regulation, if this angle is small, then I can afford to neglect this veridical components of this voltage drop. So, this angle, if I try to write what this angle is, this angle will be $\phi_{eq} - \phi_L$. So, sign of $\phi_{eq} - \phi_L$ this we are neglecting, this is neglected.

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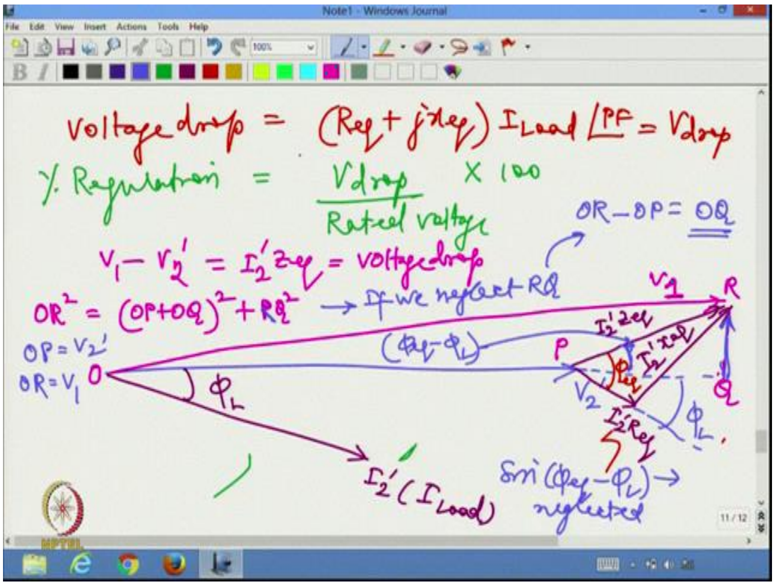

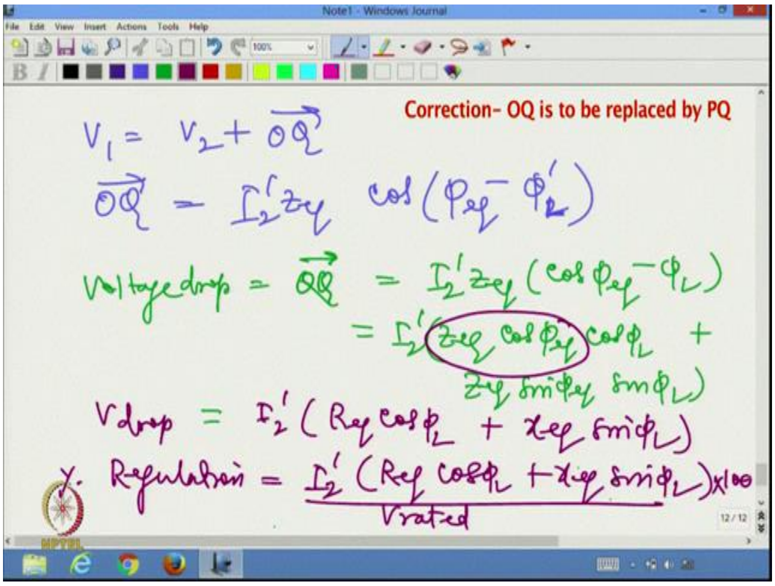

So, I can write now V_1 = V2+OQ, so I am writing basically what is OQ? So OQ, so let me write, what is OQ? OQ if you look at, $OQ = I_2^{'} Z_{eq} \cos(\phi_{eq} - \phi_L)$. So that will give me this is $OQ = I_2^{'} Z_{eq} \cos(\phi_{eq} - \phi_L)$, this is what is going to be my, rather ϕ_L or whatever, this is what is going to be my OQ. So I would say voltage drop is equal to $OQ = I_2^{'} Z_{eq} \cos(\phi_{eq} - \phi_L)$, so in which case I can write this to be $I_2^{'}Z_{eq}$, you guys know better the expansion of $\cos(\phi_{eq}-\phi_L)$, so I can write $I_2^{'}(Z_{eq}\cos\phi_{eq}\cos\phi_L + Z_{eq}\sin\phi_{eq}\sin\phi_L)$ and cos (A- B)=cosA cosB+sinA sin B.

So, we are essentially going to write this $Z_{eq}\cos\phi_L$, we wrote it already in short-circuit test also that will be essentially $R_{eq}\cos\phi_L + X_{eq}\sin\phi_L$. So, this is the voltage drop, $\%\text{regulation} = \dfrac{I_2^{'}(R_{eq}\cos\phi_L + X_{eq}\sin\phi_L)}{V_{rated}} \times 100$, this is what is regulation, this is an approximate value of regulation.

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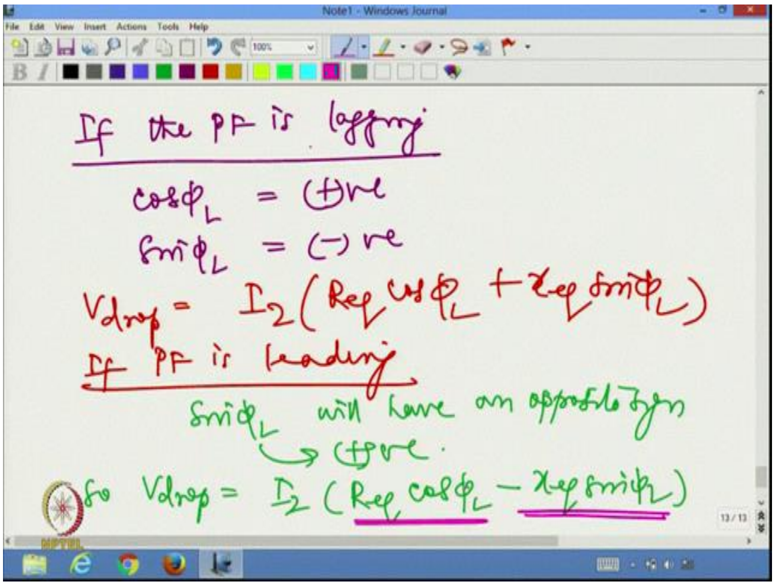

Now with this we should be able to say if the power factor is lagging $\cos\phi_L$ is positive, whereas $\sin\phi_L$ is going to be negative, And please remember this expression was derived assuming a lagging power factor, that is why we got $\cos(\phi_{eq}-\phi_L)$. So we should be able to write $V_{drop} = I_L(R_{eq}\cos\phi_L - X_{eq}\sin\phi_L)$, whereas if the power factor is leading, I am going to have $\sin\phi_L$ will have an opposite sign, that is I am going to have this to be positive, so $V_{drop} = I_2(R_{eq}\cos\phi_L - X_{eq}\sin\phi_L)$.

So whenever we have a leading power factor, it can so happen that $R_{eq}\cos\phi_L + X_{eq}\sin\phi_L$ can cancel out with each other and we may not get any drop at all, by chance, if $X_{eq}\sin\phi_L$ is dominating overall $R_{eq}\cos\phi_L$ than may be we may get a positive regulation.

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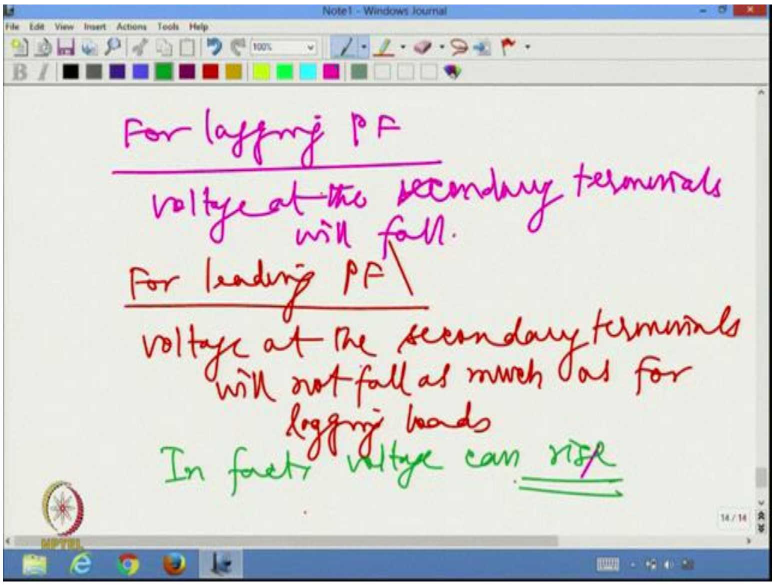

So for lagging power factor voltage at the secondary terminals will fall, whereas for leading power factor voltage at the secondary terminals will not fall as much as for lagging loads, in fact, the voltage can rise, it may not even fall it can rise. So, whenever we have capacitive load we will see that the secondary voltage rises.

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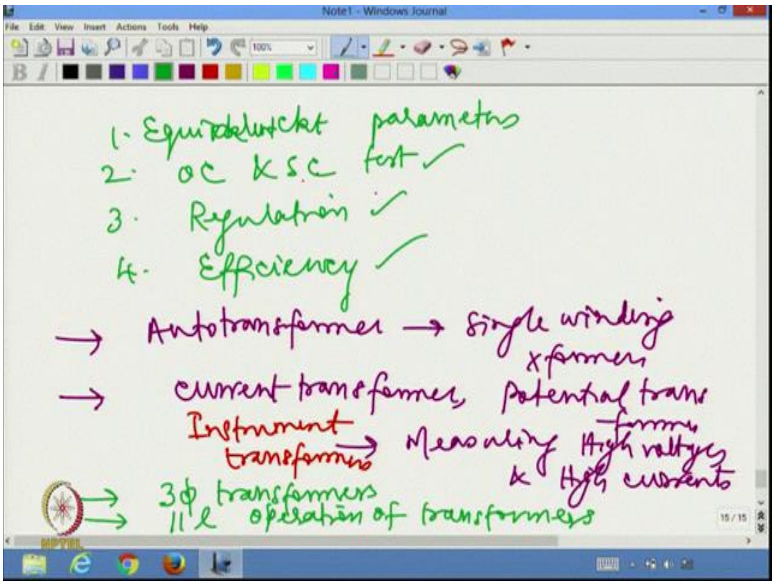

So, this completes our discussion on what we had seen so far is equivalent circuit parameters. We had seen OC and SC test, we had seen regulation, we had seen efficiency. Now a few more topics that are pending that we need to address are, autotransformer or variac, how we are going to really, connect this will be a single winding transformer, we will be looking at this, then we will also be looking at current transformer and potential transformer which we would use basically for measuring voltages and currents, so generally they are used in instrumentation so sometimes they are also known as instrument transformers.

After finishing with these things, we will also look at three-phase transformers, then last but not the least will look at parallel operation of transformers. So this may take like 3 to 4 lectures, so will be discussing those as well after this, So these four topics will take up after finishing up, this having finished equivalent circuit parameters, open circuit, short-circuit test, regulation and efficiency, we would be taking up these things, we would work out some more problems on OC, SC test, regulation and efficiency, which would make actually the understanding much clearer than what you have understood already, So, so far so good.