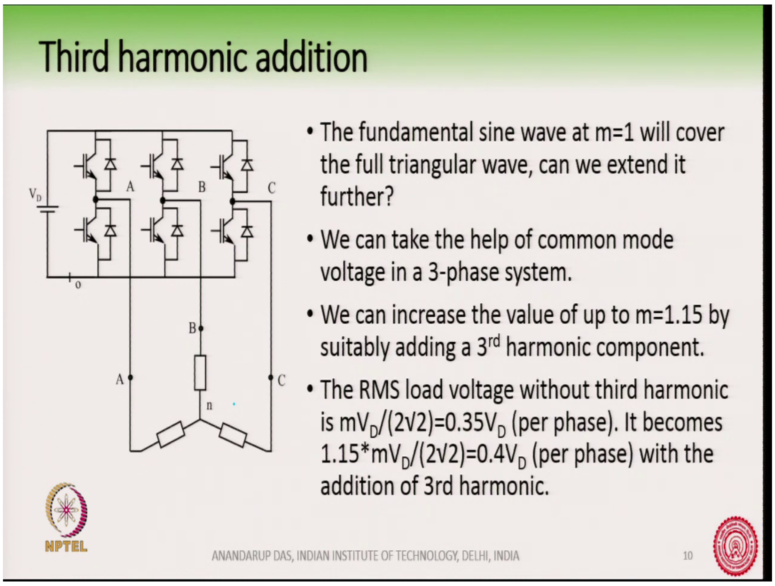
High Power Multilevel Converters - Analysis, Design and Operational Issues

Dr. Anandarup Das
Department of Electrical Engineering
Indian Institute of Technology, Delhi

Lecture – 05
Third harmonic addition in Sine PWM

(Refer Slide Time 00:18)



Next, we will study about Third harmonic addition in Sine triangle PWM. The objective of adding a third harmonic into the sine triangle PWM is to increase the DC bus voltage utilization of the converter. So, you can see here there is a 3 phase converter drawn here, and for with sine triangle PWM or with sinusoidal PWM. We get a certain value of load voltage given a certain value of DC bus voltage.

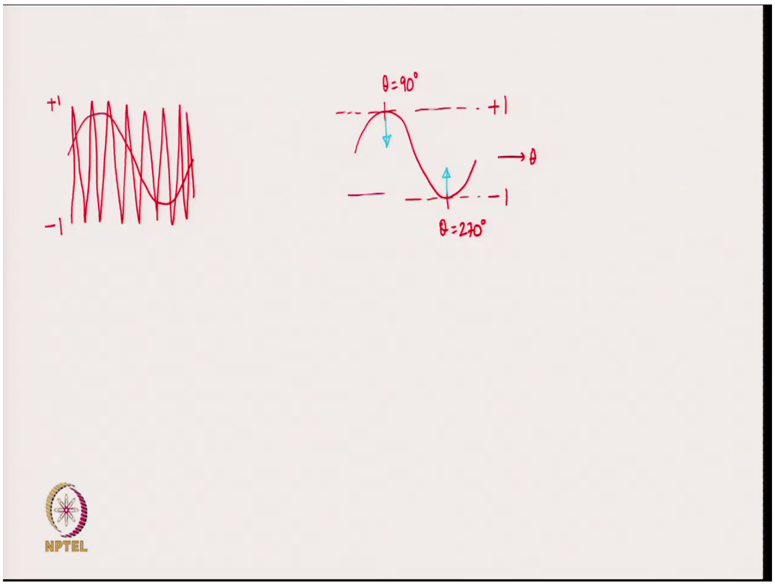
With the help of third harmonic addition in the modulating waveform; we can get more AC voltage across the load from the same DC bus. So, this is the main purpose of adding the third harmonic. We get more AC voltage on the load from the same DC bus as compared to sinusoidal PWM. So, we will see then how can we do this ok, what is the way of adding the third harmonic.

So, before we proceed, I should clarify that; in sinusoidal PWM, we have seen earlier that the peak voltage that we can get is m times v d by 2 and with square wave mode of operation, we can get 1.27 times this value. So, in linear modulation we get, suppose if we get 1 per unit of voltage in square wave, we can get 27 percent more than that. Then, why do not we go into square wave or even why do not we go into over modulation? So, between 1 and 1.27 is the over modulation region.

So, why do not we go there? Primarily, because this modulation or square wave mode of operation generates a lot of unwanted low frequency harmonics. So, we try to avoid the going into over modulation region. So, it is a natural question is that; what is the cost of it that we lose this region between 1 and 1.27 because, square wave gives us the maximum voltage. So, the question is; can we get more voltage out of the converter? So, this is the basic question, can we get more voltage out of the converter.

In fact, it is yes, we can get by adding the third harmonic and we can get up to 1.15. So, 15 percent more than the sine triangle PWM.

(Refer Slide Time 03:28)



So, how we do it is what we will now study. So, we will first see; how we do the sine PWM. So, here we have the sinusoidal modulating waveform and we have a high frequency carrier wave right. So, the here we see that; if m is 0 the sine wave is having 0 amplitude whereas, if m equal to 1. So, if this the height of the triangle wave is between plus 1 and minus 1.

So, when the sine wave reaches the value of 1, then it touches the carrier wave. Beyond which we go into over modulation. Below which mean we are into the linear modulation range. So, if I can draw the sine wave once more and I can draw this peak of the sine wave, then I can say this is plus 1 and minus 1 and if this is theta then we know that at theta equal to 90 degree and theta equal to 270 degrees. We get the magnitude of the sine wave becomes 1 or minus 1.

Now, if you analyse this carefully; you will see that as the amplitude of the sine wave increases and when it becomes equal to 1, the sine wave touches this plus 1 and minus 1 the ceiling plus 1 and minus 1 heights only at theta equal to 90 degree and theta equal to 270 degrees. So, but other for other regions for other points, for example here or here in these regions; the sine wave is much below than the plus 1 or minus 1.

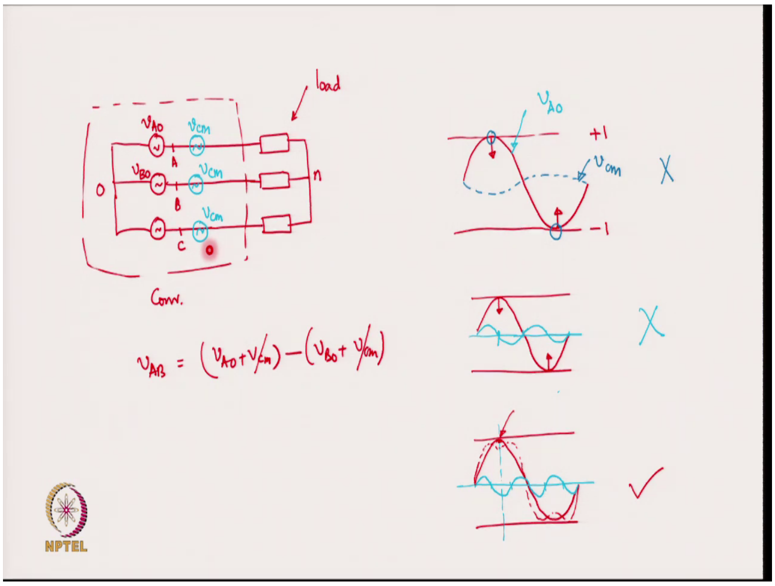
So, only because the sine wave is touching the plus 1 at a particular theta equal to 90 degree, we say that; we have reached the end of the linear modulation range and we then stop, increasing the sine wave any further because, we do not want to have the unwanted lower order harmonics. But this is a very disadvantageous situation, because only at 1 or only at theta equal to 90 and theta equal to 270; we touch the waveform we touch the upper and lower barriers and then we stop increasing the sine wave any further.

Now, therefore, what is the alternative solution? If somehow we can the sine wave down. For example, if we can push this part down and if we can push this part up some way, by some way if we can push this down and that part up. Then it may be possible to keep the sine wave within the plus 1 and minus 1 region ok. Then we are still in the linear modulation region.

Now, how can we do that? We and do that by adding another waveform. We can add another waveform to the original sine wave or to the original sinusoidal modulating waveform. So, this another waveform we will see is the third harmonic that we are going to add. Now, why third harmonic, why not any other harmonic?

So, let us see, if what will happen if we try to add any harmonic before we go into that we must first try to see the equivalent circuit of this converter ok. So, we have a 3 phase converter and this is consisting of three half bridges V A o, V B o, V C o are the voltage produced by this converter. So, what is the equivalent circuit? The equivalent circuit of this converter is something like this.

(Refer Slide Time 08:15)



So, this is the point A B and C and this is the o and this is the n. So, this is the load here and this side is the converter. So, this here is the converter. This is the converter equivalent circuit and this is the load side, the right hand side is the load. So, this voltage here is V Ao and similarly V B o and V C o. Now, when we add any voltage to V Ao or the when we add. So, when we said that we will add some voltage to the original sine wave will add some voltage to the original sine wave.

So, as to push this point down and that point up ok. We are trying to push the point down, now whatever extra modulating wave we are going to add remember that must be a common mode voltage ok. It must be a common mode voltage. Why it should be a common mode voltage? Because, if we add a suppose add the common mode voltage. So, we are generating this additional voltage here.

So, we say V c m; that is the common mode voltage ok. So, we are adding the common mode voltage the same voltage to all the three phases it must be done because the common mode voltage, even if you add to the original V Ao waveforms. So, this is the; this is the V Ao modulating waveform ok. V Ao reference waveform and to which we are add going to add a common mode voltage.

Now, the common why the it is a common mode voltage? Because, in the line; the common mode voltage will vanish and hence across the load there will not be any common mode voltage right. So, because suppose, we have added V C m as a common mode voltage to V Ao V Bo V Co.

So, then we can see that suppose this is VBo then the line voltage VAB will be equal to V Ao plus V C m minus VB o plus V C m and hence you can see that; the common mode voltage cancels out. So, whatever we are going to add here in this sine wave. So, as to push the theta. So, as to push the sine wave down here and up here it must be a common mode voltage.

So that it does not appear in the line and if it does not appear in the line; it will not appear across the load. Because, the point n and o will be isolated this is isolated. So, no current can flow between the point n and o. So, this voltage V common mode. So, what can be; what can we add to this voltage. so, as to push the sine wave up and down.

So, let us start with the first option. So, we can start say by saying that ok. We add this voltage ok, because we want to push this part of the sine wave. We want to push this part of the sine wave down and this part of the sine wave up. So, this waveform which we have added is same frequency waveform, fundamental waveform; however, you can understand that although this waveform, this waveform here whatever we have added will push down the a phase waveform stroop it will push it down from its peak. But, for the other two phases B phase and C phase which are somewhere here and here.

This voltage this V c m is going to push them up. So, although, we can push the A phase down we will be pushing the B and C phases up and beyond there plus 1 and minus 1 limits which is not acceptable, because they will go into over modulation region. So, adding this V C m as the fundamental is not allowed. So, this will not be possible to do. So, let us see the second option.

So, again we have the sine wave here and we want to push this down and down. Let us add a second harmonic. So, how does the second harmonic look like? So, we can add a second harmonic like this right. We can add a second harmonic like this, but you see here the second harmonic is not helping us in pushing the peak down or this lower peak up because its 0 crossing is at the same place. So, second harmonic addition is not possible.

Now let us next we add the third harmonic. So, this is our fundamental voltage, and let us add the third harmonic this is the 0. And so, now we see that; yes indeed at this point here theta equal to 90 degrees at this point theta equal to 90 degrees, this third harmonic is pushing the fundamental down. So, if you if the if you add the V C m to the 3 phases, all 3 phases because it is a third harmonic.

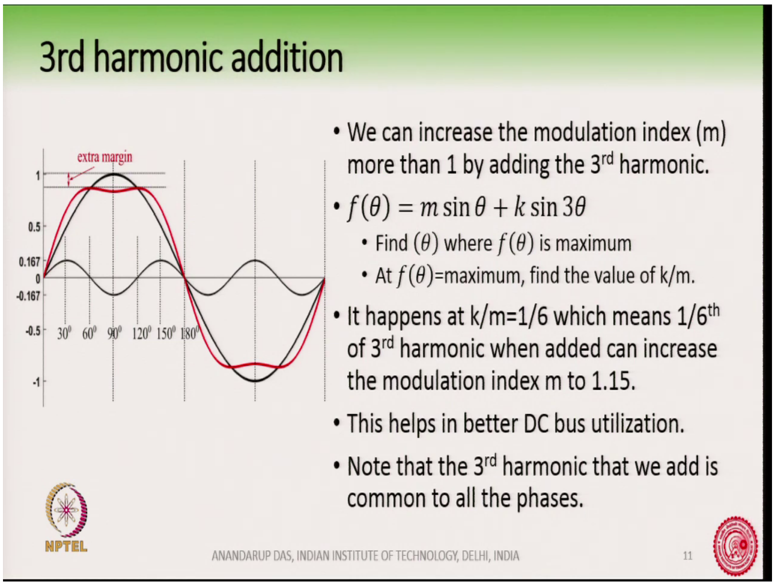
So, it is symmetrical about the 120. So, all 3 phases their peaks will go down at theta equal to 90 degrees A phase B phase and C phase ok. The peak of B phase will also go down exactly at theta equal to 90 and 270 degrees for their own reference. So, we see that; the third harmonic, if we add to this waveform then it is possible to reduce this peak and so, this waveform will be somewhat like this. The resultant wave form.

The resultant wave form will be somewhat like this. We will show you the resultant wave form little later. So, we see that in this circuit; the addition of the third harmonic here. The addition of the V C m as a third harmonic helps us to push the resultant wave form a little bit down from reaching the plus 1 and minus 1 at modulation index 1. In fact, this is the reason; where we can go a little bit higher than m equal to 1 and with there we can see we will see the mathematics now.

With the help of this third harmonic addition; the modulation index can in fact, be extended up to m equal to 1.15 ok. So, the RMS load voltage without third harmonic, we have seen it before is m V D by 2 root 2 that is 0.35 V D. This is without third harmonic addition and with third harmonic addition the m value can go to 15 percent more and so, we can get 0.4 V D with the addition of third harmonic.

So, summarize m equal to 1; that is the end of sine triangle modulation or sinusoidal PWM and with the help of the third harmonic; we can go up to m equal to 1.15. Beyond which we go into over modulation and eventually ending up in square wave mode of operation where we get the maximum voltage possible from the converter that is 1.27 times what we get in sinusoidal PWM, but with the penalty of lower frequency harmonics.

(Refer Slide Time 19:03)



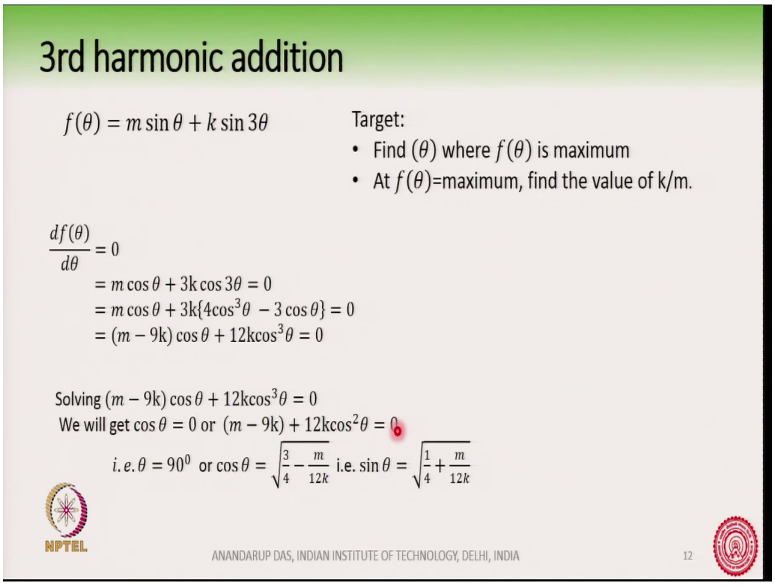
So, how do we know now, what is the exact magnitude of third harmonic we can add? What is the exact magnitude of third harmonic that we should add so as to push or to get the maximum out of the sinusoidal PWM. So, here we see that there are 2 waveforms; this is the fundamental waveform and then we also have the third harmonic which we have added. And the red coloured waveform is the a resultant waveform after adding the third harmonic.

So, we can write the equation of this waveform f theta equal to m sine theta plus k sine three theta. Where, m is the amplitude of the fundamental and k is the amplitude of the third harmonic. So, what are we trying to find is; find the value of theta where f theta is maximum and at f theta equal to maximum find the value of k by m.

So, we are trying to find out what is the ratio of k by m. So that, we will know that given a value of m, how much third harmonic I can inject. So, that I will never cross the plus 1. So, we our aim is we will not never cross this plus 1 here by adding the third harmonic. So, we are trying to find out the value of k by m.

So, the result is already written here it will happen net k by m equal to 1 by 6. Which means, 1 6th of the fundamental is what we should add to increase the modulation index to 1.15. Again I must say that the third harmonic that we add is common to all the phases it is a common mode voltage.

(Refer Slide Time 21:11)

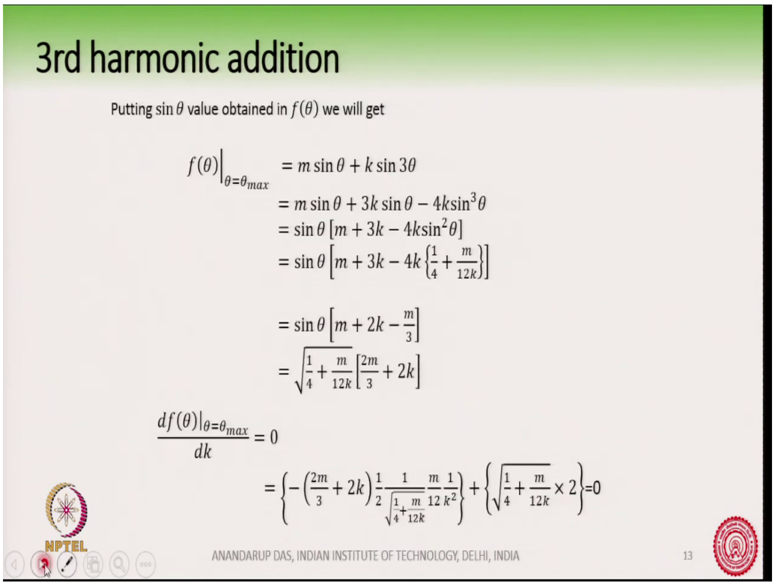


So, let us do the mathematics here. So, f theta is equal to m sine theta plus k sine 3 theta and our target is to find out where f theta is maximum first. So, if I differentiate it d f d theta and equal to 0 and differentiate this expression which is shown in some few steps, we come to this equation here m minus n k cos theta plus 12 k cos cube theta.

So, if I equate this to 0 and then solve it; we will get cos theta equal to 0 or m minus n k plus 12 k cos square theta equal to 0. So, if cos theta is equal to 0 which means theta equal to 90 degree and we can see from this waveform that at theta equal to 90 degree basically we are getting a minimum of the resultant. So, we will not take theta equal to 90 degree ok.

We will take the other solution m minus n k plus 12 k cos square theta equal to 0 and then we will first solve the value of cos theta and sine theta here which we will utilize here.

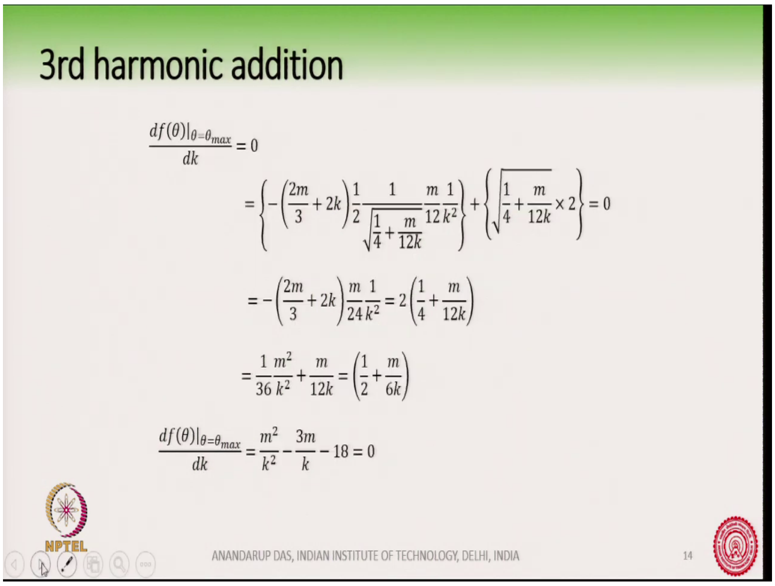
(Refer Slide Time 22:27)



We will put it back at theta equal to maximum; the value of f theta which is m sine theta plus k sine 3 theta and expanding we get something like this here and we have already obtained the value of sine theta at theta equal to maximum earlier and we substitute that value of sine theta here and so, we get this expression here.

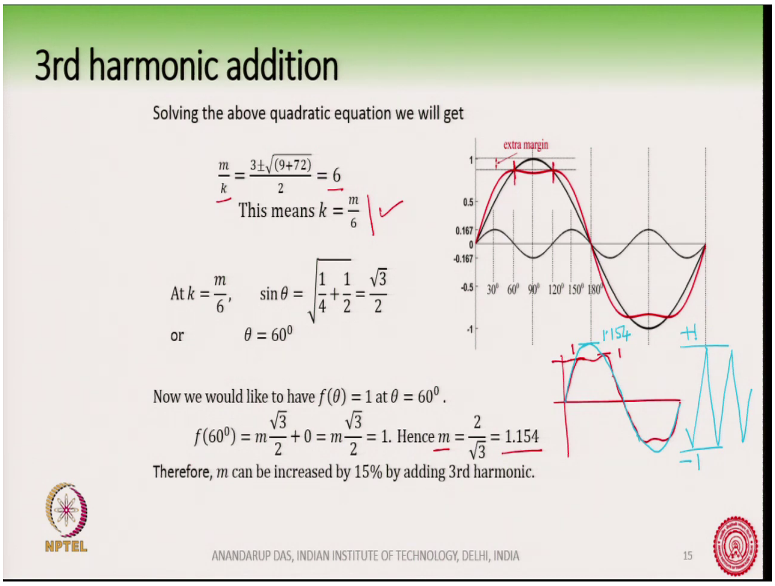
Now, this expression; we will differentiate once more to find out what value of k will maximize this. So, we will d f theta by d k at theta equal to theta max we will put it 0 and we will differentiate it once more which is shown here.

(Refer Slide Time 23:15)



From which by several steps; we get to an equation here like this which ultimately comes up to be like this, m square by k square minus 3 m by k minus 18 equal to 0 ok. So, this is the expression which gives us the value of k ok.

(Refer Slide Time 23:45)



So, if you get to this equation, then by solving this equation; we see that m by k, we see that m by k is equal to 6 which means k equal to m by 6 right. So, it means that the magnitude of the third harmonic that we add is one sixth of the magnitude of the fundamental that will be taken and so, at k equal to m by 6 the value of sine theta comes out to be theta equal to 60 degree.

So, which means that this maximum will happen at theta equal to 60 degree as you can see in this curve; that at theta equal to 60 degree we see and also at theta equal to 120 degree that is here. We see that the resultant wave form is actually maximizing ok. Now, what value of this resultant wave form we should have so, as to keep us in the linear modulation.

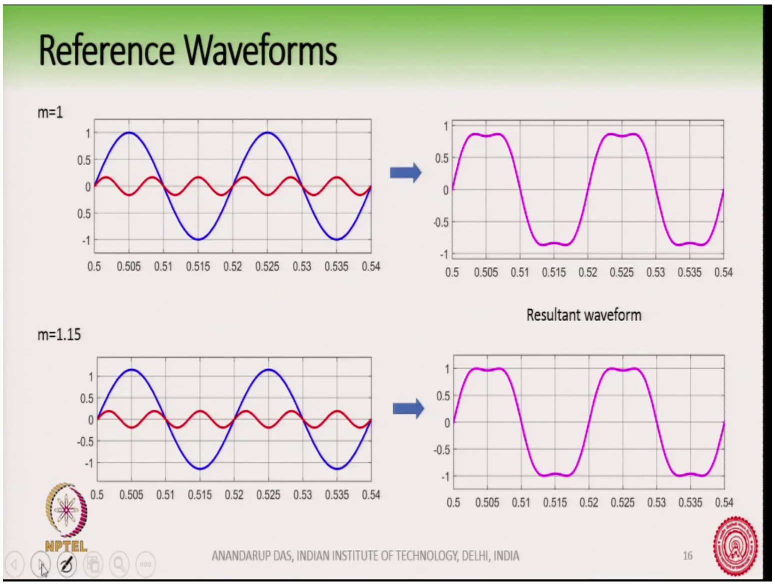
Of course we would like to touch the plus 1 at this maximum value of the resultant wave form ok. So, we would like to have at theta equal to 60 degree f theta equal to 1 because that is the maximum we can get ok. So, if we equate f theta equal to 1 at theta equal to 60 degree then by substituting, we find that m equal to 1.154.

So, it means that; this value the resultant value, the resultant value here, the resultant value here will touch 1. This is what we expect the resultant value should touch 1 here this 1 will touch at 1 when m equal to 1.154; that means, the sine wave is having an amplitude of 1.154. So, we see that the sine wave is now more than 1 in fact, it is 15 percent more.

But with the help of the third harmonic; we have made the resultant inside the plus 1 and minus 1, because our carrier is inside here. This is where our carrier is. We do not want to go into over modulation, we will always keep ourselves between plus 1 and minus 1, but the fundamental is actually more than that. So, earlier we had also said that whatever is the fundamental voltage in the pole voltage appears across the load.

So, if we have 15 percent more in the fundamental in the pole voltage it will appear across the load. So, therefore, with the help of this third harmonic; we are able to get 15 percent more voltage from the same DC bus into the load. So, that is the whole idea.

(Refer Slide Time 27:27)



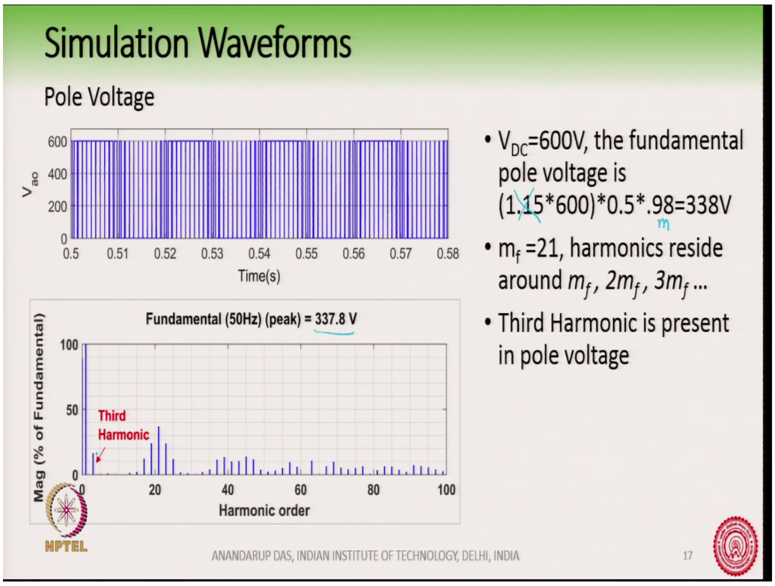
So, here I here we are showing you the two cases where first one is m equal to 1 and the second case is m equal to 1.15. These are the two cases we are going to see. So, if m equal to 1, we can add one sixth of the third harmonic which I have shown. This is the one sixth of m equal to 1. So, that is about 0.166.

So, when we add this to the resultant wave form you can see is like this, but interestingly you see this value here. This value the peak of the resultant is less than 1. So, there is something which we are getting additionally ok. This is the extra voltage that we are getting by adding the third harmonic. So, if we get this extra voltage, of course, I would like to increase the modulation index further.

So, I have increased the modulation index further to 1.15 and now the fundamental voltage is now more than 1 here and the resultant wave form you see is touching 1 and 1 here ok. So, we are basically able to go 1.15 times the modulation index. Again I it is important for you to understand that this is a common mode voltage. At each instant of time, the same voltage is being applied to A phase B phase and C phase. It is a common mode voltage.

So, it will not allow any current to flow from the load neutral to the source or the inverter neutral here. So, let us see some of the simulation waveforms for this addition of the third harmonic.

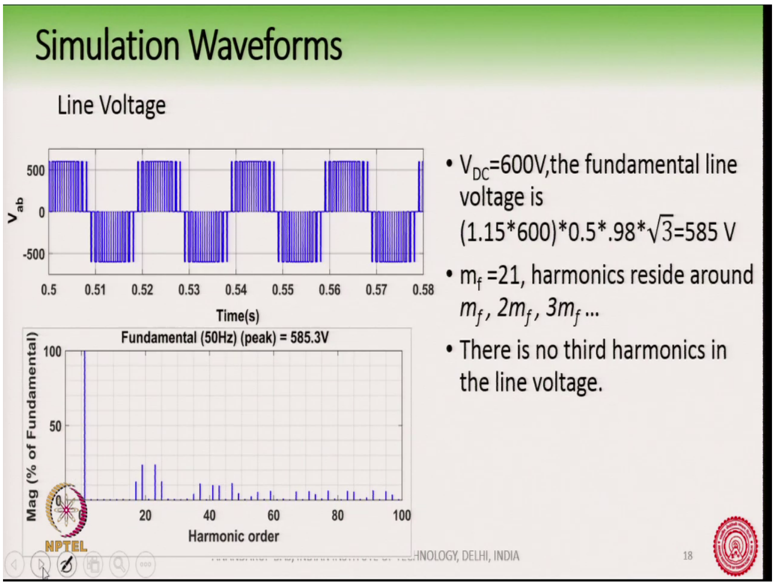
(Refer Slide Time 29:38)



So, at first; we will see the pole voltages. So, we have V DC equal to 600 volt and the fundamental voltage, fundamental pole voltage that we can get with the addition of the third harmonic will be 1.15 times VD by 2; that is 600 into 0.5 into 0.98. 0.98 because, we have taken in this example m equal to 0.98.

So, this is m here. So, m equal to 0.98 and so we get about 338 volt. If there was no third harmonic then we would not have got this factor here. We would have got 15 percent less. So, if you see the harmonic spectrum of this pole voltage we see that the fundamental voltage is 337.8, it is almost like 338 volt and we see that there is a third harmonic here. This third harmonic is one sixth of the fundamental, but it is present in the pole voltage, it is present here this is the third harmonic.

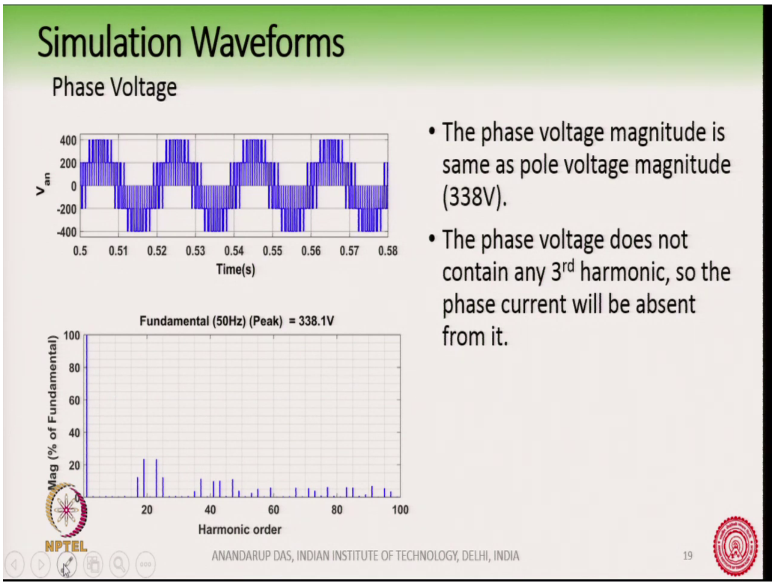
(Refer Slide Time 31:09)



But let us see in the line voltage; in the line voltage which is again the waveform is shown here. We see there is no third harmonic here. The third harmonic in this place; the third harmonic which was earlier present here. The third harmonic is has vanished in the line voltage which is what we desire. We do not want the third harmonic to come across the load.

So, with this V DC at 600 volt, the fundamental line voltage is root 3 times what we obtained from the pole voltage and will be close to 585 volt. And so, there is no third harmonics in the line voltage.

(Refer Slide Time 32:00)



And what will happen in the phase voltage? We see that in the phase voltage the phase voltage magnitude, the fundamental phase voltage magnitude is same as the pole voltage magnitude you see here, the fundamental phase voltage is again 338 volt here. But since the line voltage does not contain any third harmonic it cannot contain any third harmonic, because it will always cancel out.

So, the phase voltage on the load phase voltage will also not contain any third harmonic as you can see from the harmonic spectrum. So, in this way; we see that by cleverly adding the third harmonic in the modulating waveform, we can get a third harmonic voltage in the pole wave pole voltage waveform but it will disappear in the line voltage and eventually it will disappear in the phase voltage, but what is the benefit?.

The benefit is that; the we are getting more fundamental voltage from the same DC bus by adding the third harmonic technique.