

High Power Multilevel Converters - Analysis, Design and Operational Issues
Dr. Anandarup Das
Department of Electrical Engineering
Indian Institute of Technology, Delhi

Lecture – 06
Introduction to Space Vectors

Today, we will talk about Space Vector PWM. Space vector PWM is one way of or one technique of switching the converter.

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Space vectors

- The origin of space vectors lies in rotating mmf in machines.
- The resultant mmf for a three phase system is a rotating mmf having a fixed magnitude and direction at every instant of time.
- Space vector is a mathematical concept which is useful for visualizing the effect of three phase variables in space.

The diagram shows three vectors labeled A, B, and C originating from a common point. Vector A is horizontal to the right, B is at an angle above A, and C is at an angle below A. A resultant vector is shown as a red arrow pointing upwards and to the right. The NPTEL logo is in the bottom left, and the IIT Delhi logo is in the bottom right. The text 'ANANDARUP DAS, INDIAN INSTITUTE OF TECHNOLOGY, DELHI, INDIA' and the number '2' are at the bottom center.

So, before we talk about space vector PWM let us understand, what are space vectors. So, space vectors the origin of the space vectors the origin lies in rotating mmf in machines. The concept of space vectors has come from the rotating m m f in machines. So, we have seen, we

know from our basic understanding of machines that; suppose, we have three sets of windings in for example, the stator of the of an induction motor.

If we have three sets of windings which are spatially displaced by 120 degrees, for example, let me give you an example here. So, we have three sets of winding for example, like this these may be termed as the A phase, B phase and C phase. These three windings are spatially displaced by 120 degrees. If we send a current through these three windings which are 120 degree apart in time.

In that case, so the current through each of these winding produces its own mmf which is oscillating in nature. However, the resultant of the three mmf is something which is a rotating mmf here, the resultant. So, the resulting mmf are or is having a fixed magnitude and deduction ok. So, it has a magnitude and it will rotate at the same frequency at which A B C phase is are excited.

This we know from the basic understanding of electrical machines. The space vector concept also comes from there. What is the advantage of this rotating mmf concept? If you see it conceptually it is very advantageous to have a single quantity representing the three the variations happening in the three phases, three axis. So, space vector is a concept in which we are kind of like representing three variables by a single space vector.

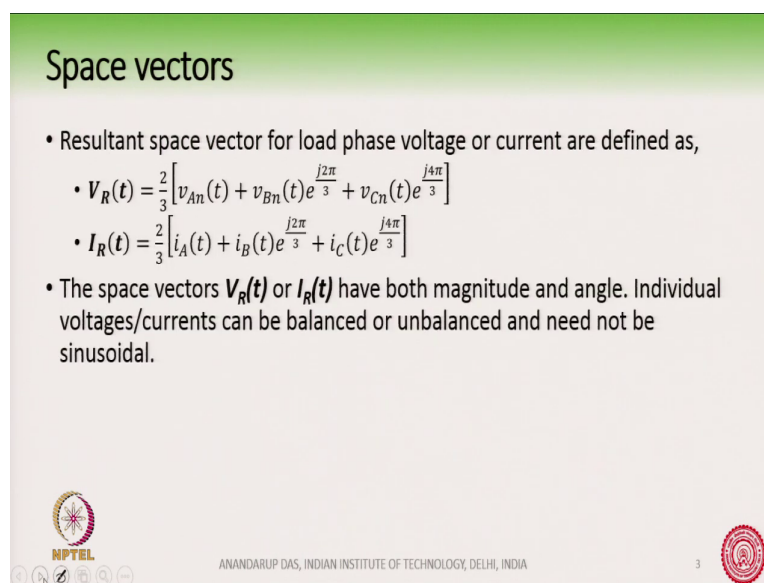
So, this is useful in many many applications. First and most useful is that you are no longer going to deal with three different quantities in three different phases. So, you can represent it by a single quantity and additionally it is very easy to visualise this quantity. For example, we can easily visualize the resultant air gap resultant flux is rotating in the air gap of the machine.

So, this is a single quantity rotating in the machine air gap. So, similarly space vector is also a very useful way of a representing three variables and it is very helpful in visualisation of the resultant of the three phases working together. But, space vector is basically a mathematical concept the, for example, the flux in a machine is really a physical quantity; which is residing

in the air gap of the machine. But, space vector is not necessarily a physical quantity, it is a mathematical concept.

So, for example, we can define the space vector of voltage, we can define the space vector of current, but they are not necessarily a physical quantity represented in an actual space. So, these are mathematical concept.

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Space vectors

- Resultant space vector for load phase voltage or current are defined as,
 - $V_R(t) = \frac{2}{3} \left[v_{An}(t) + v_{Bn}(t)e^{\frac{j2\pi}{3}} + v_{Cn}(t)e^{\frac{j4\pi}{3}} \right]$
 - $I_R(t) = \frac{2}{3} \left[i_A(t) + i_B(t)e^{\frac{j2\pi}{3}} + i_C(t)e^{\frac{j4\pi}{3}} \right]$
- The space vectors $V_R(t)$ or $I_R(t)$ have both magnitude and angle. Individual voltages/currents can be balanced or unbalanced and need not be sinusoidal.

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So, let us see what we, how do we define the space vector. So, space vector for example, of load phase voltage is shown here. This is the resultant space vector or the space vector is defined. So, it is a combination of three quantities and these three quantities; since, we are talking about load phase voltage space vector.

So, $v_{A n}$ is the A phase load voltage $v_{B n}$ is the B phase load voltage and $v_{C n}$ is the C phase load voltage. And V_R multiplying $v_{B n}$ and $v_{C n}$ by this operator here. And this operator basically helps us to represent the three quantities by a single uniform vector; that is $V_{R t}$ ok. Basis basically, this $e^{j 2 \pi / 3}$ corresponds to the axis B phase axis and $e^{j 4 \pi / 3}$ corresponds to the C phase axis.

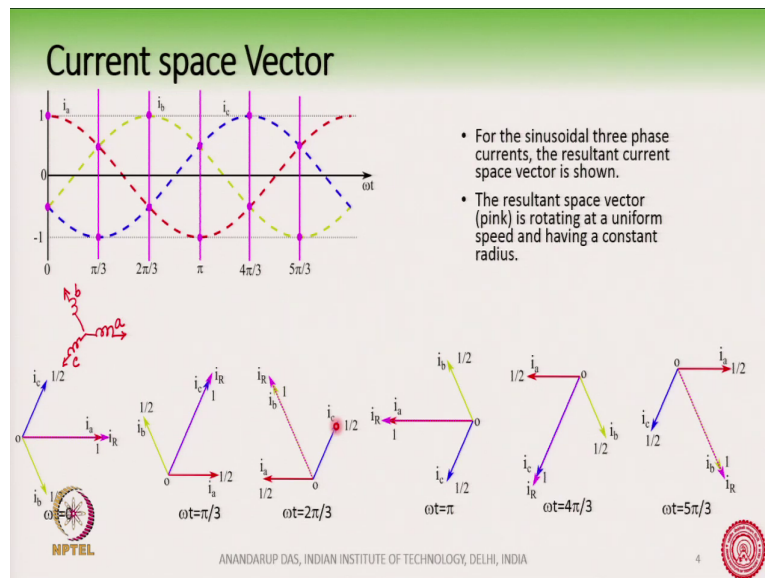
So, basically, we are defining the resultant space vector as the instantaneous magnitudes along the three phases; phase A, phase B, phase C multiplied by the rotation of this axis into one single phase ok. So, this is what defines space vector. Additionally, we have multiplied, we have a multiplying factor of 2 by 3 in the front and that 2 by 3 is sometimes, people use 2 by 3 and sometimes some researchers or some books you will find that they have not used 2 by 3.

So, 2 by 3; in this course, we are using this 2 by 3 because this is a power invariant transformation. The power from the space vector, when you multiply voltage space vector with the current space vector then you get a power invariant transformation. So, that is why we are using the 2 by 3.

So, in a similar way like we have defined the space vector of say load phase voltage; we can also define the space vector of current ok. The space vector of current is the instantaneous values of the three phase currents $i_{A t}$ $i_{B t}$ and $i_{C t}$ and then $i_{B t}$ and $i_{C t}$ are multiplied by $e^{j 2 \pi / 3}$ and $e^{j 4 \pi / 3}$.

So, this resultant $v_{R t}$ and $i_{R t}$ have both magnitude and angle. So, they are called space vectors. But, the individual quantities this is for example, i_A i_B i_C they can be balanced or unbalanced and they need not to be sinusoidal. So, it can be sinusoidal, but it is not necessary to have this quantities three quantities a sinusoidal. So, we see that space vector is kind of like a mathematical concept ok, but it is very useful in visualising ok. How the resultant of three phase systems is appearing on the plane ok.

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So, in order to understand or in order to visualise it better; we have taken this current space vector here ok. So, here, on the top we see that; there are three currents for example, load currents and these are the these are we have taken sinusoidal sinusoidally varying currents i_a i_b i_c , three phase current.

Now, the result in space vector for this three phase current is shown by this diagrams here at different time instant ok. For example, if we see that at ωt equal to 0 that is at 0 time here, ωt equal to 0. We see that; i_a affected has the i_a the instantaneous value of i_a at t is having the maximum value while i_b and i_c are negative.

But, they are equal negative values. So, if we can visualise the three axis, which can be written as. So, we can visualise that this is the a phase axis. So, this is the a phase axis, this is the b phase axis and this is the c phase axis ok. So, if we have the currents; which at ωt

equal to 0 is something like this that i_a is maximum while i_b and i_c are negative equal values. So, at that point then i_a will have the maximum value along this axis.

Whereas, i_b and i_c are both negative. So, i_b has a negative half amplitude on this direction downward. Because, i_b axis in this direction. So, i_b is downward and i_c is again negative, but in this direction, because the c axis is like this ok. So, we see that the resultant. When you apply the resultant; which is found out from this formula here $\frac{2}{3} i_a t + i_b t e$ to the power $j 2 \pi$ by 3 and $i_c t e$ to the power $j 4 \pi$ by 3. If we apply the formula; the resultant is this i_R here, which is having the same amplitude as i_a ok.

Now, if we take another time instant, say ωt equal to π by 3 then we see here that; i_a has reduced to lower value while i_b has increased and i_c has reached the negative maximum. So, the resultant space vector so, i_a has reduced in magnitude. So, it is still in this direction i_b is positive and it has become like this while i_c is negative peak.

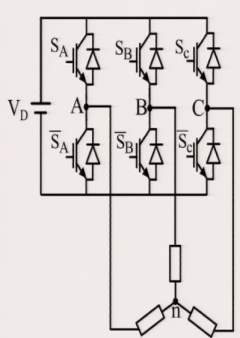
So, which means c axis is in this direction c is in this direction. So, i_c peak is like this here. And therefore, the resultant i_R is lying here. So, at ωt equal to π by 3 the space vector i_R initially was here at ωt equal to 0 and has moved by π by 3 angle to this position. The magnitude of i_R remains the same as before. So, in this way if we go on doing the analysis at other points of time.

We will see that; the resultant vector is basically rotating in a counter clock wise direction having or keeping the same amplitude all the time and rotating at the same frequency by which this is dictated by the frequency of the $i_a i_b i_c$ signals. And you can understand from here that; this concept here is very similar to the rotating mmf concept in the electrical machines.

So, we are basically now, representing $i_a i_b i_c$, these three quantities by a single space vector which is i_R having magnitude at an angle at different points of time. So, this is the major advantage of having this space vector concept ok.

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Space vectors



- The pole voltage of one phase of the converter has two switching states: 1 ($=V_D$) and 0 ($=0$).
- The converter has total eight switching states ($2*2*2=8$). These are: (000,111,100,110,010,011,001,101).
- There are six active vectors and two zero vectors.
- What is the load phase voltage space vector for 100 combination?

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Now, for a converter which is shown here; we can also have the space vector of pole voltages space vector of line voltage, we can have space vector of load voltage and etcetera and load space vector of current etcetera. So, space vector can be any three phase variable. Now, here when we are talking about the space vector we are basically interested into the load voltage space vector; that is the voltage applied on the load that is V_a V_b V_c .

So, this is primarily, the space vector in which we are interested in, because this is what we are actually extracting out of the converter and get an impressing on the load. So, then we analyse the space vector of the load phase voltages; first let us understand that what can the converter produce ok. Now, the pole voltage of one phase of the converter has basically two switching states. As we have seen here, this any phase S_A and this upper and lower switches are complementary.

So, this voltage V_{AO} with respect to the O point which is the negative of the DC bus the V_{AO} voltage can be either V_D or 0. So, we define two switching states of this phase A of the converter. So, V_D is taken as 1 and 0 is taken as 0. So, there are two switching states from each phase of the converted as the two switches top and bottom switches are switching. So, therefore, if we since; we have three phases. So, the converter will be having 2 into 2 into 2, 8 switching states here ok. So, these we can also write is as like this like 0 0 0, 0 0 0 means, A phase is having a switching state of 0 B 0 C 0.

While if you take 1 0 0, which means a phase V_{AO} is 1 and V_{BO} and V_{CO} are 0 and 0. 1 corresponds to V_D voltage. So, this state for example, that this switching state 1 0 0 will be will happen when the upper switch that is S_A in phase A is turned on while $S_{B\bar{}}$ here and $S_{C\bar{}}$ here will be turned on.

The lower two switches in B and C phases will be turned on and we can get this 10 switching state ok. Now, we can also understand that. So, that will be, there will be total 8 switching states out of these 8 switching states the 000 and 111, these are called the zero vectors or zero switching states ok. We will talk about this six active vectors and two zero vectors a little bit later when we talk about the space vector diagram. Now, what is the load phase voltage space vector for 100 combination.

So, these are the eight switching states possible from the converter and let us find out what is the load phase voltage space vector for any one of this combination.


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Space vector for 100 combination


- $v_{AO}(t) = VD$, $v_{BO}(t) = 0$, $v_{CO}(t) = 0$
- $v_{An}(t) = \frac{2}{3}v_{AO}(t) - \frac{1}{3}v_{BO}(t) - \frac{1}{3}v_{CO}(t) = \frac{2}{3}VD$
- $v_{Bn}(t) = \frac{2}{3}v_{BO}(t) - \frac{1}{3}v_{CO}(t) - \frac{1}{3}v_{AO}(t) = -\frac{1}{3}VD$
- $v_{Cn}(t) = \frac{2}{3}v_{CO}(t) - \frac{1}{3}v_{AO}(t) - \frac{1}{3}v_{BO}(t) = -\frac{1}{3}VD$

- $V_R(t) = \frac{2}{3} \left[v_{An}(t) + v_{Bn}(t)e^{\frac{j2\pi}{3}} + v_{Cn}(t)e^{\frac{j4\pi}{3}} \right] = \frac{2}{3}VD e^{j0}$

• Similarly we can deduce the resultant space vector for other combinations.



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So, when I say 100, 100 means, A phase is 1; that means, V D that is what is written here V D and B and C are both 0 ok. Now, in one of the previous classes, we had derived that $v_{An}(t)$ is two third $v_{AO}(t)$ minus one third $v_{BO}(t)$ minus one third $v_{CO}(t)$. So, if you substitute these values, we see that; the v_{An} is two third V D here. And similarly, v_{Bn} ; if you also substitute these values v_{Bn} is two third v_{BO} minus one third v_{AO} minus one third v_{CO} . We see that v_{Bn} is minus one third V D.

And v_{Cn} is similarly, minus one third V D ok. So, these are the instantaneous load phase voltages v_{An} , v_{Bn} and v_{Cn} . And their values are two third minus one third and minus one third V D. So, the resultant load phase voltage space vector for 100 switching state combination will be two third v_{An} plus $v_{Bn} e^{j2\pi/3}$ plus $v_{Cn} e^{j4\pi/3}$.

power $j 4 \pi$ by 3 and if you substitute these values into this equation, then we see that; $V_R t$ is two third $V_D e$ to the power $j 0$.

Which means that; the resultant load phase space vector for the switching state 1 0 0 is line with the A phase axis because it is e to the power $j 0$ and its magnitude is two third V_D . So, similarly for all the other eight switching state combinations; we can reduce the resultant space vector.

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Space vector for all combinations

Space Vector	Switching States	Resultant space vector ($V_R(t)$)	
V0	000	$\vec{V}_0 = 0$	Zero Vector
V1	100	$\vec{V}_1 = \frac{2}{3} V_D e^{j0}$	Active Vector
V2	110	$\vec{V}_2 = \frac{2}{3} V_D e^{j\pi/3}$	
V3	010	$\vec{V}_3 = \frac{2}{3} V_D e^{j2\pi/3}$	
V4	011	$\vec{V}_4 = \frac{2}{3} V_D e^{j3\pi/3}$	
V5	001	$\vec{V}_5 = \frac{2}{3} V_D e^{j4\pi/3}$	
V6	101	$\vec{V}_6 = \frac{2}{3} V_D e^{j5\pi/3}$	
V7	111	$\vec{V}_7 = 0$	Zero Vector

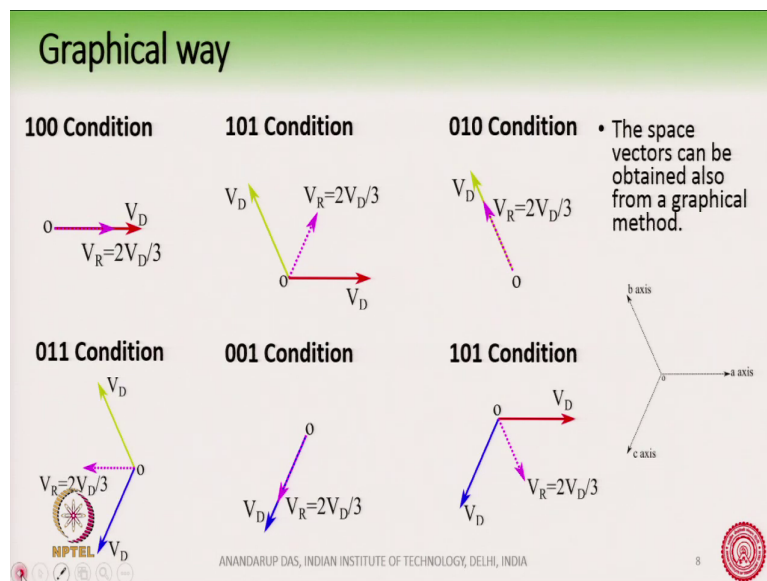
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And this is shown here in this table. So, for example, V 1 is this 100 and the resultant space vector is two third $V_D e$ to the power 0 and 0, when you have switching state 0 then the resultant space vector is also 0. Similarly, when you have 111, the resultant space vector is also 0 ok. For any other combination; you can c 110 for example, if you can put it into the formula, then the resultant space vector is two third $V_D e$ to the power $j \pi$ by 3 ok.

Now, so, we can for this eight switching states; we find that there are two combinations 000 and 111; they are producing the resultant space vector as 0. And so, they are termed as zero vectors ok. The resultant load phase voltage space vector is having zero magnitude for 000 and 111 these two switching states.

So, this is called as the zero vector. Whereas, for other combinations, you can have all the six active vectors which are termed V_1 V_2 V_3 V_4 V_5 and V_6 ok. And we can also so, all these active vectors are having the same magnitude of two third V_D , but the angle is changing ok.

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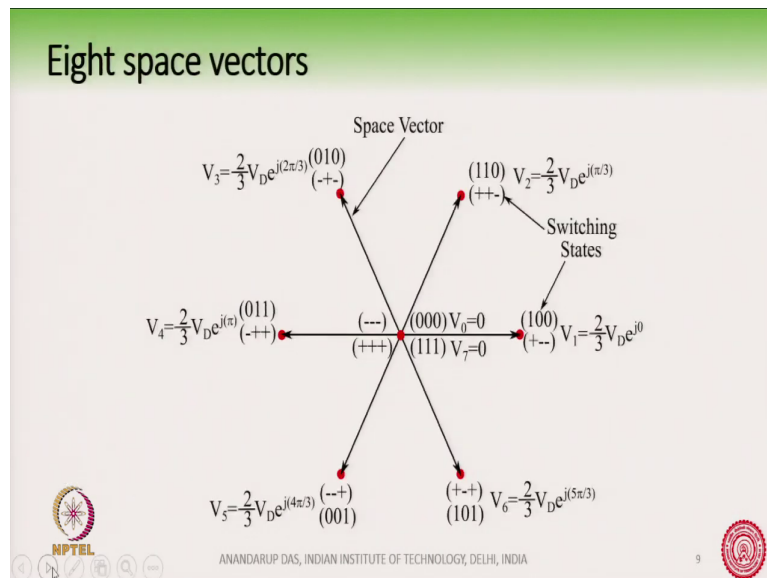
So, we can now plot it; we can now plot it in the plane, in the space vector plane. The space vectors can also be obtained in a graphical wave ok. And it is quite simple because, suppose you have 100 combination. So, in 100 switching state combination; you can have a space

vector of 1 of magnitude 1 on the A phase axis and no magnitudes on B and C phase axis. So, therefore, the resultant is two third V D here.

So, when you have a 110 combination; then you will have 1 here and B phase also 1 here and the resultant is here two third V D. Similarly, you can take up the other combinations like 011 combination. If you have 011 combination; then you have no means A phase does not contribute any vector while B phase has 1 and C phase has V D. And the resultant is two third V D along this.

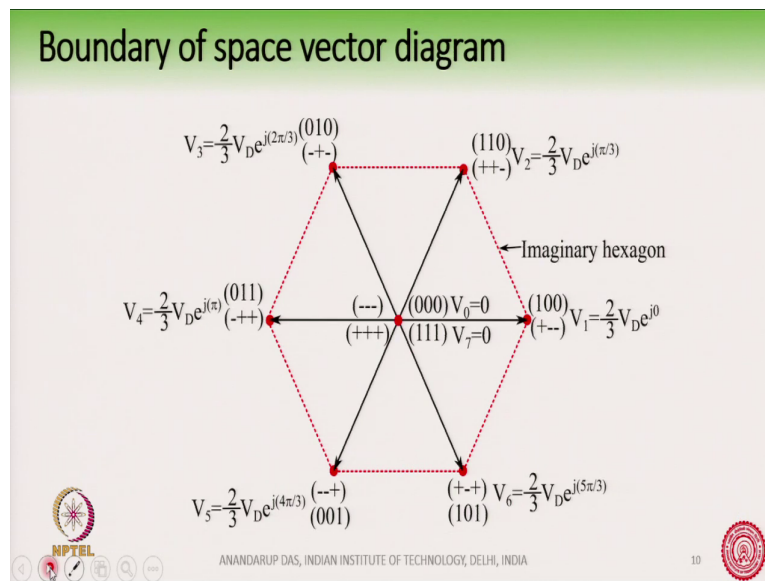
So, we see that the instead of all the time going back to the formula; we can also find out the resultant space vector using this simple graphical method where here, a b c axis are shown in this.

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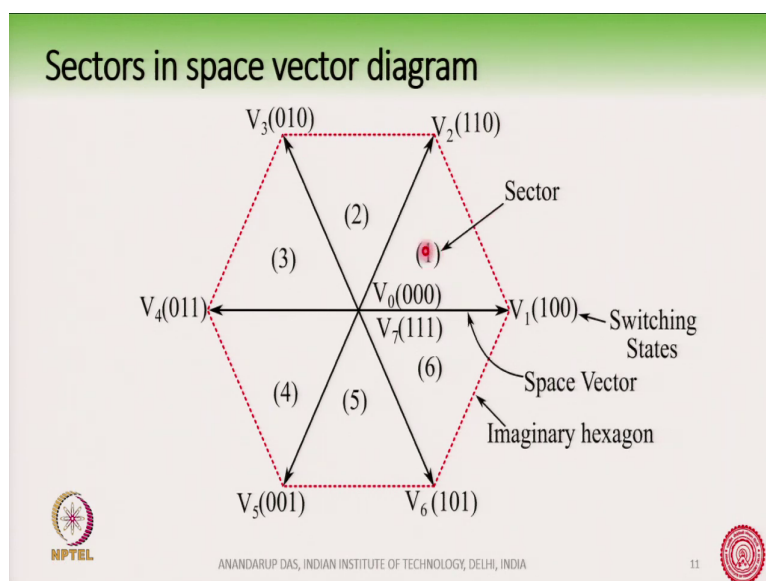
So, the resultant space vector. So, we see that there are six result in space vectors with their magnitude two third V_D and they are at an angle of 0 degree, 60, 120, 240, 300 and 360 degree. And these are the switching states which are producing these resultant vectors. So, there are six resultant vectors and two zero vectors ok. These are the vectors in this space.

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These six vectors or the tip of the tip of these six vectors can be joined with an imaginary hexagon or this is the boundary. So, you can see the boundary of the space vector diagram is this imaginary hexagon ok. And with this imaginary hexagon and the vectors which we have drawn here, we can see that; the whole space vector diagram which is actually a regular hexagon is divided into six sector; sector 1 2 3 4 5 6.

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So, we see in this space vector diagram; the boundary is a regular hexagon there are six sectors and there are eight switching states which are indicated in this space vector diagram, six active vectors and two zero vectors now. So, this is the complete space vector diagram. Where, we can see that; the six active vectors and the two zero vectors are shown along with the boundary of the space vector diagram and also we see that this space vector diagram is divided into six sectors ok.

These triangular regions are called sector. So, therefore, there are six sectors 1 2 3 4 5 6, in this space vector diagram. This sectors are useful, when we do this switching on the space vector diagram. We will see that sometime later ok.

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Space Vector PWM

- How to switch the eight vectors so that the correct voltage is impressed on the load?
- Space vector PWM is an extension of sine triangle PWM. Here the PWM is done by using space vectors.



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Now, how to switch? So, now, we come to the next topic; which is the space vector PWM ok. Now, how to switch these eight vectors so that the correct voltage is impressed on the load? So, this is the question that we are asking that ok, we have got a good understanding of the space vector diagram; that it is a eight there are eight vectors forming the hexagon.

But, now the question is, how do we switch this vectors. Basically, we will use one of the very established principle that is the volt second balance of the principle of volt second balance that concept we will use here also. Later, you will understand that; the space vector PWM is nothing, but extension of the sine triangle PWM which we have already studied earlier.

This technique of switching the eight space vectors in such a way that the resultant voltage or current is realised is called the space vector PWM. So, it is a switching strategy; by which we can appropriately switch the vector for certain time durations.

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Space Vector PWM

- The space vectors are switched for certain duration of time in a cycle so as to produce the resultant vector.
- $V_R T_S = V_1 T_1 + V_2 T_2 + V_0 T_0 = V_1 T_1 + V_2 T_2 + V_0 T_{01} + V_0 T_{07}$
- $T_S = T_1 + T_2 + T_0$
- In space vector PWM, $T_{01} = T_{07} = T_0/2$

$V_0 \rightarrow 000 \rightarrow T_0/2$	$V_6 \rightarrow 000 \rightarrow T_0/2$
$V_1 \rightarrow 100 \rightarrow T_1$	$V_5 \rightarrow 001 \rightarrow T_1$
$V_2 \rightarrow 110 \rightarrow T_2$	$V_4 \rightarrow 011 \rightarrow T_2$
$V_7 \rightarrow 110 \rightarrow T_0/2$	$V_7 \rightarrow 111 \rightarrow T_0/2$

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So, the space vectors are switched for certain duration of time in a cycle to produce the resultant vector. In space vector PWM, the space vector are switched for particular duration of time in a cycle. And it is dictated by this equation here, which is; $V_R T_S = V_1 T_1 + V_2 T_2 + V_0 T_0$. So, what are this? So, $V_R T_S$ is the resultant space vector, which is applied for a duration during the switching timing interval T_S , this total switching interval or switching period is T_S . And the resultant space vector is V_R which is applied for this time T_S .

Now, T_S during the time T_S in order to realise this resultant vector or in order to produce this resultant vector. We are switching the three nearest vectors which is enclosing this resultant vector ok. So, for example, if this is the resultant vector; we see that it is lying in sector 1. If it is lying in sector 1, then this vector, this vector and this vector that is V_0 or V_7 and V_1 and V_2 , these three are the vectors which are enclosing this resultant vector V_R ok.

This so, as so we will switch the these three vectors; this one, this one, and this one, for a certain duration of time in a switching cycle T_S ok. Now basically, it means that; we can visualise it as if like adding some weights to the three corners of this triangle here ok. So, if we add weights; the resultant will move, depending on how much weightage we add to the three corners of the triangle. For example, what is this weightage? This weightage is nothing but, the duty cycle for example, if V in a switching cycle T_S in a switching period T_S .

If we apply this vector for more duration of time and these two vectors for less duration of time, then the resultant vector will come very close to here ok; resultant vector will come very close to here. On the other hand; if we apply this vector for a long duration of time within the time period T_S ; that means, this vector is being applied for the largest fraction of the total time period T_S , then the resultant vector will move here, closer to this one. It will be far away from these two vectors V_1 and V_2 .

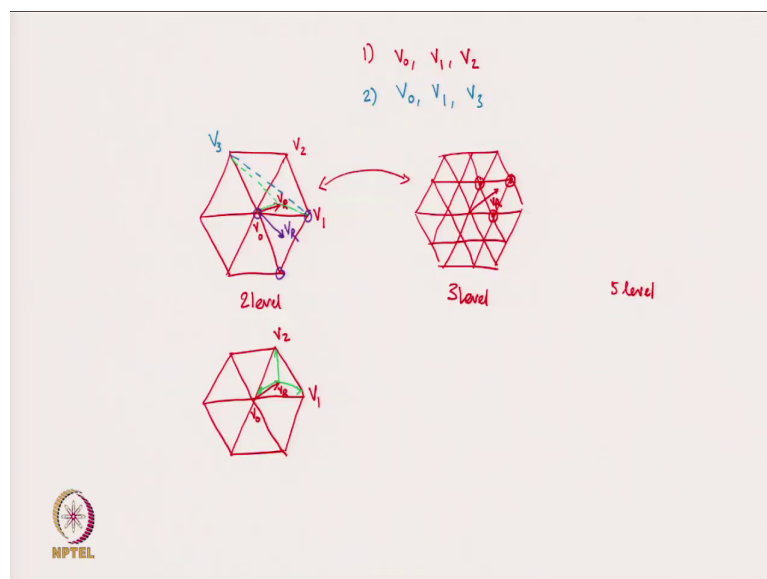
So basically, we are as if like we are adding weights to the three corners of the triangle and this weights are what these weights are the time for which these vectors are applied. So, for example, T_1 by T_S is the vector is the weight which is T_1 by T_S is the weight which is being applied on the vector V_1 .

Similarly, T_2 by T_S is the weight of V_2 and T_0 by T_S is the weight for the zero vectors. So, the resultant V_R will accordingly move depending on; how much at the weight to this three corners. These additionally, we see that the 0 period actually is divided into two 0 periods. Why this is called as 0 period T_0 ? Because we are applying the zero vector during this time ok. So, we are applying this zero vector for T_0 time, we are applying the V_1 vector for T_1 time and V_2 vector for T_2 time ok.

Additionally this zero vector T_0 is divided into T_{01} and T_{07} , these two periods. We will see; why it is done. This is done to reduce the number of transitions in switching, but we have divided T_0 into T_{01} and T_{07} . Of course T_S , T_S will be the total time period of switching is equal to T_1 plus T_2 plus T_0 , which means; the sum of the time for which this vector, this vector and this vector are switched will be equal to the total time period total switching time period T_S .

Now one more point to observe is that; in this particular case, we have taken these three vectors V_0 or V_7 , V_1 , V_2 to realise the vector V_R . Now, it may not be necessary or it may not be it may be possible, not to use these three vectors rather we can also use four vectors or we can use V_3 , V_1 and V_0 . This is also possible for example, if the reference vector is here V_R .

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So, for example, let me take an example for example, this is the space vector diagram and there is a resultant vector which is here ok. The resultant vector which we want to produce by switching V_1 , V_2 and V_0 ok. So, we have, we can take these three as the three nearest vectors; which are enclosing the resultant vector V_R . But, we may also instead switch V_0 , V_1 and V_3 . We can switch them because, these three vectors are also enclosing the reference vector V_R .

So, one option, so this is the second option. So, the first option was, the first option was; switching V_0 , V_1 and V_2 . And the second option is switching V_0 , V_1 and V_3 . Now, which one we will choose? Because, both these are actually enclosing the reference vector V_R . When we choose the vectors, remember that; we must choose the three nearest vectors which are enclosing the reference vector V_R .

If we do not choose the three nearest vectors, then the instantaneous error between the vectors and the reference vector will be large. For example, in this case. In this case this is V_R . And what is the instantaneous error? So, this is the V_1 , V_2 and V_0 . So, this is the instantaneous error ok. What is the instantaneous error? See we are trying to produce the resultant vector V_R ok.

But there is always an instantaneous error between, what we want to produce and what we are actually producing from the converter which means; that V_R is the resultant vector which we want to produce which we wish to produce from the converter. However, we have only V_0 , V_1 and V_2 in our hands, because the converter can only produce these eight vectors. However, I would like to produce V_R ; which is far away from all these three vectors ok.

So, there will always be an instantaneous error between; what we wish to produce and what we are actually producing from the converter. And this instantaneous error is the green arrows which I have shown in this diagram. This instantaneous error is the source of harmonics. You see here the instantaneous error; if I switch V_0 , V_1 and V_3 instead of switching V_0 , V_1 and V_2 . If I switch V_0 , V_1 and V_3 the instantaneous error will be more.

For example, if you draw it here, the green the instantaneous error is more when we switch V_0 , V_1 and V_3 . And so, we come to the conclusion that; if we switch three vectors which are enclosing three nearest vectors, which is enclosing the V_R vector, then we produce the minimum error where as if you choose any other vector then this instantaneous error will be more than the previous case. And therefore, the harmonic performance on the total magnitude of harmonics will be more, if we use those V_0 , V_1 and V_3 vectors.

So, therefore, we will always try to choose vectors which are the three nearest ones to V_R and which is enclosing ok. If we choose a vector which is non enclosing one for example, if I choose if my resultant vector which I want to produce V_R is here and I switch V_0 , V_1 and V_2 . What will happen? One of the timing, as per the formula; one of the timing will come out to be negative, indicating that V_R cannot be realised by switching V_0 , V_1 and V_2 , it will not be possible.

So, when we choose, when we want to realise V_R , we have to choose the V_R which I have shown here, we must choose for example, these three. Which are the three nearest enclosing vectors able to realise V_R . One more thing that; we can we will see later is that if the, this is the space vector diagram for a two level converter.

Later, we will see some other converter which are called say, multilevel converters; where this space vectors become more and more dense ok. For example, in some other converters we can have, this is the boundary of the space vector diagram of this multilevel converter. But, we can have many more space vectors. For example, this is the space vector diagram of a three level converter. If you see these two diagrams; the boundaries remain the same in both the cases almost ok. The boundaries are same here.

But now, you have space vectors which are dense, which are closer to each other as compared to the two levels. So, this is the two level space vector diagram, this is a three level space vector diagram; which will see later, how this diagram comes. We see that the space vectors are becoming denser and denser. For example, if you go to five level converter it will even become more and more dense.

So, you will have a lot of space vectors and what will happen there is the instantaneous error. Now there, if you have the instantaneous error suppose, you are switching this V_R , the instantaneous error. So, of course, we will use this one, this one and this one. These are the three nearest enclosing vectors. Instantaneous error is smaller. As the instantaneous error is smaller; it indicates that the harmonic performance of the three level converter will become superior as compared to the two level converter.

So, the instantaneous error is what control the harmonics and we would always like to have the instantaneous error as small as possible. Ideally, the instantaneous error should be 0. Because, then we produce the V_R as a perfect sine wave. Ideally, it should be than 0, but unfortunately we do not have so many space vectors available in our hand because of the limitation in the converter.

If we had space vector diagram where these space vectors are infinitely close to each other then, we will get a perfect sine wave out of the converter. Coming back to this discussion about this space vector PWM, we see therefore, that V_R into this is the V_R into T_S is V_1 into T_1 plus V_2 into T_2 plus V_0 into T_0 which is again divided into V_0 into T_{01} and V_0 into T_{07} .

So, the zero vector; both the zero vectors are used ok. And they are used half of the time during the zero period. So, T_{01} is one zero vector one zero vector timing duration while T_{07} is the other zero vector timing duration and they are usually made equal; that is T_0 by 2 ok.

In this example, V_R is the reference vector you want to produce and you will switch V_1 that is the space vector 100 and you will switch it for T_1 duration of time and then you will switch V_2 for the which has a switching combination of 110 for T_2 time and we you will also switch the, you will also switch the V_0 vector. For example, 000 combination for T_{01} by 2 time and V_7 vector having switching combination of 111 for T_{07} by 2 time ok.

So, this is how we will switch these four vectors for this time duration in order to realise the V_R vector. Let us see that if suppose V_R is in sector 4 this is sector 4. So, if I want to realise V_R ; I have to switch V_0 , V_4 , V_5 and V_7 right. So, I can switch, say V_0 . I can switch that is the 000 vector, I can switch for T_0 by 2 time ok. And then I can switch V_5 , which is 001 and this is switched for say T_1 time and then I will switch V_4 which is 011 which is made by the combination 011 and it is switched for T_2 time and then again I will switch V_7 which will be done by the switching combination 111 and I will switch for T_0 by 2 time.

Note that, when we do this kind of a switching; then, we are always when we go from one switching state to another switching state, there is only one switching transition ok. For example, in this case you see that when we go from 000 to 100 transition then, only A phase is changing its state B and C phases are not changing any state.

So, B and C phases; the switches will not change their position in A phase there will be a switching change. Similarly, when we go from 100 to 110 or when we go from 110 to 111, sorry this is 111. When we go from 110 to 111, then also we see that each time we have only one switching transition. This is always maintained in space vector PWM.

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Mathematical expression of timings

- $\frac{OA}{\sin(\frac{\pi}{3}-\theta)} = \frac{OB}{\sin(\theta)} = \frac{OC}{\sin(\frac{2\pi}{3})}$
- $\frac{V_1 T_1}{\sin(\frac{\pi}{3}-\theta)} = \frac{V_2 T_2}{\sin(\theta)} = \frac{V_R T_S}{\sin(\frac{2\pi}{3})}$
- $T_1 = \sin(\frac{\pi}{3}-\theta) \frac{V_R}{V_1} \frac{2}{\sqrt{3}} T_S = \sin(\frac{\pi}{3}-\theta) \frac{V_R}{V_D} \sqrt{3} T_S$
- $T_2 = \sin \theta \frac{V_R}{V_1} \frac{2}{\sqrt{3}} T_S = \sin \theta \frac{V_R}{V_D} \sqrt{3} T_S$
- $T_0 = T_S - T_1 - T_2$
- What happens at $\theta = 0$, and $V_R = 2/3 V_D$?

ANANDARUP DAS, INDIAN INSTITUTE OF TECHNOLOGY, DELHI, INDIA

Now, so far we have said that we will switch a vector for T 1 period another vector for T 2 period and a zero vector for the T 0 period, but how to calculate this timing duration? How much is T 1, how much is T 2 etcetera? In order to do that we have to get the timing mathematical expression of the timings of this vector. So, here we have taken an example in sector one.

We see that O C is the V R vector or the reference vector; which we are trying to switch. Now, in order to get the timing we can very easily do it by these geometrical analysis. So, we see that this O C vector is nothing but, the vectorial sum of O A and O B ok. O C is nothing but, O A and O B and they are forming this triangle ok.

So, in this triangle, we can apply this formula sides of the triangle that is $O A O$. So, the angles at this angle is $2\pi/3$ and this angle is for example, θ that is the reference vector angle. The reference vector is making an angle of θ with the V_1 axis.

And so therefore, this angle will be $\pi/3$ minus θ . So, here we can say that $O A$ divided by $\sin \pi/3$ minus θ will be equal to $O B$ in this triangle; we see that in this triangle $O A$ divided by this angle will be equal to $A C$ which again is equal to $O B$, $A C$ divided by $\sin \theta$. And will be equal to $O C$ divided by $\sin 2\pi/3$; that is what we have written here ok that from the basic geometrical analysis.

So, $O A$ by $\sin \pi/3$ minus θ is equal to $O B$ by $\sin \theta$ is equal to $O C$ by $\sin 2\pi/3$. Now, what is $O A$? $O A$ is the, as I told you; if you apply the V_1 vector for T_1 duration, the net volt second is represented by $O A$. Similarly, if you apply the vector V_2 for a timing duration of T_2 then the net volt $O B$ is representing the net volt second and $O C$ is representing the net V_{ref} into T_S , $O C$ is representing that length V_{ref} into T_S .

So, therefore, substituting $O A$ as V_1 into T_1 , $O B$ as V_2 into T_2 and $O C$ as V_R into T_S . We get these two equations from which it is possible for us to calculate the value of T_1 as this, this expression here. Here we can say that V_1 is having a magnitude of two third V_D . So, V_1 is having a magnitude of two third V_D . V_2 is also having a magnitude of two third V_D .

So, if you substitute here V_1 , the expression of V_1 you substitute here and then you will see that we get up to this expression. Similarly, you can get T_2 as this expression here and T_0 , that is the remaining of the time for which the zero vector will be applied. So, zero vector will not cause any addition to the volt second, because it is a zero vector it is a magnitude is 0. So, V_0 into T_0 will not cause any volt second here. So, therefore, T_0 is equal to T_S minus T_1 minus T_2 ok.

Now, so, these are the timing durations for which the vector V_1 V_2 and V_0 will be applied, in order to realise the reference vector V_R . So, you can see here, say; two cases very

interesting is, what happens at theta equal to 0. Theta equal to 0 means; this green vector is now line here right.