Micro Wave Engineering

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Lecture 10:

Scattering Matrix (S-Parameters) Part-2

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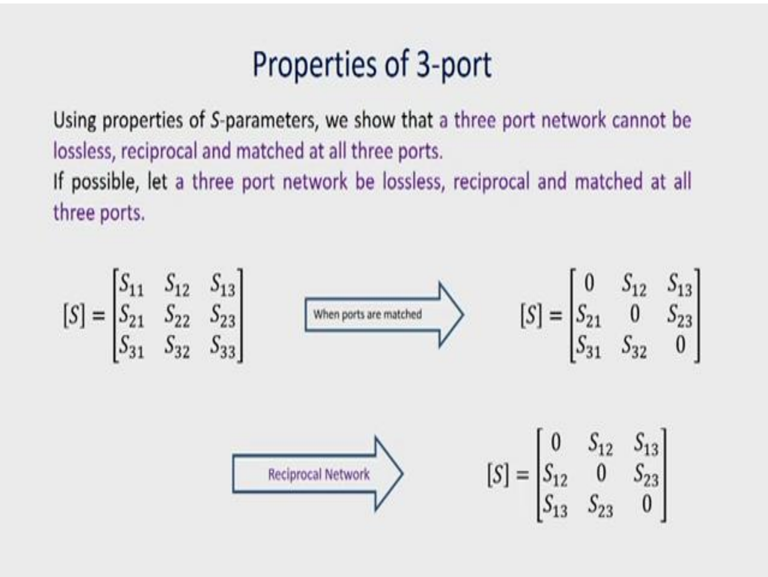
Using properties of S-parameters, we show that a three port network cannot be lossless, reciprocal and matched at all three ports.

If possible, let a three port network be lossless, reciprocal and matched at all three ports.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \quad \text{(when ports are matched)}$$

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \quad \text{(reciprocal network)}$$



Let us continue our discussion over S-Parameters. We look into the properties of 3 port network, so using the properties of S-Parameters, we show that a 3 port network cannot be lossless reciprocal and matched at all 3 ports. That means we cannot have a 3 port, which is lossless, which is reciprocal, and it is matched at all 3 ports. So, let us see how we can demonstrate this in order to prove this. We start with the assumption that it is possible to have a 3 port network which is lossless, reciprocal, and matched at all 3 ports. And then we will show that such assumption will . We lead to contradiction.

So a 3 port S-Parameter can be written in this form S_{11}, S_{12}, S_{13}, S_{21}, S_{22}, S_{23}, S_{31}, S_{32}, S_{33}. Now if we consider that all the ports are matched, then we get an S-Parameter for the diagonal elements are zero. S_{11} is 0, S_{22} is 0, S_{33} is zero. Now next if we consider it to be reciprocal then, the S matrix is symmetric so, we will have S_{21} equal to S_{12}, S_{31} will become S_{13}. Similarly S_{32} will become S_{23}. So this is the form of S matrix where it is assumed that it is matched at all 3 ports. And also it is reciprocal, now let us bring in the other issue that means lossless of this, of the network represented by such S matrix.

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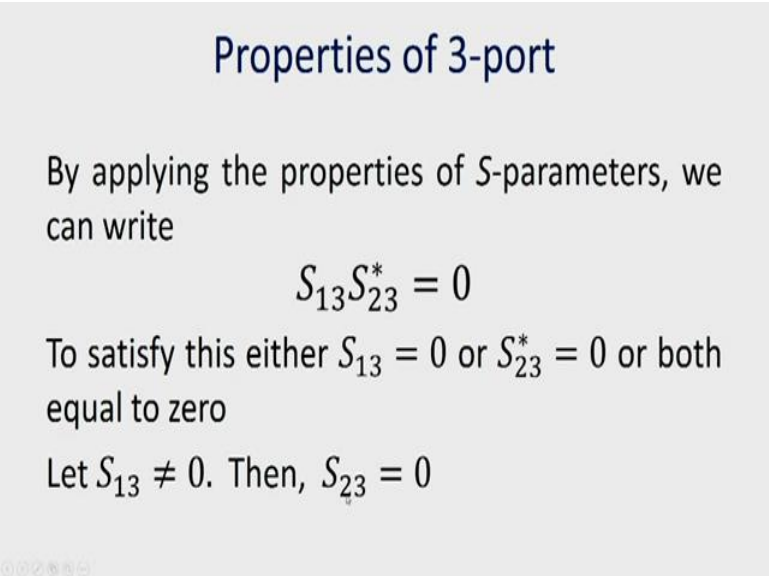
By applying the properties of S-parameters, we can write

$$S_{13}S_{23}^* = 0$$

To satisfy this either $S_{13} = 0$ or $S_{23}^* = 0$ or both equal to zero

Let $S_{13} \neq 0$. Then, $S_{23} = 0$



So, we know that if we apply the properties of S-Parameters for a lossless junction, we can write S_{13} into S_{23} is equal to zero. This comes from S_{13} into S_{23} conjugate is equal to zero. Now to satisfy this, either S_{13} has to be zero, or S_{23} conjugate is zero, or both have to be equal to 0. So let us assume that S_{13} is not equal to zero. Then, S_{23} will be equal to zero. The lossless S matrix will also satisfy the unity property, and considering our network; we find that we will have mod of S_{12} square plus mod of S_{13} square equal to 1, like that for all the columns.

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Further, we have $|S_{12}|^2 = 1 - |S_{13}|^2 = 1 - |S_{23}|^2$

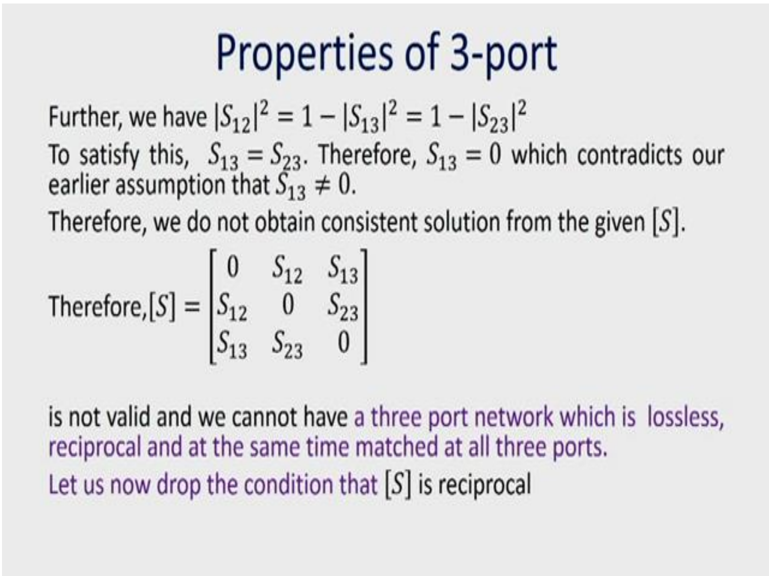
To satisfy this, $S_{13} = S_{23}$. Therefore, $S_{13} = 0$ which contradicts our earlier assumption that $S_{13} \neq 0$.

Therefore, we do not obtain consistent solution from the given $[S]$.

Therefore, $[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$

is not valid and we cannot have a three port network which is lossless, reciprocal and at the same time matched at all three ports.

Let us now drop the condition that $[S]$ is reciprocal



So if we consider this type of relations, we can write mod of S_{12} square equal to 1 minus mod of S_{13} square, and it can be made equal to 1 minus mod of S_{23} square. Now if the above equation is to be satisfied then we must have S_{13} equal to S_{23}, and this will lead to S_{13} equal to zero because we have earlier said that S_{23} equal to 0, but S_{13} equal to zero, contradicts our earlier assumption that S_{13} not equal to zero.

Therefore, we cannot obtain a consistent solution with the given form of S matrix, and therefore this S matrix which is reciprocally matched at all 3 ports, cannot be lossless and we make the statement a 3 port network which is lossless, reciprocal at the same time matched at all 3 ports, we cannot have a solution for the S matrix. What we can do, we can drop the condition that S is reciprocal. So, once we drop this condition that S is reciprocal.

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$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

$$S_{31}^{*}S_{32} = 0$$

$$S_{12}^{*}S_{13} = 0$$

$$S_{23}^{*}S_{21} = 0$$

$$|S_{21}|^2 + |S_{31}|^2 = 1$$

$$|S_{12}|^2 + |S_{32}|^2 = 1$$

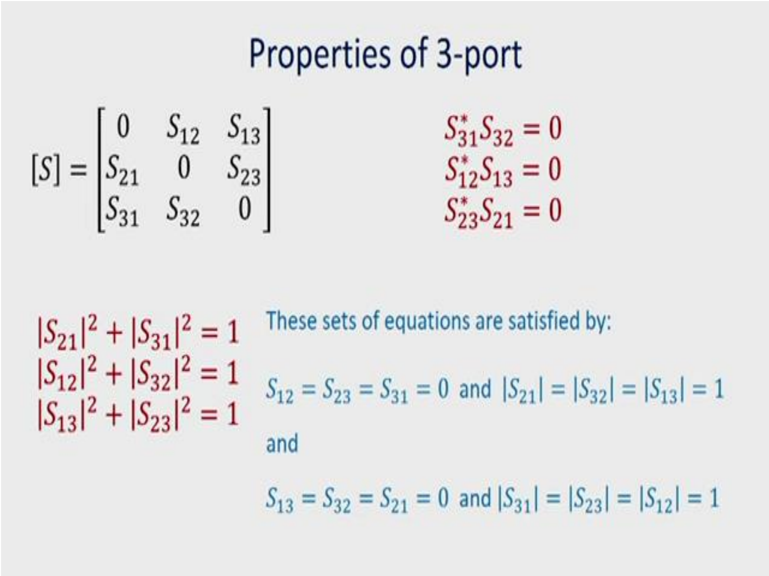
$$|S_{13}|^2 + |S_{23}|^2 = 1$$

These sets of equations are satisfied by:

$S_{12} = S_{23} = S_{31} = 0$ and $|S_{21}| = |S_{32}| = |S_{13}| = 1$

and

$S_{13} = S_{32} = S_{21} = 0$ and $|S_{31}| = |S_{23}| = |S_{12}| = 1$



Then we get this form of S matrix, and this, if it is lossless, that means this s matrix is representing a lossless junction, then by the properties of S matrix, we will have S_{13} conjugate S_{23} equal to 0, S_{12} conjugate S_{13} equal to 0, S_{23} conjugate S_{21} equal to 0, and also if we look at the individual columns, magnitude of S_{21} square plus magnitude of S_{31} square equal to 1, and similarly for the other 2 columns.

Now this set of conditions can be satisfied if we consider S_{12}, S_{23}, S_{31} equal to zero, and S_{21} mod of S_{21} is equal to mod of S_{32} equal to mod of S_{13} is equal to 1, you can see that if we put S_{12} equal to 0, then mod S_{32} becomes 1, similarly if we put S_{23} equal to zero, then mod S_{13} becomes equal to 1, like that we can verify that we have a solution of this equation provided these relations are satisfied.

That means S_{12}, S_{23}, S_{31} is zero, then we get a solution for the remaining parameters S_{21}, S_{32}, S_{13}. In the same manner, another set of solution is possible instead of this we want either this S21 will become zero, S_{32} will become zero, S_{13} will become zero, then we will find S_{31}, S_{32}, S_{12} all will have unity magnitude, now this 2 sets of solution give rise to a device.

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The three port satisfying the condition

$S_{12} = S_{23} = S_{31} = 0$ and

$$|S_{21}| = |S_{32}| = |S_{13}| = 1$$

will give rise to a circulator which will provide clockwise circulation. With proper choice of phase references

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

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$S_{13} = S_{32} = S_{21} = 0$ and

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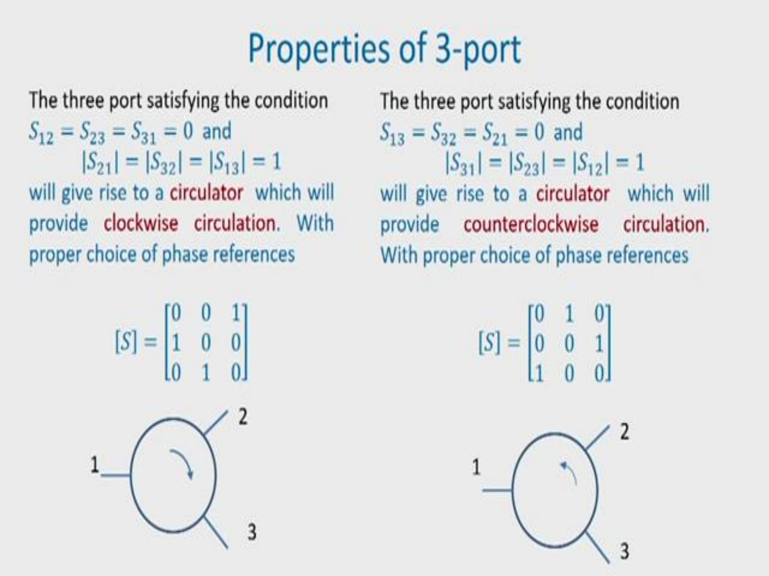
$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



Which is non-reciprocal and it is called a circulator so, what we can do, this parameters magnitude is unity, and by proper choice of the reference plane their phase can be set equal to zero, so S_{21}, S_{32}, S_{13} becomes equal to 1, and therefore, we get this matrix. Here this matrix as essentially represents a circulator which is shown here, so when you excite port 1it goes to port 2, not to port 3, and you can see that S_{21}, is 1. Similarly, when a signal is incident at port 2 then it goes to port 3, not to port 1. And therefore, we have S_{32} equal to 1, and finally, when the signal is incident at port 3, it goes to 1, so S_{13} is 1.

Please note that from 1 port to another, it rotates in a clockwise manner so, it is a clockwise circulation. If we consider the 3 port satisfying the condition S_{13}, S_{32}, S_{21} 0 and S_{31}, S_{23}, and S_{12} are equal to 1. Then this will give rise to by proper choice of phase references. We can get a S matrix of this form, and this gives rise to again another circulator, but this time, the sense of rotation or the circulation is counterclockwise. That means when we have port 3 excited then we have the signal going to port 2, not to port 1.

So, we have S_{23} equal to 1, and next S_{12} equal to 1, so from 2 it goes to 1, and finally from 1 to 3. So, it is S_{31}. This circulation is in the counterclockwise sense. Later on we will see when we discuss some of the practical microwave systems, this type of circulators find lot of use in radar system.

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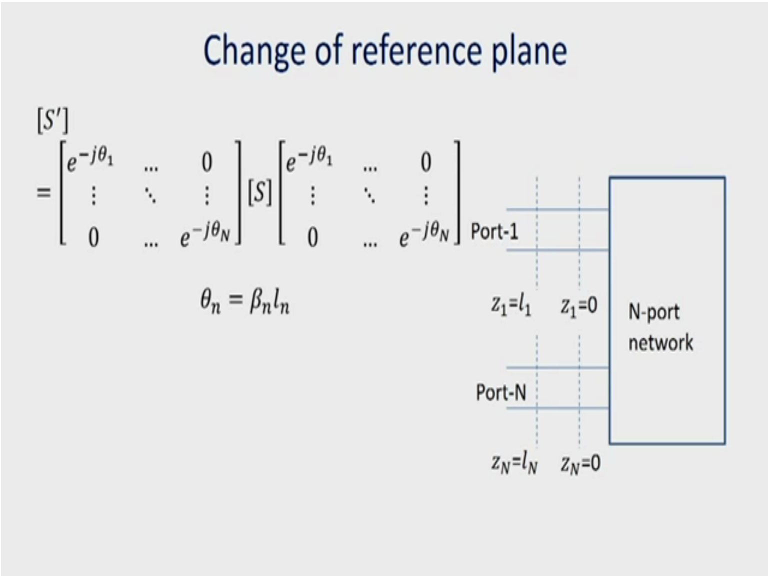
$$[S'] = \begin{bmatrix} e^{-j\theta_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & e^{-j\theta_N} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & e^{-j\theta_N} \end{bmatrix}$$

$$\theta_n = \beta_n l_n$$



We move on to another topic, which is the change of reference plane. We have already seen that when you consider an N port microwave network and find out the S-Parameters the phase reference is defined at the terminal planes. For example, Z_1 is equal to 0. We will be with respect to this terminal to this terminal plane. Similarly, for the Nth port if we take capital Nth port Zn equal to zero, will be here. Now many attempts, for example the evaluation of S-Parameters or measurement of S-Parameters not possible to be done, at the reference to where we want the S-Parameters.

So, what we can do we can modify the S-Parameters, original S-Parameters. Whether for this N port network considering this as the reference planes by change of reference planes, and let the reference plane we now move to Z_1 is equal to l1 and Z_N equal to ln, so it has been moved by this distance and then modified S-Parameter S dash becomes S pre and post multiplied by this 2 matrices where, the off-diagonal elements at zero, and diagonal elements e to the power minus j beta nln. So for the port 1 it is e to the power minus j theta 1, which is equal to e to the power minus j beta 1l1, like that all diagonal elements we will give the phase shift that is resulted due to the movement of the reference to it.

And it may be noted that the S matrix is pre and post multiplied by such matrices to get the equivalent S-Parameter matrix S dash. So we have discussed the various aspects of S-Parameters next.

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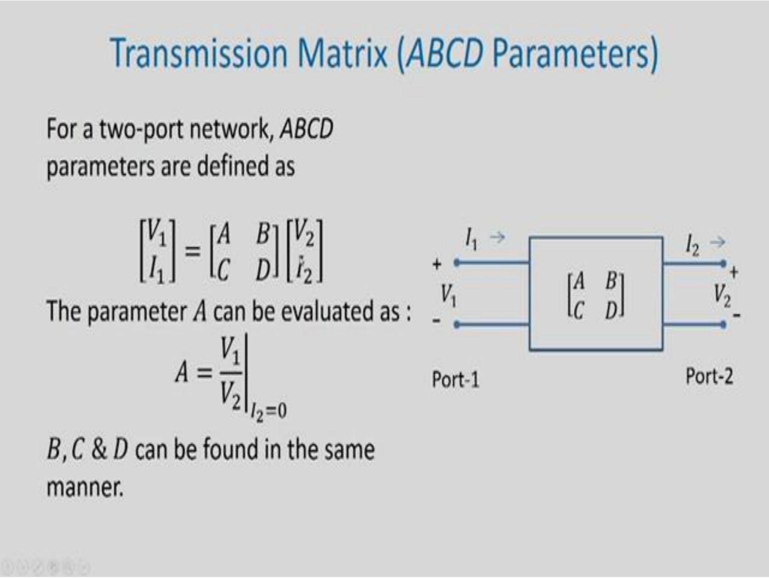
For a two-port network, *ABCD* parameters are defined as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

The parameter A can be evaluated as :

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

B, C & D can be found in the same manner.



We will discuss another set of parameters, which are known as transmission parameters or ABCD parameters. We have seen that we can use different types of parameters such as Z Y S for the representation of microwave networks, we have also seen that S-Parameters are very popular in representation of such microwave networks, because they can be measured directly.

Now we introduce another set of parameters that have called ABCD parameters or transmission parameters. These parameters as you will see, will be particularly useful when we deal with cascaded systems, so for a 2 port network the ABCD parameters are defined as, for this 2 port network we have V_1 and I_1 in the port 1, related to V_2, I_2 of port 2 by this matrix relation V_1 I_1 is ABCD V2 I_2. Now, for example, we can calculate the parameter A as V_1 by V_2 when I_2 is equal to zero. That means when port 2 is open-circuited at that time the ratio of voltage in port 1 and port 2. This will give the parameter A., In the same manner, we can find the other parameters BC and D, for example, B will be V_1 by I_2 when V_2 equal to zero.

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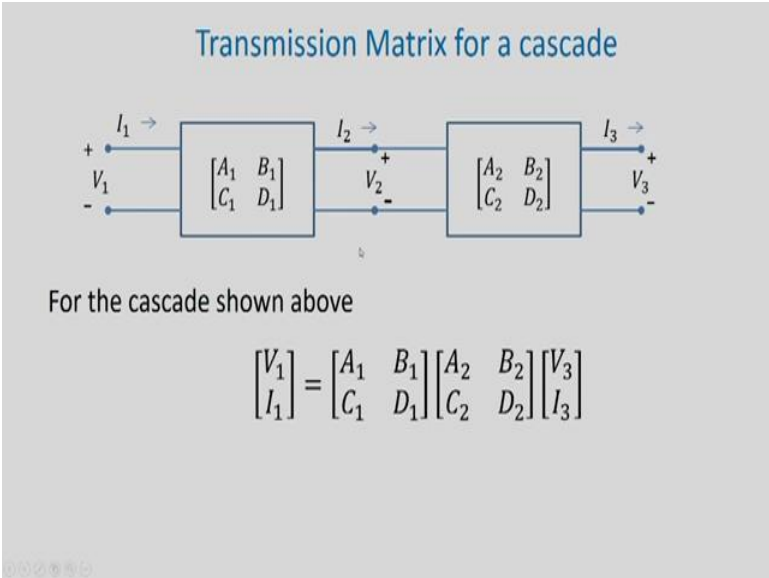
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$



Now as I told that these parameters are particularly useful when we have a cascade of 2ports and in practical microwave circuits we will have different subsystems represented as 2ports, and they are cascaded, so when we have this, type of a cascade of 2ports two or more here we are showing two. We can write the overall ABCD matrix as product of ABCD matrices of individual 2ports. So, this entire cascade can be represented as a 2port, and the voltages and currents at port 1, and port 2 will be related, and this resultant ABCD parameters can be found as product of the ABCD matrices of the individual 2 ports.

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For the two port shown, when $I_2 = 0$,

$I_1 = 0$ and $V_2 = V_1$

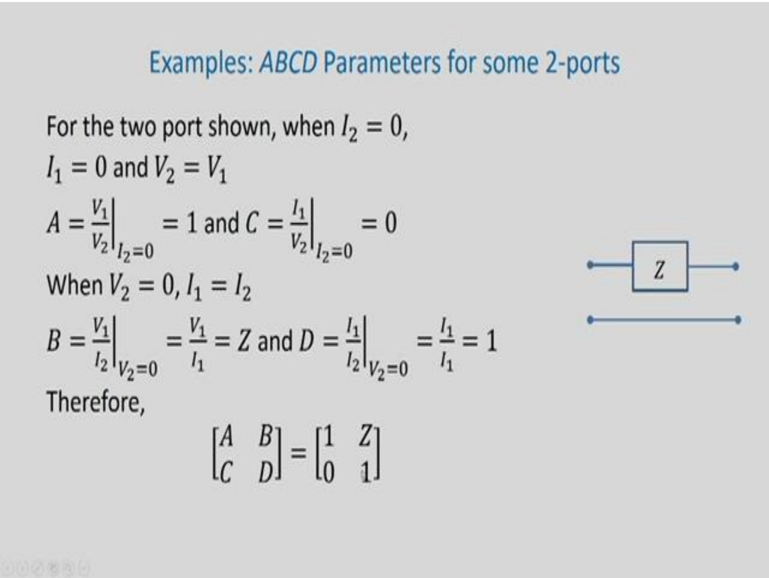
$A = \frac{V_1}{V_2}\Big|_{I_2=0} = 1$ and $C = \frac{I_1}{V_2}\Big|_{I_2=0} = 0$

When $V_2 = 0$, $I_1 = I_2$

$B = \frac{V_1}{I_2}\Big|_{V_2=0} = \frac{V_1}{I_1} = Z$ and $D = \frac{I_1}{I_2}\Big|_{V_2=0} = \frac{I_1}{I_1} = 1$

Therefore,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$



Now let us how we can evacuate this ABCD parameters for some simple 2 port network here, we consider a very simple 2 port network where, we have a single series element Z, so this Z is an impedance element, and if we find the ABCD parameters for this type of simple 2 port, we find that when I_2 will be zero, in that case I_1 also will be zero, because now an open circuit will be created here, and essentially V_2 will be equal to V_1.

So, for this type of a 2 port we will have A equal to V_1 by V_2, I_2 equal to zero, this will become 1. And since I_1 has become 0, C, which is I1 by V2under, the condition of I_2 equal to zero. This becomes zero. Next when we put V2 equal to 0, that means we short circuit here, then I1 becomes equal to I_2. And therefore, we can find B which is equal to V_1 by I_2 under the condition V_2 equal to zero, it can be written as V_1 by I_1 and which becomes equal to Z. and D which is I_1 by I_2 for V_2 equal to zero, it becomes I_1 by I_1, and it becomes one. So, for this 2 port or this series impedance element Z we can write the ABCD matrix as 1Z01.

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In this example our two-port is a section of transmission line as shown

For a transmission line, we have

$$V(Z) = V^+e^{-j\beta z} + V^-e^{j\beta z}$$

$$I(Z) = \frac{1}{Z_0}\left(V^+e^{-j\beta z} - V^-e^{j\beta z}\right)$$

Let us set $z = 0$ at port-2. Therefore, $V_2 = V^+ + V^-$ and $I_2 = \frac{1}{Z_0}(V^+ - V^-)$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

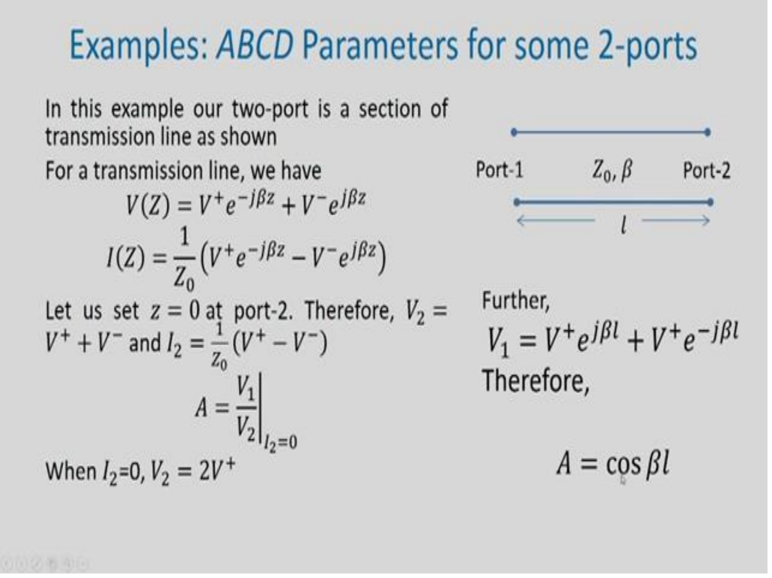
When I_2=0, $V_2 = 2V^+$

Further,

$V_1 = V^+e^{j\beta l} + V^+e^{-j\beta l}$

Therefore,

$$A = \cos\beta l$$



Let us now consider another example where we consider a section of a lossless transmission line which is having characteristic impedance Z_0 phase constant beta the length of this line is L, and this is considered as port 1, and this is port 2, so, when you want to find out the ABCD matrix for this 2 port. Now for a transmission line, in general, we can write, V_Z is equal to V plus e to the power minus j beta Z plus V minus e to the power j beta Z, and I_Z is 1Z Naught V plus e to the power minus j beta Z, minus V minus e to the power j beta Z.

So, this voltage and current excretions are valid when we have wave traveling in both plus Z and minus Z directions, now we set the reference to say Z is equal to zero, at port 2 therefore, at port 2 V_2 becomes V plus V minus and I_2 becomes 1 by Z_0 V plus minus V minus. Now let us see the definition of A, A is V_1 by V_2 when I2 equal to zero, so in order to find out the parameter A we need to calculate V_1 here at port 1 and also find out V_2, under the condition I2 equal to zero.

So, when I_2 equal to 0, V plus becomes V minus and therefore, we have V_2 equal to 2, V plus and V_1 we can calculate by replacing this Z by minus l, so V_1 becomes equal to V plus e to the power j beta l plus now we can write, V plus here, and therefore e to the power minus j beta l and now, if we take out this V plus and divide by V_2 which is 2V plus we will get, e to the power j beta l plus e to the power minus j beta l by 2. And therefore, A becomes cos beta l.

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$C = \frac{I_1}{V_2}\Big|_{I_2=0}$ We have, $I_1 = \frac{1}{Z_0}\left(V^+e^{j\beta l} - V^+e^{-j\beta l}\right)$

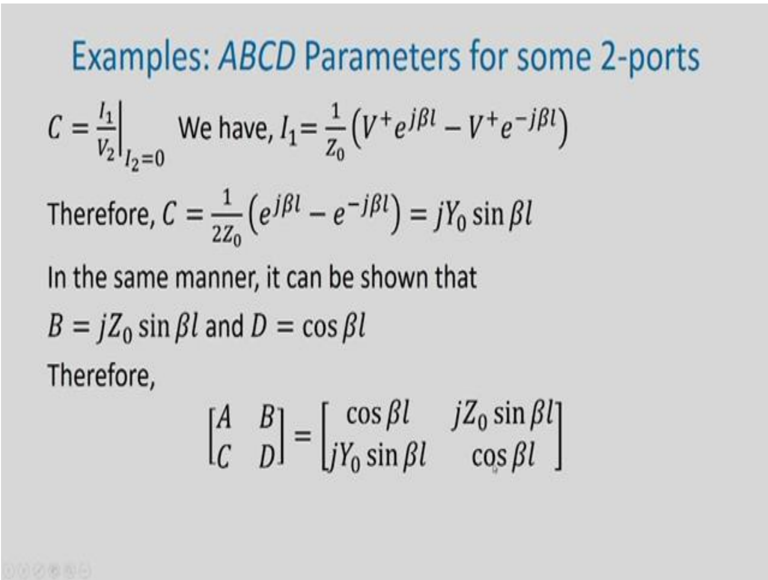
Therefore, $C = \frac{1}{2Z_0}\left(e^{j\beta l} - e^{-j\beta l}\right) = jY_0 \sin\beta l$

In the same manner, it can be shown that

$B = jZ_0 \sin\beta l$ and $D = \cos\beta l$

Therefore,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta l & jZ_0 \sin\beta l \\ jY_0 \sin\beta l & \cos\beta l \end{bmatrix}$$



Now let us see the parameter C which can we found for the same condition when we have I2 equal to zero, and C is I_1 by V_2 when I2 equal to zero, now I1 can be written as 1 by Z_0 V plus e to the power j beta l, Z has been replaced by minus l, minus V minus has been replaced by V plus e to the power j beta l, and therefore now we can write, 1 by 2 Z_0 e to the power j beta l, minus e to the power minus j beta l, because V plus when taken out we will cancel with the V plus of V_2. And this can be written as j Y_0 sin beta l where Y_0 is 1 by Z_0.

In the same manner we can show that V is equal to Z_0 sin beta l and B is equal to cos beta l. and therefore, we have ABCD parameter for a section of transmission line is given by cos beta l, j Z_0 sin beta l, j Y_0 sin beta l, and cos beta l.

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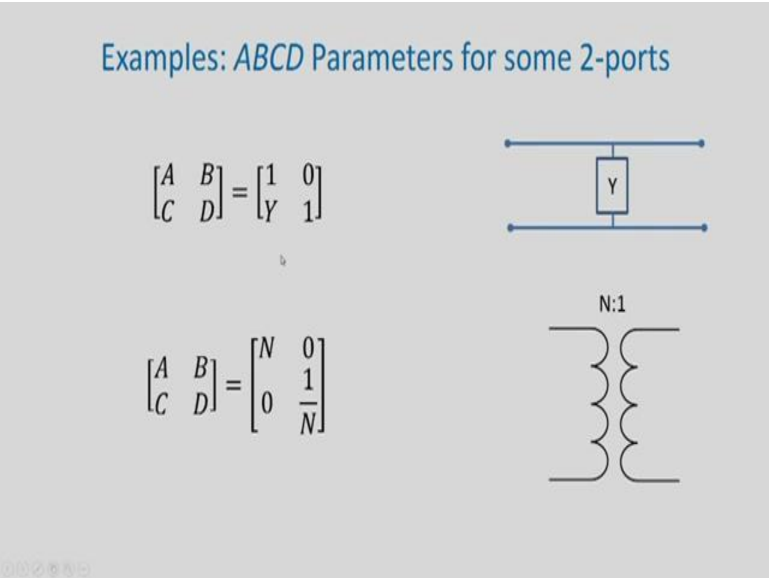
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} N & 0 \\ 0 & \frac{1}{N} \end{bmatrix}$$



Let us now consider some more examples, we have considered a 2 port in the form of series impedance, let us now consider a 2 port of the form of shunt admittance Y and for this case the ABCD parameters can be found out to be equal to 1 0 Y 1, similarly, if we have a transformer in the circuit with turns ratio N is to 1 for this we can find the ABCD parameters to be N 0 01 by N, so this can be derived once again considering the fundamental relations between the port voltages and currents with the ABCD parameters.

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S-parameters of a two-port can be related to *ABCD* parameters. We assume that both ports have characteristics impedance Z_0.

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0}$$

$V_1 = V_1^+ + V_1^-$

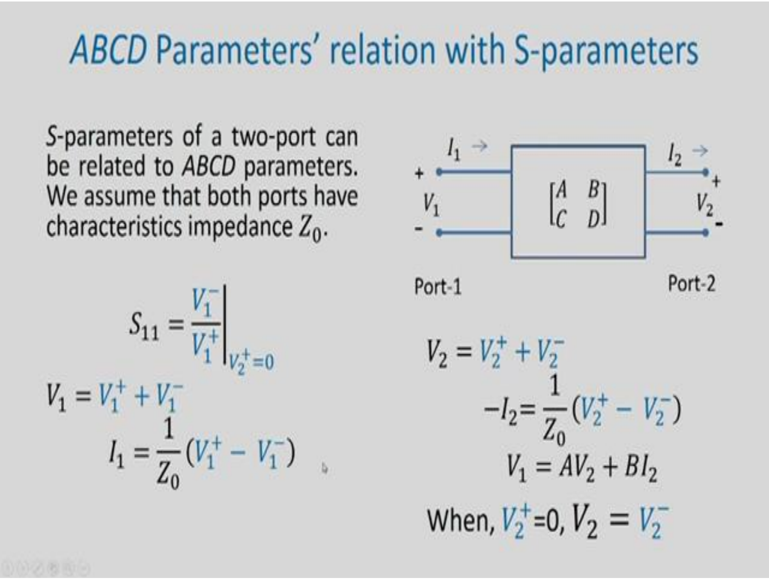
$$I_1 = \frac{1}{Z_0}(V_1^+ - V_1^-)$$

$V_2 = V_2^+ + V_2^-$

$$-I_2 = \frac{1}{Z_0}(V_2^+ - V_2^-)$$

$$V_1 = AV_2 + BI_2$$

When, V_2^+=0, $V_2 = V_2^-$



Now, let us see how we can relate the ABCD parameters, with S-Parameters in fact later on we will see that in some of the cases where the direct computation of S-parameters may be difficult we can actually find out the ABCD parameter first, and then we calculate the S-Parameters. So, for that we need to establish the relation between S-Parameters and ABCD parameters. So, let us consider a 2 port once again and let us assume that both the port impedances are same equal to Z_0.

So, under this condition when the port impedances are same S_{11} equal to V_1 minus by V_1 plus when V_2 plus is equal to zero, so let us see how we can express these voltages V_1 I_1 V_2 I_2 in terms of V_1 plus V_1 minus and Z_0, so we can write, V_1 to be equal to V_1 plus V_1 minus and I_1 to be equal to 1 by Z_0. V_1 plus minus V1 minus similarly, V_2 is V_2 plus V_2 minus but, note that this current I_2 here is outgoing current, and that is why we write, minus I2 equal to 1 by Z V_2 plus minus V_2 minus. And we have V1, from this 2 port, is equal to A V_2 plus BI_2. So, when V_2 plus equal to 0, we have V_2 is V2 minus, and I2 will become V_2 minus divided by Z_0.

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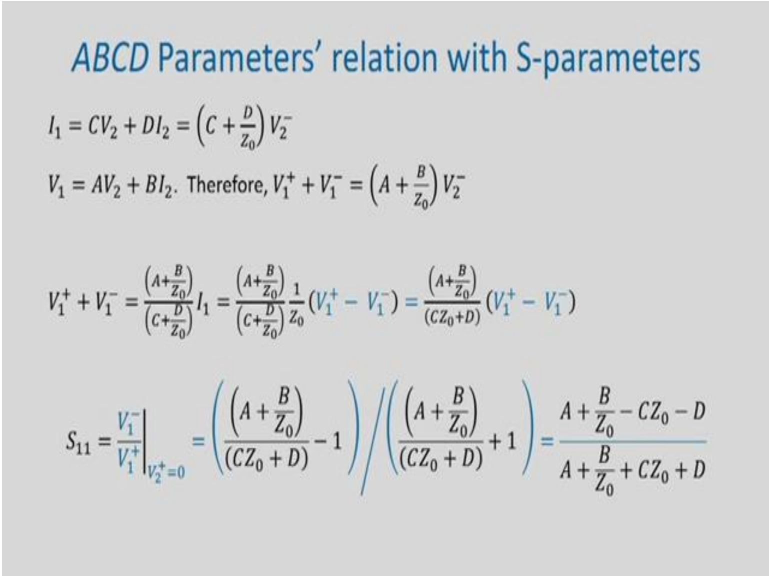
$I_1 = CV_2 + DI_2 = \left(C + \frac{D}{Z_0}\right) V_2^-$

$V_1 = AV_2 + BI_2$. Therefore, $V_1^+ + V_1^- = \left(A + \frac{B}{Z_0}\right) V_2^-$

$$V_1^+ + V_1^- = \frac{\left(A+\frac{B}{Z_0}\right)}{\left(C+\frac{D}{Z_0}\right)} I_1 = \frac{\left(A+\frac{B}{Z_0}\right)}{\left(C+\frac{D}{Z_0}\right)} \frac{1}{Z_0} (V_1^+ - V_1^-) = \frac{\left(A+\frac{B}{Z_0}\right)}{(CZ_0+D)} (V_1^+ - V_1^-)$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \left(\frac{\left(A+\frac{B}{Z_0}\right)}{(CZ_0+D)} - 1\right) \Big/ \left(\frac{\left(A+\frac{B}{Z_0}\right)}{(CZ_0+D)} + 1\right) = \frac{A + \frac{B}{Z_0} - CZ_0 - D}{A + \frac{B}{Z_0} + CZ_0 + D}$$



Now I_1 is CV_2 plus DI_2, and this can be written as in place V_2 we write V_2 minus in place of i_2 we have seen I_2 becomes V_2 minus by Z_0. So we write V_2 minus by Z_0, so we can write I_1 to be equal to C plus D by Z_0, V_2 minus. And we have V_1 to be equal to AV_2 plus BI_2, and therefore, we can write V_1 plus V_1 minus, in the same manner, we substitute V_2 is equal to V_2 minus I_2 equal to V_2 minus by Z_0, and we get A plus B by Z_0, V_2 minus.

Now, if we express V_2 minus as I1 by C plus D by Z_0, then we can write V_1 plus V_1 minus is equal to A plus B by Z_0 into I1 divided by C plus D by Z_0, now we can substitute I1 as 1 by Z_0, V_1 plus minus V_1 minus, and this can be written in this form A plus B by Z_0, C Z_0 plus D and this Z_0 will get canceled into V_1 plus minus V_1 minus, so once we have this expression now we can rearrange we can take all V_1 plus terms to the right-hand side and V_1 minus terms to the left-hand side. So if we rearrange these terms we can write, S11 which is V_1 minus divided by V_1 plus it can be written as A plus B by Z_0 , by C Z_0 plus D minus 1.

So, this divided by A plus B by Z_0 , C Z_0 plus D plus 1. Here you can see that when you take V_1 minus term this side you get 1 plus A plus V by Z_0, divided by C Z_0 plus D and once you take V plus this side you get A plus B by Z_0, C Z_0 plus D minus 1, and from that we can find V_1 minus by V_1 plus and then, you can put it in this form S_{11} equal to A plus B by Z_0, minus C Z_0 minus The, divided by A plus B by Z_0, plus C Z_0 plus D. So we have actually related S11 to ABCD parameters and Z_0 is the normalizing impedance.

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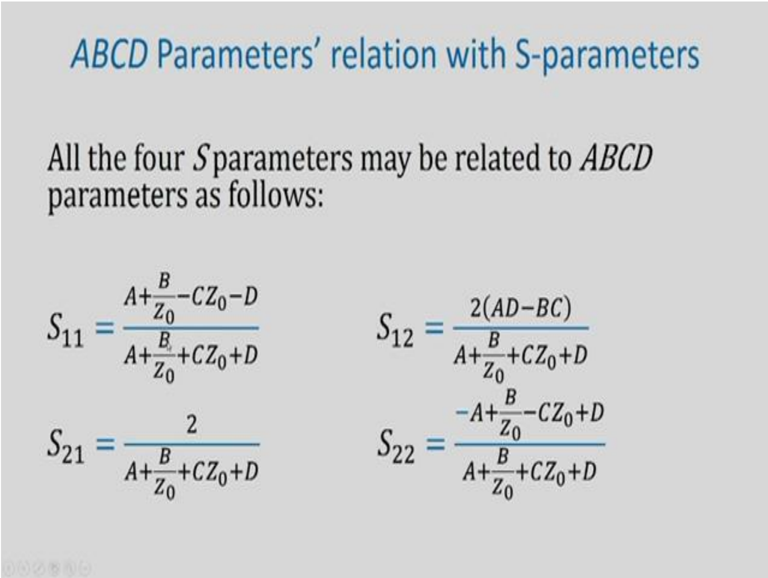
All the four *S* parameters may be related to *ABCD* parameters as follows:

$$S_{11} = \frac{A+\frac{B}{Z_0}-CZ_0-D}{A+\frac{B}{Z_0}+CZ_0+D} \qquad S_{12} = \frac{2(AD-BC)}{A+\frac{B}{Z_0}+CZ_0+D}$$

$$S_{21} = \frac{2}{A+\frac{B}{Z_0}+CZ_0+D} \qquad S_{22} = \frac{-A+\frac{B}{Z_0}-CZ_0+D}{A+\frac{B}{Z_0}+CZ_0+D}$$



We can carry out similar exercise for all the 4 S-Parameter and they may be related to ABCD parameters as, S_{11} we have already seen S_{12} is 2 AD minus BC divided by A plus B Z_0, plus C Z_0 plus D. similarly, S_{21} is 2 divided by A plus B by Z_0, C Z_0 plus D., and S_{22} is minus A plus B by Z_0, minus C Z_0 plus D divided by A B by Z_0 , plus C Z Naught plus D.

So, once we have this relationship established if we know the ABCD parameters for a 2 port we can find out the S-Parameter and this gives us the ease of calculating the S-Parameters particularly when we have the 2 ports in cascades, we can find out the ABCD parameter for the individual 2ports and then we can combine these ABCD parameters to get the equivalent ABCD parameter for the whole cascade and then we can find out the S-Parameters directly in terms of ABCD parameters, later on would will see such examples where we will have to calculate S-Parameters by first calculating ABCD parameters.

So, in summary in this module we have seen the representation of N port microwave networks we have introduced the different types of parameters Z Y S which are used to describe such N port network, we have seen how these parameters can every evaluated for simple networks, and finally we have introduced the transmission parameters or ABCD parameters, which are very helpful in finding out the S-Parameters for cascaded systems. So, in the next module, we will discuss different ways of doing impedance matching in microwave circuits.