Microwave Engineering
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Lecture 11:
Impedance Matching Using L-Section and Series Stub Networks

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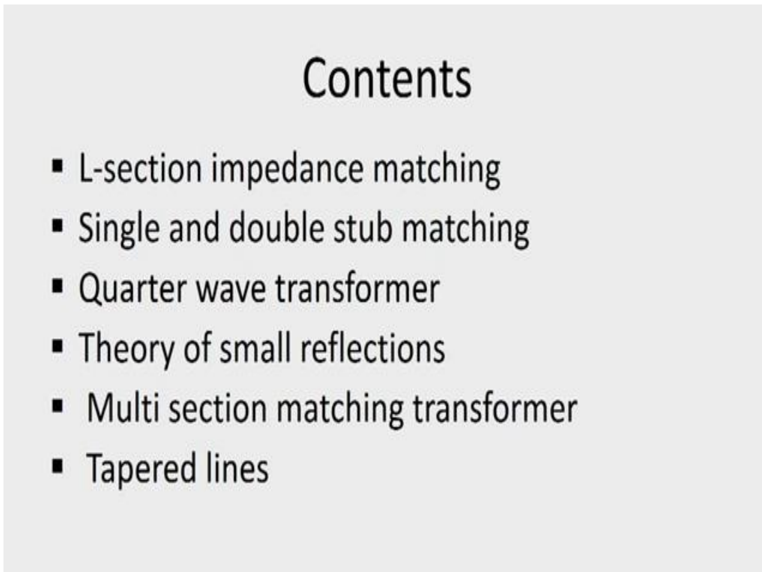

We start a new module Impedance Matching and in this particular module we are going to cover the following contents will first discuss about L section Impedance Matching then we will discuss single and double stub matching L section Impedance matching will involve lumped elements where is stub matching will involve sections of Transmission lines then we will see a very interesting matching circuit which is called a quarter-wave transformer we will discuss the theory of small reflections and then utilizing the theory of small reflections we will discuss how multi-section matching Transformers can be designed then we will consider another form of matching network which is a tapered line this type of taper lines can be easily designed using planar transmission lines.

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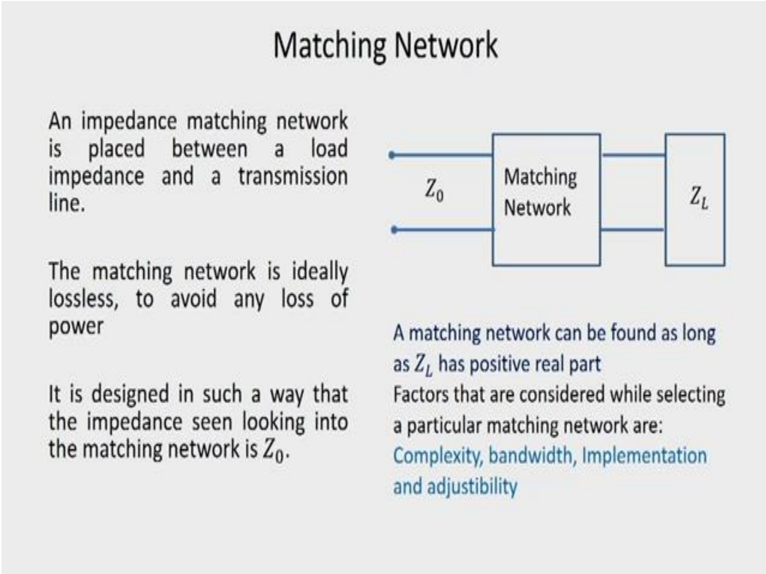

So we start our discussion on matching network first in general discussion on matching network so what is a matching network an impedance matching network is a circuit which is placed between a load impedance and a transmission line now this is the block diagram of a matching network we have a transmission line Z_0 is the characteristic impedance and Z_L is the load impedance we know that if we connect Z_L directly to this transmission line then when Z_L is not equal to Z_0 there will be reflected signal and the power will not be fully absorbed by the load it will be reflected back so what do we introduce a matching network in between Now this matching network ideally it should be lossless so that the matching network itself should not dissipate power so in order to avoid any loss of power within the matching network ideally it should be lossless.

So a matching network is designed in such a way that the impedance in looking into the matching network is Z_0 so if you look at this point then the load impedance transformed by the matching network will give an input impedance of Z_0 here. A matching network can be found as long as Z_L has a positive real part and there are several factors which are taken into consideration when we design the matching network, and these factors are Complexity, the matching network should be very simple, Bandwidth, it should offer sufficient bandwidth the matching should remain valid over a wide band of frequencies.

Implementation has to also take into consideration it should be easily implementable and adjustability because in certain cases particularly when the load is variable the matching network should provide some adjustability with the variable load, so these are broadly the factors which are taken into consideration when we go for the design of the matching network.

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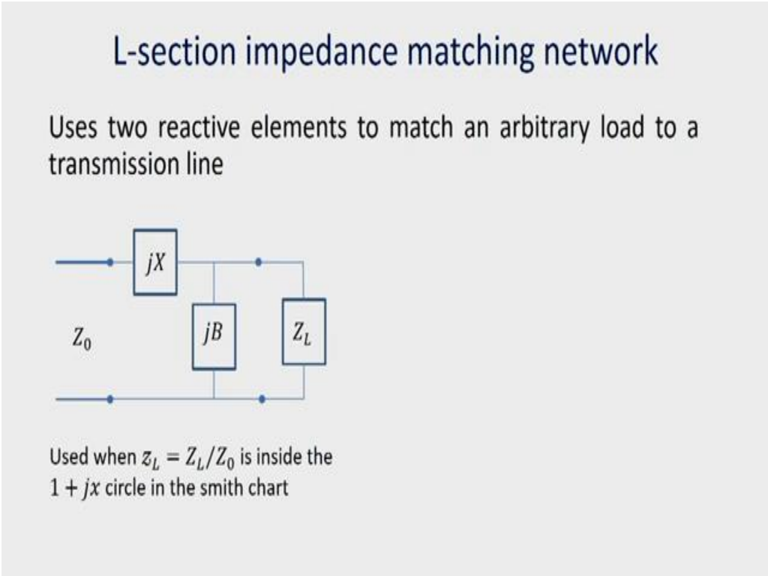

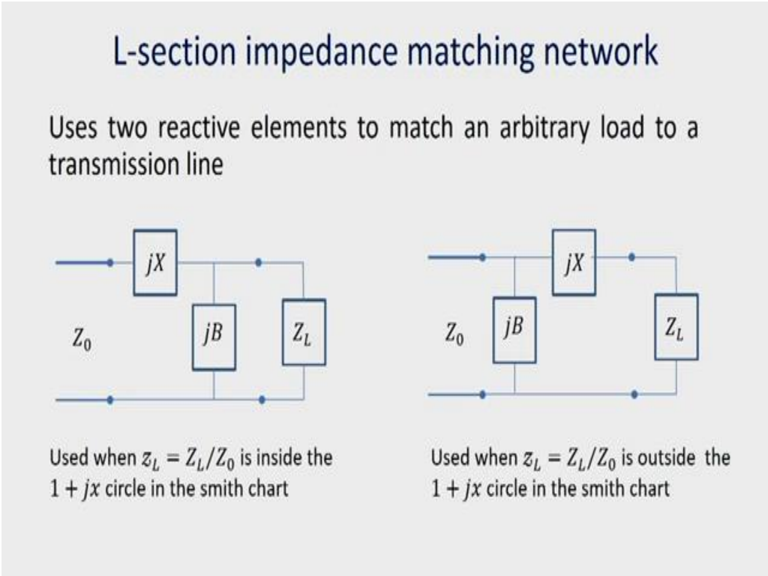

Now let us come to L-section Impedance Matching network it essentially uses two reactive elements to match an arbitrary load to a transmission line so the first matching network using L section we are showing it so we have a series element jX and a shunt element shown as jB. So depending upon the lumped element used this can be an inductive element it can be a capacitive element similarly here also we can use an inductive or a capacitive element now this form of matching network is normally used when the real part of Z_L is greater than Z_0 or in other words if you considered the normalized impedance Z_L by Z_0 it lies inside the 1 plus jx circle in the smith chart.

So in the smith chart if you consider the circle r equal to 1 so inside the circle we will have r greater than 1 and which actually indicates the real part of the load impedance is greater than the characteristic impedance Z_0 so this type of matching network or this configuration of matching network is suitable for handling the loads for which the real part of the load impedance is greater than Z_0.

This is another form of Impedance matching network using L-sections here we also have a series and a shunt element but you can see that the shunt element jB is connected first and then it is followed by the series element jX in series in series with Z_L in the earlier network jB is in parallel with Z_L here jB is in parallel with the series combination of jX and Z_L and this type of matching network is suitable when the normalized impedance Z_L is outside the 1 plus jX circle in the smith chart what does it mean that the real part of Z_L is less than Z_0.

Here also this jX can be an inductor or a capacitor this jB may be an inductor or a capacitor so you can see that for these 2 circuit configurations essentially we can have 8 different circuit combinations using the lumped elements inductors and capacitor. For example this may be inductor inductor inductor-capacitor like that 8 combinations are possible.

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For this case $R_L > Z_0$

For impedance matching, we must have

$$Z_0 = jX + \frac{1}{jB + 1/(R_L + jX_L)}$$

$$Z_0 = jX + \frac{R_L + jX_L}{jBR_L - BX_L + 1}$$

$$Z_0(jBR_L - BX_L + 1) = jX(jBR_L - BX_L + 1) + R_L + jX_L$$

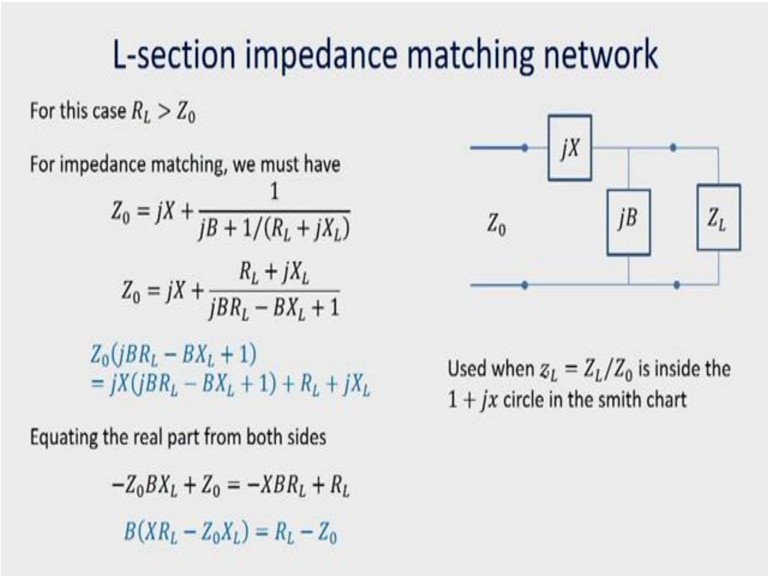
Equating the real part from both sides

$$-Z_0BX_L + Z_0 = -XBR_L + R_L$$

$$B(XR_L - Z_0X_L) = R_L - Z_0$$

Used when $z_L = Z_L/Z_0$ is inside the

$1 + jx$ circle in the smith chart



Let us considered the L-section matching network that is used for the case when R_L, the real part of the impedance is greater than Z_0. Now for this circuit in order to have the Impedance matching we must have the input impedance looking this point must be equal to Z_0 so we will have Z_0 is equal to jX plus the parallel combination of jB and Z_L and 1 by Z_L can be added to jB and then the reciprocal of this entire term can be added to jX and that has to be equal to Z_0.

So if we multiply jB by R plus jXL and also take if we multiply jB by R_L plus jX_L and also take RL plus jX_L to the numerator we get this expression and then we can multiply this denominator term with jX and Z_0, and then we get this expression and from this expression what we can do? We equate the real and imaginary part so if we equate the real and imaginary part so you can see that will have first let us equate the real part so we will have minus Z_0 BX_L plus Z_0 this will give another real-time so minus X_BR_L and then these two terms will be imaginary R_L so minus X_BR_L plus R_L and rearranging the terms we can write B_XR_L minus Z_0 X_L is equal to R_L minus Z naught, so this is the equation we get after equating the real part only.

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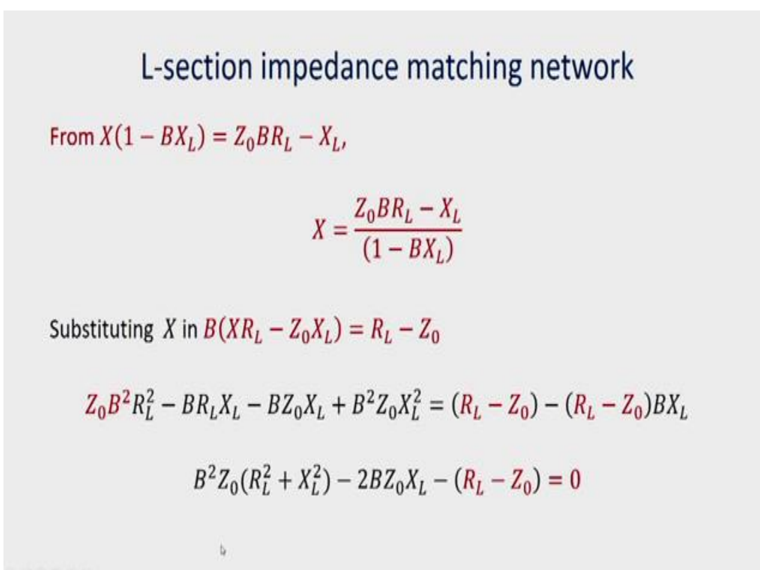
From $X(1 - BX_L) = Z_0BR_L - X_L$,

$$X = \frac{Z_0BR_L - X_L}{(1 - BX_L)}$$

Substituting X in $B(XR_L - Z_0X_L) = R_L - Z_0$

$$Z_0B^2R_L^2 - BR_LX_L - BZ_0X_L + B^2Z_0X_L^2 = (R_L - Z_0) - (R_L - Z_0)BX_L$$

$$B^2Z_0(R_L^2 + X_L^2) - 2BZ_0X_L - (R_L - Z_0) = 0$$



Now let us consider the equation that we get by comparing the imaginary parts, I have written the main equation here so if you collect the imaginary parts from both sides and equate them then we get Z_0 BR_L is equal to minus X B X_L plus X plus X_L and once again if we rearrange the terms we get X 1 minus B X_L is equal to Z_0 BR_L minus X_L.

And therefore now we have a set of two equations, one from the real equating the real parts and another by equating the imaginary parts, and from this set of two equations, we can now solve for B and X, which are the unknown. So we can solve for B and X, so from this equation we can write X_{in} terms of BX_L Z_0 and R_L so X can be written as Z_0 BR_L minus X_L divided by 1 minus BX_L now what we can de we can substitute this X_{in} other equation so we substitute this X in the equation BXR_L minus Z_0 X_L is equal to R_L minus Z_0.

So once we make this substitution and then expand we will get this equation now in this equation we can reorganize the terms, for example, this B square can be taken out from this term, and here similarly B Z_0 X_L is here and from this side we will get another B Z_0X_L and finally will left with this quadratic equation in B which can be solved.

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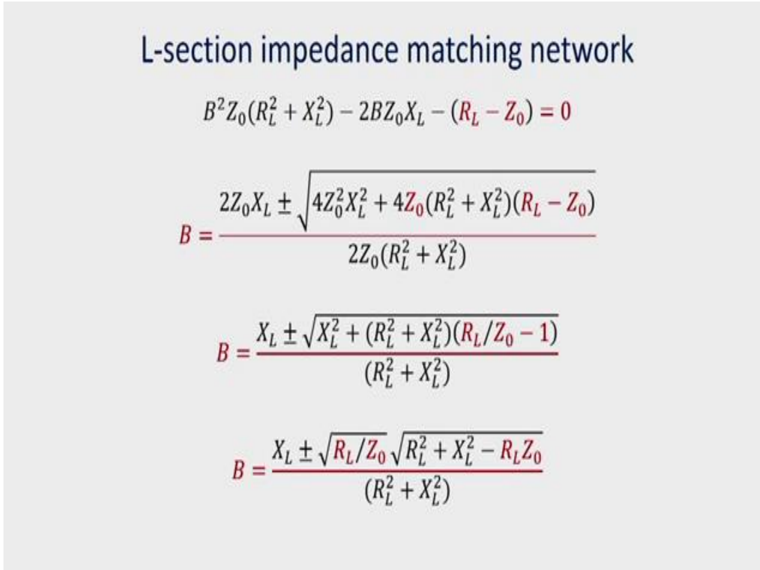
$$B^2 Z_0(R_L^2 + X_L^2) - 2BZ_0X_L - (R_L - Z_0) = 0$$

$$B = \frac{2Z_0X_L \pm \sqrt{4Z_0^2X_L^2 + 4Z_0(R_L^2 + X_L^2)(R_L - Z_0)}}{2Z_0(R_L^2 + X_L^2)}$$

$$B = \frac{X_L \pm \sqrt{X_L^2 + (R_L^2 + X_L^2)(R_L/Z_0 - 1)}}{(R_L^2 + X_L^2)}$$

$$B = \frac{X_L \pm \sqrt{R_L/Z_0}\sqrt{R_L^2 + X_L^2 - R_LZ_0}}{(R_L^2 + X_L^2)}$$



And we can write the solution of B as 2 Z_0 X_L plus minus 2 Z_0 X_L whole square plus 4 Z_0 R_L square plus X_L square into (Multiply) R_L minus Z_0 so, and the entire term is divided by 2 Z_0R_L square plus X_L square.

So this can be further simplified we can divide both numerators and denominator by 2 $Z_0$and then B can be written in this form and then here we can simply further B becomes X_L plus minus root R_L by Z_0 root R_L square plus X_L square minus R_L Z_0 divided by R_L square plus X_L square now we are considering the circuit the matching circuit for which R_L Is greater than Z_0. So, when R_L is greater than Z_0 this term is always positive because this term R_L Z_0 will be less than R_L square.

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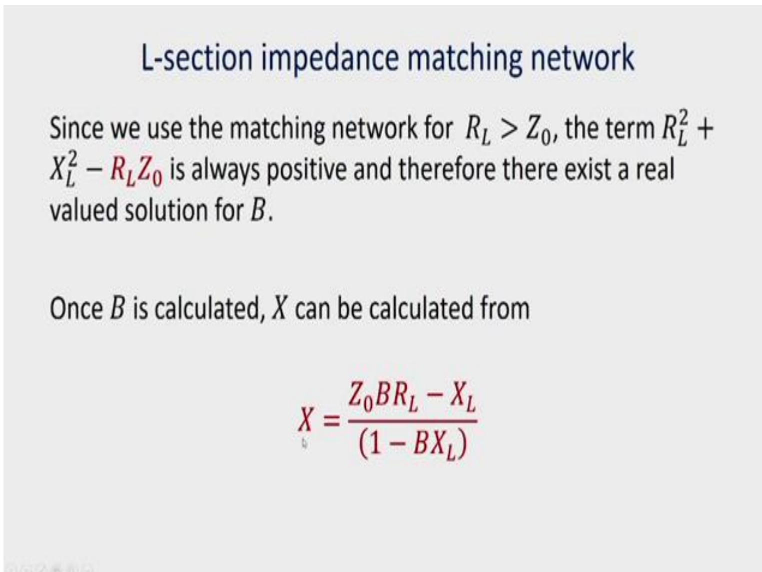
Since we use the matching network for $R_L > Z_0$, the term $R_L^2 + X_L^2 - R_L Z_0$ is always positive and therefore there exist a real valued solution for B.

Once B is calculated, X can be calculated from

$$X = \frac{Z_0 B R_L - X_L}{(1 - B X_L)}$$



And therefore what you can do we can get from here we find that we can always get a real solution for B, and once we calculate B then we can calculate X from the relationship that we have written earlier, so here B is already calculated, and other parameters are known so X_L can here B is already calculated other parameters are known, and therefore X can be calculated from this expression.

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Let an impedance of $Z_L = (100 - j50)\Omega$ is to be matched to a 50Ω line using a L-section matching network at an operating frequency of 500 MHz. Let us design the matching network.

We have

$$B = \frac{X_L \pm \sqrt{R_L/Z_0}\sqrt{R_L^2 + X_L^2 - R_L Z_0}}{(R_L^2 + X_L^2)}$$

$$B = \frac{-50 \pm \sqrt{100/50}\sqrt{100^2 + 50^2 - 100 \times 50}}{(100^2 + 50^2)} = \begin{cases} 0.0058\ \Omega^{-1} \\ -0.0138\ \Omega^{-1} \end{cases}$$

$$X = \frac{Z_0 B R_L - X_L}{(1 - B X_L)} = \begin{cases} 61.2372\ \Omega \\ -61.2372\ \Omega \end{cases}$$

Example: L-section matching

Let an impedance of $Z_L = (100 - j50)\Omega$ is to be matched to a 50Ω line using an L-section matching network at an operating frequency of 500 MHz. Let us design the matching network.

We have

$$B = \frac{X_L \pm \sqrt{R_L/Z_0}\sqrt{R_L^2 + X_L^2 - R_L Z_0}}{(R_L^2 + X_L^2)}$$

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$$X = \frac{Z_0 B R_L - X_L}{(1 - B X_L)} = \begin{cases} 61.2372\ \Omega \\ -61.2372\ \Omega \end{cases}$$

So how we utilize this solution let us explain by taking an example, so we consider an example of L-section matching network design and let an Impedance of Z_L is equal to 100 minus j50 ohm is to be matched to a 50-ohm line using an L-section matching network at an operating frequency of 500 megahertz. So let us design a matching network we already have the expressions for B and X so if we substitute the values in the expressions for B we get B is equal to minus 50 plus 100 by square plus square minus 100 into 50 everything under root divided by 100 square plus 50 square.

Now if this is computed, we will actually get two values, and these two values are 0.0058 ohm inverse and minus 0.0138 ohm inverse so we have therefore two values of B similarly if we use these values of B in calculating X we will get 2 values of X 61.2372 ohm and minus 61.2372 ohms.

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We have two solutions which are as follows:

Solution 1

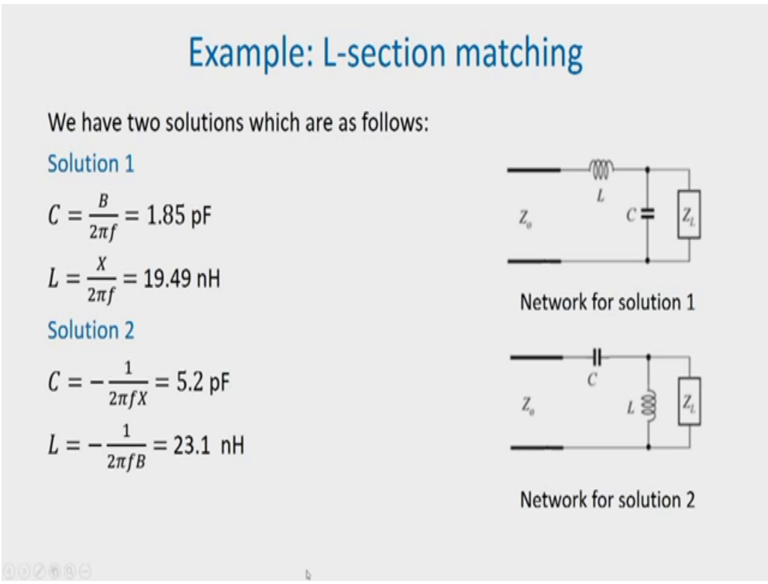
$C = \frac{B}{2\pi f} = 1.85$ pF

$L = \frac{X}{2\pi f} = 19.49$ nH

Solution 2

$C = -\frac{1}{2\pi f X} = 5.2$ pF

$L = -\frac{1}{2\pi f B} = 23.1$ nH



So what we can have now from these values of X and B we can calculate values for the lumped elements, the inductors, and the capacitors. So let us consider the first solution in this solution we have omega C is equal to B and therefore C is B by 2 pie f, f is given to be 500 megahertz B value is we have already calculated, so we take first value the positive value of B, and then we get a value of C to be 1 point 85 Pico farad similarly omega L is equal to X and therefore we know the value of X let us take the first value of X that we have calculated and then divide by 2 pie f, f is 500 megahertz then we get 19 point 49 Nano henry.

So this L and C combination actually will give a matching circuit which is shown so you have L C and load impedance Z_L and the transmission line Z naught. We can have a second solution because we have 2 values of B and 2 values of X so the second solution we find C is equal to minus 1 by 2 pie f X and again substituting the value of second value of X we get C is equal to 5 point 2 Pico farad.

And similarly L is equal to 1 by 2 pie f B so if we substitute the second value of B we get a solution of 23.1 Nano Hendry and this is the second network and here you can see the series reactants it is an inductance here the series element is a capacitance here the shunt element was a capacitor now it becomes shunt element become inductor and both the circuit will be able to match the given load impedance at the given frequency that means the load impedance of 100 minus j50 at the apparent frequency of 500 megahertz with these values of L and C used in the two circuits respectively.

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For the matching network as shown, we have $R_L < Z_0$

$$\frac{1}{Z_0} = jB + \frac{1}{jX + R_L + jX_L}$$

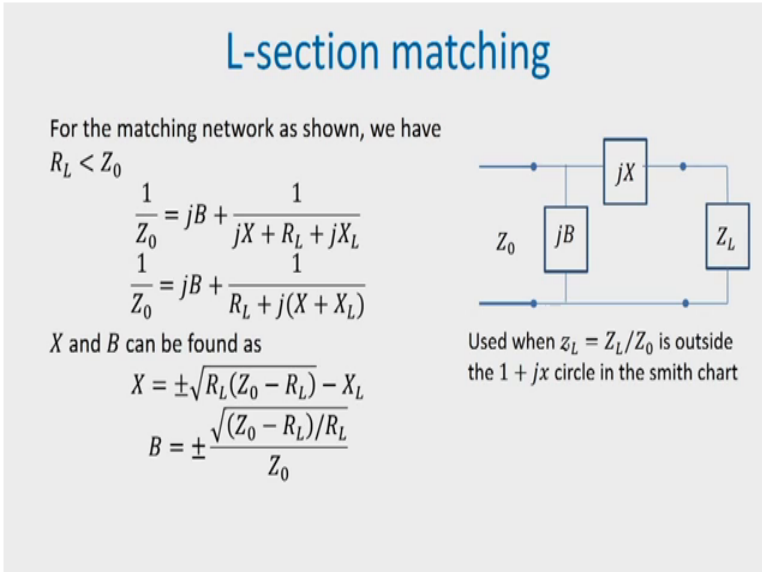
$$\frac{1}{Z_0} = jB + \frac{1}{R_L + j(X + X_L)}$$

X and B can be found as

$$X = \pm\sqrt{R_L(Z_0 - R_L)} - X_L$$

$$B = \pm\frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0}$$

Used when $z_L = Z_L/Z_0$ is outside the $1 + jx$ circle in the smith chart



Now we move to the second configuration of the L-section matching network here you can see the shunt element jB is connected first and then the load impedance Z_L is in series with the reactance jX now for this configuration we use this configuration when the normalized load impedance Z_L is outside the 1 plus jx circle on the smith chart, what does it mean that when the real part of Z_L is less than Z_0 ? So this matching network shown here will use when R_L is less than Z_0.

As before we need to find out the impedance looking at this point impedance or admittance looking at this point an equate it is easier to work with admittance here so 1 by Z_0 becomes equal to jB plus 1 by jX plus R_L plus jXL and therefore we can write 1 by Z_0 is equal to jB plus 1 by R_L plus j X plus X_L now from this equation once again we separate the real and imaginary parts and then we get a set of two equations from which X and B are solved.

When these steps are carried out and X and B are solved we get the expression for X to be equal to plus minus R_L plus minus square root R_L Z_0 minus R_L minus X_L and B is equal to plus minus square root Z_0 minus R_L divided by RL and the entire term divided by Z_0. because of these plus minus signs here once again we will get two solutions for X and two solutions for B and we can map these values of X and B to a capacitive element and an inductive element and therefore we will get two circuits with B and X been represented by lumped elements either a capacitance or an inductance.

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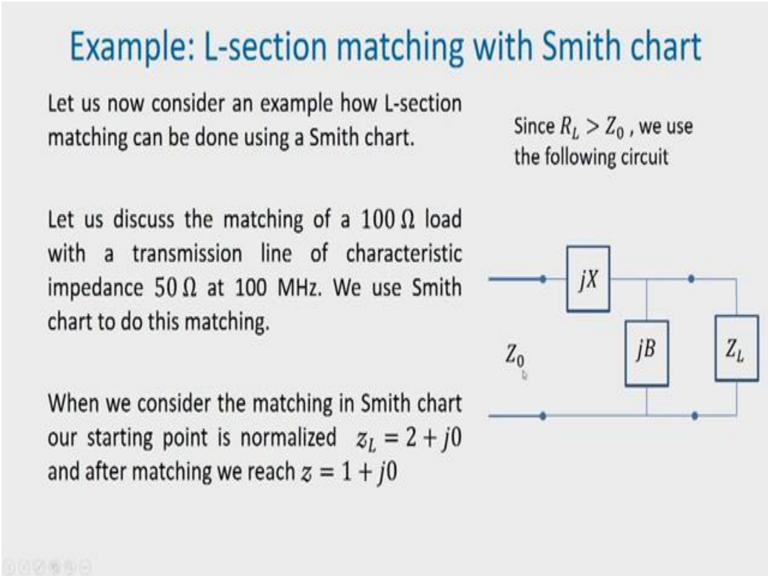

So we have seen, how we can calculate the values for the components of the L-section matching network analytically now let us consider an L-section matching design using smith chart we consider an example how the values of the L-section matching network can be found out using a smith chart let us discuss the matching of a 100 ohm load with a transmission line of characteristic impedance 50 ohm at an operating frequency of 100 megahertz and we use smith chart to do this matching so when we consider matching in smith chart our starting point is the normalized impedance ZL and which is, in this case, is 2 plus j zero and after matching we reach the point Z is equal to 1 plus j0.

So in our case since RL is greater than Z_0 we use the following circuit where we have this shunt element jB connected to Z_L first, and then the series element jX connected and this L-section network will match this load Z_L with Z_0.

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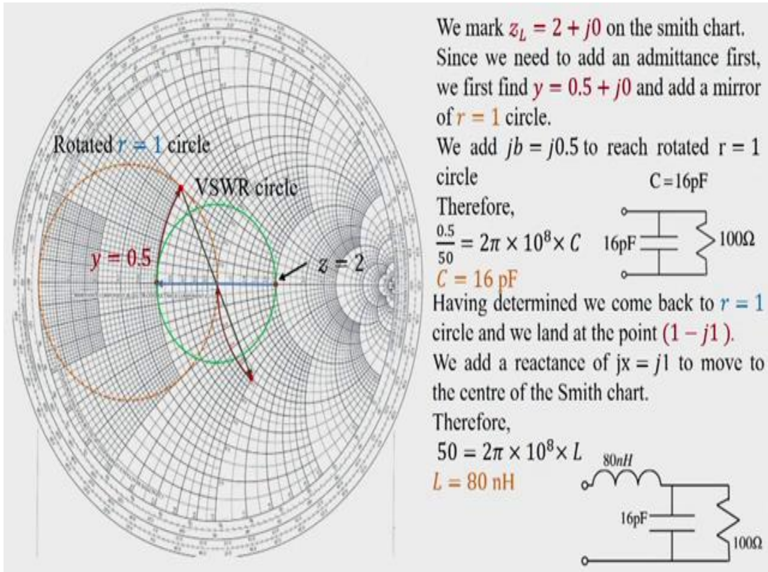

So for a smith chart solution what we do we first plot this normalized impedance z is equal to 2 on the smith chart and this is shown here next we draw the constant VSWR circle and then since we need to add an admittance first we find out y is equal to 0.5 corresponding to this impedance Z is equal to 2 next we draw a rotated r is equal to 1 circle, and we add jB equal to point 5 to reach this point in the rotated r equal to 1 circle so at this point we can calculate the capacitor value that will be needed, so our b is our small b is 0.5 so we multiply it by 1 by 50 is the value of Y naught to get b and that should be equal to 2 pi f C, and from this we find the value of the capacitor to be 16 Pico farad. So at this stage we have found out this shunt capacitor that required to be connected across this 100 ohm load, and that will give us the real part of the input admittance to be equal to 1.

Now we need to add the series element to what we come back to r equal to 1 circle that means this circle, so in order to do that we draw this line through the center of the smith chart and find the intersection point with r equal to 1 circle. Now we read the value of the normalized impedance here at this point we have the value of the normalized impedance to be 1 minus j1 that means in order to reach to the center of the smith chart we need to add a reactance of plus j and once we do that, we reach the center of the smith chart.

Now this, x is the normalized value which is having 1 so 50 into 1 we equate with 2 pi f is 10 to the power 8, L and then solve for L and we get the value of L to be equal to 80 Nano henry and therefore we found out both the elements of the L-section matching network the capacitor value is 16 Pico farad and the inductor value is 80 Nano henry, and these values can also be verified using the analytical formulation which was described previously.

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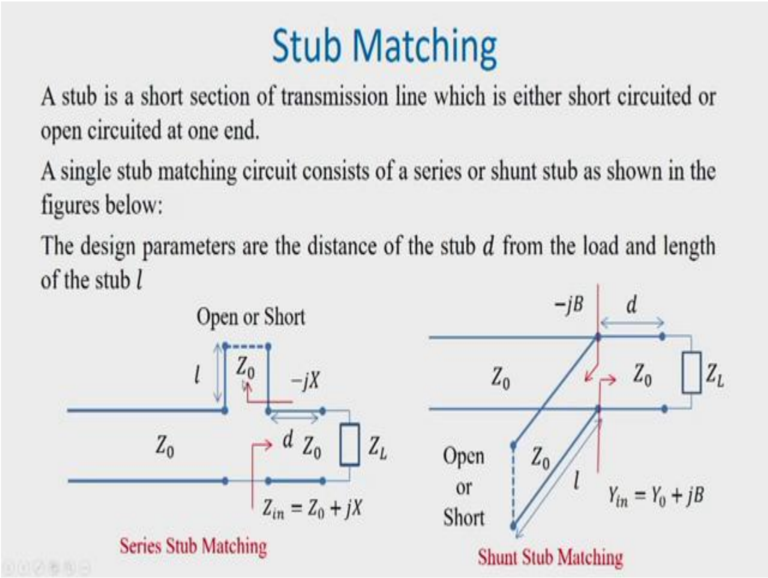

So, we start out discussion on another matching technique, which is called stub matching a stub is a short section of a transmission line which is either short-circuited or open-circuited at one end. Now a single stub matching circuit consists of a series or a shunt stub as shown in the figures. So this is the figure for a series stub matching circuit, so we have the impedance Z_L, which is to be matched to this transmission line of characteristic impedance Z_0.

So what we do we try to find out the length of the transmission line d such that the input impedance looking here it becomes Z_0 plus jX now this extra element jX is nullified by connecting this series stub of length l which can be open or short, so this is one way of designing a matching network and this type of matching network is called a series stub matching.

In another design, we can find out the input admittance in such a way that Y_n becomes equal to Y naught plus jB and then add a shunt stub open or short circuit to cancel out this remaining jB and therefore match this to the line of characterized impedance Z_0. So, if we look at both the circuits what we observe that the design parameters in this type of circuits are the distance of the stub d from the load and length of the stub l, which may be either open or sort, so various design options are available.

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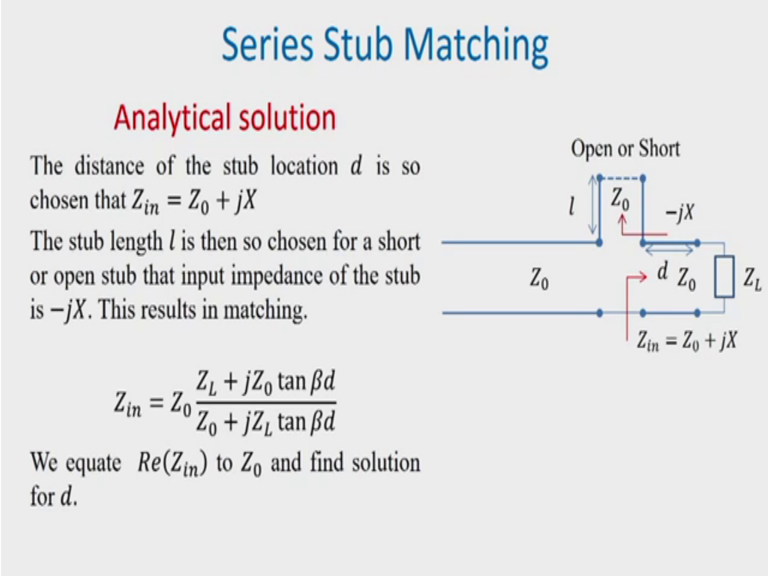
Analytical solution

The distance of the stub location d is so chosen that $Z_{in} = Z_0 + jX$

The stub length l is then so chosen for a short or open stub that input impedance of the stub is $-jX$. This results in matching.

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta d}{Z_0 + jZ_L \tan \beta d}$$

We equate $Re(Z_{in})$ to Z_0 and find solution for d.



Now let us consider an analytical solution to this series stub matching problem that means what we want to do we want to find out this distance d and also the length of the stub l as we have already mentioned that at the stub location d the input impedance Z_{in} is Z_0 plus jX and we need to choose the length l of the stub a sort circuited stub or a open circuited stub in such a way that the input impedance of the stub looking at this point has to be equal to minus jX and this will result into matching.

Now we know that the input impedance Z_{in} can be written as Z_0 Z_L plus j Z_0 tan beta d divided by Z_0 plus jZ_L tan beta d and what we do we equate the real part of Z_{in} to Z_0 then we can find a solution for d.

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Analytical solution

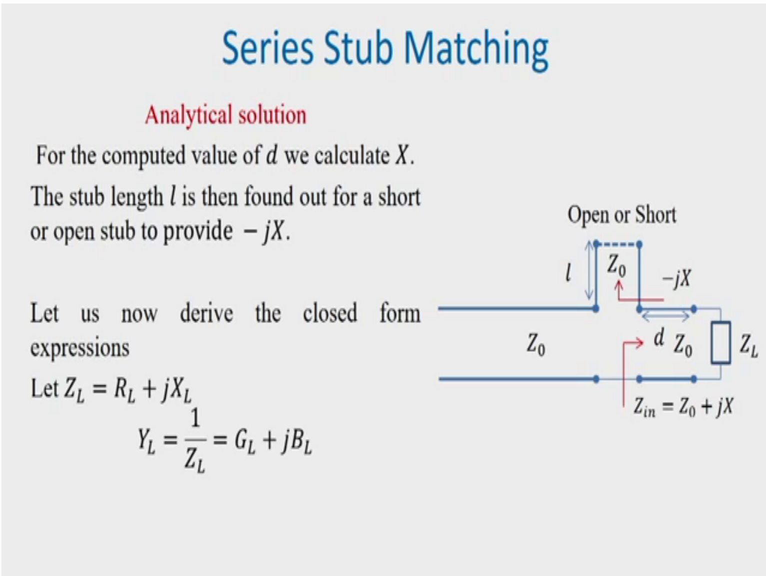
For the computed value of d we calculate X.

The stub length l is then found out for a short or open stub to provide $-jX$.

Let us now derive the closed form expressions

Let $Z_L = R_L + jX_L$

$$Y_L = \frac{1}{Z_L} = G_L + jB_L$$



And once we have calculated the, what we can do, we calculate the value of X by substituting d, and then we find out the length of the stub so is to provide minus jX. Now let us try to see how we can derive the closed-form expression for d and l. So we have Z_L equals to RL plus jX_L. We can write Y_L, which is 1 by Z_L to be equal to GL plus jB_L.

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Let $t = \tan\beta d$

$$Y_{in} = \frac{1}{Z_{in}} = Y_0 \frac{Y_L + jY_0 \tan\beta d}{Y_0 + jY_L \tan\beta d}$$

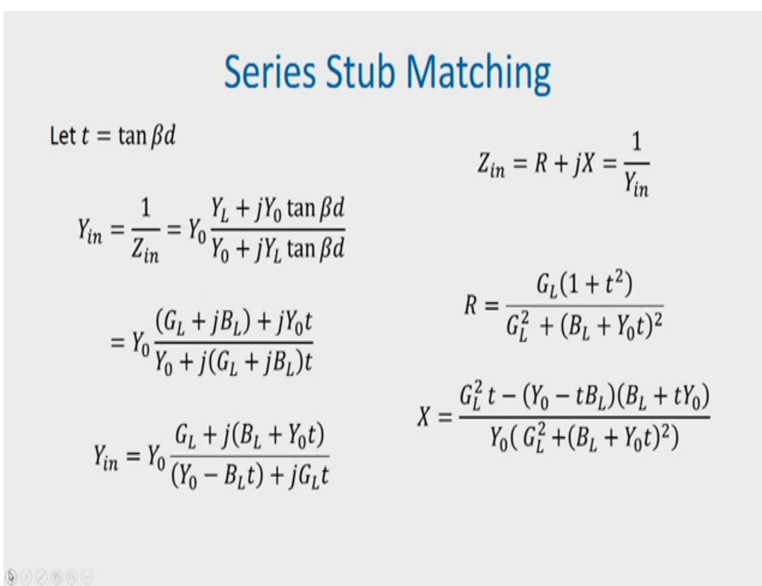
$$= Y_0 \frac{(G_L + jB_L) + jY_0 t}{Y_0 + j(G_L + jB_L)t}$$

$$Y_{in} = Y_0 \frac{G_L + j(B_L + Y_0 t)}{(Y_0 - B_L t) + jG_L t}$$

$$Z_{in} = R + jX = \frac{1}{Y_{in}}$$

$$R = \frac{G_L(1 + t^2)}{G_L^2 + (B_L + Y_0 t)^2}$$

$$X = \frac{G_L^2 t - (Y_0 - tB_L)(B_L + tY_0)}{Y_0(G_L^2 + (B_L + Y_0 t)^2)}$$



Now, let us define t equal to tan beta d so therefore we can now write Y_{in} which is 1 by Z_{in} to be equal to Y_0 Y_L plus j Y_0 tan beta d divided by Y_0 plus jY_L tan beta d and this is we substitute t here for tan beta d and we substitute Y_L equal to GL plus jB_L and then we can write Y_{in} of this form Y_0 GL plus j B_L plus Y_0 t divided by Y_0 minus BL t plus jG_Lt.

So what we can do we can now write Z in is equal to R plus jX is equal to 1 by Y_{in} and then from there we can find out the real part R is equal to G_L into 1 plus t square divided by G_L square plus BL plus Y_0 t whole square and the expression for X also can be obtained and it is given by G_L square t minus Y_0 minus tB_L into B_L plus t Y_0 divided by Y_0 into G_L square plus B_L plus Y_0 t whole square. Now we have, R and X expression for R and X we know that for our matching this R has to be equal to Z_0 and once we equate add to Z_0 G_L and B_L Y_0 these are been known we can find out the solution for t.

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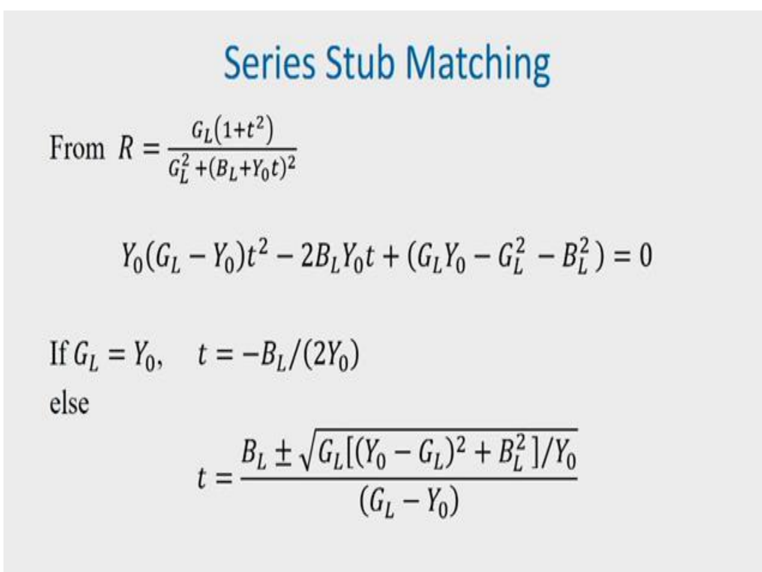
From $R = \frac{G_L(1+t^2)}{G_L^2+(B_L+Y_0 t)^2}$

$$Y_0(G_L - Y_0)t^2 - 2B_L Y_0 t + (G_L Y_0 - G_L^2 - B_L^2) = 0$$

If $G_L = Y_0$, $\quad t = -B_L/(2Y_0)$

else

$$t = \frac{B_L \pm \sqrt{G_L[(Y_0 - G_L)^2 + B_L^2]/Y_0}}{(G_L - Y_0)}$$



So from the expression of R_L we can write if we write R equal to Z_0 is equal to 1 by Y_0, if we make this substitution then we can and then rearrange the terms we can write Y_0 G_L minus Y_0 t square minus 2 B_L Y_0 t plus G_L $Y_0 G_L$ square minus B_L square equal to zero.

Now, this is a quadratic equation, and this equation can be solved to get expressions for t one particular case is when G_L equal to Y_0, in this case, this term becomes zero and here also G_L Y not so this G_L square G_L square cancels and what do we get a simple solution for t which is minus B_L divided by two Y_0. So this is specific case when G_L is equal to Y_0 if G_L is not equal to Y_0 than we can solve t to be equal to the standard solution for a quadratic equation we can write and we can write t in this form. So once t is known, we remember that t is equal to tan beta d.

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We get two solutions for d which are given by

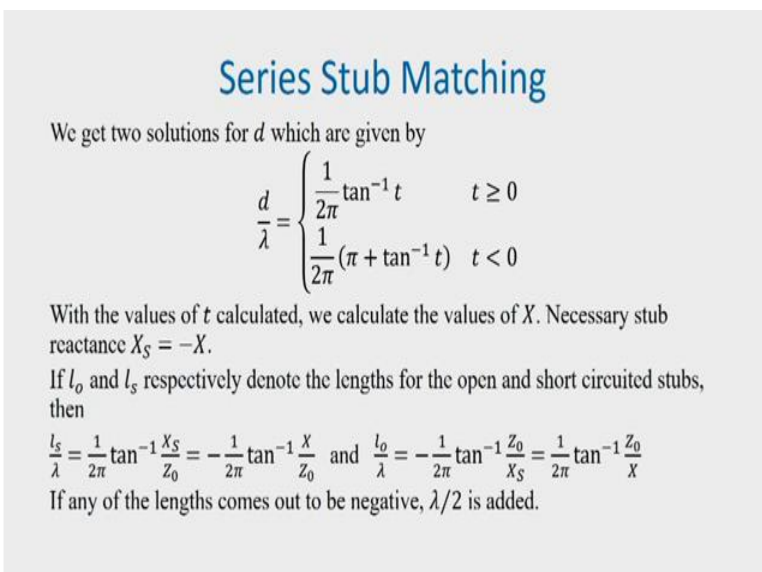
$$\frac{d}{\lambda} = \begin{cases} \frac{1}{2\pi}\tan^{-1} t & t \geq 0 \\ \frac{1}{2\pi}(\pi + \tan^{-1} t) & t < 0 \end{cases}$$

With the values of t calculated, we calculate the values of X. Necessary stub reactance $X_S = -X$.

If l_o and l_s respectively denote the lengths for the open and short circuited stubs, then

$\frac{l_s}{\lambda} = \frac{1}{2\pi}\tan^{-1}\frac{X_S}{Z_0} = -\frac{1}{2\pi}\tan^{-1}\frac{X}{Z_0}$ and $\frac{l_o}{\lambda} = -\frac{1}{2\pi}\tan^{-1}\frac{Z_0}{X_S} = \frac{1}{2\pi}\tan^{-1}\frac{Z_0}{X}$

If any of the lengths comes out to be negative, $\lambda/2$ is added.



Now we can get a solution for d, so we get two solutions for d and d by lambda can be written as 1 by 2 pi tan inverse t when t is greater than zero, and this is 1 by 2 pi into pi plus tan inverse t when t is less than zero. So we have 2 possible values of t and depending upon whether it is greater than zero or less than zero we will get 2 solutions for d or d by lambda.

So with the values of d calculated, we calculate the value of X, and once we calculate we have already shown the expression for X, and when we calculate the value of X the necessary stub reactants X_S is equal to minus X.

Now as we have already said the stub can be realized either as an open-circuited stub or it can be realized as a short-circuited stub. So let us denote l o as the length of the open-circuited stub and ls denotes the length of the short-circuited stub. So we can find out ls by lambda is equal to 1 by 2 pi tan inverse X by Z, and we know that X_S is equal to minus X so once we substitute minus X here we get minus 1 by 2 pi tan inverse X by Z_0.

Similarly, for the open-circuit case we get l open or lo by lambda is equal to minus 1 by 2 pi tan inverse Z_0 by XS and this is equal to 1 by 2 pi tan inverse Z_0 by X we are calculating the lengths of the stub so it may be noted that if any of the lengths come out to be negative we can add lambda by two to get the actual stub length.

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Let us consider an example where $Z_L = 100 + j50\ \Omega$ is to be matched to a 50 Ω line. By applying the analytical solutions we get:

$t = -0.333 = t_1$ and $t = 1.0 = t_2$. We get two solutions for d

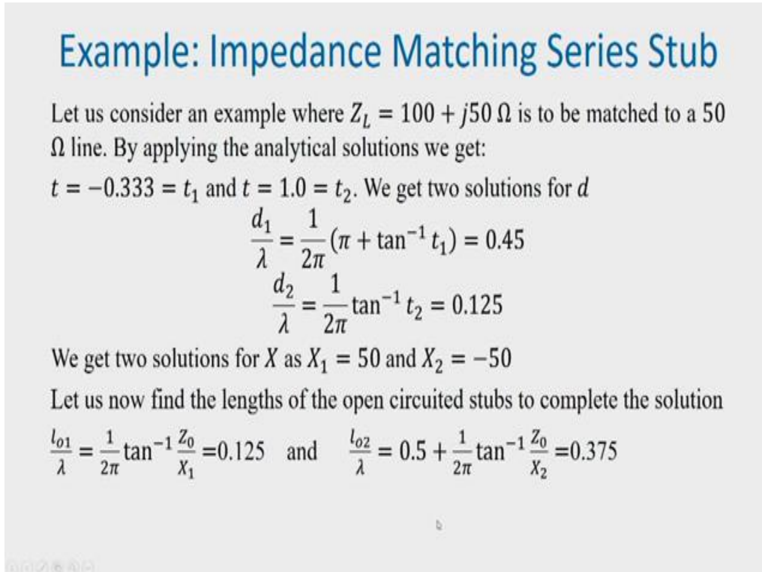
$$\frac{d_1}{\lambda} = \frac{1}{2\pi}(\pi + \tan^{-1} t_1) = 0.45$$

$$\frac{d_2}{\lambda} = \frac{1}{2\pi}\tan^{-1} t_2 = 0.125$$

We get two solutions for X as $X_1 = 50$ and $X_2 = -50$

Let us now find the lengths of the open circuited stubs to complete the solution

$\frac{l_{o1}}{\lambda} = \frac{1}{2\pi}\tan^{-1}\frac{Z_0}{X_1}$ =0.125 and $\frac{l_{o2}}{\lambda} = 0.5 + \frac{1}{2\pi}\tan^{-1}\frac{Z_0}{X_2}$ =0.375



So, let us now consider an example, and in this example we consider a load impedance Z_L to be equal to 100 plus j50 ohm, and this load impedance ZL is to be matched to a 50 ohm line. Than if we apply the analytical solutions, we get 2 values of t which are minus 0.333 which we call t_1 and t equal to 1 which we call t_2 now corresponding to this two values of t we will also get 2 solutions for d and d1 by lambda can be found out to be 0.45 and d2 by lambda can be found out to be 0.125.

Now once you substitute these values of t, we get 2 solutions of X, and we call them X_1 is equal to fifty X_2 is equal to minus 50, and when we have 2 values of X we can also have 2 values of the open-circuited here we are using open-circuit stub to do the matching so we can calculate the values for the length of open circuit at stubs, and we call it lo1 by lambda is equal to 1 by 2 pi tan inverse Z naught by X_1, and this X_1 is 50.

So it becomes tan inverse 1, and therefore it is pi by 4 so essentially $l_1$by lambda becomes 0.125 and l_2 by lambda this is 2 plus 1 by 2 pie tan inverse Z_0 by X_2, X_2 is minus 50, so it becomes 0.5 minus 0.125 which is 0.375 so we have found out the locations of the stub where the stub need to be placed from the load d1 and d_2 in terms of lambda and the corresponding values of the lengths of the stub again in terms of lambda we have calculated, so one stub is 0.125 lambda, and the other stub is 0.375 lambda and the matching can be obtained by placing this stubs at the locations what we have calculated.

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$$\frac{d}{\lambda} = 0.338 - 0.213 = 0.125$$

$$x = -1$$

$$x_S = 1$$

$$\frac{l_0}{\lambda} = 0.375$$

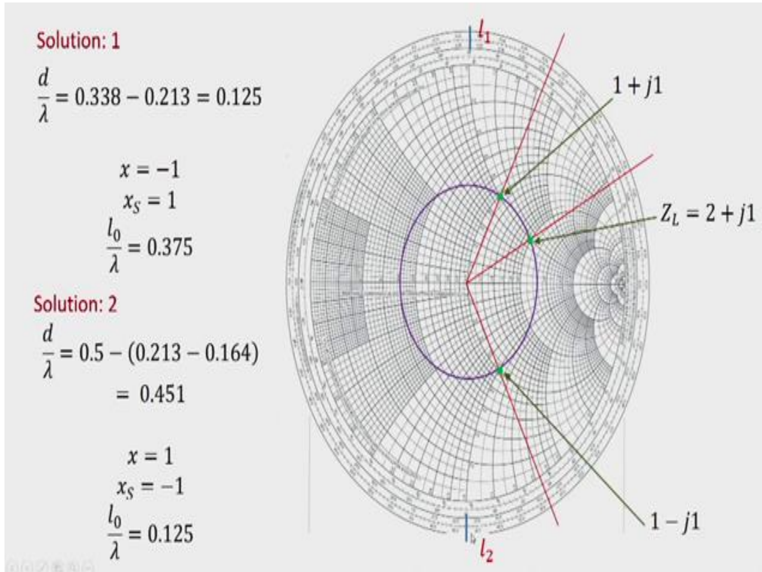
$$\frac{d}{\lambda} = 0.5 - (0.213 - 0.164)$$

$$= 0.451$$

$$x = 1$$

$$x_S = -1$$

$$\frac{l_0}{\lambda} = 0.125$$



So let us now move on to how we can do this matching L section matching using a smith chart in the smith chart we first locate the point normalise impedance 2 plus j1 and then we pass a line from the centre of the smith chart through this point also we draw the constant VSWR circle and we find that as we move from the load over this VSWR circle. Towards the generator, for the first time we intersect R is equal to one circle at this point and we pass a line through this point.

Now, the wavelength value here can be read out similarly the wavelength value here can be read out and the difference of these two will give us the distance d one of the solutions by which we should move from the load to place the stub. And at this point we find that the input impedance is 1 minus j1 so in order to do the matching for this point, we need to add 1 plus j1 if we continue our journey this way we find that we intersect the r equal to 1 circle.

Once again, at this point, which is given by 1 plus j1 we draw another line, so now we will have to move from this point over this circle, and this will give us another solution for the distance d, and here the normalized impedance is 1 plus j1. So the stub must provide a reactance of minus 1 so we can find out the lengths we know that this is the open circuit point and start from here if we move across the periphery of the smith chart at this point we find plus one j corresponding to this solution and this gives us the length l1 and similarly from this open-circuit point we move towards the generator this is the point we get minus 1 reactance, and this is the length l_2 and this length l_1 and l_2 can be seen to be 0.125 lambda, and this one will be 0.375 lambda.

And therefore we now calculate the values of d in terms of lambda for the solution of 1 at this point. We have the value point 213, and at this point we have 0.338. So the difference of these two gives us 0.125 as the length of the transmission line d, and after traversing this 0.125 lambda we reach this point where we need to apply axis is equal to 1 for which l open by lambda is 0.375. The second solution we have is d by lambda is this is point 5 minus this distance that means 0.213 minus 0.164 this arc length, and this comes out to be d by lambda equal to 0.451 and for this solution we require X_S to be equal to minus 1 that means the length of the stub in terms of lambda is given by 0.125 where we get minus 1 X_S as we travel down the smith chart from the open-circuit point.

So, this completes our solution we have found out the lengths of the stub we have found out the location of the stub, and from our analytical calculation, we find that these values agree with the smith chart calculated values.