Microwave Engineering
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Lecture 16
Waveguide Resonators

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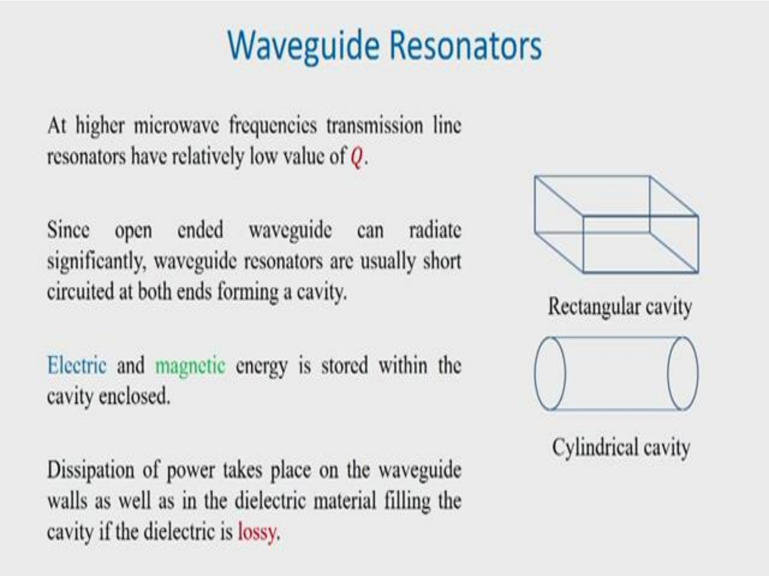

We have seen different types of transmission line resonators. Let us now consider Waveguide Resonators. At higher microwave frequencies the transmission line resonators have a relatively low value of Q. Since open-ended waveguide can radiate significantly, waveguide resonators are usually short-circuited at both ends, forming a cavity. Electrical and magnetic energy is stored within the cavity.

So, a rectangular cavity and a cylindrical cavity is shown in the figure. So, you can see that this rectangular cavity is essentially a rectangular waveguide, and with both the ends it is now short-circuited. Similarly it is a circular waveguide with end caps at both ends. Now, the dissipation of power takes place on the waveguide walls as well as in the dielectric material feeling the cavity if the dielectric is lossy.

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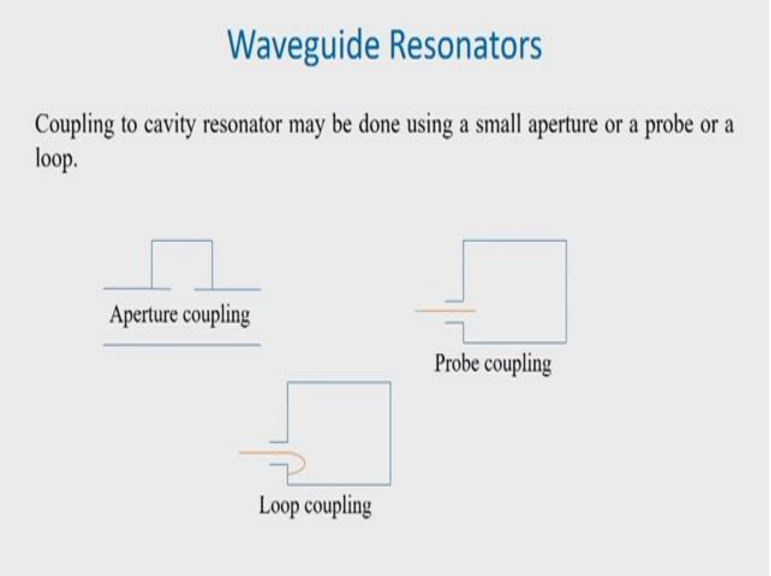

Coupling to cavity resonator may be done using a small aperture or a probe or a loop. So, this is an aperture coupling where, through this aperture, the coupling with the cavities is achieved. This is a probe coupling where the central probe goes inside the cavity, and this is a loop coupling where the central conductor of the coax forms a loop.

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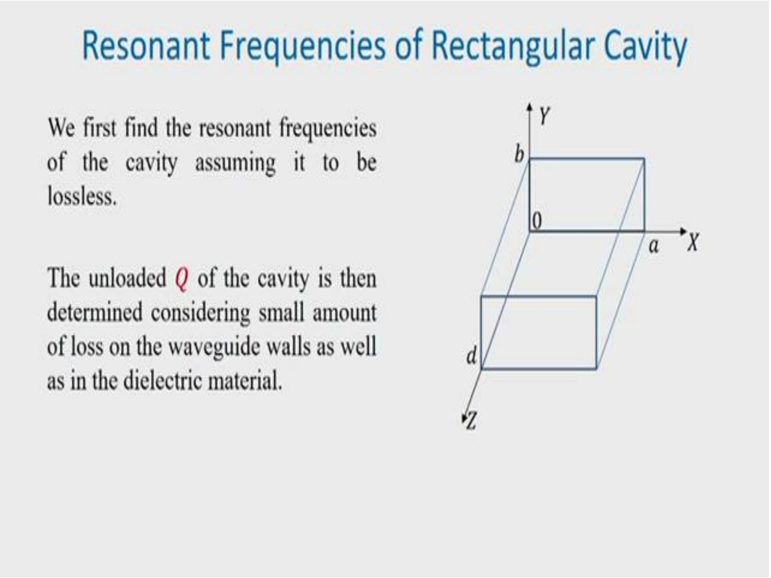

So, we start our discussion by determining the resonant frequencies of rectangular cavity. So, here we show a rectangular cavity which has sides a, b, and d. So, what we do? We first find the resonant frequencies for such cavity assuming that the cavity walls are lossless and also the dielectric material if any present within this cavity it is also lossless.

The unloaded Q of the cavity is then determined considering the small amount of loss on the waveguide walls as well as in the dielectric material. So, initially we find the resonant frequency considering lossless condition.

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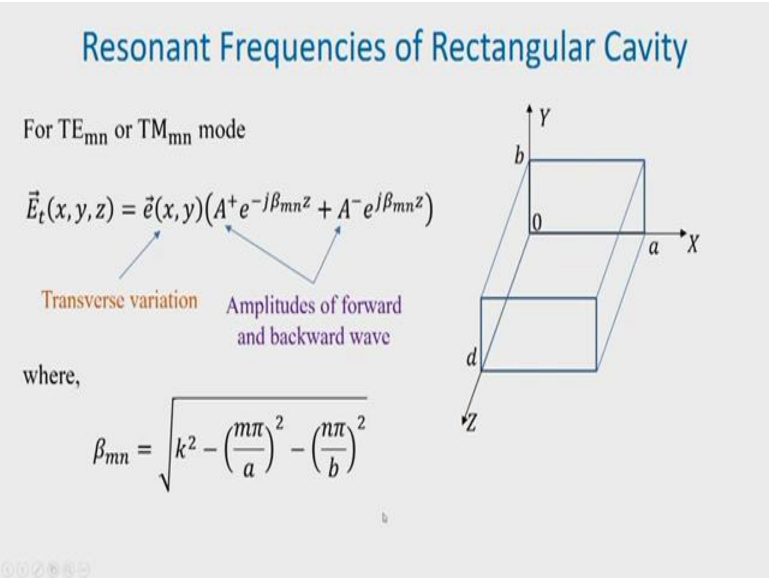
For TE_{mn} or TM_{mn} mode

$$\vec{E}_t(x,y,z) = \vec{e}(x,y)\left(A^+e^{-j\beta_{mn}z} + A^-e^{j\beta_{mn}z}\right)$$

where,

$$\beta_{mn} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$



So, for TE mn or TM mn mode we can write the transverse electric fields in the form of e bar x, y which takes into account transverse variation, variation with respect to x and y and this bracketed term gives the longitudinal variation and since the cavity is closed at both ends. We have also considered the reflected wave and therefore A plus and A minus this is at the amplitudes of the forward and backward wave.

And beta mn this is given by k square minus m pi by a whole square plus n pi by b whole square and everything under root. So, what happens? Now, we have at Z is equal to 0, and Z is equal to d we have perfectly conducting walls.

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$\vec{E}_t = 0$ at $z = 0$ $\quad\quad \Rightarrow A^+ = -A^-$

$\vec{E}_t = 0$ at $z = d$

$$\therefore \vec{E}_t(x, y, d) = -\vec{e}(x, y)A^+ 2j \sin\beta_{mn} d = 0$$

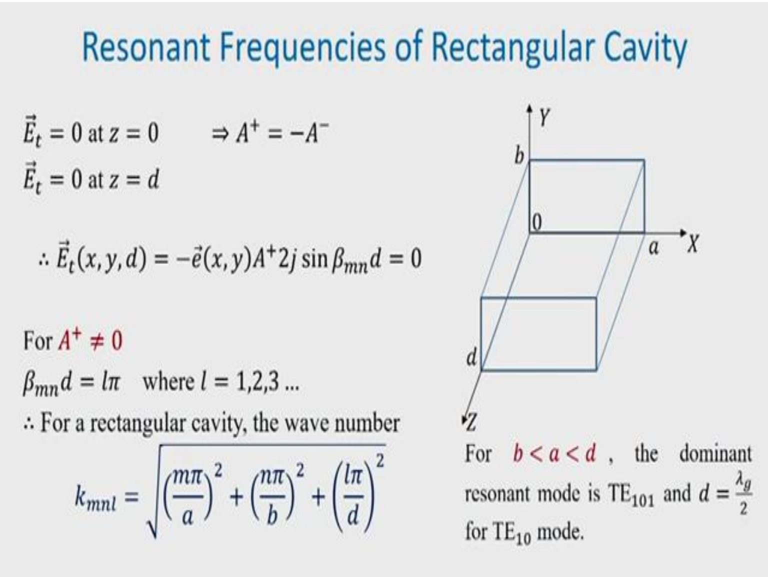
For $A^+ \neq 0$

$\beta_{mn} d = l\pi \quad$ where $l = 1,2,3 \ldots$

$\therefore$ For a rectangular cavity, the wave number

$$k_{mnl} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

For $b < a < d$, the dominant resonant mode is TE_{101} and $d = \frac{\lambda_g}{2}$ for TE_{10} mode.



And therefore, the tangential component E t tangential field has to be 0 at Z is equal to 0 and this condition will give A plus equal to minus A minus. E t is also 0 at Z is equal to d here and this condition once we substitute A minus to be equal to minus of A plus and divide throughout by 2j here in this term if we substitute A minus to be equal to minus of A plus then we can take A plus outside and divide numerator and denominator by 2j.

In that case we get E t x, y, d to be equal to minus e x, y A plus 2j sin beta mn d and that must be equal to 0 here and sin beta mn d equal to 0 this implies that beta mn d is equal to l pi where l is equal to 1, 2, 3. And therefore for a rectangular cavity now we can write the wavenumber

k_{mnl} which is equal to m pi by a whole square, n pi by b whole square plus l pi by d whole square.

And therefore, for a rectangular cavity we can write the wavenumber k_{mnl} as under root m pi by a whole square plus n pi by b whole square plus l pi by d whole square. Now, whenever we have this b less than a and a less than d the dominant mode will be TE_{101} and this dimension d will be lambda g by 2 for the TE_{10} mode.

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For TE_{10l} mode we can write the field components as follows:

$$E_y = A^+ \sin\frac{\pi x}{a}\left(e^{-j\beta z} - e^{j\beta z}\right)$$

$$H_x = -\frac{A^+}{Z_{\mathrm{TE}}}\sin\frac{\pi x}{a}\left(e^{-j\beta z} + e^{j\beta z}\right)$$

$$H_z = \frac{j\pi A^+}{k\eta a}\cos\frac{\pi x}{a}\left(e^{-j\beta z} - e^{j\beta z}\right)$$

We have seen that for TE_{10} mode

$$H_z = A_{10}\cos\frac{\pi x}{a}e^{-j\beta z}$$

$$E_y = \frac{-j\omega\mu a}{\pi}A_{10}\sin\frac{\pi x}{a}e^{-j\beta z}$$

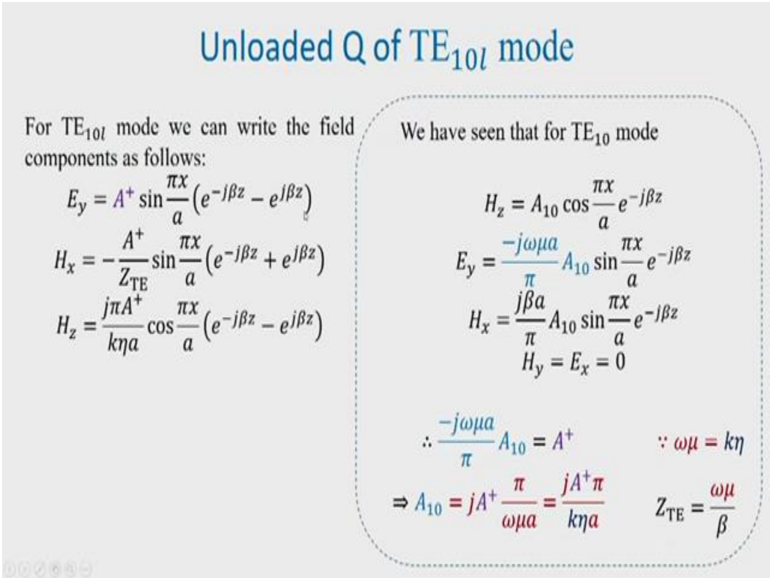
$$H_x = \frac{j\beta a}{\pi}A_{10}\sin\frac{\pi x}{a}e^{-j\beta z}$$

$$H_y = E_x = 0$$

$$\therefore \frac{-j\omega\mu a}{\pi}A_{10} = A^+$$

$$\Rightarrow A_{10} = jA^+\frac{\pi}{\omega\mu a} = \frac{jA^+\pi}{k\eta a}$$

$$\because \omega\mu = k\eta$$



Now, let us calculate the unloaded Q of TE_{10l} mode. For TE_{10l} mode we can write the field components E y component we can write A plus sin pi x by a e to the power minus j beta z minus e to the power j beta z. so, this is the wave that is traveling in the plus z and this is in the minus z-direction. And similarly we can write H x is equal to A plus by z TE sin pi x by a e to the power minus j beta z plus e to the power j beta z and we can write H z in this particular form.

Now, how we can put these forms? Because we have seen TE_{10} mode, for TE_{10} mode we had H z is equal to A 10 cos pi x by a e to the power minus j beta z and E y minus j omega mu a divided by pi A 10 sin pi x by a e to the power minus j beta z and H x is equal to j beta a by pi A 10 sin pi x by a e to the power minus j beta z. These expressions we have already seen in our discussion on dominant TE 10 mode of a rectangular waveguide.

Now, what we do? We are writing A plus for this term minus j omega mu a by pi if you compare E y from here and E y from here and therefore A 10 becomes j A plus pi by omega mu a and this can be written as j A plus pi by k eta a. Since we have omega mu is equal to k eta and now you can see that here in this expression H z is A 10 and A 10 in terms of A plus it is j pi A plus by k eta a.

So, this is how we are writing the same field equations what we studied earlier in slightly different form. Similarly we can see that H x can be put in the form of minus A plus divided by Z TE sin pi x by a e to the power minus j beta z plus e to the power j beta z and this Z TE is actually omega mu by beta. So, this expressions we have already seen earlier and using this expressions we can write the E y, H x and H z field components in the form shown here.

It may be noted that in this expressions we are only considering the waves which are traveling in the plus Z direction, in cavity we consider the waves which travel in plus Z as well as minus Z direction.

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$$E_y = A^+ \sin\frac{\pi x}{a}\left(e^{-j\beta z} - e^{j\beta z}\right)$$

$$H_x = -\frac{A^+}{Z_{\text{TE}}}\sin\frac{\pi x}{a}\left(e^{-j\beta z} + e^{j\beta z}\right)$$

$$H_z = \frac{j\pi A^+}{k\eta a}\cos\frac{\pi x}{a}\left(e^{-j\beta z} - e^{j\beta z}\right)$$

Substituting $E_0 = \frac{2A^+}{j}$, we get

$$E_y = E_0 \sin\frac{\pi x}{a}\sin\frac{l\pi z}{d}$$

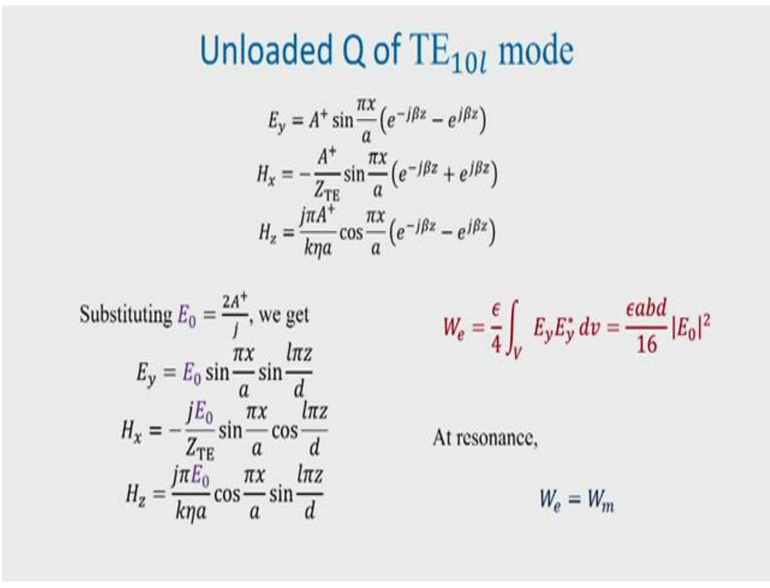
$$H_x = -\frac{jE_0}{Z_{\text{TE}}}\sin\frac{\pi x}{a}\cos\frac{l\pi z}{d}$$

$$H_z = \frac{j\pi E_0}{k\eta a}\cos\frac{\pi x}{a}\sin\frac{l\pi z}{d}$$

$$W_e = \frac{\epsilon}{4}\int_V E_y E_y^* \, dv = \frac{\epsilon abd}{16}|E_0|^2$$

At resonance,

$$W_e = W_m$$



So, we have E y, H x and H z components defined. So, if we substitute E naught is equal to 2A plus by j that means A plus is equal to E naught j by 2 then we can write E y to be equal to E naught sin pi x by a and this can be written as sin l pi z by d because we have seen that beta becomes equal to l pi by d. Similarly, we can write H x to be equal to minus j E naught by Z TE sin pi x by a cos l pi z by d and H z is j pi E naught divided by k eta a cos pi x by a sin l pi z by d.

So, once we have these field components written in this form what we can do? We can find out the electric energy that is stored within the cavity, inside this electric field E y and that can be computed as epsilon by 4 an it is volume integral E y E y conjugate dv and when this expressions for the E y field component is substituted and we evaluate this integral over the volume of the cavity that means 0 to a, 0 to b and 0 to d respectively being the variation of a x, y and z we get this expression. And we know that at resonance stored electrical energy W e is same as stored magnetic energy W m.

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Case I. The dielectric is perfect but cavity walls are slightly lossy

The power loss on the conducting walls can be found as

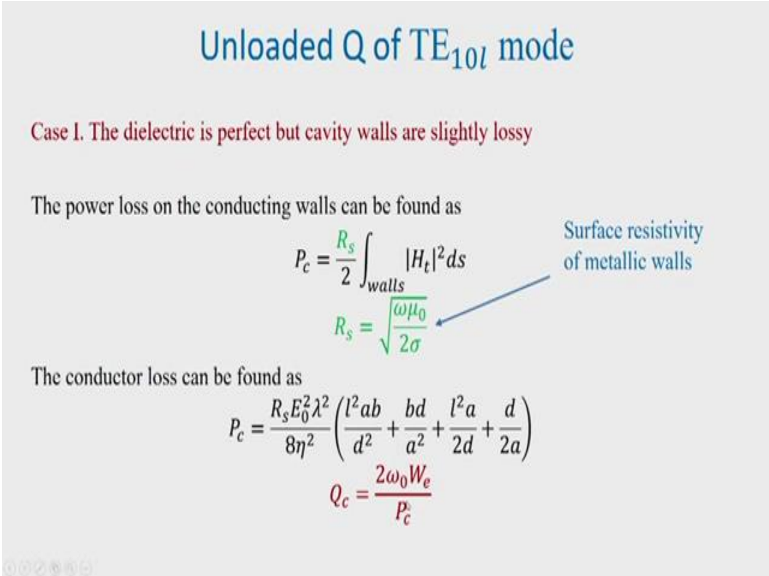
$$P_c = \frac{R_s}{2}\int_{walls} |H_t|^2 ds$$

$$R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}}$$

The conductor loss can be found as

$$P_c = \frac{R_s E_0^2 \lambda^2}{8\eta^2}\left(\frac{l^2 ab}{d^2} + \frac{bd}{a^2} + \frac{l^2 a}{2d} + \frac{d}{2a}\right)$$

$$Q_c = \frac{2\omega_0 W_e}{P_c}$$



Now, we consider the case when the dielectric is perfect but the cavity walls are slightly lossy. So, what we can do? We can find out the power loss on the conducting walls and that can be found out as P c is equal to R s by 2 integrated over the walls we have six walls and at each wall we find out mod of H t square ds and R s the sheet resistance for the conductors it is given by omega mu not by 2 sigma it is the surface resistivity of the metallic walls and we can find the conductor loss.

So, when we substitute these field components, tangential field components when we substitute tangential magnetic field components in this expression and find out the power loss in the individual walls and then add them together this is the expression for the conductor loss and we can now define Q c which is equal to 2 times W e this is the energy stored multiplied by omega naught and divided by P c the power dissipated in the conductors.

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Case II. The dielectric is lossy but cavity walls are perfectly conducting.

$$\epsilon = \epsilon' - j\epsilon'' = \epsilon_0\epsilon_r(1 - j\tan\delta)$$

Power dissipated within the dielectric volume is

$$P_d = \frac{1}{2}\int_V \vec{J}.\vec{E}^* dv = \frac{\omega\epsilon''}{2}\int_V |E|^2 dv$$

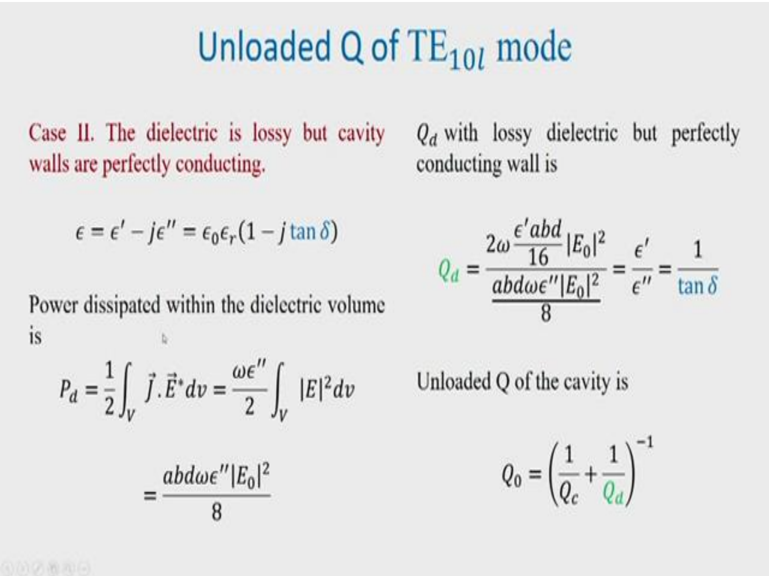
$$= \frac{abd\omega\epsilon''|E_0|^2}{8}$$

Q_d with lossy dielectric but perfectly conducting wall is

$$Q_d = \frac{2\omega\frac{\epsilon' abd}{16}|E_0|^2}{\frac{abd\omega\epsilon''|E_0|^2}{8}} = \frac{\epsilon'}{\epsilon''} = \frac{1}{\tan\delta}$$

Unloaded Q of the cavity is

$$Q_0 = \left(\frac{1}{Q_c} + \frac{1}{Q_d}\right)^{-1}$$



In the second case we consider the dielectric to be slightly lossy but the cavity walls are perfectly conducting and in that case we can write epsilon to be equal to epsilon dash minus j epsilon double dash and this we can write as epsilon naught epsilon r 1 minus j tan delta where tan delta is the loss tangent and power dissipated within the dielectric volume can be found out as P d equal to half volume integral of J dot E conjugate dv and this is becomes omega epsilon double prime divided by 2 volume integral of mod of E square dv.

And from there, if we substitute the expression for electric field we get abd omega epsilon double prime E naught magnitude square by 8. So, Q d with lossy dielectric but perfectly conducting wall can then be found out. So, 2 epsilon prime abd by 16 mod of E naught square this gives the total stored energy multiplied by omega and we have found out P d so we substitute the expression for P d.

And then we get a very simple relation for Q d which is equal to epsilon prime by epsilon double prime and this becomes equal to 1 by tan delta. So, we have seen for the two cases when the dielectric is lossless but the waveguide walls are lossy and in the second case when the waveguide walls are perfectly conducting but the dielectric is slightly lossy.

So, we can find the overall unloaded Q, Q naught as we have seen in the earlier case 1 by Q naught will become 1 by Q c plus 1 by Q d and therefore Q naught will be 1 by Q c plus 1 by Q d whole rest to the power minus 1.

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Since the dominant mode of circular waveguide is TE_{11}, the dominant mode of the circular waveguide cavity is TE_{111}.

For TM modes, the mode with the lowest cut off frequency is TM_{01} mode.

The resonant frequencies of TE_{nml} and TM_{nml} modes of the circular waveguide cavities can be found as follows:

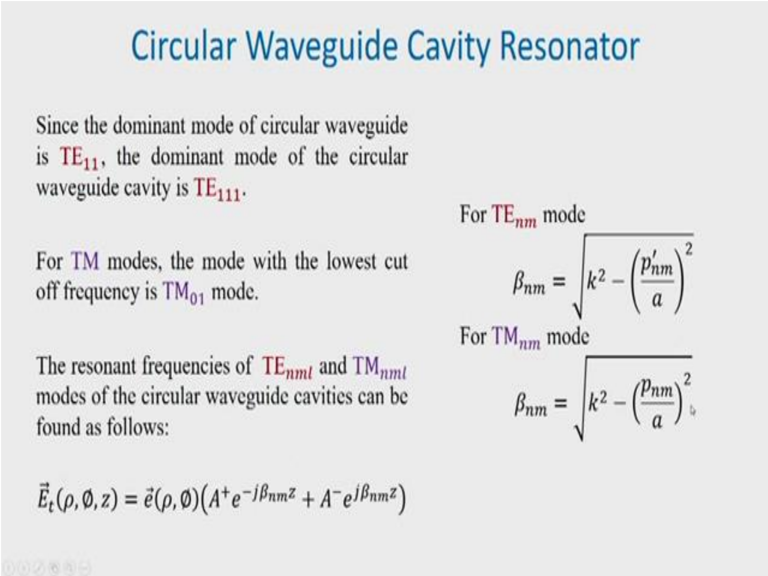
$$\vec{E}_t(\rho, \emptyset, z) = \vec{e}(\rho, \emptyset)\left(A^+ e^{-j\beta_{nm}z} + A^- e^{j\beta_{nm}z}\right)$$

For TE_{nm} mode

$$\beta_{nm} = \sqrt{k^2 - \left(\frac{p'_{nm}}{a}\right)^2}$$

For TM_{nm} mode

$$\beta_{nm} = \sqrt{k^2 - \left(\frac{p_{nm}}{a}\right)^2}$$



Let us now discuss about the circular waveguide cavity resonator. Since the dominant mode of circular waveguide is TE $_{11}$, the dominant mode of circular waveguide cavity is TE $_{111}$. For TM modes, the mode with the lowest cut off frequency is TM $_{01}$ in a circular waveguide. Now, we can write the tangential component of the electric field in the same manner in terms of the transverse coordinates rho and phi and also a wave traveling along plus z and minus z and we use this to find out the resonant frequencies for the TE $_{nml}$ and TM $_{nml}$ modes of the circular waveguide.

For TE $_{nm}$ mode, we have beta nm is equal to k square minus P prime nm by a whole square we remember that this term it is the roots of the derivative of Bessel function and for TM $_{nm}$ mode we have beta nm is equal to under root k square minus P nm by a whole square.

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$\vec{E}_t = 0$ at $z = 0$

We have

$$A^+ = -A^-$$

$\vec{E}_t = 0$ at $z = d$

We have

$$\sin \beta_{nm} d = 0$$

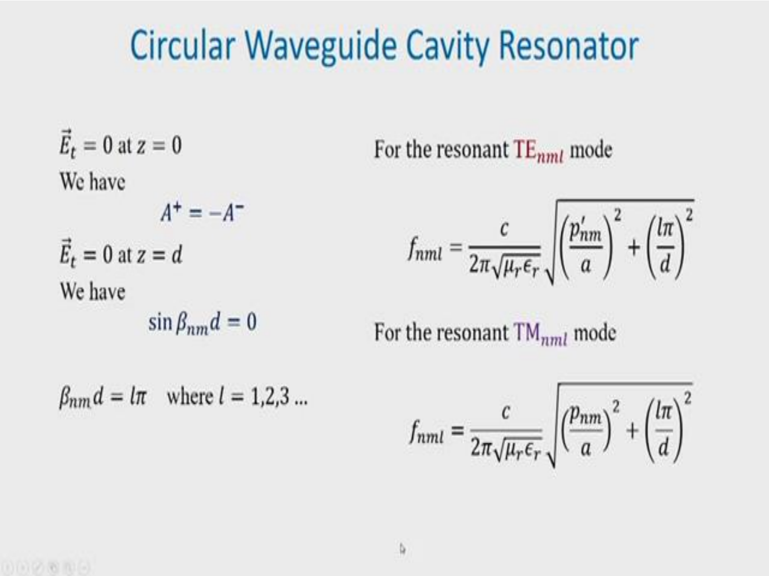
$\beta_{mn} d = l\pi$ where $l = 1,2,3 \ldots$

For the resonant TE_{nml} mode

$$f_{nml} = \frac{c}{2\pi\sqrt{\mu_r \epsilon_r}} \sqrt{\left(\frac{p'_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

For the resonant TM_{nml} mode

$$f_{nml} = \frac{c}{2\pi\sqrt{\mu_r \epsilon_r}} \sqrt{\left(\frac{p_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$



From the condition that E tangential is equal to 0 at z is equal to 0 we get A plus is equal to minus A minus. Now, when this condition is substituted for the expression of the tangential electric field we will get a term which is sin beta z and we know that E t will have to 0 at z is equal to d and that means sin beta nm d should be equal to 0 and from there we get beta nm should be equal to l pi.

And therefore the resonant TE_{nml} modes, we have the resonant frequency given by f_{nml} is equal to c by 2 pi root mu r epsilon r P prime nm by a whole square plus l pi by d whole square. And that for the TM_{nml} mode we have f_{nml} is equal to c by 2 pi under root epsilon mu r P nm by a whole square plus l pi by d whole square.

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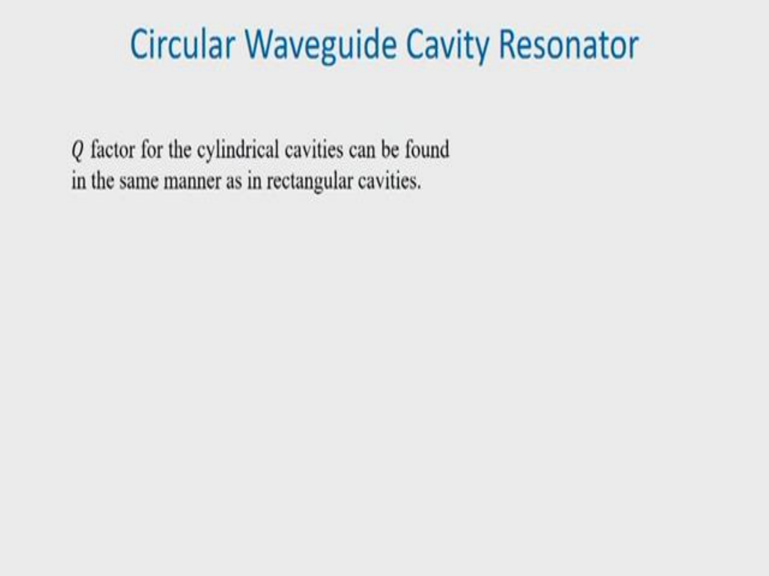

The Q factor for the cylindrical cavities we can find out adopting the same procedure and considering first lossless dielectric and the cavity walls are lossy and then we can consider the Q factor for the cylindrical cavities can be found in the same manner as in the rectangular cavities and we can first consider, the case when the dielectric is lossless but the cavity walls are slightly lossy. And then for the second case when the cavity wall is perfectly lossless whereas the dielectric is slightly lossy and then find out the unloaded Q as in the case of a rectangular cavity.

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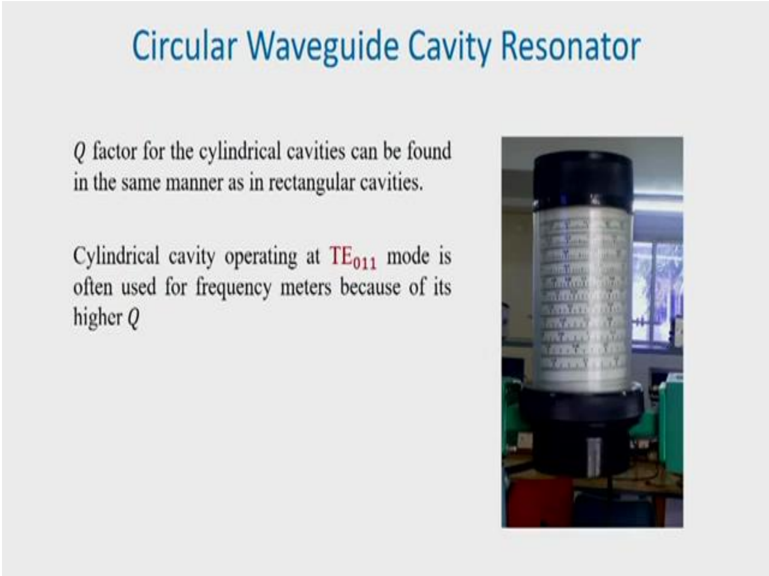

The cylindrical cavity operating at TE_{011} mode is often used in the frequency meters because of it is higher values of Q, this type of frequency meters are used in as direct reading type microwave frequency meters. So, we have seen different types of resonators the transmission line and waveguide type resonators, and we have seen how near resonant frequency. We can find an equivalent RLC representation for a transmission line type resonator.

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Power Dividers, Directional Couplers and Filters

In the next module we will discuss about power dividers, directional couplers and filters.