Microwave Engineering
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Lecture 17 - Power Dividers, Directional Couplers and Filters

We start a new module: power dividers, directional couplers, and filters.

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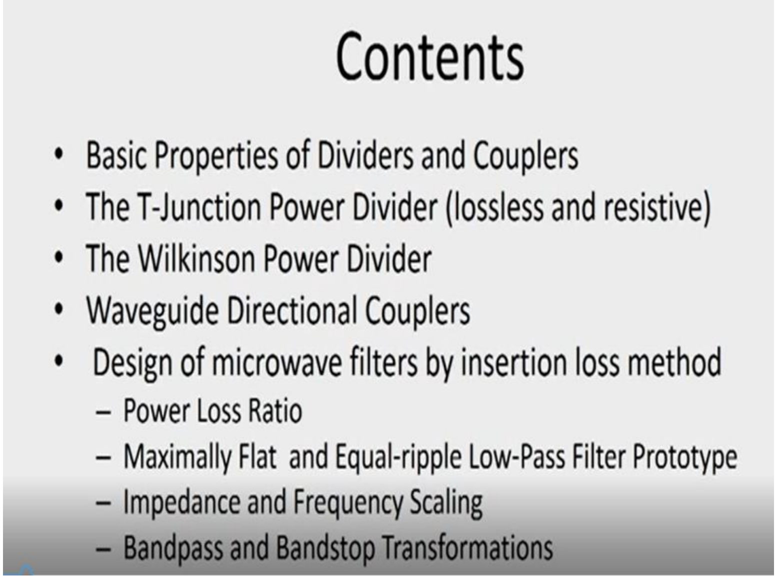

The contents of this module are basic properties of dividers and couplers, then we will discuss the T-junction power divider, we will discuss both the variants, lossless and resistive, then we will discuss a special form of power divider which is called Wilkinson power divider. This will be followed by a discussion on waveguide directional couplers, and then we will study the design of microwave filters by insertion loss method.

We will introduce what power loss ratio is, we will see how we can design maximally flat and equal ripple, low-pass filter prototype, and then how the impedance and frequency scaling can be done and then from the low pass prototype we will get bandpass and bandstop filters by making suitable transformations.

So let us start our discussion with the basic properties of dividers and couplers.

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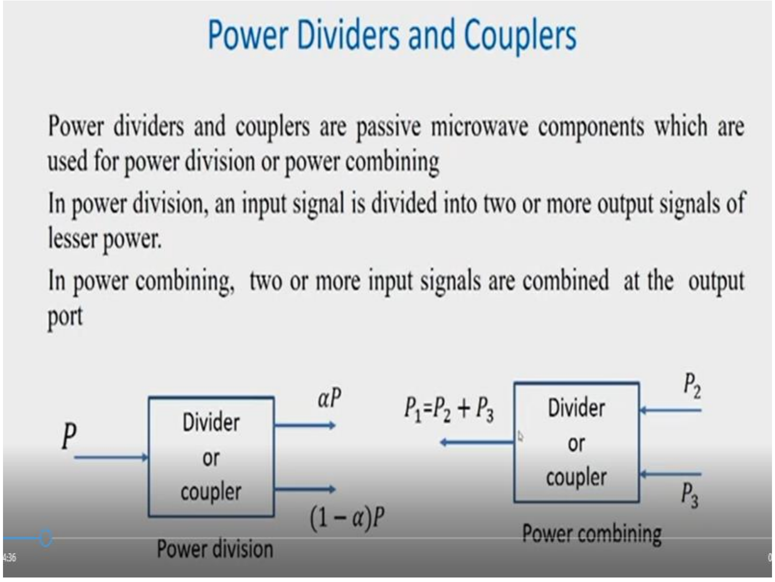

Power dividers and couplers are passive microwave components which are used for power division or power combining. In power division, an input signal is divided into two or more output signals, and each signal will have lesser power, whereas in power combining, two or more input signals are combined at the output port of the combiner, and this figure shows the power division. So, we have a divider; we are showing only two output ports. In fact, the divider can have more than two output ports.

Here the input power is P, and this power P is divided as Alpha P and 1 minus Alpha P, Alpha is less than 1. When alpha is equal to half we get an equal power divider. Otherwise, we will get unequal power division. We consider now the combiner action. So, in power combining, we will have inputs, two or more inputs; here we are showing two, P_2 and P_3 are the input powers, and at the combiner output, we have P_1, which is equal to P_2 plus P_3.

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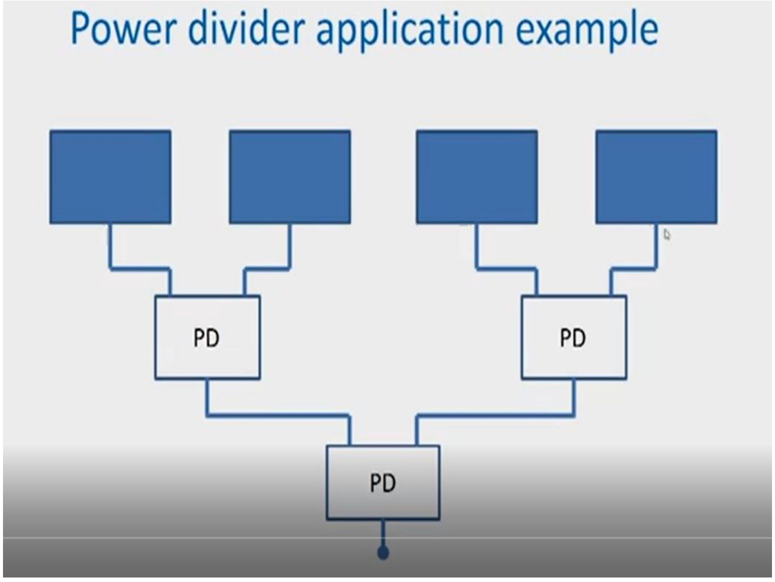

Now, let us see an example of an application of power divider. This shows an array antenna. The array of printed elements and assume that the aerial elements are to be uniformly excited from a single input source. So, what we can do? We can equally split the power between the two output ports and we maintain the same phase and then we go for the second level of splitting and in this manner, we have been able to split the power and excite individual antenna elements.

If we want the elements to be excited not uniformly having different excitation levels at individual antenna elements and also phase 2 change, this type of power divider circuits with unequal power division can be made and we can use additional phase shift. So we see that in practical systems this type of such as an array antenna, this type of power dividers are extremely useful.

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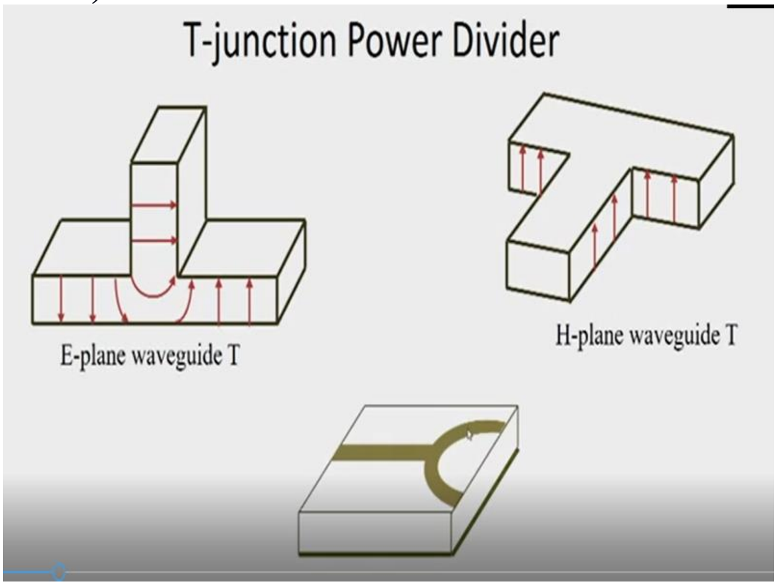

One particular form of power divider is very useful. It is called a T-junction power divider. Let us see first the waveguide version. So this particular waveguide T, it is called E-plane waveguide T. So we can see that these are the field lines shown for the electric field, so the wave propagates in this direction and at this junction you can see that the wave splits and gets coupled to the two output ports, but the nature of coupling is such that they are in opposite phase. So they will have equal power but will have a phase shift between the two ports.

So this type of T-junctions are called E-plane waveguide T. Another T-junction can be made from waveguides, and this is called H-plane waveguide T, and here we see the input signal once again splits between the output port but this time in phase. So, we have an H-plane T it also splits the power equally but in the same phase, unlike E-plane waveguide T where a phase shift of pi occurs.

Let us consider another form of T-junction, a microstrip version where we have, this is the ground plane, this is the dielectric material and this is the microstrip line which will be the input port, and the power will be divided between these two output ports, and this division can be designed to be equal or unequal.

Now we have seen that these T-junctions are essentially three-port junctions. While studying S- parameters we have already studied the properties of such three-port junctions. In fact, we have seen that a three-port junction cannot be reciprocal, lossless, and matched at all three ports. So, this is something we need to note that if we want to make power divider as a three-port lossless junction, in that case, we will not be able to match all the three ports simultaneously.

Suppose we make an arrangement and match the input port, then the output ports will remain unmatched. Moreover, we will see that the two output ports in case of a lossless T-junction or three-port junction will not remain isolated.

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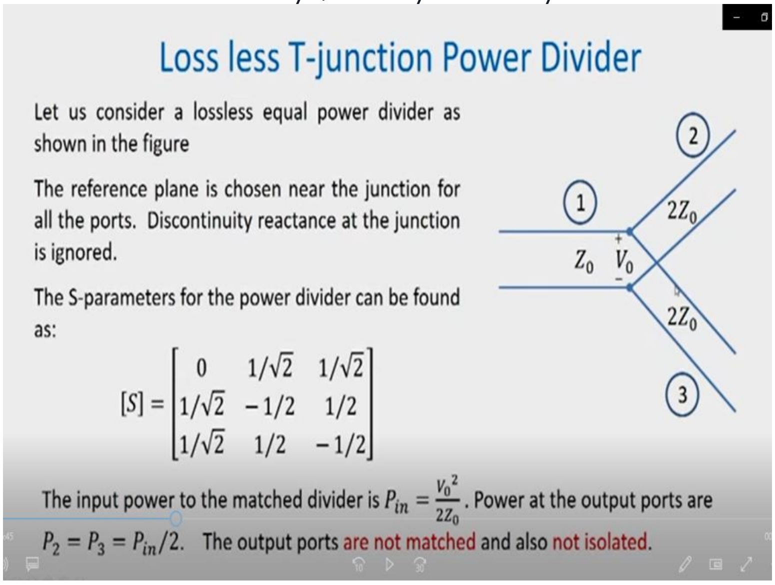
The S-parameters for the power divider can be found as:

$$[S] = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/2 & 1/2 \\ 1/\sqrt{2} & 1/2 & -1/2 \end{bmatrix}$$



So we now consider lossless T- junction power divider. So, let us consider a lossless equal power divider, and the circuit is shown in the figure. So, here we see that we have three ports, 1, 2, and 3 and in order to match the input port we will have to have characteristic impedance of the transmission lines constituting the port 2 and 3 will be of 2 Z naught so that the parallel combination of these two will be Z naught, and therefore, the port 1 will be matched. The reference plane here we have considered on the junction itself and further when this type of junctions are formed by connecting transmission lines we have a small reactance which is ignored.

Now, this power divider is an equal power divider, and the port 1 is matched. So, if we find the S- parameters for this three-port network, we find that S is given by 0, 1 by root 2, 1 by root 2, S_{12} is 1 by root 2 and it is a symmetrical network so we will have S21 also 1 by root 2 and we find that S_{22} and S_{33} they are not 0.

In fact, we can calculate S_{22}. This 2 Z naught will come in parallel with Z naught, and therefore, we will get the equivalent of Z naught parallel 2 Z naught as the load here for this port 3 and similarly for port 2, Z naught parallel 2 Z naught, it will appear as a load here and if we substitute those values we will get a reflection coefficient of minus half in ports 2 and 3.

This is a lossless junction and you can see that power balance is there if you take any column, so half plus half that means S_{21} square plus S_{31} square this becomes 1. Similarly S_{12} square becomes half, S_{22} square becomes one-fourth, S_{32} square becomes one-fourth, so, if we sum them up we get 1. Since it is a lossless network, we find that power balance is maintained, but it is not matched at all three ports.

Moreover, once we calculate the S parameters, we find that S_{23}, and S_{32} are not 0. That means port 2 and 3 are not isolated. The input power to the matched divider is P in is equal to V naught square by 2 Z naught whereas this V naught is common to all the three ports, therefore, the output powers will be P2 equal to P3 which is equal to P in by 2 because this characteristic impedance is 2 Z naught. So, the output ports are not matched and they are not also isolated. So, for this lossless power divider, we find that the input port is matched. It is an equal power divider. So we have same power going into 2 and 3, output ports are not matched and they are also not isolated.

We can also have unequal power division from this configuration, only the port impedances are to be chosen appropriately to make the power division unequal.

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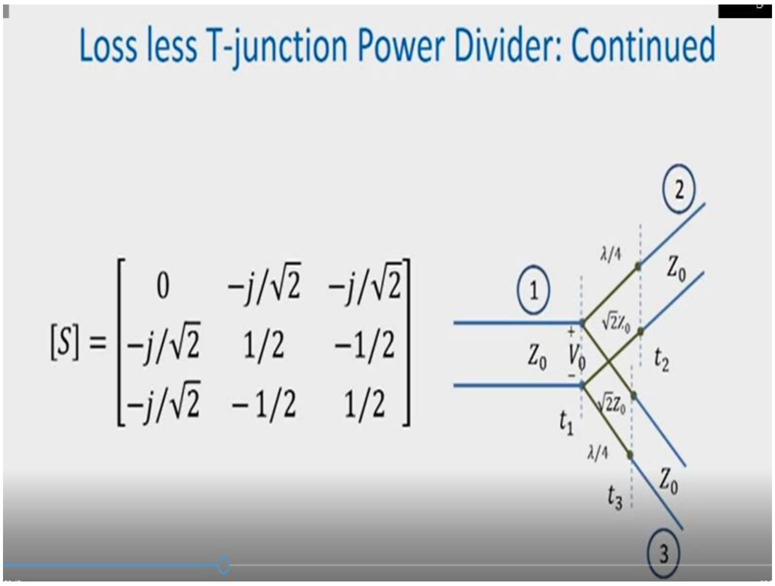
$$[S] = \begin{bmatrix} 0 & -j/\sqrt{2} & -j/\sqrt{2} \\ -j/\sqrt{2} & 1/2 & -1/2 \\ -j/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$$



It may be noted that in the previous lossless divider we had the port impedances different, for port 1 it was Z naught, port 2 and 3 it was 2 Z naught. Now here we can have same impedance at all the three ports. What we can do? We can use a Lamda by 4 transformers with a characteristic impedance root 2 Z naught. So, this root 2 Z naught characteristic impedance of the Lambda by 4 transformers what it will do? It will transform these impedance Z naught here to 2 Z naught. Similarly, this Lambda by 4 sections with characteristic impedance root 2 Z naught will transform this Z naught of port 3 to 2 Z naught here at the terminal plane of port 1.

Please note that we are showing the terminal planes t_1, t_2, and t_3 here for the phase references. In the previous example, we considered all the terminal planes are to be located at the junction. So, we have this Z naught transformed here to 2 Z naught, similarly this Z naught transformed here to 2 Z naught here and finally, we will get parallel combination of these 2 Z naughts giving Z naught here. So if we find the S matrix for this junction we will get S_{11} 0 as before and we can see that because of the shifting of the terminal planes now we have minus j 1 by root 2, minus j 1 by root 2 as the values of S_{21} and S_{31}.

Once again the reflection coefficient at the other two ports 2 and 3, they are non-zero because of the symmetry half and half, and therefore this power divider is also not matched to the output ports. Only the thing is that now the output port impedance is Z naught instead of 2 Z naught in the earlier case and we see that the isolation is also not there because S_{23} and S_{32}, the isolation between the output ports are not there as S_{23} and S_{32} they are non-zero.

So, if we summarize, we can use a three-junction power divider for equal or unequal power division but we will not have the two output ports matched and moreover, the ports will not be isolated.

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$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3}$$

$$Z_{in} = \frac{Z_0}{3} + \frac{2Z_0}{3} = Z_0$$

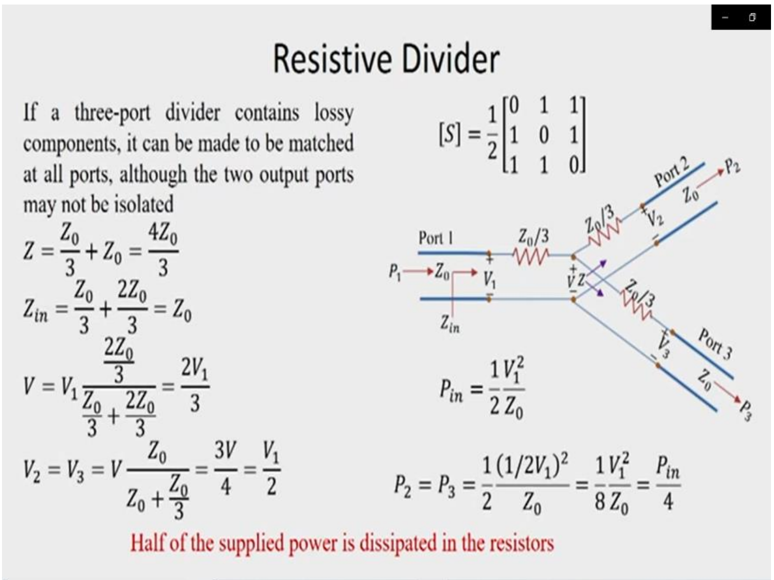
$$V = V_1 \frac{\frac{2Z_0}{3}}{\frac{Z_0}{3} + \frac{2Z_0}{3}} = \frac{2V_1}{3}$$

$$V_2 = V_3 = V \frac{Z_0}{Z_0 + \frac{Z_0}{3}} = \frac{3V}{4} = \frac{V_1}{2}$$

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$P_{in} = \frac{1}{2}\frac{V_1^2}{Z_0}$$

$$P_2 = P_3 = \frac{1}{2}\frac{(1/2V_1)^2}{Z_0} = \frac{1}{8}\frac{V_1^2}{Z_0} = \frac{P_{in}}{4}$$



Let us now move onto another type of power divider where we have a T-junction but this T-junction is not lossless rather we have resistive elements present at the junction which makes the junction lossy and once we have this junction lossy we will see that we will be able to match all the three ports. So if a three-port divider contains lossy components, it can be made to be matched at all three ports, although the two output ports may not be isolated.

So, let us see the structure of the resistive power divider. So, here we have three transmission line sections, each of characteristic impedance Z naught and they are connected in the form of a T, and also at the junction we have three resistances having values Z naught by 3 each. Here we are making equal power divider. If we have unequal power dividers, these values will change.

Now let us see how this power divider operates. If we look at port 2 or port 3 from this junction, then we see an impedance Z, which is given by Z naught by 3, the value of the resistance plus the port impedance Z naught. So it becomes Z naught by 3 plus Z naught equal to 4 Z naught by 3 and when we look from this port, port 1, both these port 2 and port 3 they will appear as load and therefore Z_{in} when looking from this side, we will see an impedance of Z naught by 3 here plus parallel combination of these two.

These two ports having the same impedance Z as we look from here. They are equal, so their parallel combination will be Z by 2, and therefore we get this parallel combination to be 2 Z naught by 3. So, Z naught by 3 plus 2 Z naught by 3, this gives us Z_{in} equal to Z naught, and therefore we now see that port 1 is matched because the port 1 transmission line characteristic impedance is also Z naught and from port 1 we are looking into the whole network. We see an impedance Z_{in} equal to Znaught, and therefore there will not be any reflection and in fact, this network if you look at this network is a symmetrical network.

So you take port 2 then also you will find if we look from this port 2 the input impedance will be Z naught and therefore port 2 also will be matched, similarly port 3 also will be matched. Let us now see how the power gets divided. Let V be the voltage at this junction and V1 it represents the voltage at the input port 1. So by circuit theory now we can write we have already seen if we look from this point, the input impedance is Z parallel Z, that means Z by 2 that means 2 Z naught by 3 and the voltage division takes place V equal to V_1 into 2 Z naught by 3 divided by Z naught by 3 plus 2 Z naught by 3 and this becomes equal to 2 by 3 V_1.

So V is two-third of the voltage at port 1 that means two-third of V1 and we see that because of symmetry V2 is equal to V3. Now we can find V2 and V3. If voltage here is V then V2 will be Z naught divided by Z naught plus Z naught by 3 into V, so V into Z naught divided by Z naught plus Z naught by 3 and this comes out to be 3 V by 4 and if you substitute V it becomes V1 by 2. So at both port 2 and port 3 we essentially have half V1, the voltage at the port 1.

Now we can find out the power. So P in is half V1 square by Z naught. This is the input power and we have already said that the power divider is matched at its input port. So, the entire power goes into the power divider and then P2 equal to P3 and that becomes equal to half V2 square or V3 square. Now V2 and V3 they are V1 by 2 and therefore it becomes half V1 square by Z naught multiplied by half and it becomes 1 by 8 V1 square by Z naught and that means it is P in by 4 that means one-fourth of the input power.

So this port 2 it finds one-fourth of the input power, port 3 also finds one-fourth of the input power. So we can see that the power that is appearing at port 2 and port 3 these are equal. However, we do not get the input power divided between port 2 and 3. The power gets dissipated in these resistances. So, as it is a reciprocal network we can write the S parameters, we have already said that S_{11} will be 0 and also S_{22} will be 0, S_{33} will be 0 and if you consider S_{21} for example power to port 2 from 1 so that will be mod S_{21} square and this will be 1 by 4 that means P2 by P in will be equal to 1 by 4, so this entry will be half.

Similarly, P3 by P in also will be 1 by 4. So we will have S_{31} is equal to half, and being a reciprocal circuit we will have S_{12} equal to S_{21}, S_{13} is equal to S_{31}. One can calculate that S_{23} and S_{32} these parameters also will behalf. Now we find that power gets divided equally between the two output ports but that is not the input power, half the power gets dissipated in the resistors and remaining half-power gets equally divided between the two output ports. We also find that S_{23} and S_{32} are also half that means the isolation between these ports' complete isolation is not there.

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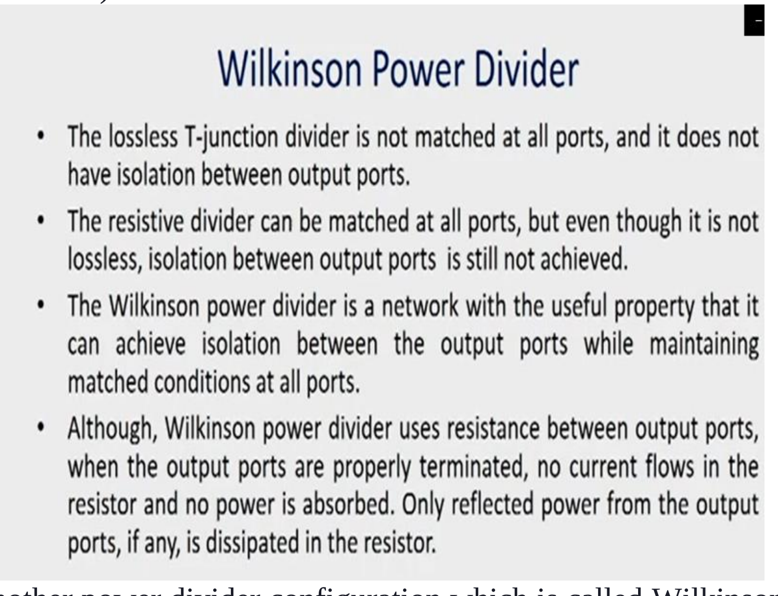

We move onto another power divider configuration which is called Wilkinson power divider. So a lossless T-junction divider is not matched at all ports, and it does not have isolation between the output ports. Similarly, we have discussed a resistive power divider. Because a resistive power divider is a lossy divider, it can be matched at all ports but isolation between output ports is still not achieved.

So, Wilkinson power divider it is a network with the useful property that it can achieve isolation between the output ports while maintaining matched conditions at all three ports. Now, Wilkinson power divider we will see it will use resistance between the output ports, but when the output ports are properly terminated, no current will flow in this resistor, so essentially there will not be any power dissipation in this resistor when the ports are properly terminated. Only if there is reflected power from the output ports they will be dissipated in the resistor.

So there have been many versions of Wilkinson power divider because it is widely used. The wideband version has been developed, we will see that Wilkinson power divider basic form uses A Lambda by 4 sections. Wilkinson power divider in the basic form uses Lambda by 4 sections of transmission line. So miniaturized version of Wilkinson power divider has also been developed which are suitable for integrated circuits.

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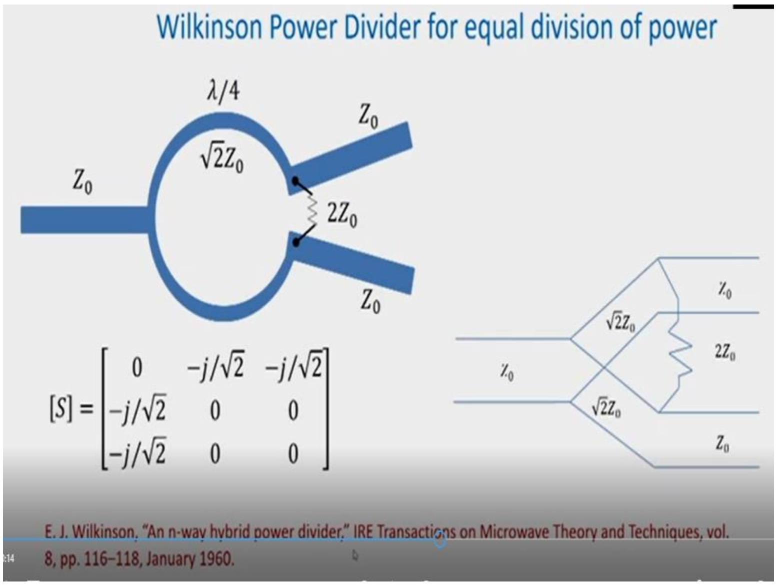
$$[S] = \begin{bmatrix} 0 & -j/\sqrt{2} & -j/\sqrt{2} \\ -j/\sqrt{2} & 0 & 0 \\ -j/\sqrt{2} & 0 & 0 \end{bmatrix}$$



So, in the basic form here we are showing a planar version of a Wilkinson power divider. We have all three ports. It is an equal power divider, we have three ports. So, this is the input port and these two are the output ports and all three ports have a characteristic impedance of Z naught. Now, these two sections are Lambda by 4 sections and each of these sections is having characteristic impedance root 2 Z naught.

Now we can see that between the two output ports we have a resistance 2 Z naught connected. Please note that these values mentioned here are for equal power dividers. Wilkinson power divider can also be designed for arbitrary power division ratio. It can also be multiport, we are showing here only two output ports and this is another version with transmission line sections. So once again we see that resistance of 2 Z naught is connected here and we have three ports all having characteristic impedance Z naught and we have this Lambda by 4 sections having root 2 Z naught as the characteristic impedance.

Now if we consider the S parameters for this type of equal power divider this type of Wilkinson equal power divider, we get the S matrix as S_{11} 0, S_{22} 0, S_{33} 0. So Wilkinson power divider gives all the three ports. So in Wilkinson power divider, all three ports are matched. We find that power division is equal, and half the power goes to port 2, and because mode of S_{21} square is half and similarly mode of S_{31} square is half, so equal power goes to port 2 and 3 and also see that S_{23} and S_{32} are 0. So, the two output ports of the Wilkinson power divider are isolated. Please note that we have a resistance connected between the output ports.

So when the output ports are under perfect matching condition, no current will flow through this resistor and there will not be any power dissipation. So under the perfect matching condition in spite of having a lossy element this power divider will be virtually lossless. However, if there is a mismatch and there are reflected power this reflected power will be absorbed in that resistance.

Wilkinson power divider is in the name of E. J. Wilkinson and he introduced an n-way hybrid power divider in 1960 in the IRE Transactions on Microwave Theory and Techniques and thereafter different versions of Wilkinson power divider has been developed and this power divider is still considered a very useful component in the design of Rf and microwave circuits.

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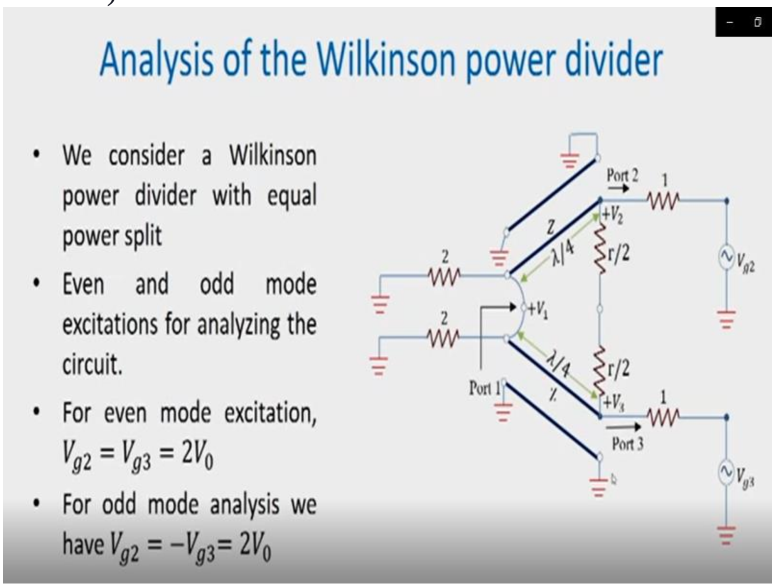

Let us now see how we can analyze such a Wilkinson power divider and get the S matrix as we have shown. For that, we consider a Wilkinson power divider with equal power split and this is shown here. So what we do, we take advantage of the symmetry of this network and we perform what is known as even and odd mode analysis and to do that even and odd mode analysis we represent the impedances here by their normalized value. So, this port was having impedance Z naught so it can be represented a parallel of two resistances having values, normalized values 2 each so that their parallel combination gives 1.

Similarly, this Z, it is the normalized characteristic impedance of this Lambda by 4 sections. The resistance between the two output ports normalized resistance is R and which has been written in this form R by 2 and R by 2. Similarly, these port impedances are normalized values 1 each and we have supposed we have these 2 ports 2 and 3 we excite using two generators denoted by V_{g2} and V_{g3} and port 1 has been kept matched terminated. Now when I consider even mode excitation we have V_{g2} is same as V_{g3} and we take V_{g2} equal to V_{g3} equal to 2V naught.

Similarly, for odd mode analysis, we consider V_{g2} equal to 2 V naught, but V_{g3} equal to minus 2 V naught. So in the old mode what will happen because the excitation at these ports is the same no current will flow through this branch, and we will have an open circuit condition created here. For the odd mode analysis, this V_{g3} will be minus 2 V naught, V_{g2} will be, V_{g2} will be plus 2 V naught, so in this central line or in the plane of symmetry, we will have 0 voltage. So having a 0 voltage here is essentially having a short circuit.

So what we find that in the even mode case, we have an open circuit created here, and for odd mode excitation a short circuit will be created here. We next show the two equivalent circuits, but these are symmetrical, so when an open circuit is created, the two halves are symmetrical, and we can only analyze one section and find out the voltages and also the S parameters.

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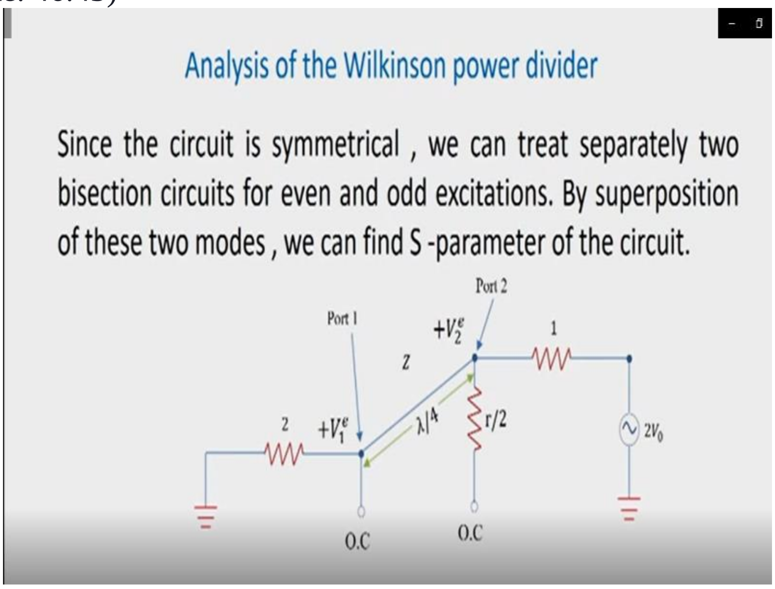
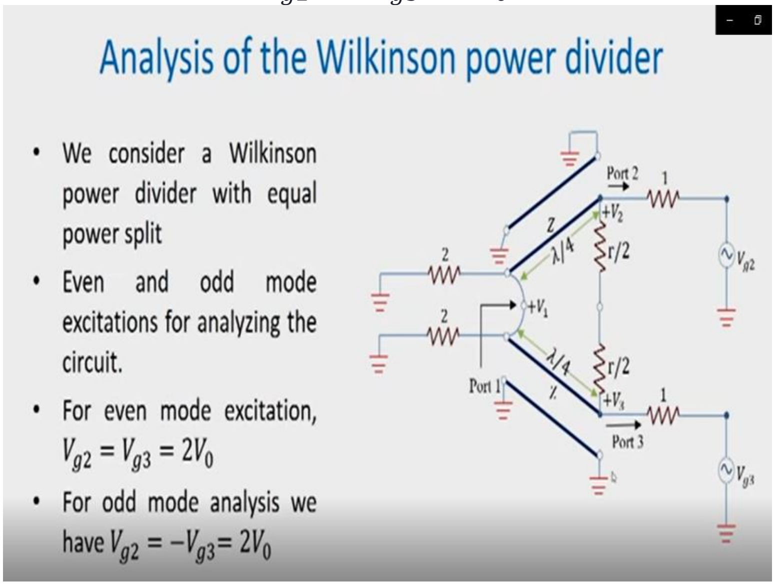


$$V_{g2} = V_{g3} = 2V_0$$
$$V_{g2} = -V_{g3} = 2V_0$$



Since the circuit is symmetrical, we can treat separately two bisection circuits for even and odd mode excitation. By superposition of these two modes, we can find S-parameter of the circuit. So we find out the solution for the even mode, we find out the solution for the odd mode and then superpose these two solutions and then we get the S parameters for the circuit when we do it for individual ports.

So when we consider even mode excitation we have seen that we can analyze only this half of the circuit and this is shown here. So we have an open circuit here, an open circuit here, and a normalized 2 ohm resistance is connected to port 1, in port 2 we have connected V_{g2} which is 2 Volt and we have a normalized value of resistance 1 representing Z naught and we have a Lambda by 4 transmission line section, the other conductor is not shown here and it has a normalized characteristic impedance of Z.

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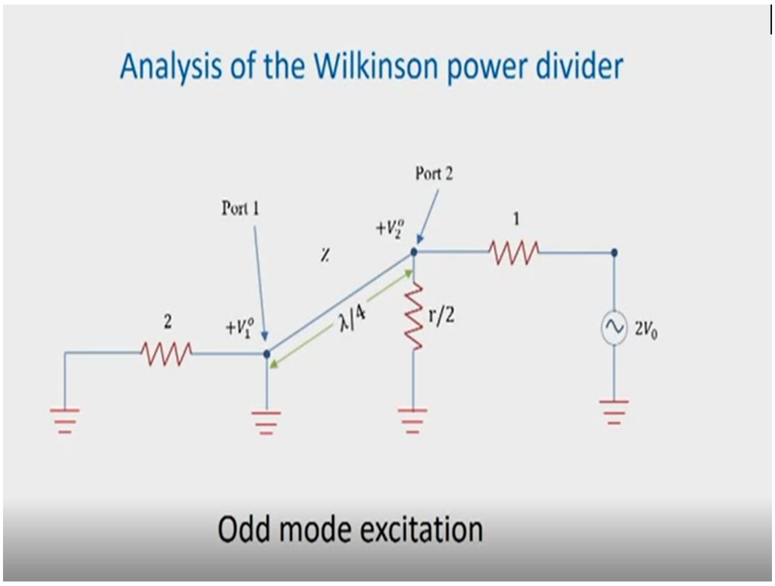

Similarly, if we consider the circuit, one half of the circuit under odd mode excitation, in that case, we will get these two points short-circuited, and this is 2 ohms normalized; this is the Lambda by 4 sections having normalized impedance Z. Now this r by 2 resistance earlier it was floating in case of even mode, now it becomes grounded, and we have once again 2 V naught as the input for the upper half and minus 2 V naught will be the input for the lower half, and this one is the normalized resistance of the port.

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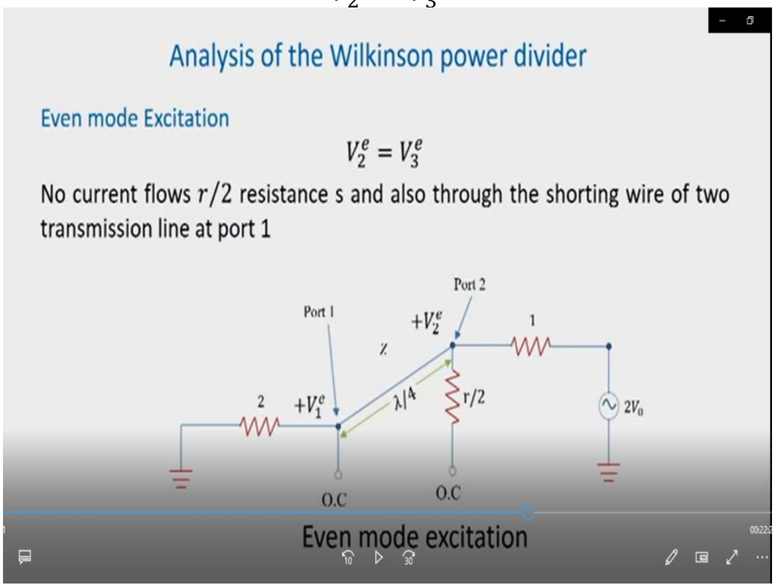
$$V_2^e = V_3^e$$



Since the power divider is the symmetrical network, so we can go for even and odd mode excitation, and we have seen that in the even mode excitation case we have an open circuit in the plane of bisection and for odd mode excitation it creates a short circuit, and we can actually consider only one part of such bisected circuit for carrying out the analysis. So, let us start with the even mode excitation case.

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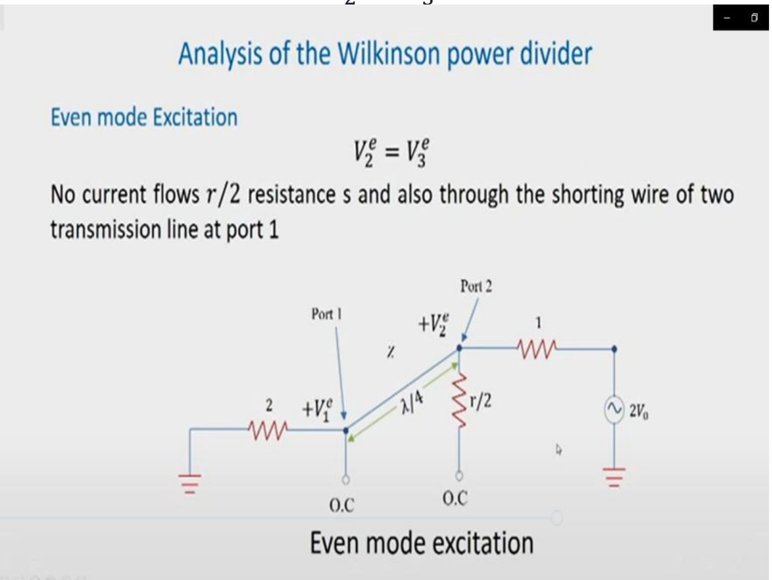
$$V_2^e = V_3^e$$



Now, this is the circuit shown for even-odd mode excitation and here we can say these are the open circuits. Further, we have V_2^e that means the voltage at this port 2 under even mode excitation condition will be same as V_3^e and when we have V_2^e same as V_3^e, so they are in the same potential and no current flows through r2 resistance because it has become open here and also through the shorting wire of two transmission lines at port 1. So here we can see it is an open circuit and we have V_{g2} equal to 2 V naught.

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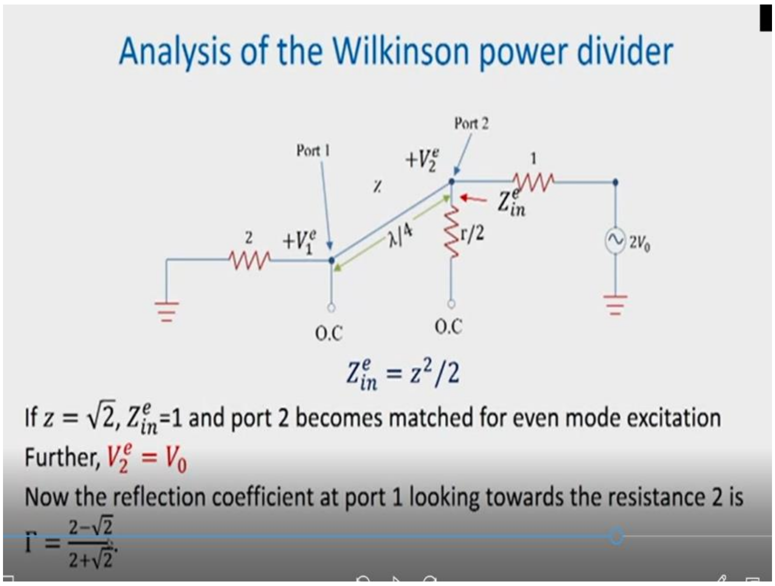
$Z_{in}^e = z^2/2$

If $z = \sqrt{2}$, Z_{in}^e=1 and port 2 becomes matched for even mode excitation

Further, $V_2^e = V_0$

Now the reflection coefficient at port 1 looking towards the resistance 2 is $\Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}}$.



Now, let us see what is the impedance we see if we look from this point when we have this circuit and these two points are open-circuited. In that case, we see that the input impedance seen here, this r by 2 will be a floating resistor, so it will not contribute to the impedance at this point. The impedance at this point will be this normalized 2 ohm resistance being transformed by this quarter-wave transformer having characteristic impedance Z, and therefore we can write Z_{in} e into 2 is equal to Z square or Z in e is equal to z square by 2.

Now, this Z_{in} e for matching has to be equal to 1 and therefore we find that if Z is equal to root 2, normalized impedance is root 2, then Z_{in} e equal to 1 and port 2 becomes matched for even mode excitation. So we have found out the value of the characteristic impedance of the quarter-wave section and once Z_{in} e becomes 1 in that case by voltage division V_2^e becomes 2 V naught by 2 into 1 which becomes equal to V naught.

Next, if we consider the reflection coefficient that we see from port 1 looking towards this 2 ohm resistance then our Z is now root 2, so, therefore, gamma will become 2 minus root 2 divided by 2 plus root 2.

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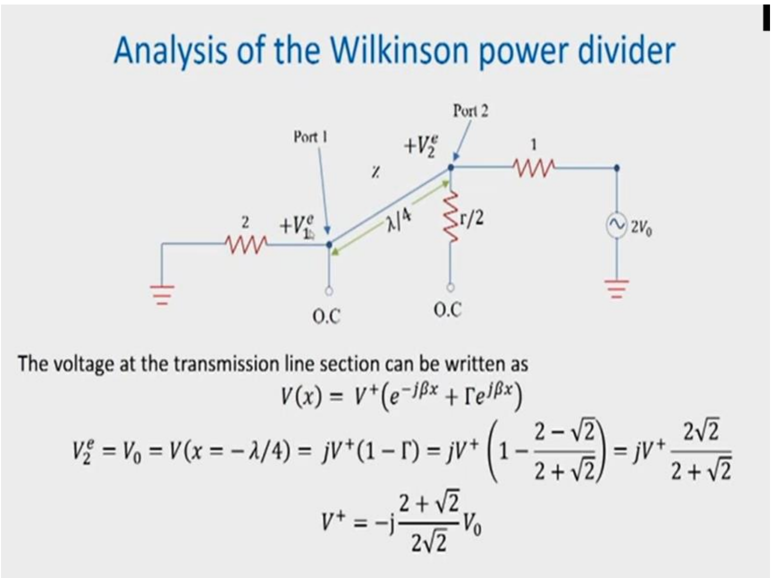
The voltage at the transmission line section can be written as

$$V(x) = V^+\left(e^{-j\beta x} + \Gamma e^{j\beta x}\right)$$

$$V_2^e = V_0 = V(x = -\lambda/4) = jV^+(1-\Gamma) = jV^+\left(1 - \frac{2-\sqrt{2}}{2+\sqrt{2}}\right) = jV^+\frac{2\sqrt{2}}{2+\sqrt{2}}$$

$$V^{+} = -\mathrm{j}\frac{2+\sqrt{2}}{2\sqrt{2}}V_0$$



So, we have found out V 2 e. Let us see how we can now find out V 1 e. Now on this transmission line section, we can write voltage at any distance x is V plus e to the power, voltage at any distance x V x is equal to V plus into e to the power minus j Beta x plus Gamma into e to the power j Beta x. Now what we do, we take the reference to be here. So our x is equal to 0 is taken here. So for that case, if we find out at port 2 so V 2 e which we know already V naught and this must be equal to V x is equal to minus Lambda by 4. So if we substitute x is equal to minus Lambda by 4 here, we get e to the power j 2 Pi by Lambda, Lambda by 4 which becomes e to the power j pi by 2 and e to the power j pi by 2 is j.

Similarly, this will give e to the power minus j Beta Lambda by 4 and it will become minus j into Gamma. So, if we take out j then we get j V plus 1 minus Gamma and now we substitute the value of Gamma which is 2 minus root 2 divided by 2 plus root 2 as we have seen. Then V 2 e which is equal to V naught also becomes equal to j V plus 2 root 2 divided by 2 plus root 2. So from here we can find out V plus and therefore V plus will be minus j 2 plus root 2 divided by 2 root 2 V naught. So once we have found out V plus, now we can find out V 1 e to be equal to V at x equal to 0.

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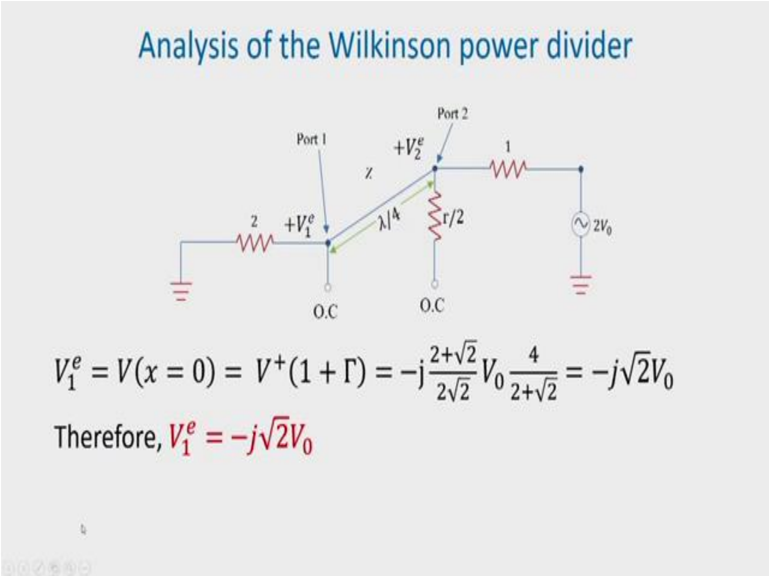
$V_1^e = V(x=0) = V^{+}(1+\Gamma) = -\mathrm{j}\frac{2+\sqrt{2}}{2\sqrt{2}}V_0\frac{4}{2+\sqrt{2}} = -j\sqrt{2}V_0$

Therefore, $V_1^e = -j\sqrt{2}V_0$



So, V 1 e is equal to V x equal to 0, this becomes V plus 1 plus Gamma and if we substitute the expression for V plus it becomes minus j 2 plus root 2 divided by 2 root 2 into V naught and 1 plus Gamma becomes 4 divided by 2 plus root 2. So finally we get V 1 e to be equal to minus j root 2 V naught. So now we have found out V 2 e to be V naught and V 1 e to be minus j root 2 V naught.

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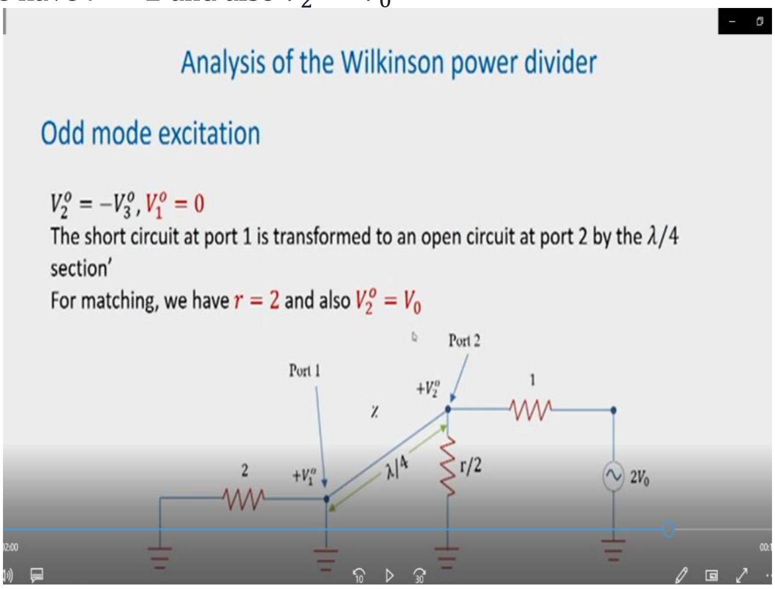
$V_2^o = -V_3^o, V_1^o = 0$

The short circuit at port 1 is transformed to an open circuit at port 2 by the $\lambda/4$ section'

For matching, we have $r = 2$ and also $V_2^o = V_0$



Let us now see what happens for the odd mode excitation. We have seen that for odd mode excitation this bi-section line here it becomes short circuit, r2 becomes short-circuited and therefore we can see that V 1 o since this point is grounded is 0.

Similarly for odd mode excitation here we connect 2 V naught and in port 3 we will connect minus 2 V naught and therefore V 3 o it will be negative of V 2 o, next when we calculate the input impedance here at port 2, this short circuit at port 1 this is anyway short-circuited so it will not contribute, this 2 ohm resistance, this point is short-circuited, so we have essentially a transmission line with Z is equal to root 2 and it is short-circuited and therefore it will get transformed to an open-circuited at the location of port 2 and therefore this entire part of the circuit will not contribute anything here. So we will be left with the remaining part of the circuit which is 1 and r by 2 and for matching we must have now r by 2 equal to 1 and therefore we will have r is equal to 2 and in the same manner when r by 2 equal to 1 by voltage division voltage at port 2, V 2 o this also becomes V naught.

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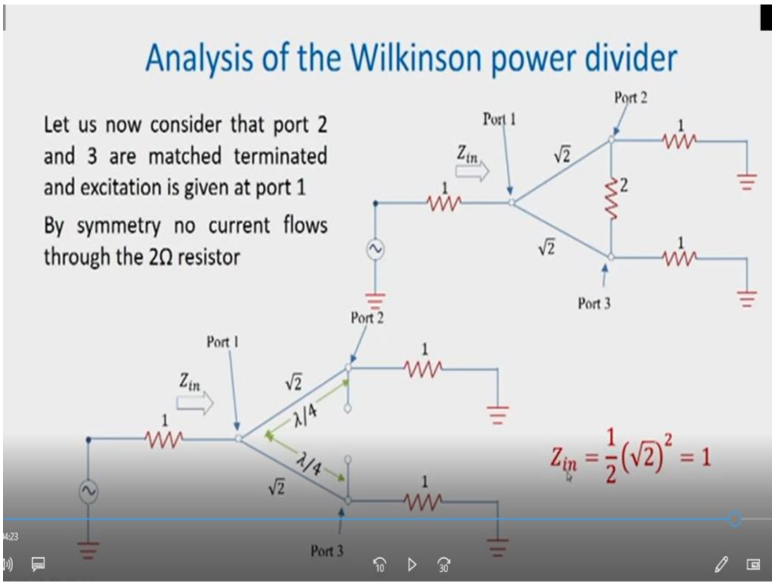
$$Z_{in} = \frac{1}{2}\left(\sqrt{2}\right)^2 = 1$$



So let us now summarize. So in the previous analysis, even and odd mode analysis we assumed port 1 matched to be terminated and we excited from ports 2 and 3. So if we consider this particular case where port 2 and 3 they are matched terminated and then we put an excitation over here we find that because of the symmetry of the circuit, the voltage here at port 2 and port 3 will be same and essentially no current will flow in this branch connecting port 2 and 3. So no current will flow through this 2-ohm resistance.

So when there is no current flow through these we can see that it is essentially an open circuit and now we can find out Z_{in} at port 1. So, Z_{in} at port 1 can be calculated like this. This one will be transformed to port 1 by this quarter wavelength transformer of characteristic impedance Z equal to root 2 to a value, which is root 2 square by 1 that means 2.

Similarly, this 1-ohm resistance also will be transformed by this quarter-wave transformer to a value 2. Now this 2-ohm resistance is here at port 1, will come in parallel, and this will become 1, so Z_{in} becomes 1, and therefore we will get Z_{in} is half root 2 square is equal to 1. So once we have found out Z_{in} equal to 1 that means S_{11} will be 0.

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$S_{11} = 0$ (Z_{in}=1 at port 1)

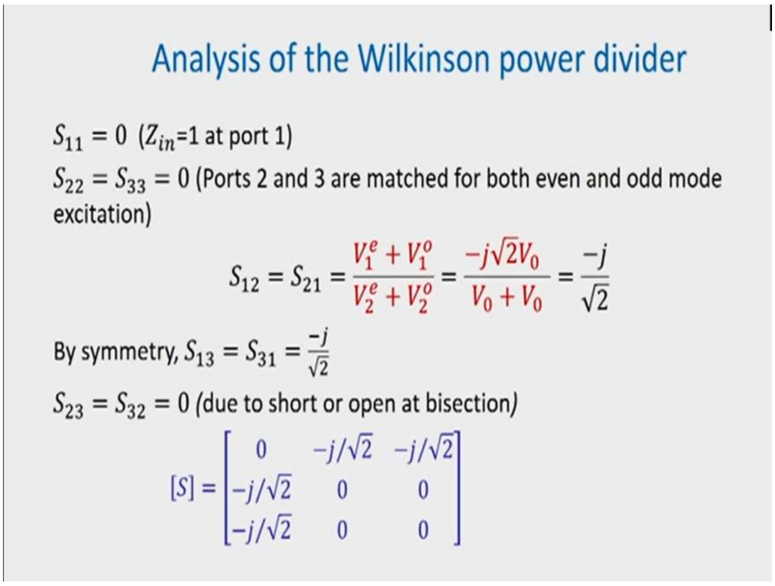
$S_{22} = S_{33} = 0$ (Ports 2 and 3 are matched for both even and odd mode excitation)

$$S_{12} = S_{21} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = \frac{-j\sqrt{2}V_0}{V_0 + V_0} = \frac{-j}{\sqrt{2}}$$

By symmetry, $S_{13} = S_{31} = \frac{-j}{\sqrt{2}}$

$S_{23} = S_{32} = 0$ (due to short or open at bisection)

$$[S] = \begin{bmatrix} 0 & -j/\sqrt{2} & -j/\sqrt{2} \\ -j/\sqrt{2} & 0 & 0 \\ -j/\sqrt{2} & 0 & 0 \end{bmatrix}$$



So, we can now summarize since Z_{in} is equal to 1 at port 1, we have S_{11} equal to 0. We will have S_{22} and S_{33} equal to 0. This is because we have seen port 2 and port 3 they are matched for both even and odd mode excitation. So, S_{22} and S_{33} become equal to 0. S_{12} and S_{21} we can calculate. S_{12} and S_{21} can be written as the ratio of the voltages V_1^e plus V_1^o that means superposition of the voltages under even and odd mode excitation at port 1 divided by V_2^e plus V_2^o.

So we have seen V_1^o is 0, V_1^e was minus j root 2 V naught, and V_2^e and V_2^o both were V naught, so once we substitute these values we get S_{12} is equal to S_{21} which is minus j by root 2. So we have found out S_{11}, we have found out S_{22}, we have found out S_{33}, S_{12}, and S_{21}. We are left with S_{32} and S_{23}. Now by symmetry we can also write S_{13} is equal to S_{31} is equal to minus j by root 2. So all the S parameters other than S_{23} and S32 are now found out. We find that at the bi-section we have either open or short, so this condition will keep the port 2 and port 3 isolated.

So whenever it is open-circuited no current will flow. Whenever the bi-section point is short-circuited the signal will be diverted to the ground, and therefore, S_{23} and S_{32} will be equal to 0. So all the S parameters for this Wilkinson power divider are now evaluated and we can write the S matrix to be 000 diagonal elements S_{11}, S_{22}, S_{33}, S_{12}, S_{21} or S_{13}, S_{31} they are minus j by root 2 and S_{32} and S_{23} are also 0.

So please note that for a Wilkinson power divider it is having a lossy element, a resistance 2 Z naught is connected between port 2 and 3 but even with the presence of this lossy element the resistor we achieve the impedance match at all three ports but the device actually behaves like a lossless power divider as far as power divisions are concerned and this is true when the ports remain properly matched.

So we have seen in this lecture, we have introduced the basics of power divider and combiner, then we have seen lossless T-junction power divider, the issues with the lossless T-junction power divider. We considered a power divider, resistive divider, a power divider containing resistance. We have seen that it can provide matching, but it also dissipates power. Then finally we have discussed Wilkinson power divider, which also has a lossy element, but it can behave as a lossless divider, and it can also be matched at all ports. We have discussed only an equal power division case with a Wilkinson power divider; we can have unequal divider also in the Wilkinson form. In the next lecture, we will consider directional couplers.