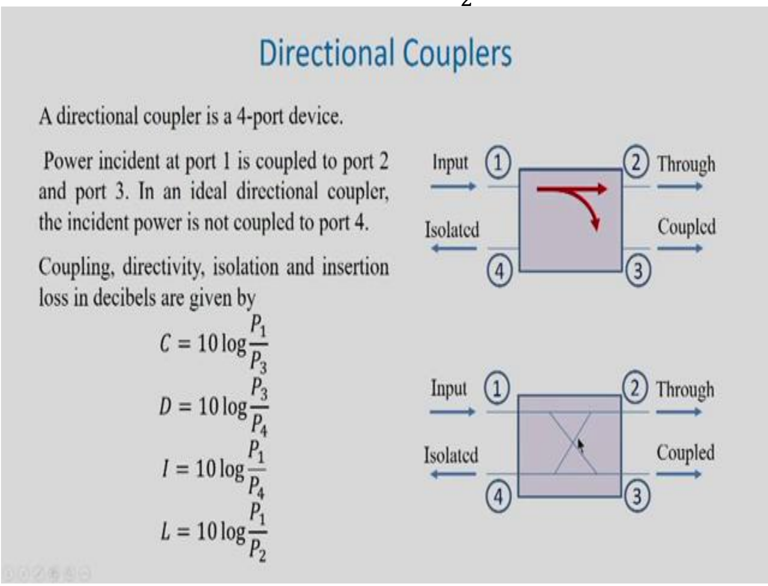
Microwave Engineering
Professor Ratnajit Bhattacharjee
Department of Electronics and Electrical Engineering
Indian Institute of Technology, Guwahati
Lecture 18 – Directional Couplers

(Refer Slide Time: 0:34)

$$C = 10\log\frac{P_1}{P_3}$$
$$D = 10\log\frac{P_3}{P_4}$$
$$I = 10\log\frac{P_1}{P_4}$$
$$L = 10\log\frac{P_1}{P_2}$$



Let us now consider another topic, the directional couplers. So a directional coupler is a 4-port device. So we can see that a directional coupler has an input port, a through port, a coupled port, and an isolated port. The power incident in port 1 is coupled to port 2 and port 3. In an ideal directional coupler the incident power is not coupled to port 4. And therefore this port is called isolated port. Power gets divided between port 2 and port 3. Now, the parameters which are defined for a directional coupler are coupling, directivity, isolation and insertion loss. And usually they are expressed in dB or decibels. And we can write coupling C is equal to 10 Log, this is log base 10 P_1 by P_3.

That means we take the ratio of the input power and coupled power input 3, and when it is expressed in dB, we get C is equal to 10 Log P_1 by P_3. Similarly, directivity gives the ratio of power between port 3 and 4. Isolation is between Port 1 and 4. And I is 10 log P1 by P4. The insertion loss or L it is 10 log P_1 by P_2. So, this is one symbol which is used to represent directional coupler. Another symbol which is also commonly used in representing directional coupler is shown here.

(Refer Slide Time: 3:34)

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

For an ideal lossless directional coupler, we have

$$S_{23} = S_{14} = 0$$

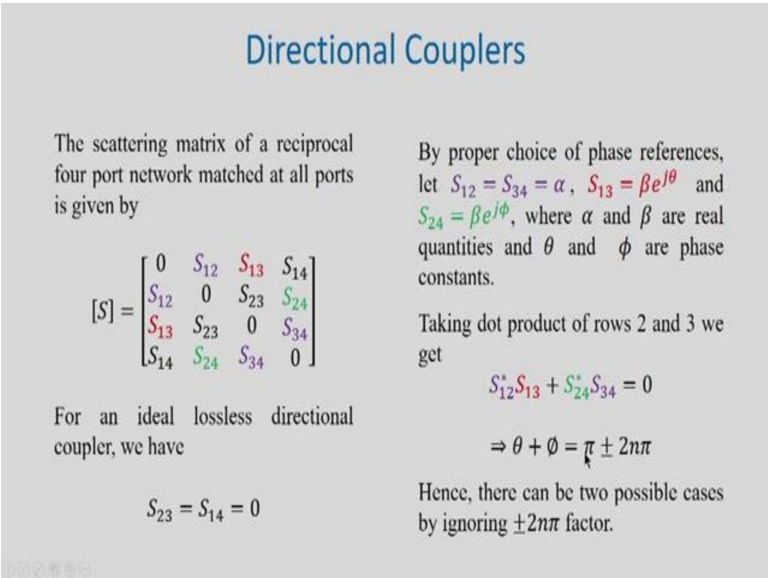
Let $S_{12} = S_{34} = \alpha$, $S_{13} = \beta e^{j\theta}$ and $S_{24} = \beta e^{j\phi}$, where α and β are real quantities and θ and ϕ are phase constants.

Taking dot product of rows 2 and 3 we get

$$S_{12}^{*} S_{13} + S_{24}^{*} S_{34} = 0$$

$$\Rightarrow \theta + \emptyset = \pi \pm 2n\pi$$

Hence, there can be two possible cases by ignoring $\pm 2n\pi$ factor.



The scattering parameter of a reciprocal four-port network matched at all ports, it is given by S equal to the diagonal elements are 0, because this four-port network is matched at all 4 ports. And since it is reciprocal, so we have S_{21} equal to S_{12}, S_{31} equal to S_{13}, and so on.

Now, when the directional coupler is ideal and it is lossless, we have port 1 and port 4 and port 2 and port 3. They are completely isolated, and therefore S_{23}, S_{14} is equal to 0. Now, what we can do by proper choice of phase references, let us set S_{12} and S_{34} to be a real quantity alpha. Alpha is less than 1. So this S_{12} and S_{34} set equal to alpha. And S_{13} equal to beta e to the power j theta and S_{24} is magnitude is same as S_{13}, but it is a different phase. So S_{24} is beta e to the power j phi. And alpha and beta are real quantities. Both less than 1 and theta and phi these are phases associated with these S parameters and defined with respect to the terminal planes. Now if you take dot product of row 2 and row 3 then what do we get? S_{12} conjugate S_{13} plus S_{24} conjugate S_{34} is equal to 0.

We get the same from columns 2 and 3 as well. And if we now substitute these values, so S_{12} conjugate will be alpha, S_{13} will be beta e to the power j theta. So this time will be alpha beta e to the power j theta. S_{24} will be beta e to the power minus j phi, and S_{34} is alpha. So it is alpha-beta e to the power minus j phi, and therefore we now get e to the power J theta will be equal to minus e to the power minus j phi. So this minus can be written as e to the power j phi and therefore we will get theta equal to Pi minus phi. And therefore theta plus phi will be equal to Pi and in fact this relationship will be true for theta plus phi is equal to pi plus minus 2n pi. And what we do, we ignore this factor plus minus 2n pi. We only consider theta plus phi is equal to pi.

(Refer Slide Time: 8:14)

Let $S_{12} = S_{34} = \alpha$, $S_{13} = \beta e^{j\theta}$ and $S_{24} = \beta e^{j\phi}$, where α and β are real quantities and θ and ϕ are phase constants.

Taking dot product of rows 2 and 3 we get

$$S_{12}^{*}S_{13} + S_{24}^{*}S_{34} = 0$$
$$\Rightarrow \theta + \emptyset = \pi \pm 2n\pi$$

Hence, there can be two possible cases by ignoring $\pm 2n\pi$ factor.

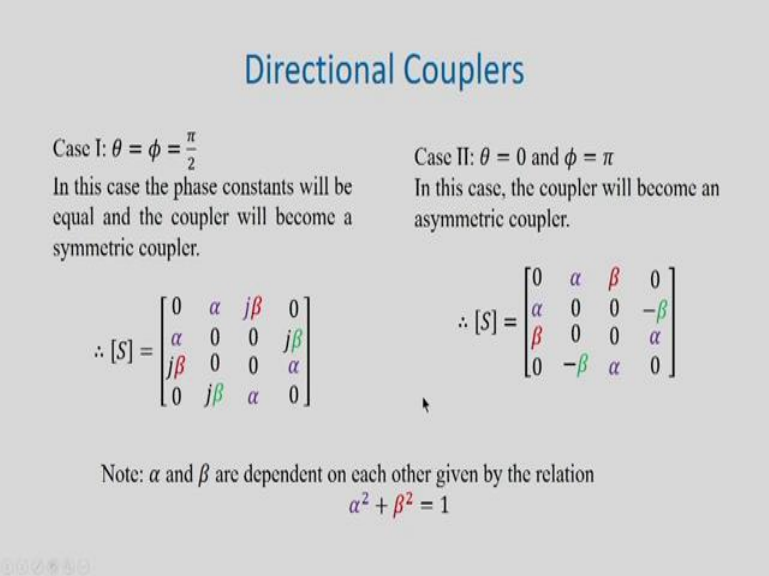
Case II: $\theta = 0$ and $\phi = \pi$

In this case, the coupler will become an asymmetric coupler.

$$\therefore [S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Note: α and β are dependent on each other given by the relation

$$\alpha^2 + \beta^2 = 1$$

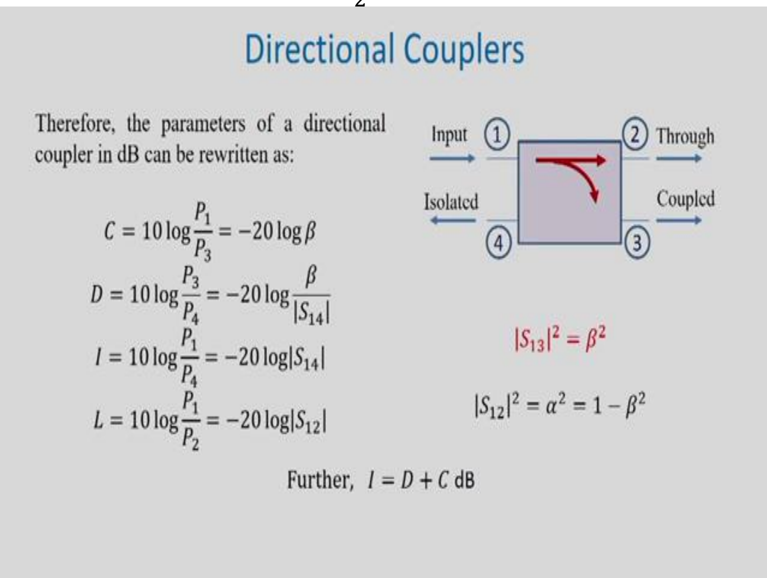


Now we consider two cases. Case 1: When theta equal to phi is equal to Pi by 2. In that case, the phase constants are equal, and the coupler will be a symmetric coupler and for which the S parameters will be given by 0 alpha j beta 0, alpha 00 j beta, j beta 00 alpha and 0 j beta alpha 0.

Another case, when theta is equal to 0 and phi is equal to pi, this case it is an asymmetric coupler. And we can see that these two betas S_{13} and S_{31} are equal to beta. But as S_{24} and S_{42} are minus beta and Alpha and Beta these are dependent on each other. And it is given by alpha square plus beta square is equal to 1. We take any column. Then if you multiply with the complex conjugate of the corresponding elements and add them then we get alpha square plus beta square equal to 1. In both cases this relationship is valid.

(Refer Slide Time: 10:18)

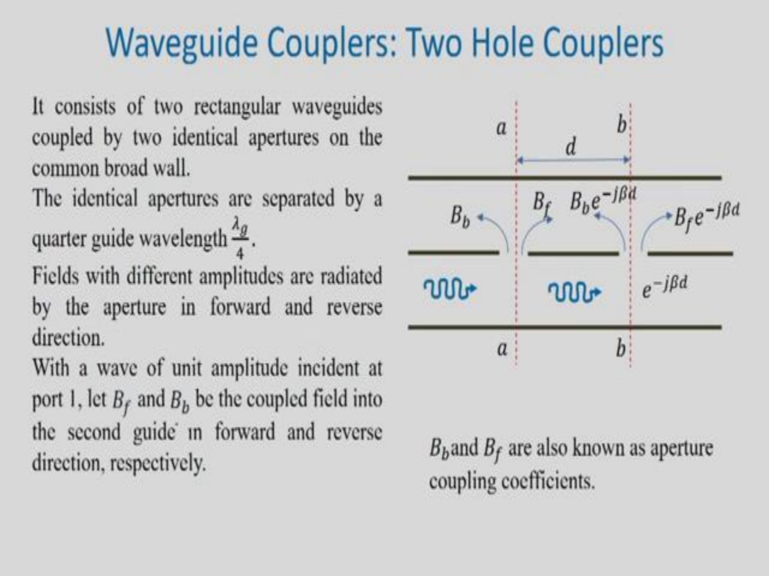
$$C = 10\log\frac{P_1}{P_3} = -20\log\beta$$
$$D = 10\log\frac{P_3}{P_4} = -20\log\frac{\beta}{|S_{14}|}$$
$$I = 10\log\frac{P_1}{P_4} = -20\log|S_{14}|$$
$$L = 10\log\frac{P_1}{P_2} = -20\log|S_{12}|$$



So we have a magnitude of S_{13} square is equal to beta square, and then S_{12} square is alpha square and that will be equal to 1 minus beta square. And we can write the parameters of the directional coupler in dBs. Now, C becomes the coupling, 10 log P_1 by P_3, and P3 by P_1 is mod S_{13} square, which is beta square. So once we substitute here 1 by beta square we get C is equal to minus 20 log beta.

Similarly, D the directivity it is 10 log P_3 by P_4 ,and this can be this P_3 by P_4 can be written as P_3 by P_1 and P_1 by P_4. So P_3 by P_4, P_3 by P_1 can be written as 1 by beta square, and similarly P_1 by P_4 can be written as 1 by mod of S_{14} square. And therefore D can be written as minus 20 log beta by mod of S_{14}. Isolation I we can write directly 10 log P_1 by P_4, and this will become minus 20 log mod S_{14} and insertion loss 10log P_1 by P_2. This is minus 20 log mod S_{12}. And from here we find that S_{12} modulus is essentially alpha or under root 1 minus beta square. We further note that if we add C and D then we get 10 log P_1 by P_3 into P_3 by P_4. So P_3 terms will get canceled and we will be left with 10 log P_1 by P4, which is I, and therefore I is equal to D plus C in dB. So we have defined different parameters of a directional coupler.

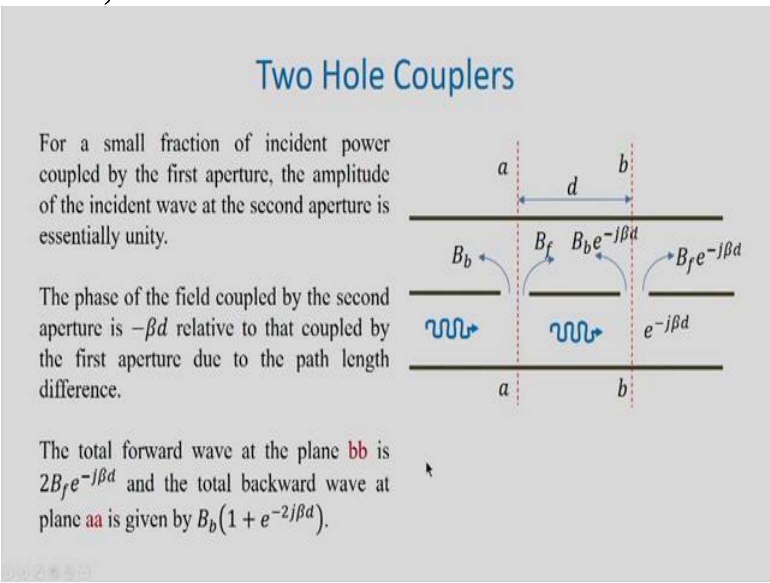
(Refer Slide Time: 14:10)



Let us turn our attention to waveguide couplers. Waveguide couplers appear in various forms. Common ones are Bethe-hole coupler, multi-hole coupler. The coupling can be made between two waveguide sections. Either the coupling between two waveguide sections joined either on the broad wall or at the narrow wall, can be made by drilling suitable aperture on these common walls. For illustration purposes, we will consider here a two-hole coupler. So in a two-hole coupler as we can see, we have two waveguides. They are stacked one over the other and therefore we have a common broad wall for both the waveguide, and we have holes made on this common wall. So it consists of two rectangular waveguides coupled by two identical apertures on the common broad wall.

The identical apertures are separated by a distance, which is a quarter of the guide wavelength that is lambda g by 4. Fields with different amplitudes are radiated by the aperture in forward and reverse direction. So, when the signal will incident on this waveguide, a part of the power will get coupled to the other waveguide. And this coupled power will travel both in forward as well as in the backward direction. Now, this coupling or the amount of field that is coupled in the forward and backward direction are different. Now let us consider a wave with unit amplitude, which is incident at port 1. So this is port 1. And let B_b and B_f with the coupled field into the second waveguide. Now B_f is in the forward direction. And B_b is in the reverse direction. Now this B_b and B_f, these are known as aperture coupling coefficients.

(Refer Slide Time: 17:48)



So we assume that this first aperture it couples only a very small fraction of the incident power. So what happens, essentially the amplitude of the incident wave at this second aperture when the wave reaches the second aperture it remains essentially unity. Now the wave which actually travels over this distance D, now we can see that if B_f representing the coupling from the first aperture, in the second aperture because of this distance D the amount of field that will be coupled is B_f into e to the power minus j beta D. Because the wave was one here, here it becomes e to power minus j beta D.

Now this forward wave Bf also travels this distance D and appears in phase with the waves that is coupled from the second slot. So what we observed is that after in this plane B_b, we have the waves coupled waves from the first slot and the second slot. They appear in phase, and they are given by B_f, e to the power minus j beta D. Whereas regarding the reverse wave we find that here it was B_b. Now here from the second slot it will be B_b e to the power minus j beta D., And then this wave will travel again.

Another length D, so will have this second wave having a phase shift of 2 beta D. And therefore, the total forward wave at plane Bb is Bf e to the power minus j beta D from the second slot.

Again B_f e to the power minus j beta D from the first slot. And therefore it is 2 B_f e to the power minus j beta D. And at the plane Aa the total backward wave will be given by B_b plus B_b e to the power minus j 2 beta D because we get this backward traveling wave which is given by Bb e to the power minus j beta D is will suffer another phase shift because of this length D.

(Refer Slide Time: 22:01)

The coupling is given by

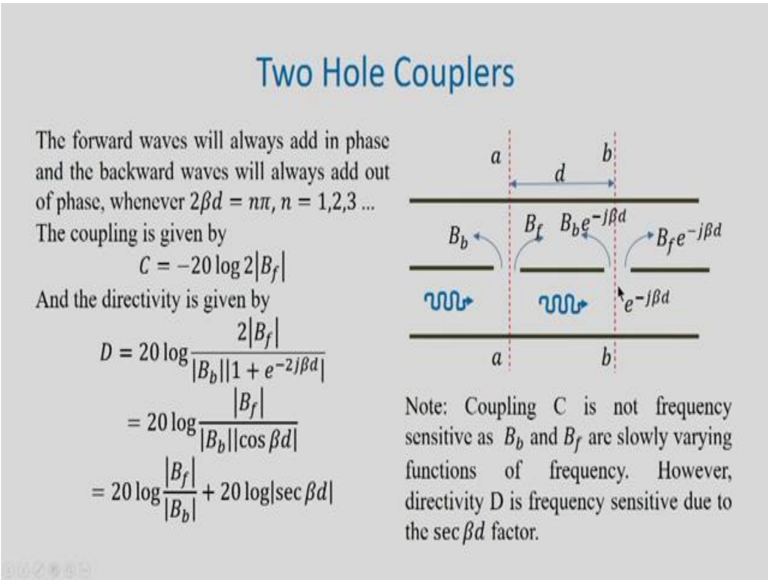
$$C = -20 \log 2|B_f|$$

And the directivity is given by

$$D = 20 \log \frac{2|B_f|}{|B_b||1 + e^{-2j\beta d}|}$$

$$= 20 \log \frac{|B_f|}{|B_b||\cos \beta d|}$$

$$= 20 \log \frac{|B_f|}{|B_b|} + 20 \log|\sec \beta d|$$



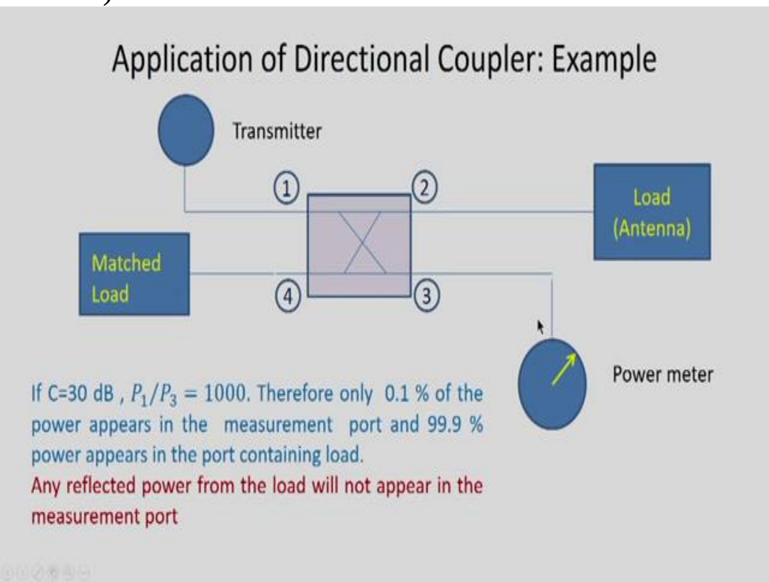
So we find the forward waves will always add in-phase and the backward waves will always add out of phase whenever 2 beta D equal to n pi and n is equal to 1 to 4. So if we have here, B_b plus B_b e to the power minus j 2 beta D, and suppose we have 2 beta D is equal to pi for n equal to 1, then we have 2 pi by lambda D is equal to pi. And therefore D by Lambda is equal to one-fourth. So if we take this D to be equal to lambda by 4 or rather lambda g by 4 where lambda D is the guide wavelength, then the backward waves can be canceled at this plane Aa. And we will be left with only the coupled signal at this plane B_b. In general, we define coupling C to be equal to minus 20 log 2 magnitude of B_f and directivity D it is given by 20 log 2 mod B_f divided by mod B_b mod of 1 plus e to the power minus 2 j beta D.

And this can be written as this term 1 plus e to the power minus 2 j beta D can be written as cos beta D, therefore D becomes 20 log mod B_f divided by mod Bb mod of cos beta D., And this can be expanded as 20 log mod B_f by mod B_b plus 20 log mod of sec beta D. We should note that the coupling C here is not frequency sensitive as Bb and Bf are slowly varying functions of frequency.

However, if we look at the directivity term, the directivity D is frequency sensitive because it has this factor sec beta D. So we can see that if we want to design a 2-hole coupler we need to know these two parameters, the coupling coefficients B_f and B_b. And once we have these values we can design a directional coupler of the specified coupling and directivity. Now, the values of these coupling coefficients depend upon the radius of these apertures, and these coupling coefficients for this type of waveguide geometrics can we estimated analytically. However, will not consider this here. We assume that these coupling coefficients are available with us while designing this type of 2-hole directional coupler.

This concept what we have discussed can be extended to multi-hole couplers because if you notice here the parameter D is very very frequency sensitive. And therefore, this type of 2-hole coupler will have a relatively narrow bandwidth. And if we go for broadband if we want broadband operation, we can go for design of multi-hole couplers and where we can actually design coupler for a specified bandwidth by controlling the coupling coefficients at different aperture locations.

(Refer Slide Time: 28:51)



Directional couples have lot of applications. We discussed here one example of such an application. So we are employing this directional coupler for monitoring the power that is supplied to this antenna element, which is the load by this transmitter. Normally, this transmitter will send large amount of power to the load, and by employing this directional coupler here, we sample a small fraction of the power in this port 3 where we perform the measurement by connecting a power meter, and we put a matched load in the port 4. For example, if we have C is equal to 30 dB then P_1 by P_3 is 1000 that means P_3 is 1000 of P_1 which is actually 0.1 percent.

So this measurement port will get 0.1 percent of the power, whereas the load will get 99.9 percent of the total power. We are assuming port 4 is isolated perfectly isolated and does not get any power from the transmitter. So by monitoring the level of power here in this port we can know how much power is being delivered to the load.

So we are essentially measuring a very small factor of power to know a large amount of power that is being delivered to the load. It should be noted that because of some load mismatch if any power is reflected from the load then it will appear at port 2. Now port 2 and 3 they are isolated; therefore this reflected power will not appear in the measurement port and will affect the measurement. This reflected power will get divided between port 1 and 4, and in port 1 it may be absorbed by the impedance of the transmitter, and whatever power goes into the port 4 it gets dissipated in the matched load connected here.

So this is how we can use a directional coupler in monitoring of power. So we have discussed different aspects of directional couplers. In the next lecture, we will consider another useful microwave device, a microwave filter.