Microwave Engineering
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Lecture 19 – Microwave Filters Part-1

In the previous lectures, we discussed the power dividers and directional couplers. In this lecture we will discuss microwave filters.

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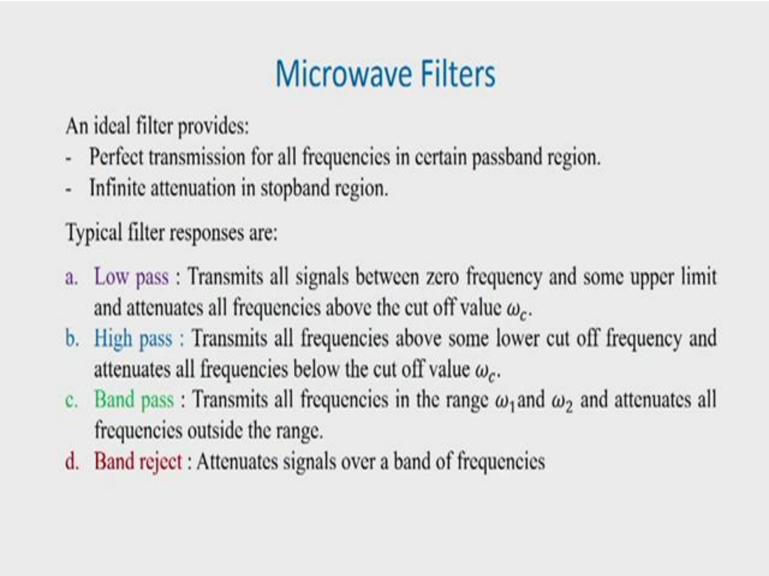

An ideal filter provides perfect transmission for all frequencies in certain passband regions and infinite attenuation in stopband region. Typical filter responses, different types of filter responses are low pass. So in a low pass filter signals between 0 and some upper limit of frequency are transmitted and all frequencies above the cut-off frequency, they are attenuated. Another response is high pass response. And high pass filter transmits all frequencies above some lower cut-off frequency and attenuates all frequencies below the cut-off frequency.

Bandpass filter transmits all frequencies within a range of two frequencies, omega 1 and omega 2, and attenuates all frequencies outside this range. And a band-reject filter attenuates signals over a band of frequencies. So filter characteristics can belong to any of these typical responses, either low pass, high pass, bandpass or band-reject.

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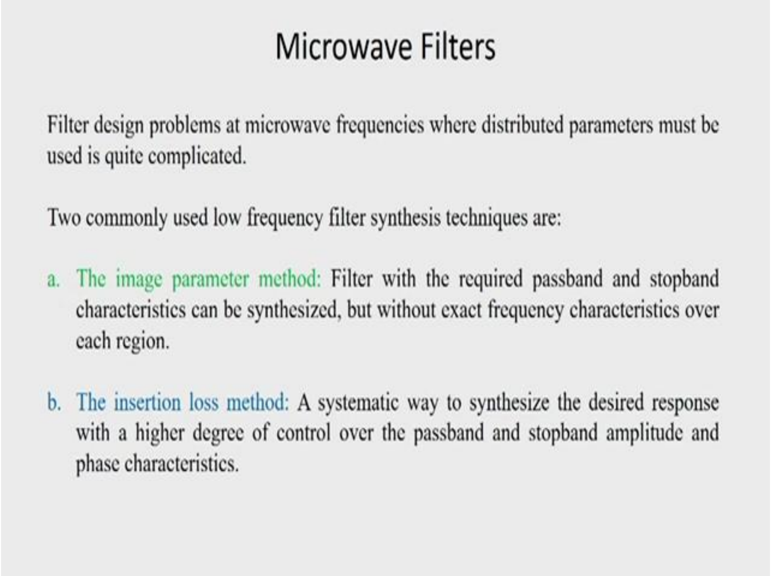

Filter design problems at microwave frequencies where distributed parameters are required to be used are quite complicated. Two commonly used low-frequency filter synthesis techniques are image parameter method. In this approach, filter with required passband and stopband characteristics can be synthesized but without exact frequency characteristics over each region.

Another approach of designing filter is by insertion loss method, which provides a systematic way to synthesize the desired response with a higher degree of control over the passband and stop band amplitude and phase characteristics. In our discussion we will consider the insertion loss method.

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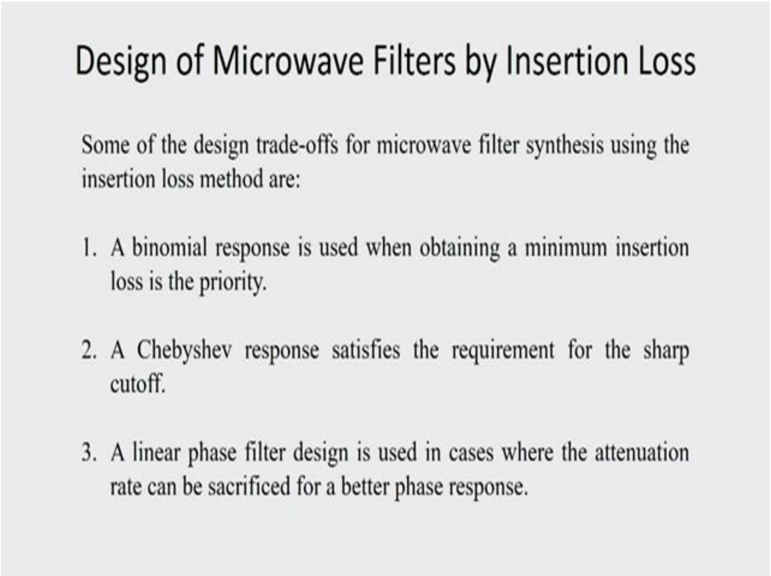

Some of the design trade-offs for microwave filters synthesis using the insertion loss method are binomial responses used when obtaining a minimum insertion loss is the priority. So we go for a binomial response when obtaining a minimum insertion loss. A Chebyshev response satisfies the requirement of sharp cut-off, that means filter response beyond the cut-off frequency if it is to fall very sharply then we go for Chebyshev response.

A linear phase filter design is used in cases where the attenuation rate can be sacrificed for a better phase response. We will see that a binomial response or a Chebyshev response, we essentially specify the magnitude response of the filter. Whenever we require some specified phase response, the attenuation rate can be sacrificed, and a linear phase filter design methodology can be adopted.

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The insertion loss or the power loss ratio in a filter network can be defined as:

$$P_{LR} = \frac{\text{Power available from the source}}{\text{Power delivered to the load}}$$

$$= \frac{P_{in}}{P_{Load}} = \frac{1}{1 - |\Gamma(\omega)|^2}$$

The insertion loss in dB is given by

$$IL = 10 \log P_{LR}$$

Since $|\Gamma(\omega)|^2$ is an even function of ω. We can express $|\Gamma(\omega)|^2$ as a polynomial in ω^2.

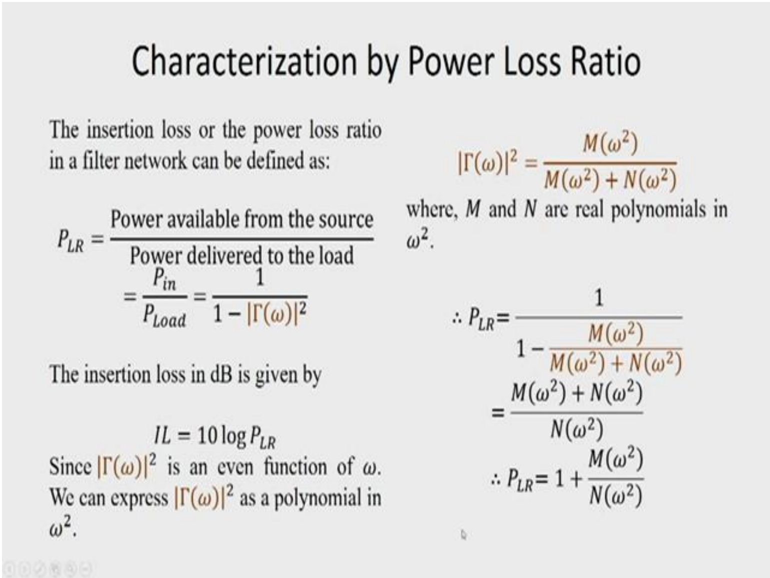
$$|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}$$

where, M and N are real polynomials in ω^2.

$$\therefore P_{LR} = \frac{1}{1 - \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}}$$

$$= \frac{M(\omega^2) + N(\omega^2)}{N(\omega^2)}$$

$$\therefore P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$



Now we introduce one parameter, which is called Power Loss Ratio. The power loss ratio in a filter network we can define as P_{LR} is equal to power available from the source divided by power delivered to the load, and therefore it can be written as P in by P load which is equal to 1 by 1 minus mod gamma omega square because we know that P load is P in into 1 minus mod gamma omega square.

And insertion loss in dB is given by IL is equal to 10 log P_{LR}. Since magnitude of gamma omega square is an even function of omega, we can express gamma omega square as a polynomial in omega square and magnitude of gamma omega square can be written as M of omega square divided by M of omega square plus N of omega square where we have this M and N are real polynomials in omega square and therefore P_{LR} the power loss ratio it becomes 1 by 1 minus M of omega square divided by M omega square plus N omega square and this can be written as M omega square plus N omega square divided by N omega square. And finally PLR is equal to 1 plus M omega square divided by N omega square.

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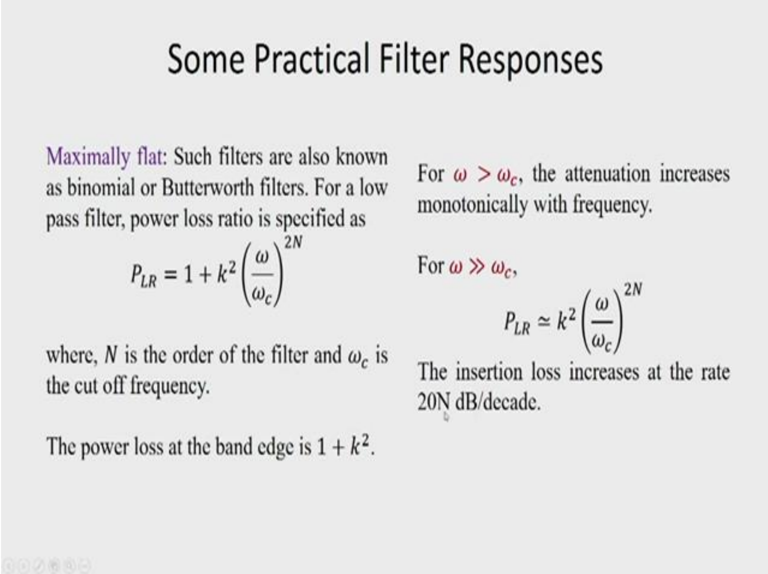
$$P_{LR} = 1 + k^2 \left(\frac{\omega}{\omega_c}\right)^{2N}$$

For $\omega > \omega_c$, the attenuation increases monotonically with frequency.

For $\omega \gg \omega_c$,

$$P_{LR} \simeq k^2 \left(\frac{\omega}{\omega_c}\right)^{2N}$$

The insertion loss increases at the rate 20N dB/decade.



Now this power loss ratio we have written in the form of polynomials in omega square M and N. Now let us see some practical filter responses. The first one we consider it is called a maximally flat filter. Such filters are known as binomial or Butterworth filter. And for a low pass filter the power loss ratio is specified as P_{LR} is equal to 1 plus k square into omega by omega c raised to the power 2N. Here capital N is the order of the filter and omega c is the cut-off frequency. So at omega equal to omega c P_{LR} value becomes 1 plus k square. When omega is larger than omega c, attenuation increases monotonically with frequency. As the frequency is increased, the attenuation increases, and when omega is very very large compared to omega c, we can approximate P_{LR} to be equal to k square omega by omega c raised to the power 2N.

And therefore by insertion loss increases at the rate of 20N dB per decade when omega is much much larger compared to omega c. So 20N dB per decade means as the frequency changes frequency becomes 10 times larger, the attenuation increases by 20N dB, N is the order of the filter.

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$$P_{LR} = 1 + k^2 T_N^2\left(\frac{\omega}{\omega_c}\right)$$

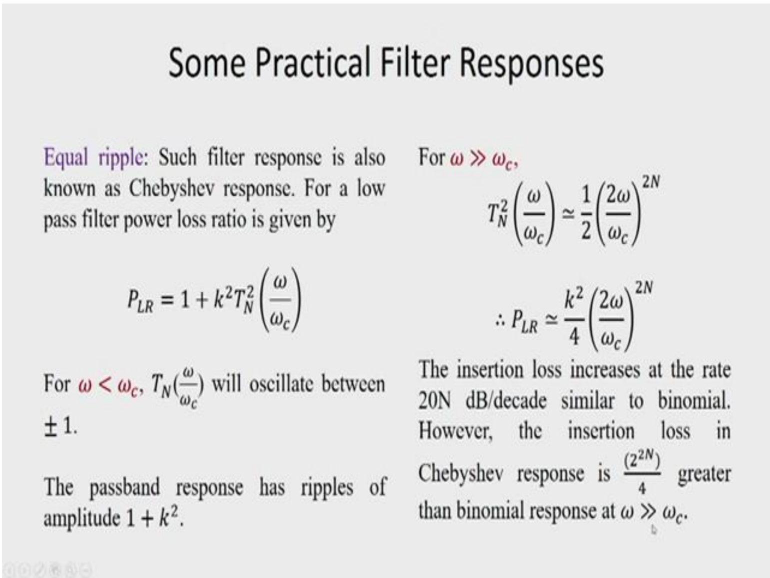
For $\omega < \omega_c$, $T_N^2(\frac{\omega}{\omega_c})$ will oscillate between ± 1.

The passband response has ripples of amplitude $1 + k^2$

For $\omega \gg \omega_c$,

$$T_N^2\left(\frac{\omega}{\omega_c}\right) \simeq \frac{1}{2}\left(\frac{2\omega}{\omega_c}\right)^{2N}$$

$$\therefore P_{LR} \simeq \frac{k^2}{4}\left(\frac{2\omega}{\omega_c}\right)^{2N}$$



Another response which is used in the design of filters is equal ripple. Such filter response is also known as Chebyshev response. And for a low pass filter power loss ratio is given by PLR is equal to 1 plus k square T_N square omega by omega c. Now, this T_N, these are Chebyshev polynomial, and for omega less than omega c, T_N will oscillate between plus minus 1. The passband response has ripples of amplitude 1 plus k square.

For omega much larger than omega c, T_N square omega by omega c this becomes approximately half 2 omega by omega c raised to the power 2N and P_{LR} becomes k square by 4, 2 omega by omega c raised to the power 2N, so here also insertion loss increases at the rate of 20N dB per decade, but the insertion loss in Chebyshev response is greater by a factor 2 to the power 2N by 4 than binomial response at omega much much larger compared to omega c.

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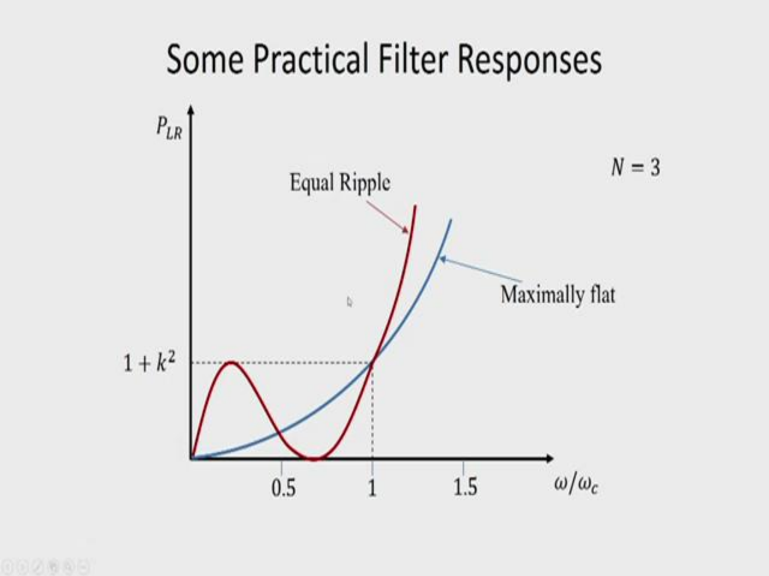

So this is shown here for the filter order of 3. Here we can see that Y-axis is the power loss ratio PLR. Now for a maximally flat filter at omega by omega c equal to 1 it becomes 1 plus k square. Also for the equal ripple case also at this passband edge it becomes 1 plus k square but here we see oscillation within the passband whereas maximally flat filter increases, the power loss ratio increases very gradually and after the passband edge that means for omega by omega c greater than 1 we find that the attenuation becomes much much sharper or stiffer for equal ripple response as compared to maximally flat.

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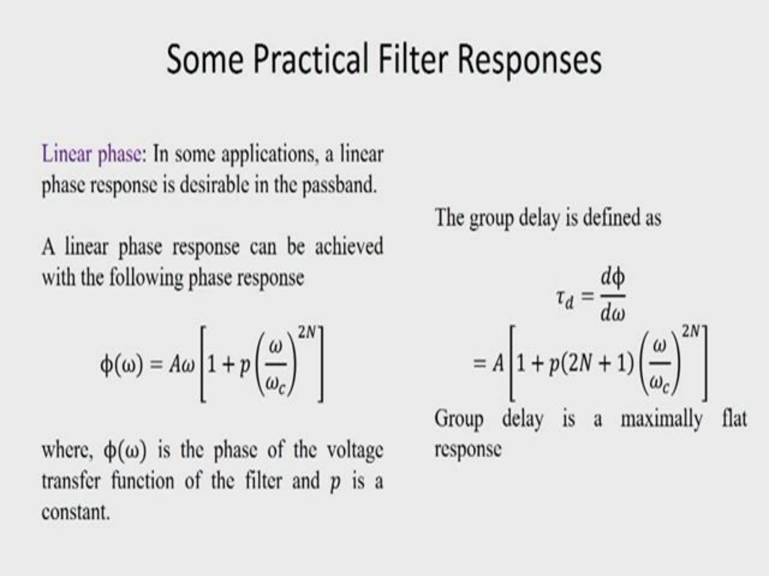
$$\phi(\omega) = A\omega\left[1 + p\left(\frac{\omega}{\omega_c}\right)^{2N}\right]$$

The group delay is defined as

$$\tau_d = \frac{d\phi}{d\omega}$$

$$= A\left[1 + p(2N+1)\left(\frac{\omega}{\omega_c}\right)^{2N}\right]$$

Group delay is a maximally flat response



We talked about linear phase although we will not discuss this in detail. For some applications, a linear phase response is desired in the passband. And a linear phase response can be achieved using the phase response given by phi omega is equal to A omega 1 plus p omega by omega c raised to the power 2N. Here phi omega is the phase of the voltage transfer function of the filter and p is a constant.

We define one parameter, which is group delay tau d is equal to d phi by d omega, and if we find out d phi by d omega from this expression then tau d becomes A into 1 plus p 2N plus 1 omega by omega c raised to the power 2N. Group delay thus becomes maximally flat response, and in a linear phase filter, the phase distortion will be kept under control while designing the filter.

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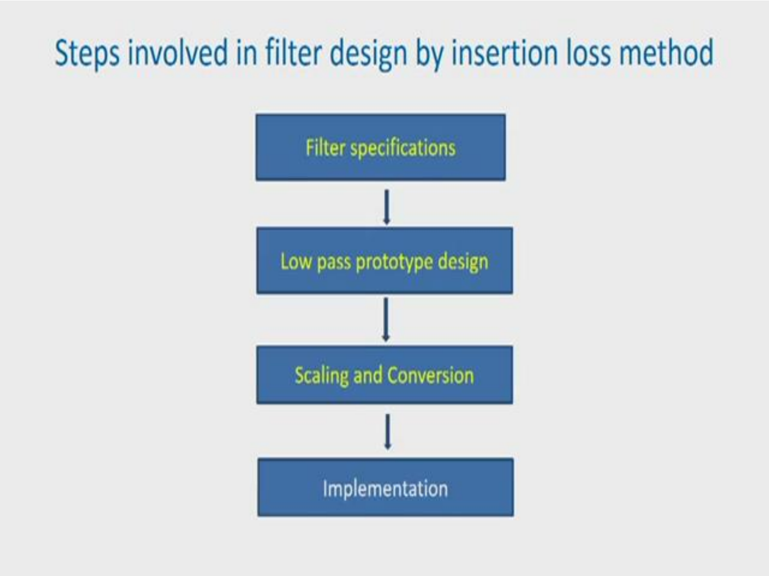

The steps involved in filter design by insertion loss method are we start with some filter specification. In filter specification we specify the cut-off frequency, if it is a low pass or a high pass filter we specify the frequencies of the passband edges for a bandpass filter, and also we specify outside the passband how the insertion loss will happen and we will see that insertion loss specification outside the passband at some frequency, this information will be used in designing the order of the filter.

So once we have the filter specification, we know the order of the filter that would be needed. We go for a low pass prototype design and for that prototype design we can find out the parameters of the prototype filter. This prototype is a low pass filter with unity cut-off frequency, the source resistance is normalized to unity and the load impedance also in many cases is normalized to unity.

So, therefore, after finding the prototype filter we need to do scaling. We need to scale the cut-off frequency of the filter to its actual cut-off frequency value, and once you do the scaling the reactive elements, inductor, and capacitor values calculated from the low pass prototype they will change.

Similarly, we need to do impedance scaling because our source resistance in the prototype is unity. So once we do all such scaling and then depending upon the type of the filter, we intend to design. We can also perform transformation from low pass to high pass, low pass to bandpass, from low pass to band-reject. After this comes the actual implementation phase. In this implementation phase we need to decide how we are going to implement the filter.

It may be using lambda element. It may be using transmission line sections, maybe microstrip lines, or it may be using waveguides. So this implementation of the design filters is quite involved, and we will not attempt it here.

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Let the source impedance be 1Ω and $\omega_c = 1$ rad/sec.

For $N = 2$,

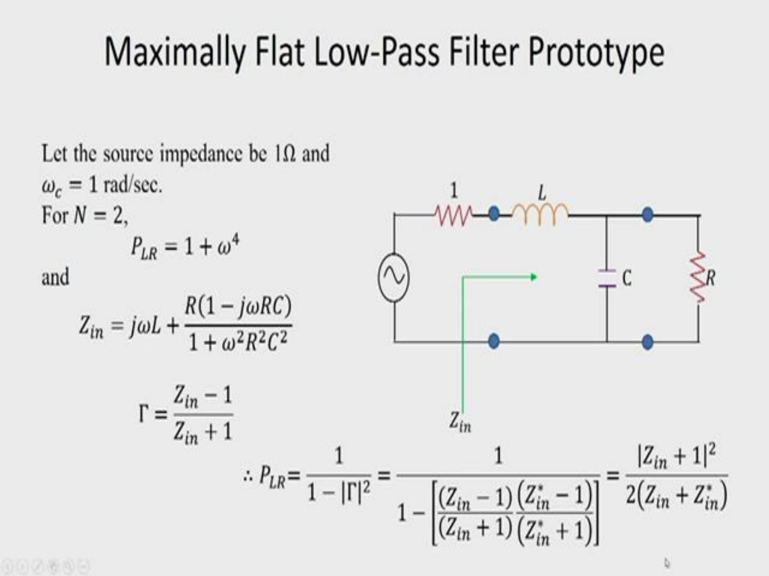
$$P_{LR} = 1 + \omega^4$$

and

$$Z_{in} = j\omega L + \frac{R(1 - j\omega RC)}{1 + \omega^2 R^2 C^2}$$

$$\Gamma = \frac{Z_{in} - 1}{Z_{in} + 1}$$

$$\therefore P_{LR} = \frac{1}{1 - |\Gamma|^2} = \frac{1}{1 - \left[\frac{(Z_{in} - 1)}{(Z_{in} + 1)}\frac{(Z_{in}^* - 1)}{(Z_{in}^* + 1)}\right]} = \frac{|Z_{in} + 1|^2}{2(Z_{in} + Z_{in}^*)}$$



To get an idea of how we go ahead with the design of the low pass filter prototype, let us consider a circuit shown in the figure. Here the source as the impedance of 1 ohm and let us consider the cut-off frequency to be 1 radian per second, and it essentially represents a second-order filter where we have this element L, series element L, and shunt element C and this filter is connected to a load resistance R.

So for a second-order filter when the cut-off frequency is 1, in that case P_{LR} becomes equal to 1 plus omega to power 4. For this circuit shown here Z_{in} can be found j omega L the reactance in series with parallel combination of R and C which is given here and also once we have Z_{in}, the gamma looking into this network since our source impedance is normalized to unity, so gamma will be Z_{in} minus 1 divided by Z_{in} plus 1. And therefore we can write P_{LR}, which is equal to 1 by 1 minus mod gamma square, and here we can substitute gamma into gamma conjugate, and finally it will become mod of Z_{in} plus 1 whole square divided by 2 Z in plus Z in conjugate.

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Now

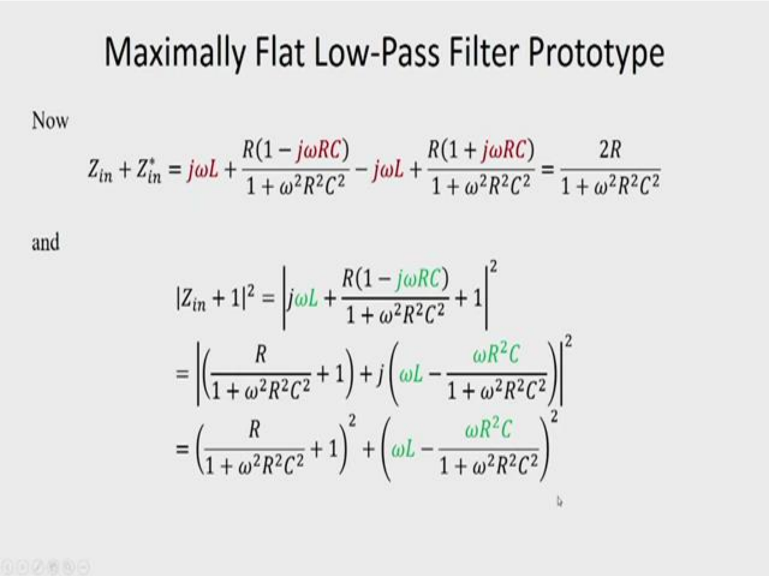
$$Z_{in} + Z_{in}^* = j\omega L + \frac{R(1 - j\omega RC)}{1 + \omega^2 R^2 C^2} - j\omega L + \frac{R(1 + j\omega RC)}{1 + \omega^2 R^2 C^2} = \frac{2R}{1 + \omega^2 R^2 C^2}$$

and

$$|Z_{in} + 1|^2 = \left|j\omega L + \frac{R(1 - j\omega RC)}{1 + \omega^2 R^2 C^2} + 1\right|^2$$

$$= \left|\left(\frac{R}{1 + \omega^2 R^2 C^2} + 1\right) + j\left(\omega L - \frac{\omega R^2 C}{1 + \omega^2 R^2 C^2}\right)\right|^2$$

$$= \left(\frac{R}{1 + \omega^2 R^2 C^2} + 1\right)^2 + \left(\omega L - \frac{\omega R^2 C}{1 + \omega^2 R^2 C^2}\right)^2$$



Now if we substitute the expressions for Z_{in} and Z_{in} n conjugate, we get Z_{in} plus Z_{in} conjugate is equal to 2R divided by 1 plus omega square R square C square. And mod of Z_{in} plus 1 square can also be evaluated by substituting the expression for Z_{in} and it comes out to be R divided by 1 plus omega square R square C square plus 1 whole square, plus omega L minus omega R square C divided by 1 plus omega square R square C square whole square.

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$$\therefore P_{LR} = \frac{|Z_{in}+1|^2}{2(Z_{in}+Z_{in}^*)} = \frac{1+\omega^2R^2C^2}{4R}\left[\left(\frac{R}{1+\omega^2R^2C^2}+1\right)^2+\left(\omega L-\frac{\omega R^2C}{1+\omega^2R^2C^2}\right)^2\right]$$

$$= \frac{1}{4R}[R^2+2R+1+\omega^2R^2C^2+\omega^2L^2+\omega^4L^2R^2C^2-2\omega^2LCR^2]$$

$$= 1+\frac{1}{4R}[(1-R)^2+\omega^2(R^2C^2+L^2-2LCR^2)+\omega^4L^2R^2C^2]$$

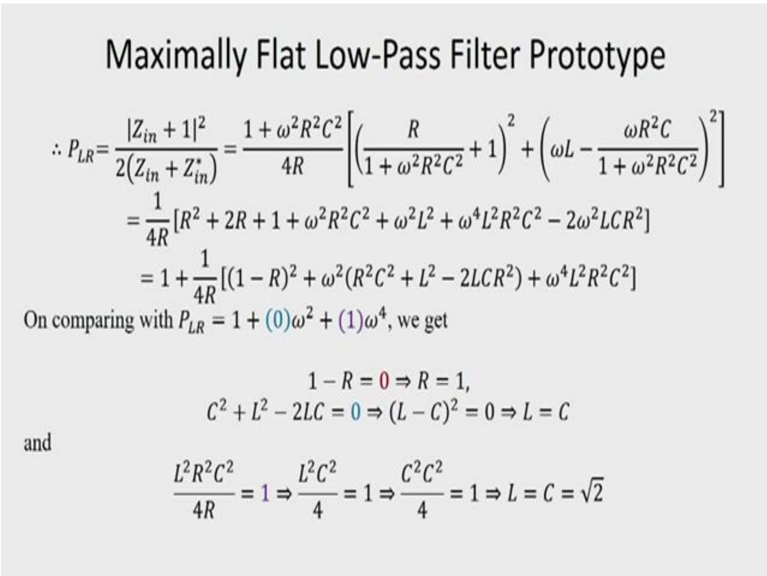
On comparing with $P_{LR} = 1+(0)\omega^2+(1)\omega^4$, we get

$$1-R=0 \Rightarrow R=1,$$

$$C^2+L^2-2LC=0 \Rightarrow (L-C)^2=0 \Rightarrow L=C$$

and

$$\frac{L^2R^2C^2}{4R}=1 \Rightarrow \frac{L^2C^2}{4}=1 \Rightarrow \frac{C^2C^2}{4}=1 \Rightarrow L=C=\sqrt{2}$$



So with mod Z in plus 1 square calculated and also Z in plus Z in conjugate calculated, once we substitute these 2 terms we get the expression for P_{LR}, which is shown here. It is 1 plus omega square R square C square divided by 4 R into R by 1 plus omega square R square C square plus 1 whole square plus omega L minus omega R square C divided by 1 plus omega square R square C square the whole square.

Now, this can be further simplified and put in the form as shown, so the P_{LR} is expressed in the form shown here. We know that P_{LR} is an even polynomial in omega square and we saw that in our case when N is equal to 2, it is 1 plus omega to the power 4, so we can write it in this form and now if we make a term by term comparison because here for the expression for P_{LR} we have omega square, we have actually 1 plus 1 by 4 R 1 minus R whole square and then a term which is the coefficient of omega square and another term L square R square C square divided by 4R which will be the coefficient of omega to the power 4.

So we can equate them, and therefore this term has to be equal to 1, which implies that 1 minus R has to be equal to 0 and R equal to 1 because then only we get this term to be equal to 1. So our load resistance is R equal to 1. The other term when R equal to 1 we get C square plus L square minus 2 LC equal to 0 and therefore L minus C whole square becomes 0, which implies L equal to C.

The last term L square R square C square by 4 R this becomes 1, and since R is equal to 1, we get L square C square by 4 equal to 1, and we have seen L equal to C. So we can solve for L equal to C equal to root 2. So we can see that by comparing the expression for the P_{LR} obtained from the prototype circuit with that of the P_{LR} for a binomial response, we can calculate R. We can calculate L and C, and this we have carried out for a filter order 2. And equal to 2, second-order filter.

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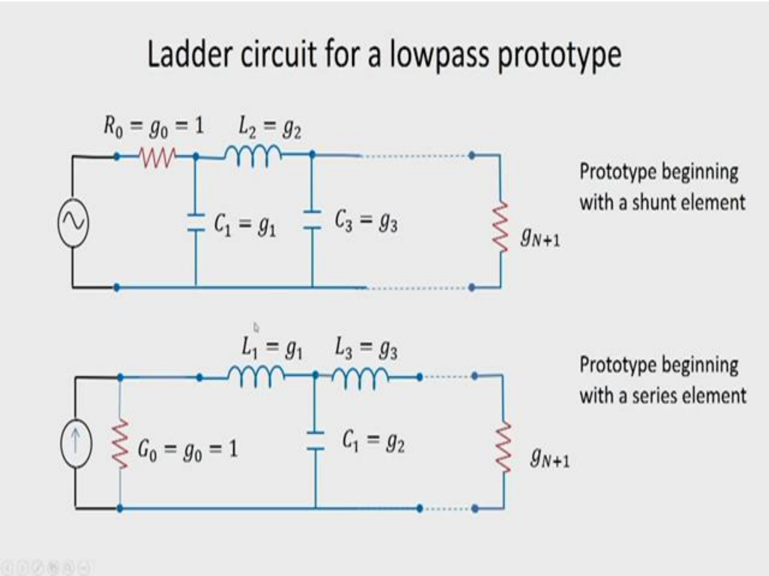

Now this can be done for a generic low pass prototype where we have the prototype, this prototype here we have R not equal to g not equal to 1 then it begins with the shunt element, then we have a series element, another shunt element, so we have this ladder network and finally g N plus 1 this is the load. And when the prototype begins with a series element, we can draw it as shown, and here also if required we can represent it in terms of a current source and a parallel source resistance.

So these are the 2 commonly used prototypes for which the tabulated values for these g's, that means g can be either C or L depending upon the prototype. It can be a series element, or it can be a shunt element, and the values of these g's are tabulated.

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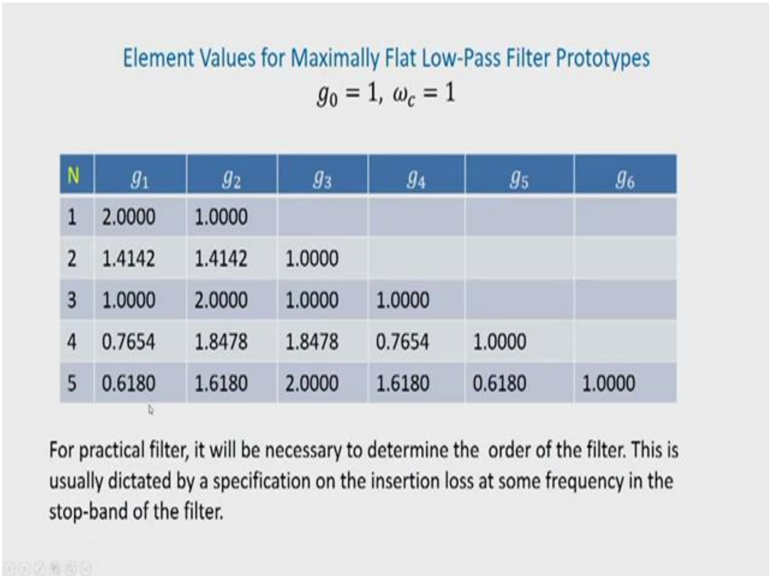

Element Values for Maximally Flat Low-Pass Filter Prototypes

$g_0 = 1,\ \omega_c = 1$

N	g_1	g_2	g_3	g_4	g_5	g_6
1	2.0000	1.0000				
2	1.4142	1.4142	1.0000			
3	1.0000	2.0000	1.0000	1.0000		
4	0.7654	1.8478	1.8478	0.7654	1.0000	
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000

For practical filter, it will be necessary to determine the order of the filter. This is usually dictated by a specification on the insertion loss at some frequency in the stop-band of the filter.

So this table shows the element values for maximally flat low pass filter prototype with g naught equal to 1, omega C equal to 1 and we can see that we found out in our second-order filter L equal to C equal to root 2 which is 1.4142 and R equal to 1, so g_3 is equal to 1. So, essentially resolved by our prototype circuit this row of the table. So for other values of N we can directly use these values for series and shunt elements, which means the g's from this table.

Here we are showing up to filter order 5, but values are also available for higher-order filter. Now in a practical filter, it will be necessary to determine the order of the filter. And as already mentioned, that usually it is dictated by the specification on the insertion loss at some frequency in the stopband of the filter. Suppose at 1.5 times the cut-off frequency we want an insertion loss of 20 dB.

So from this type of specification we can determine what order N is required and once we can know N then we can choose these values g_1, g_2 up to g_N plus 1 from the table and then we can find out the corresponding values for the inductors and capacitances for providing the desired cut-off frequency and also when the source resistance is scaled to the desired value of the source resistance.

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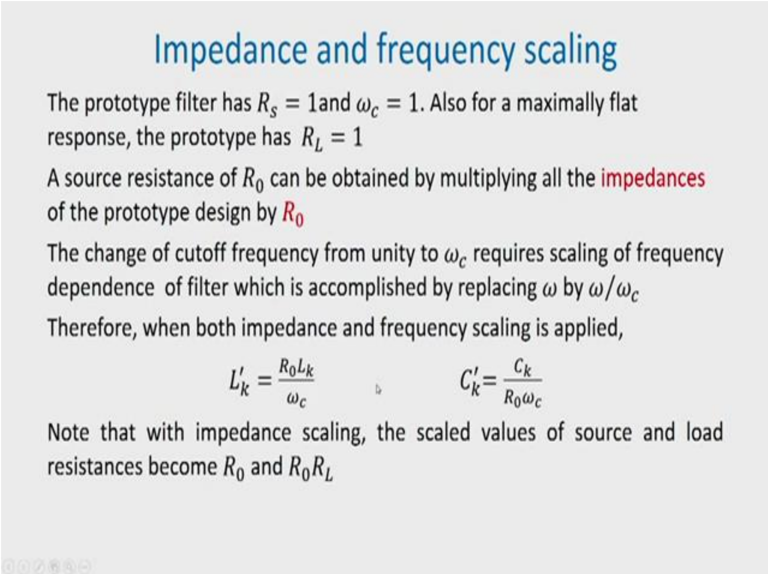
$L'_k = \frac{R_0 L_k}{\omega_c}$ $C'_k = \frac{C_k}{R_0 \omega_c}$



So let us see what do we mean by this impedance and frequency scaling. The prototype filter has R_s equal to 1, omega c equal to 1, and also for maximally flat response we have seen that RL equal to 1. A source resistance of R naught can be obtained by multiplying all the impedances.

Please note that this L and C, they will provide impedance values of omega L 1 by omega C and if we multiply the impedance values provided by the elements in the prototype design by R naught, then essentially we get the solution for those inductors and capacitors when the source resistance is R naught instead of 1 but still our cut-off frequency is 1. So next what we do? We change the cut-off frequency from unity to the actual cut-off frequency omega C for a low pass filter.

So if we do that, we require to scale the frequency dependence of the filter and this is accomplished by replacing omega by omega C. So if we do both impedance and frequency scaling then the scaled values of the inductance become R naught L_k by omega C, so this is the series element we used and scaled value of the capacitance C_k becomes C_k by R naught omega C.

And with the impedance scaling, the scaled value of the source and load resistances now become R naught and R naught R_L. So if our original prototype we have g N plus 1 as 1 that means RL equal to 1 then now new value will be R naught into 1 equal to 1. So let us see how we utilize these concepts in designing a low pass filter, and that will actually clarify whatever we have discussed so far.

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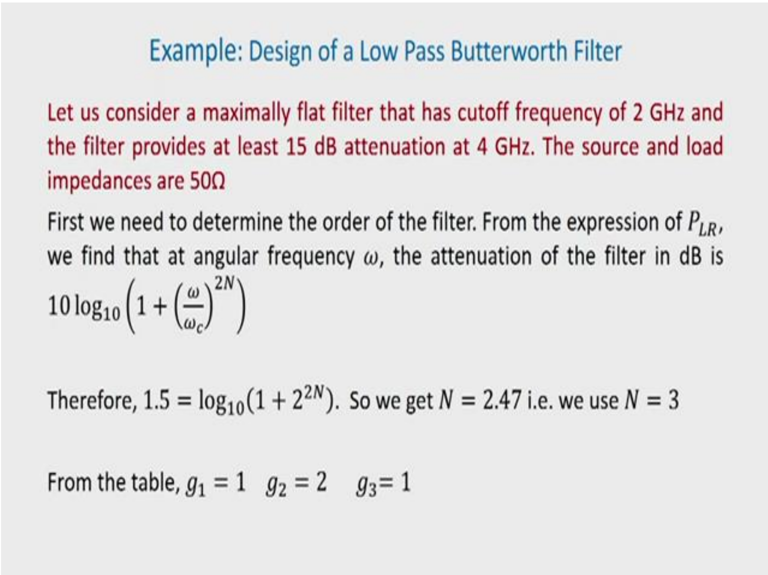
$$10\log_{10}\left(1+\left(\frac{\omega}{\omega_c}\right)^{2N}\right)$$

Therefore, $1.5 = \log_{10}(1 + 2^{2N})$. So we get $N = 2.47$ i.e. we use $N = 3$

From the table, $g_1 = 1 \quad g_2 = 2 \quad g_3 = 1$



So we consider the example of a design of a Low Pass Butterworth Filter. So let us consider a maximally flat filter that has cut-off frequency of 2 gigahertz, and the filter provides at least 15 dB attenuation at 4 gigahertz. The source and load impedances are 50 ohm. So first thing we need to do is to determine the order of the filter. So from the expression for PLR we have seen that at an angular frequency omega, the attenuation of the filter in dB is 10, log 10 1 plus omega by omega c raised to the power 2N. So this is the attenuation of the filter, and therefore in our case, omega c is 2 gigahertz, and we want 15 dB attenuation at 4 gigahertz.

So we can write 15 by 10, which is 1.5 is equal to log of 1 plus omega by omega c become 2, so 2 to the power 2N. If we solve for capital N, we get N is equal to 2.47 and therefore we use

N equal to 3, that means a third-order filter and the table which we have just discussed from there we see that g1 is equal to 1, g_2 equal to 2 and g_3 equal to 1 for a third-order filter.

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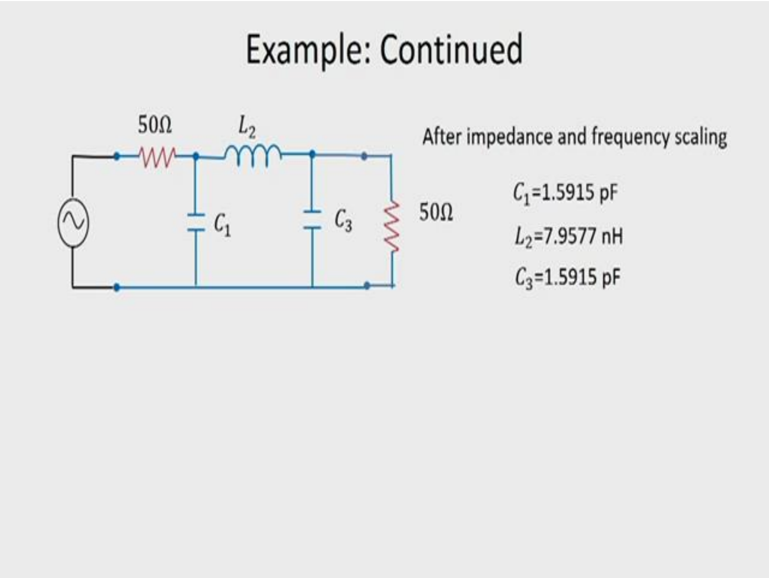

And therefore we can draw the circuit of the filter with 50 ohms source resistance and 50 ohm load resistance, and by applying frequency and impedance scaling, we can find out C_1 to be equal to 1.5915 pF. So the shunt capacitor, first capacitor is and L_2 the series inductance it is 7.9577 Nano Henry, and since we have g1 equal to 1 equal to g_3, C_3 the next shunt capacitor is also 1.5915 pF. So this completes the design of the filter determining the values of inductances and capacitances that would be required to get an attenuation of 15 dB, at least 15 dB at 4 gigahertz.

Now how do we realize this capacitor and inductor values that come under implementation? So we have seen how we can design maximally flat low pass filters. In the next lecture we will discuss how we can design equal ripple filters and the procedures involved in designing the prototype equal ripple filters. Once the low pass filter prototypes are designed, then depending upon the requirement, if we want a high pass filter we need to perform low pass to high pass transformation.

Similarly, low pass to bandpass or bandstop transformation. So in our next lecture, we will see how these transformations are performed.