Microwave Engineering
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Lecture 02
Introduction to Microwave Engineering and Transmission line theory

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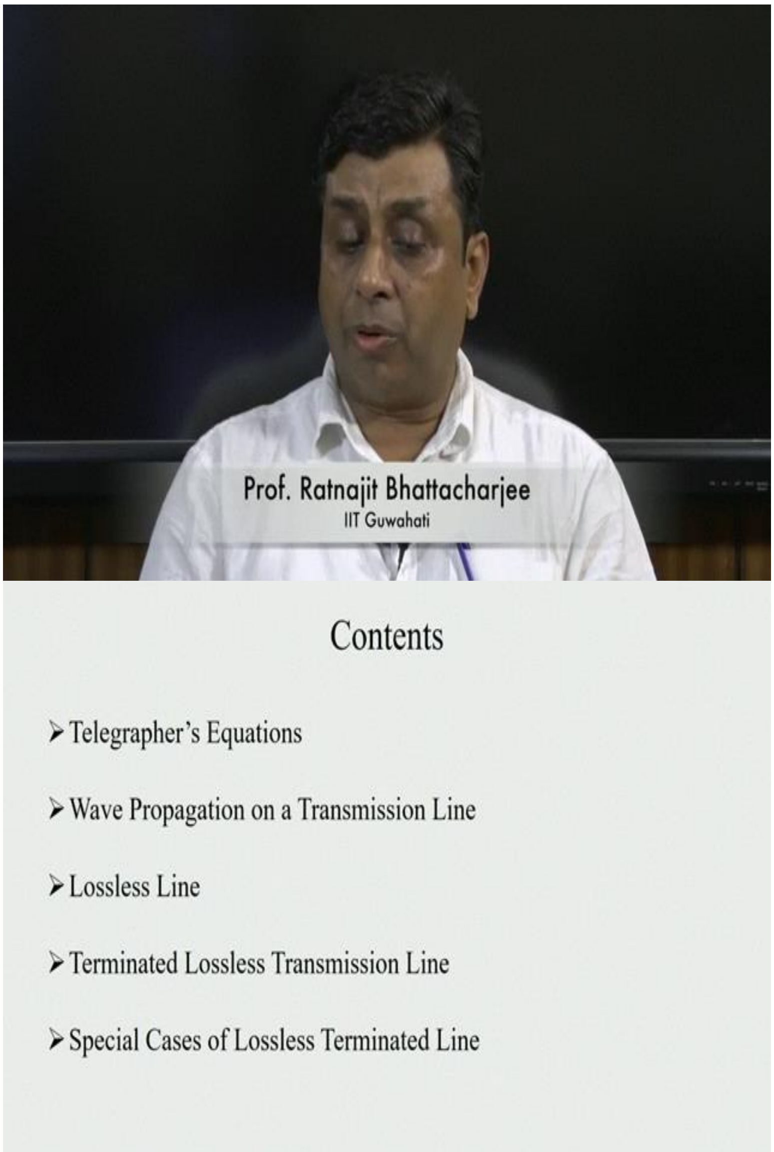

In the previous class, we discussed about some basics of transmission line and we have seen that a short section of transmission line can be represented in terms of its lumped equivalent circuit. In this lecture will cover the following- the Telegraphers equation, the wave propagation on a transmission line, then we will cover the lossless transmission line and then terminated lossless transmission line and finally special cases of terminated lossless lines.

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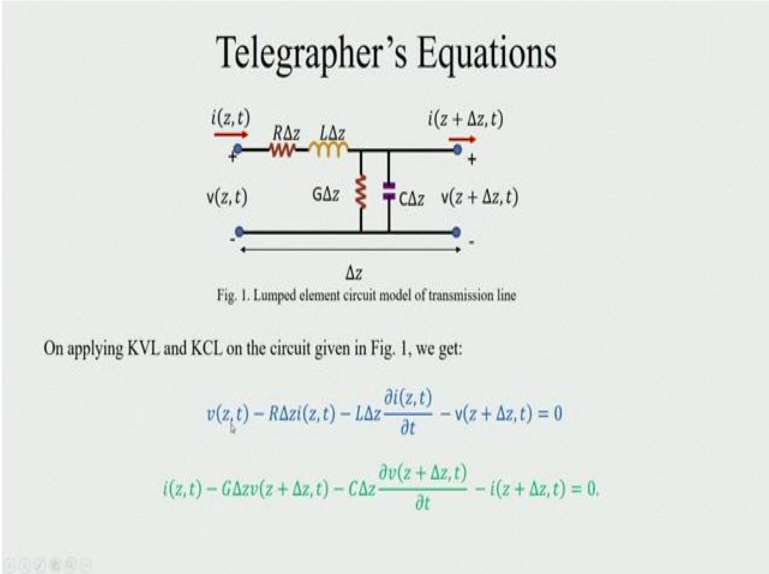

So, we start with Telegraphers equations, how he developed this equations. We have seen that the lumped elements circuit model of a short section of transmission line can be represented by the equivalent circuit as shown here, if we apply Kirchhoff's of voltage law and current law on this circuit, then we get a set of equations relating the current and voltage on the line. So, this is the equation where voltage is related to the current and the resistance, inductance, capacitance, and conductance of the equivalent circuit model.

Similarly, we can, if we apply KCL here we can get another equation. Please note that both the equation we get after the application of the KVL and KCL, these equations represents voltages and currents which are both function of distance Z and time T.

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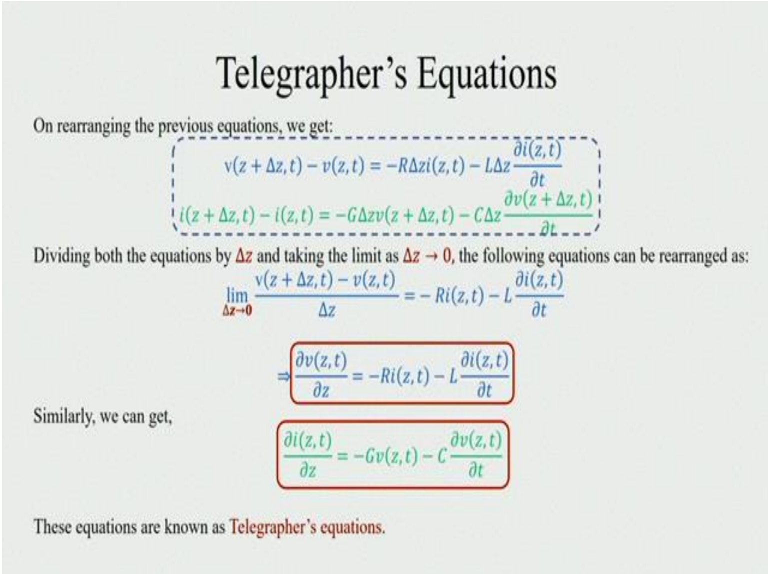

If you rearrange the previous equations we get, and similarly for the current equation, these two sets of equation now can be divided on both sides by Δz and as $\Delta z \to 0$, we get the following sets of equations. Now these equations are known as Telegrapher's equation. It may be noted that the special derivative of the voltage in the first equation is related to the time derivative of current. Similarly, the space derivative of the current in the second equation is related to the time derivative of voltage. So this, in one way these equations, they are coupled and we have both voltage and current present in this equation.

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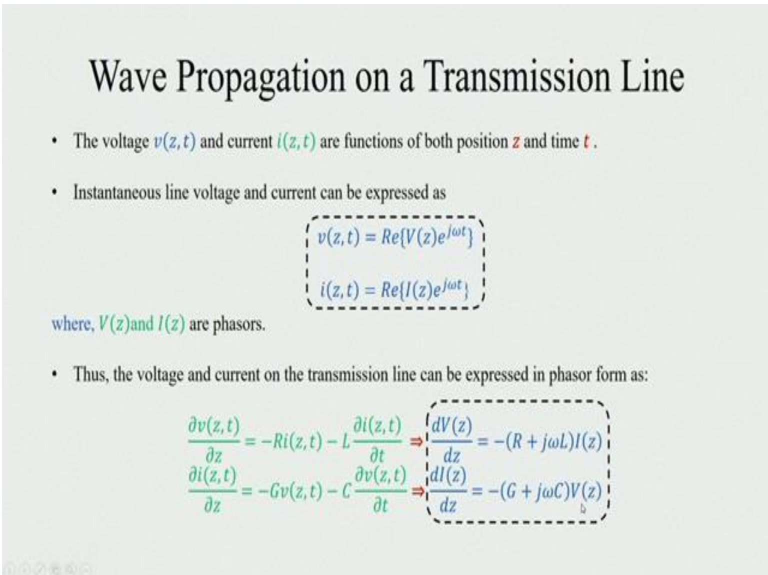

Now let us see how we can infer wave propagation on a transmission line. The voltage v and i which are function of distance z and time t can be expressed in terms of phasors, the instantaneous line voltage and current can be a expressed as $v(z,t)$ is equal to real part of a voltage phasor which is a function of position z multiplied by $e^{j\omega t}$. Similarly, $i(z,t)$ is the real part of a current phasor $I(z)$ multiplied by $e^{j\omega t}$.

Now the voltage and current on the transmission line can be expressed exclusively in the form of phasors and we note that the $\frac{\partial}{\partial t}e^{j\omega t}$ contribute this term $j\omega$. So from these two equations we get the following phasor equations

$$\frac{dV(z)}{dz} = -(R + j\omega L)I(z)$$

and

$$\frac{dI(z)}{dz} = -(G + j\omega C)V(z)$$

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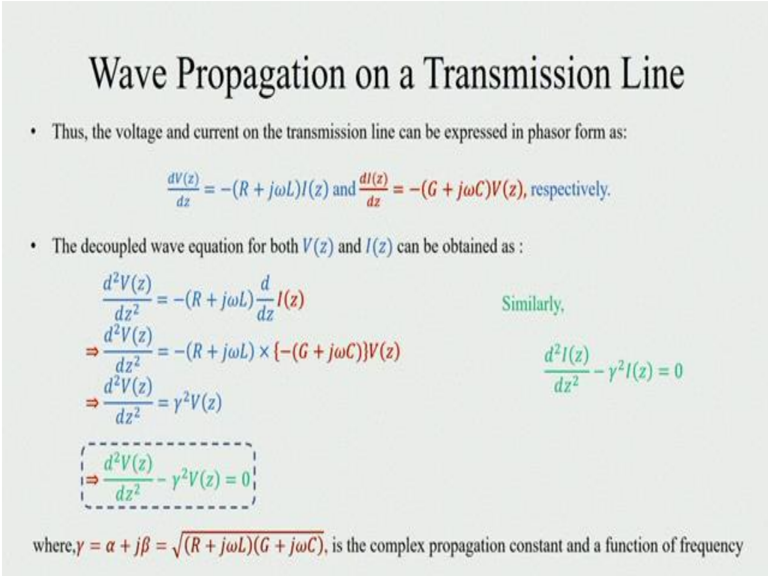

Now if you notice here the space derivative of the voltage is related to current phasor, similarly the special derivative of the current phasor is related to the voltage phasor. Now we can decoupled these two equation to form the wave equation for the voltage and current. So if you take derivative on both sides of this equation and substitute $\frac{dI(z)}{dz}$ from here, then we can write $\frac{d^2V(z)}{dz^2}$ is $\gamma^2 V(z)$.

So in this equation, we find that it is only the voltage phasor V is involved and rearranging the term, we can write of this form, $\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0$, where γ which is equal to $\sqrt{(R + j\omega L)(G + j\omega C)}$ is the complex propagation constant and can be written as $\alpha + j\beta$. We can also in the same manner form the wave equation for the current.

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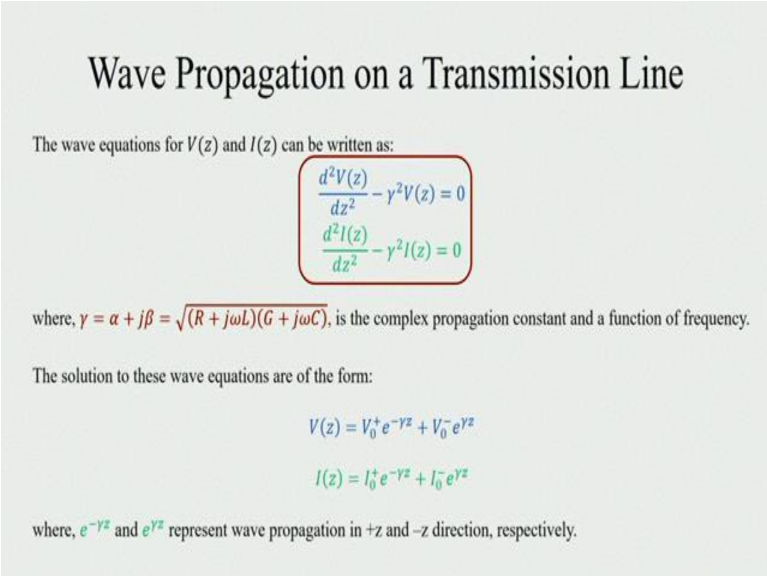

Now solution to this wave equations are of the form $V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$. Similarly, $I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$. Here we find that $e^{-\gamma z}$ and $e^{\gamma z}$ represent wave propagation in positive z and negative z direction, respectively.

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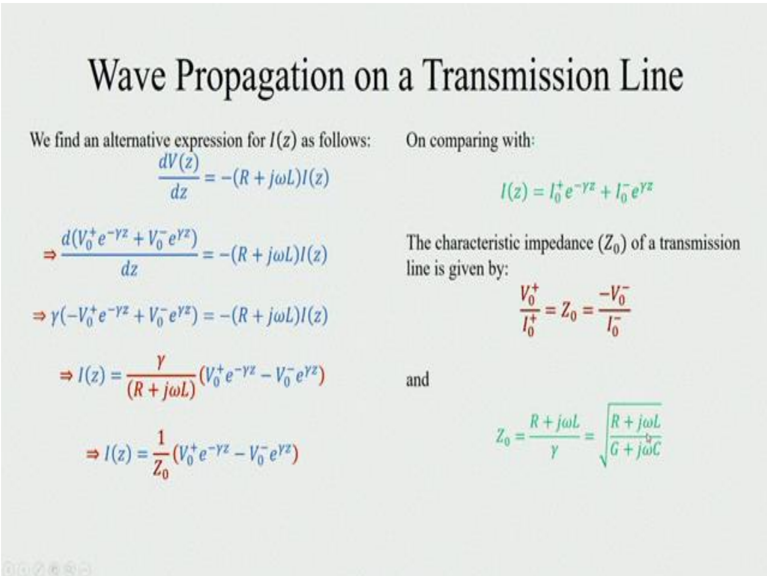

We can also find an alternative form of $I(z)$, which is found as follows, we have $\frac{dV(z)}{dz} = -(R + j\omega L)I(z)$. If you substitute the equation for $V(z)$ and rearrange the terms, then we get $I(z) = \frac{\gamma}{(R+j\omega L)}(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z})$. And this can be written as $I(z) = \frac{1}{Z_0}(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z})$, where Z_0 we are defining as $\frac{R+j\omega L}{\gamma}$.

If we compare this expression for $I(z)$ with the earlier expression that we derived, that means $I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$, then this characteristic impedance Z_0 of the transmission line can be defined as $\frac{V_0^+}{I_0^+} = Z_0 = \frac{-V_0^-}{I_0^-}$. Now in terms of the parameters of the transmission line $Z_0 = \frac{R+j\omega L}{\gamma} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$, if we substitute the expression for γ. It may be noted that Z_0 is a function of the operating angular frequency ω.

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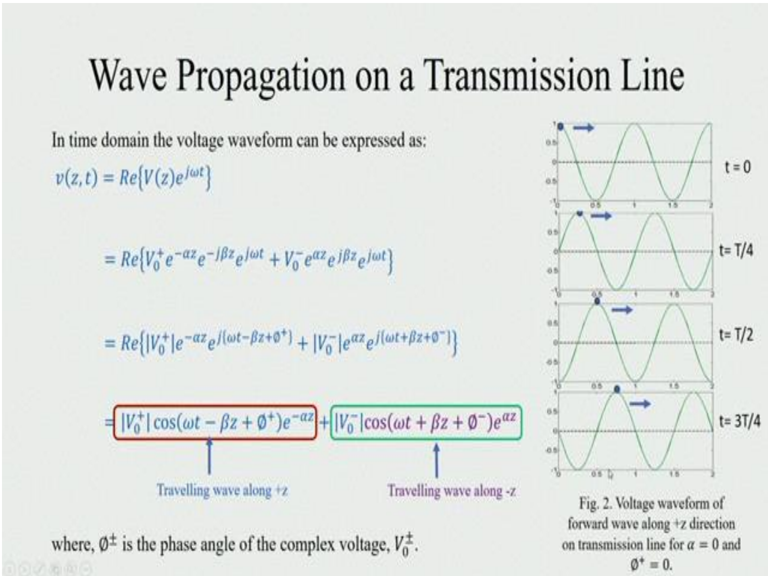

If we come back to the time domain form, then we can write V as a function of z and t as real part of voltage phasor $V(z)e^{j\omega t}$, and substituting expression for $V(z)$, writing $\gamma = \alpha + j\beta$ we get this equation, and then expressing V_0^+ and V_0^- in their magnitude and phase form, we get this form of representation of $v(z,t)$, retaining only the real part of this equation, we can write the voltage on the transmission line as a function of z and t given by $|V_0^+|\cos(\omega t - \beta z + \emptyset^+)e^{-\alpha z} + |V_0^-|\cos(\omega t + \beta z + \emptyset^-)e^{\alpha z}$.

$\emptyset^+$ and $\emptyset^-$ is the phase angle of the complex voltage V_0^+ and V_0^-. The first term represents a wave which is travelling in the +z direction. Similarly, the second equation represents a wave travelling in the -z direction. We consider a simple case where we assume that $\alpha = 0$, that means the wave does not attenuate as it propagates, further V_0^+ phase angle $\emptyset^+$ is 0.

So with this assumption, let us now plot this first equation, we can see that t equal to 0, this equation as a function of z can be plotted as shown and let us follow the moment one particular point on this wave which is marked here. So at t equal to T/4, we find that the point moves towards +z direction. Similarly at equal to T/2, that means half the time period, the point has moved further and it continues. So this equation essentially represents a wave which is travelling in the +z direction and here in this case we have considered the wave does not attenuate.

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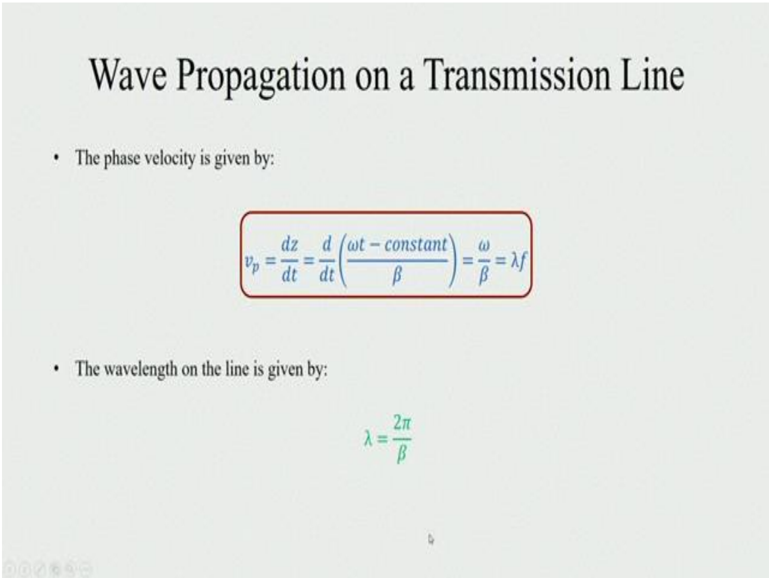

The phase velocity of the wave is given by $\frac{\omega}{\beta}$ and this is equal to λf and also the wavelength on the line is given by $\frac{2\pi}{\beta}$ which we see from this relation.

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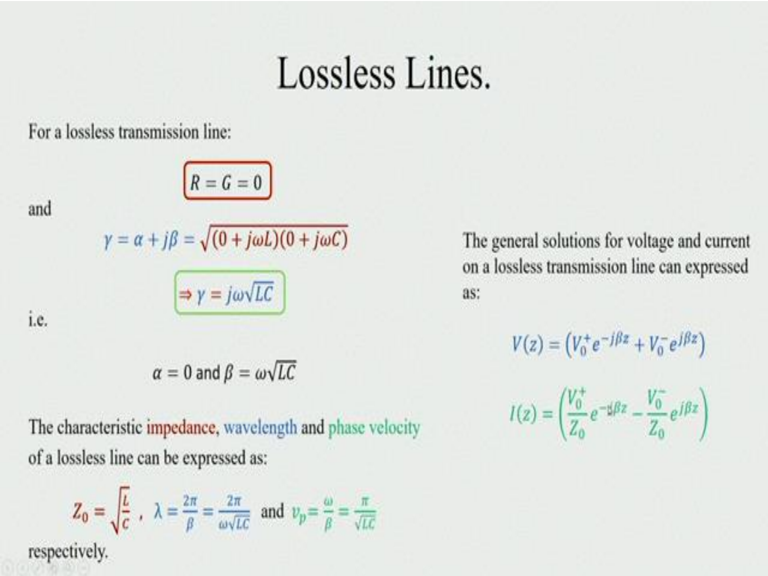

Now we consider a special case of a lossless line, for a lossless transmission line will have $R = 0$, $G = 0$ and if we put this values, then in the expression for the propagation constant we get gamma become $j\omega\sqrt{LC}$. Therefore, α, the attenuation constant becomes 0 and β, the phase constant becomes $\omega\sqrt{LC}$. The characteristic impedance wavelength and phase velocity of the lossless line can be expressed as Z_0 becomes $\sqrt{\frac{L}{C}}$ and now it is not a function of ω, λ is $\frac{2\pi}{\beta}$ which can be written as $\frac{2\pi}{\omega\sqrt{LC}}$ and phase velocity v_p, which is $\frac{\omega}{\beta}$ becomes $\frac{\pi}{\sqrt{LC}}$ Now for a lossless transmission line when $\alpha = 0$, the voltage and current phasors can be written as shown in these two equations.

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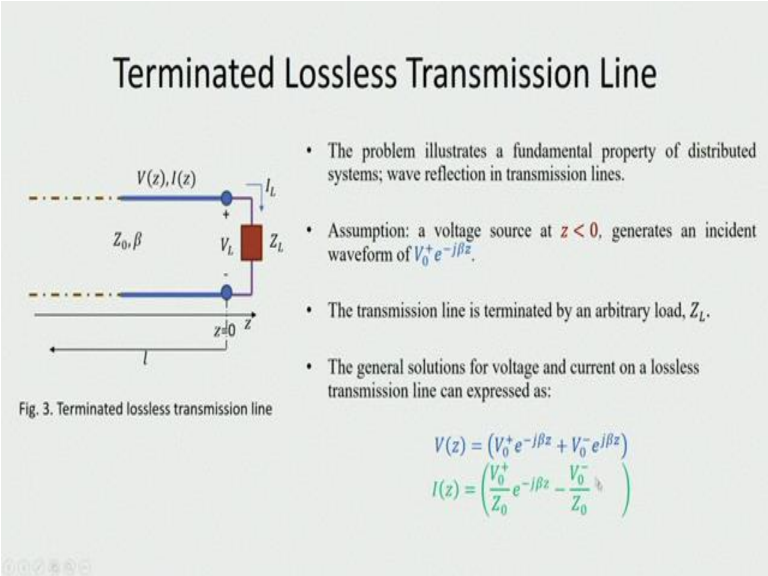

We consider a particular case of a terminated lossless transmission line, here we see that the lossless transmission line having characteristic impedance Z_0 and phase constant β is terminated to a load impedance Z_L, V_L represents the voltage across the load and I_L is the current flowing through the load, please note that the reference distance z is equal to 0 is considered to be at load location and then the distances are calculated from the load towards the source which excites the line.

Now this particular problem illustrates a fundamental property of distributed system, the wave reflection in the transmission. So we assume the voltage at z less than 0 is incident from a source and it sets up a waveform $V_0^+ e^{-j\beta z}$ propagating in the +Z direction. So this voltage when moves from this line, finally reaches the load and it sees a difference in the impedance and this results into a reflected wave, which travels back towards the source. So we have seen that the voltage on a transmission line and the current can be written in the forms as shown.

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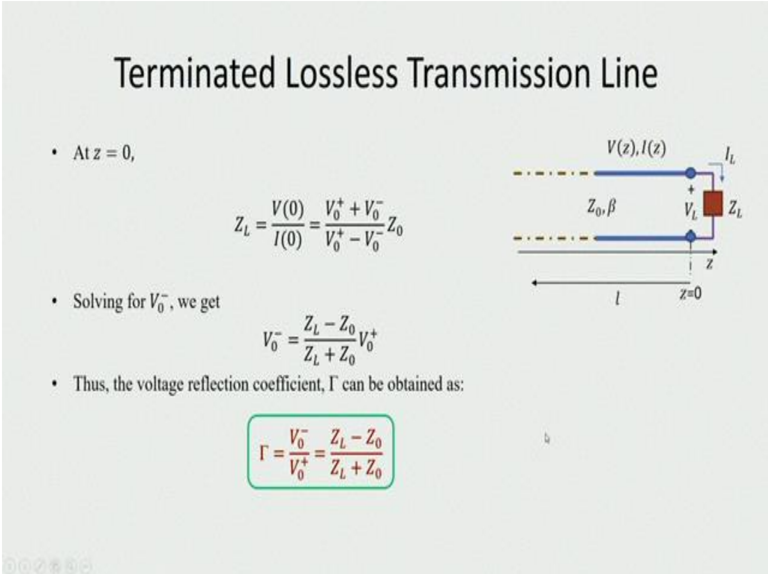

Now if you consider at z is equal to 0, then the load impedance Z_L becomes $\frac{V_0}{I_0}$ and substituting z is equal to 0 in the expression for the voltage and current, we get $\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} Z_0$. And from this equation if we solve for V_0^- then we get V_0^- is $\frac{Z_L - Z_0}{Z_L + Z_0} V_0^+$. Thus the voltage reflection coefficient, we defined as $\frac{V_0^-}{V_0^+}$, this is the reflected voltage at the load and this is the incident voltage at the load and their ratio of the reflected voltage to incident voltage is the reflection coefficient and we can find this gamma reflection coefficient to be $\frac{Z_L - Z_0}{Z_L + Z_0}$. So given the load and the characteristic impedance which and find out the reflection coefficient at the load.

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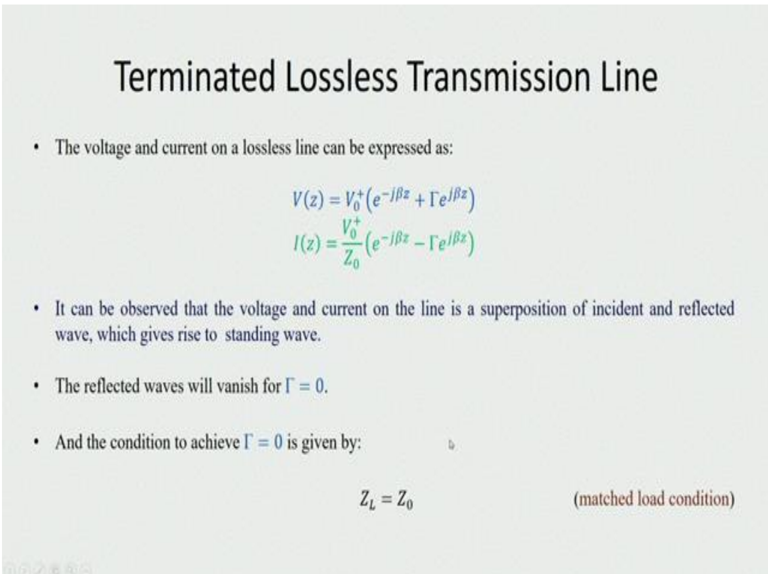

Now, once we introduced this reflection coefficient gamma, we can rewrite the expression for the voltage and current, where V_0^- is replaced by V_0^+ into Γ and V_0^+ is taken out. So we have the expression for the voltage and current as shown, now we can see that the voltage and current on the line is the superposition of incident and reflected wave and when both incident and reflected wave are present on the line it give rise to standing wave.

Please note that the reflected wave will vanish for Γ equal to 0, and the condition to achieve this is Z_L is equal to Z_0, then Γ will become $\frac{Z_L - Z_0}{Z_L + Z_0}$ equal to 0 and this particular condition, when the load impedance is same as that of the characteristic impedance of the line, this condition is called the matched condition.

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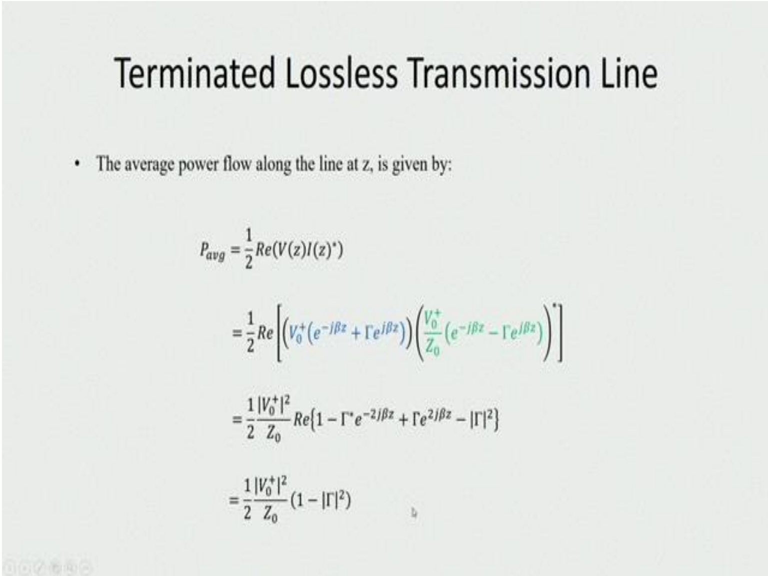

The average power flow along the line at any distance z is given by $P_{avg} = \frac{1}{2}Re(V(z)I(z)^*)$ and once you substitute the expression for the voltage and current phasors in terms of the forward and the incident and the reflected waves, we can write this equation, and then we can further simplify it and get a form which is shown here. Please note that in this expression the two middle terms when combined they give rise to an imaginary term and this term can be dropped and retaining only the real part we get average power to be equal to $\frac{1}{2}\frac{|V_0^+|^2}{Z_0}(1 - |\Gamma|^2)$.

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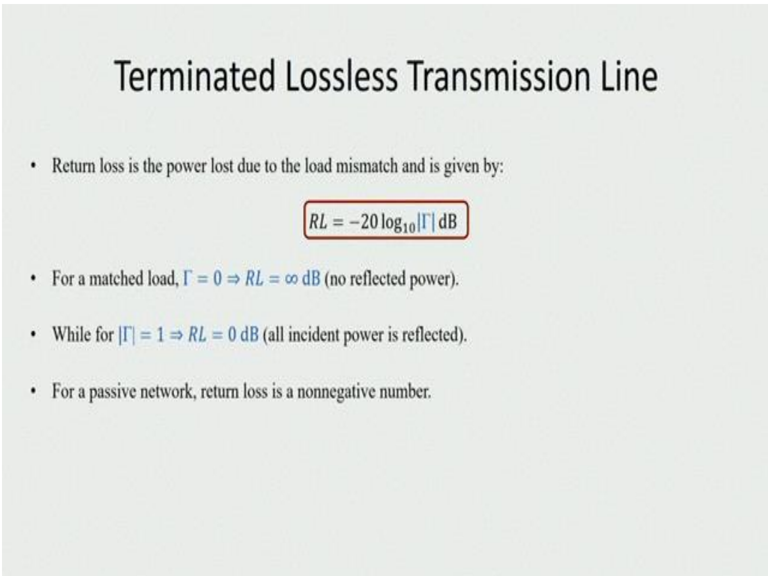

Return loss is the power lost due to the load mismatch and it is given by $RL = -20\log_{10}|\Gamma|\,\text{dB}$. For a matched load Γ is equal to 0 and RL is infinity, there is no reflected power. Similarly, when Γ is equal to 1, that means the entire power is reflected, written loss is 0 dB. For a passive network, the return loss is non-negative number.

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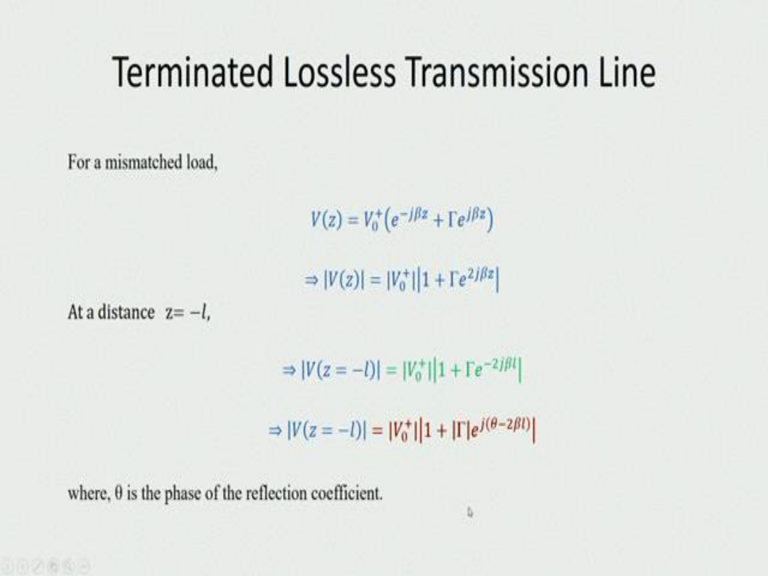

For a mismatched load, we can write $V(z)$ is equal to $V_0^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$ and if you take the magnitudes of voltage, then we can write magnitude of $V(z)$ is equal to $|V_0^+|\left|1 + \Gamma e^{2j\beta z}\right|$. Now if you consider at a distance z is equal to $-l$, then we can substitute this z by $-l$ and we get this expression.

And finally when gamma is replaced by its magnitude and phase term, we get magnitude of V at a distance z is equal to $-l$ to be $|V_0^+|\left|1 + \Gamma e^{-2j\beta l}\right|$, where theta is the phase angle of the reflection coefficient.

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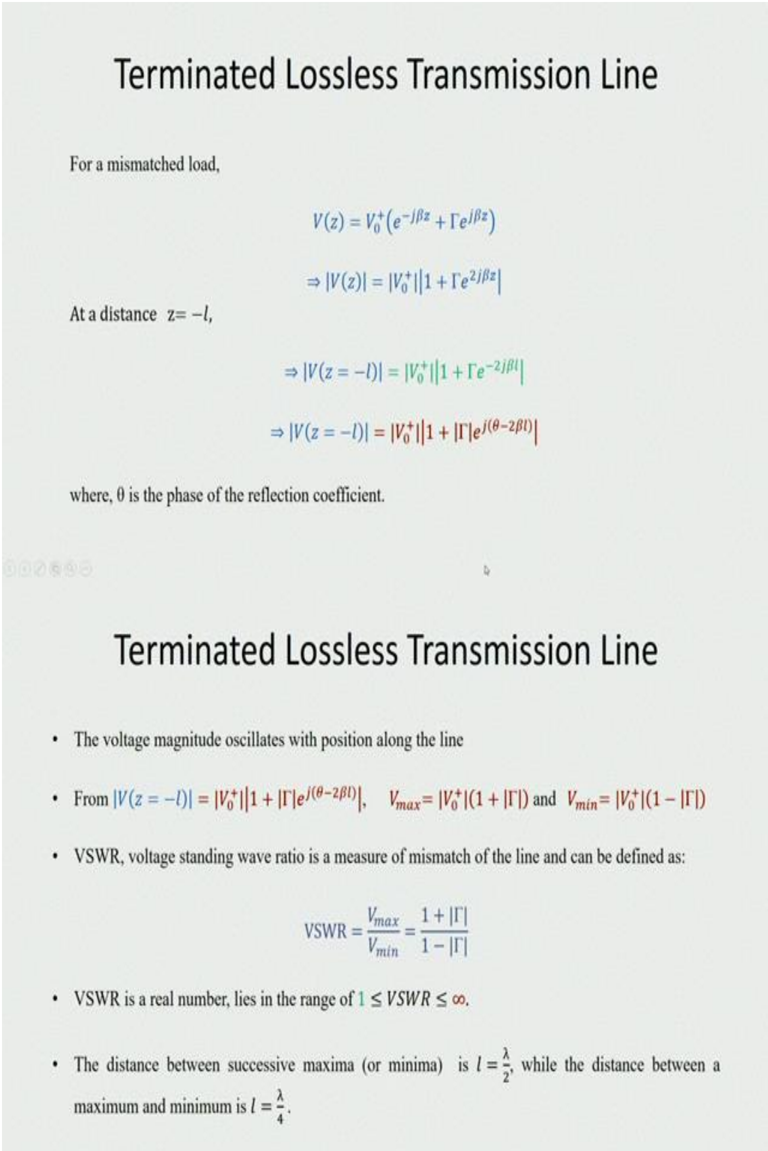

We can see that from the previous equation, the voltage magnitude it will vary with z and the voltage magnitude oscillates with position along the line and from this equation $|V(z = -l)|$ is equal to $|V_0^+||1 + |\Gamma|e^{j(\theta - 2\beta l)}|$. We find that when the phase term becomes 0, in that case, we get the maximum value of voltage and we denote it by V_{max} and $V_{max} = |V_0^+|(1 + |\Gamma|)$.

Similarly, when this term it becomes π, this phase term will become -1 and will get V_{min} is equal to $|V_0^+|(1 - |\Gamma|)$. So the voltage depending upon the position on the line we will keep changing between V_{max} and V_{min}, please note that the magnitude of the reflection coefficient becomes 1, when the load is a short-circuit or it is an open circuit, for both this cases the reflection coefficient magnitude will become 1, but the differences that when it is a short-circuit the voltage minima will be 0 because magnitude of reflection coefficient is 1 and it will be formed on the load location, whereas for an open circuited line a voltage maxima will be formed on the load location.

We introduced a term which is called voltage standing wave ratio and this is a measure of the mismatch of the line and we define VSWR is equal to V_{max} by V_{min} which is given by $\frac{1+|\Gamma|}{1-|\Gamma|}$. VSWR is a real number. It lies in the range 1 to infinity, VSWR is equal to 1 corresponds to perfect matching, there is no reflected wave that means gamma is 0 and VSWR tending to infinity gives perfect mismatch, that means all the power is reflected.

Please note that, a line terminated to a short-circuit or an open-circuit represents such conditions when VSWR becomes infinity because in both the cases mod gamma will become 1 and VSWR will approach infinity. So when we have voltage variation on the line between this V_{max} and V_{min}, the distance between the successive maxima or minima on the line is $\frac{\lambda}{2}$, while the distance between a maximum and minimum on the line is $\frac{\lambda}{4}$.

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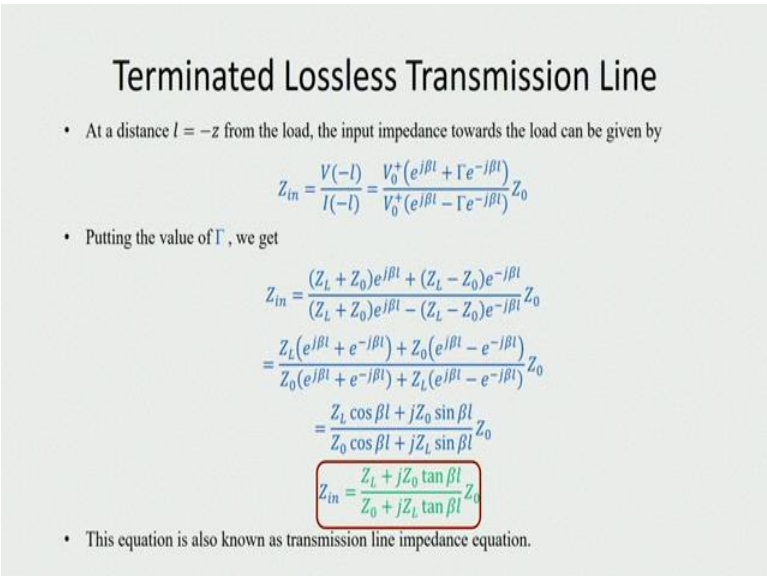

So at a distance is $l = -z$ from the load, the input impedance towards the load is given by if you substitute V with z is equal to $-l$ and then we take the ratio of voltage at z is equal to $-l$ and current and z is equal to $-l$, then we get this expression. Now we substitute the value of reflection coefficient gamma, then we can write Z_{in}, in this form and when this terms are arranged we get the final form of the equation as $Z_{in} = \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} Z_0$. So this is a very important equation and it gives the input impedance that will be seen at a distance l from the load looking towards the load and this is the transmission line impedance equation.

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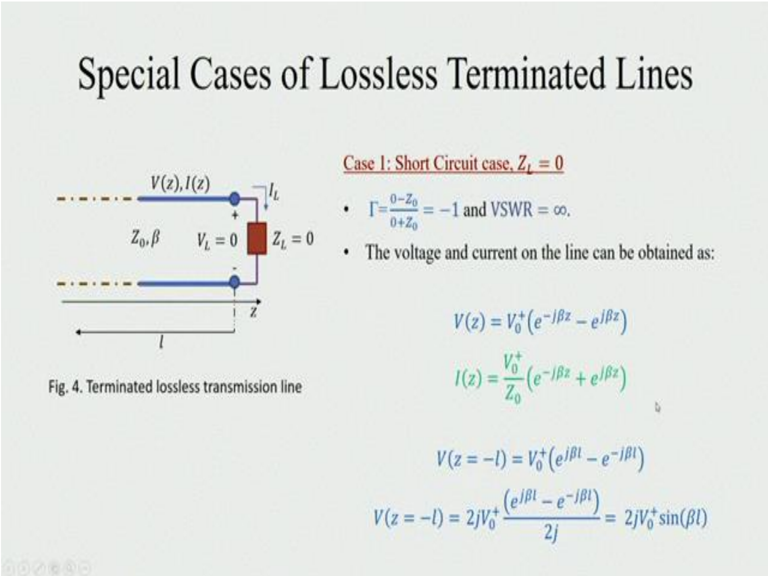

Let us now consider some special cases of terminated lines. As shown here we have Z_L set to 0, this particular case is the case of short-circuited transmission line and in this case, the reflection coefficient gamma will become minus 1, VSWR on the line is infinite. So we can obtain the voltage and current on the line as once we replaced the reflection coefficient gamma is equal to minus 1, the voltage expression becomes like this, similarly the current expression becomes as shown in the figure.

Now if you consider at a distance $-z$ is equal to l, so we can write $V(z = -l) = V_0^+\left(e^{j\beta l} - e^{-j\beta l}\right)$ and this term can be rearranged and finally we find that voltage as a function of distance l can be written as $2jV_0^+\sin(\beta l)$.

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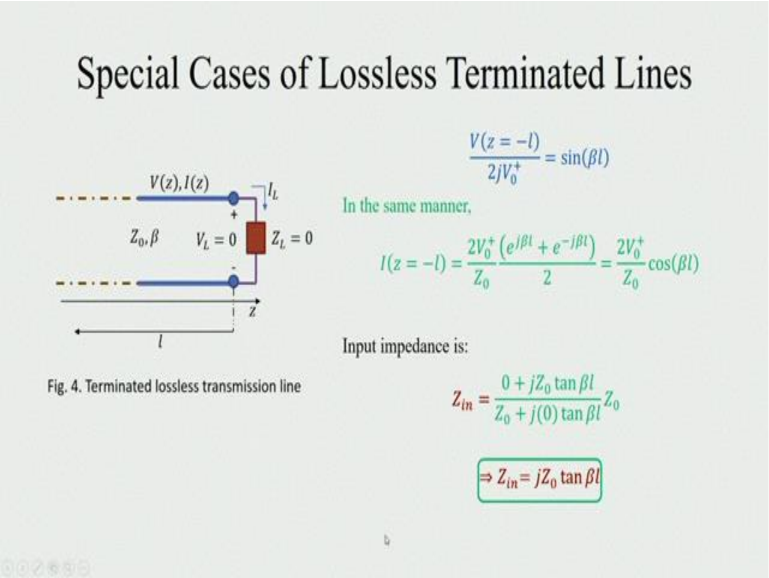

Therefore, the voltage V divided by $2jV_0^+$ is equal to $\sin(\beta l)$. In the same manner we can find out the expression for the current at a distance l from the load and this current at a distance l from the load can be written as $\frac{2V_0^+}{Z_0}\cos(\beta l)$. And therefore the input impedance seen at a distance l from the load can be written as Z_{in} is equal to $jZ_0 \tan\beta l$ and you can see that the same expression if you of the impudence if you get, if you divide voltage expression at a distance of l from the load divided by the corresponding current expression.

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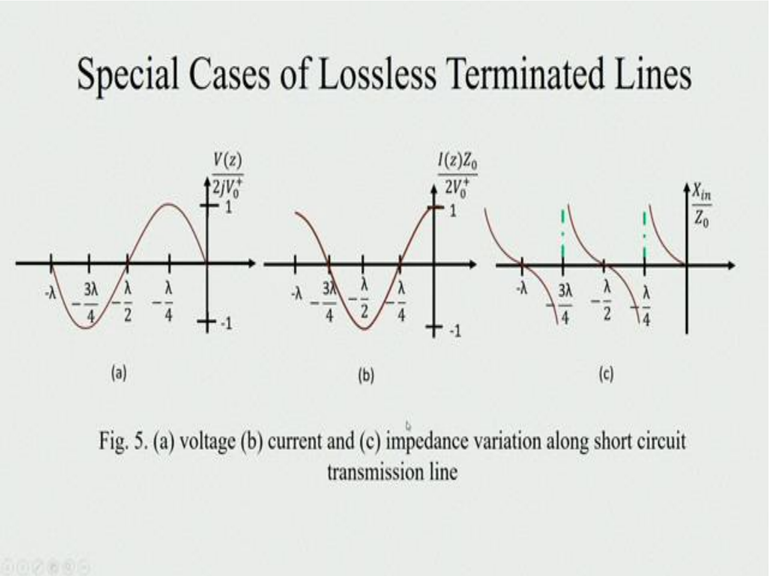

Now this voltage current and input impedance variation is plotted along the line for different distances from the load. So we find that the voltage variation gives a $\sin(\beta l)$ variation and therefore you can see that the voltage minima since it is a short-circuited line occurs at the locations of Z_L and then voltage reaches its maxima and then at a distance of $\frac{\lambda}{2}$, it becomes minimum again. Whereas if you look at the current expression, the current on the line at a distance l from the load varies as $\cos(\beta l)$.

So the short-circuit point, that means at the load location we have the maximum current and then it decreases to its minimum value at a distance of $\frac{\lambda}{4}$ from the load and it can be seen that when the voltage is maximum the current that is its minima. The impedance variation is given by of the form of $\tan(\beta l)$ and if we write input impedance is purely imaginary and if we write X in by Z_0, then we see that the variation it is initially inductive till the length is $\frac{\lambda}{4}$, that it becomes capacity, then it becomes inductive again. So this is how a short-circuit terminated line will behave in terms of voltage, current, and impedance variation.

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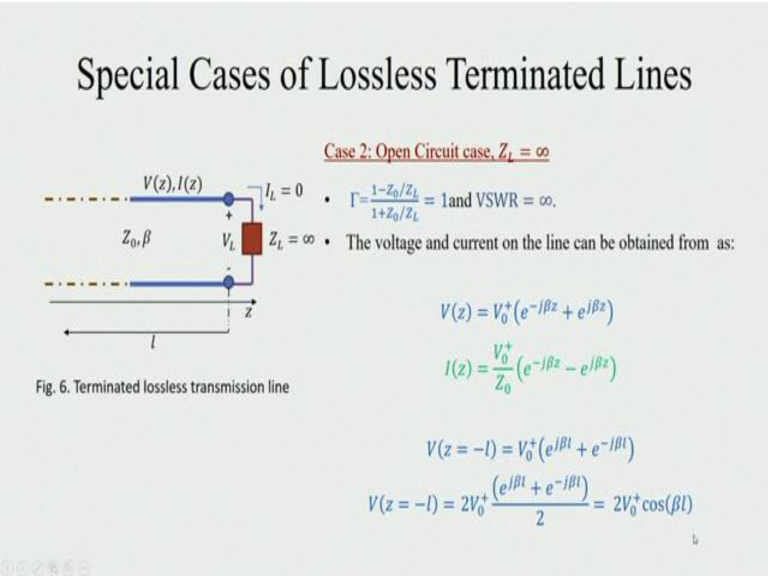

Let us discuss another case of terminated line. In that case we considered Z_L to be infinity, that means it is the case when the transmission line is transmitted to an open circuit. And in this case we can write the reflection coefficient at the load Γ to be equal to $\Gamma = \frac{1-Z_0/Z_L}{1+Z_0/Z_L}$ and when Z_L tends to ∞ in the limiting case the gamma will become 1, once again, the VSWR will become ∞.

The voltage and current on the line now we can obtain as, here we can see that if the Γ has been replaced by plus 1 here and rewriting this expressions for a distance L from the load, we can find that V at a distance of l from the load is $2V_0^+\cos(\beta l)$.

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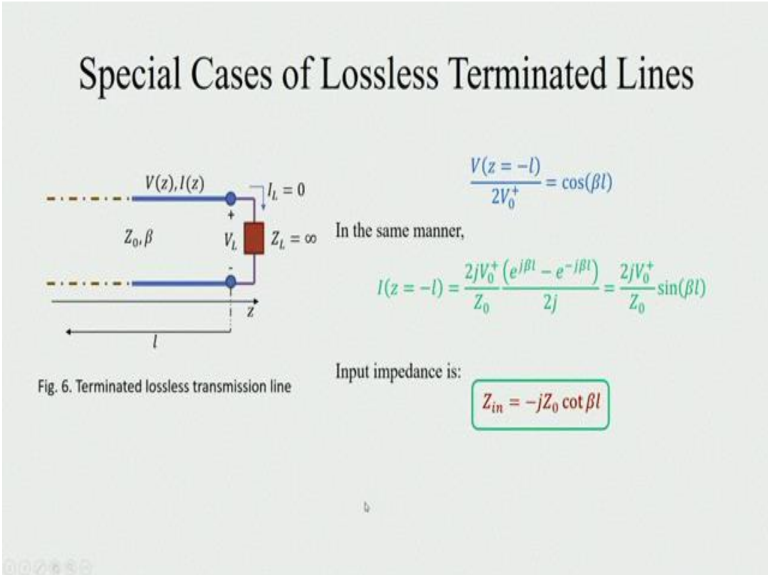

And similarly, if we find out the current, the current at a distance l from the load can be found to be $\frac{2jV_0^+}{Z_0}\sin(\beta l)$. And the input impedance once again, if you calculate V/I at a distance l from the load z in becomes $-jZ_0\cot\beta l$.

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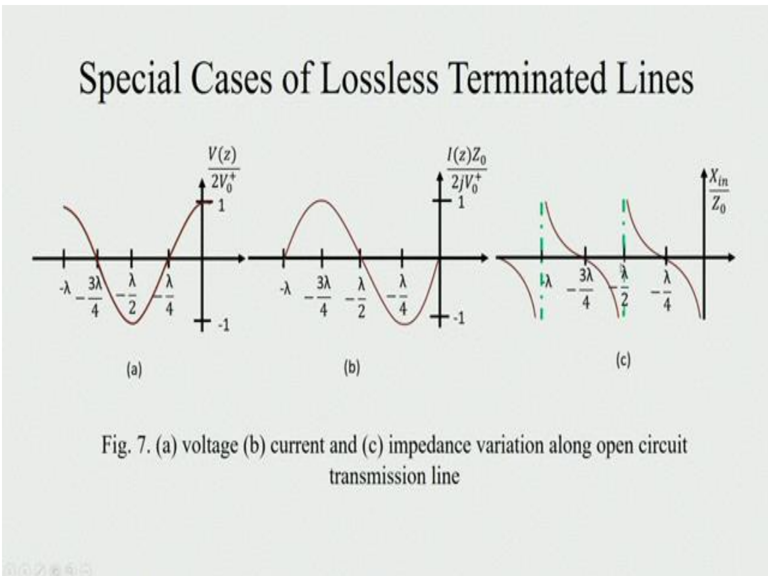

As before, we plot the voltage, current and impedance variation along the line when the line is terminated to an open circuit and we find that in this case, the voltage variation is given by $\cos(\beta l)$ and a voltage maxima occur at the load location, whereas a current minimum occurs at the load location because no current flows through the open circuit and the impedance becomes purely imaginary and X in by Z_0 as you can see for line length less than $\frac{\lambda}{4}$, it is negative. So it is capacitive and then it becomes inductive from $\frac{\lambda}{4}$ to $\frac{\lambda}{2}$ and this varies periodically over the length of the line.

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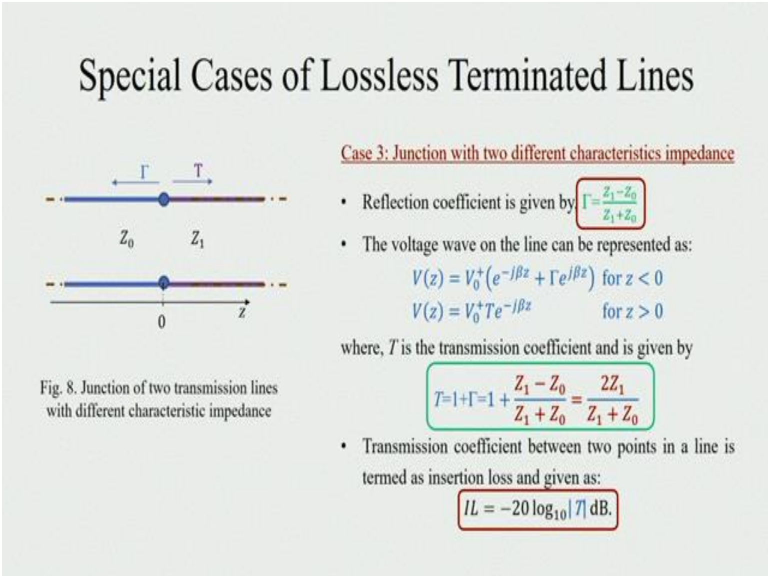

Let us now come to other special case where we consider a junction two transmission lines with different characteristic impedances. So we can see that the line shown in blue has a characteristic impedance Z_0 and it is connected to another line having a characteristic impedance Z_1. So the reflection coefficient at the junction if we compute looking from this line it will be given by $\frac{Z_1 - Z_0}{Z_1 + Z_0}$ and the voltage wave on the line now can be written as.

So will have the incidence voltage $V_0^+ e^{-j\beta z}$ and the reflected voltage $V_0^+ e^{j\beta z}$ which is travelling in the $-z$ direction and both this waves will exist in this part of the line for $z < 0$. When it comes to the wave propagation in the other part of the line, we find that the part of the voltage that is transmitted to the second line, that means for $z > 0$ can be written as $V(z)$ is equal to $V_0^+ e^{-j\beta z}$ into a transmission co-efficient T.

And if we consider the fact that the voltages at the two sides of these junction has to be same, that means at z is equal to 0, then we will get $1 + \Gamma$ will be equal to T and if we substitute the expression for gamma in that case, we get T is equal to $\frac{2Z_1}{Z_1+Z_0}$. Now it is also termed as insertion loss and it is given by insertion loss is equal to $-20\log_{10}|T|$ dB.

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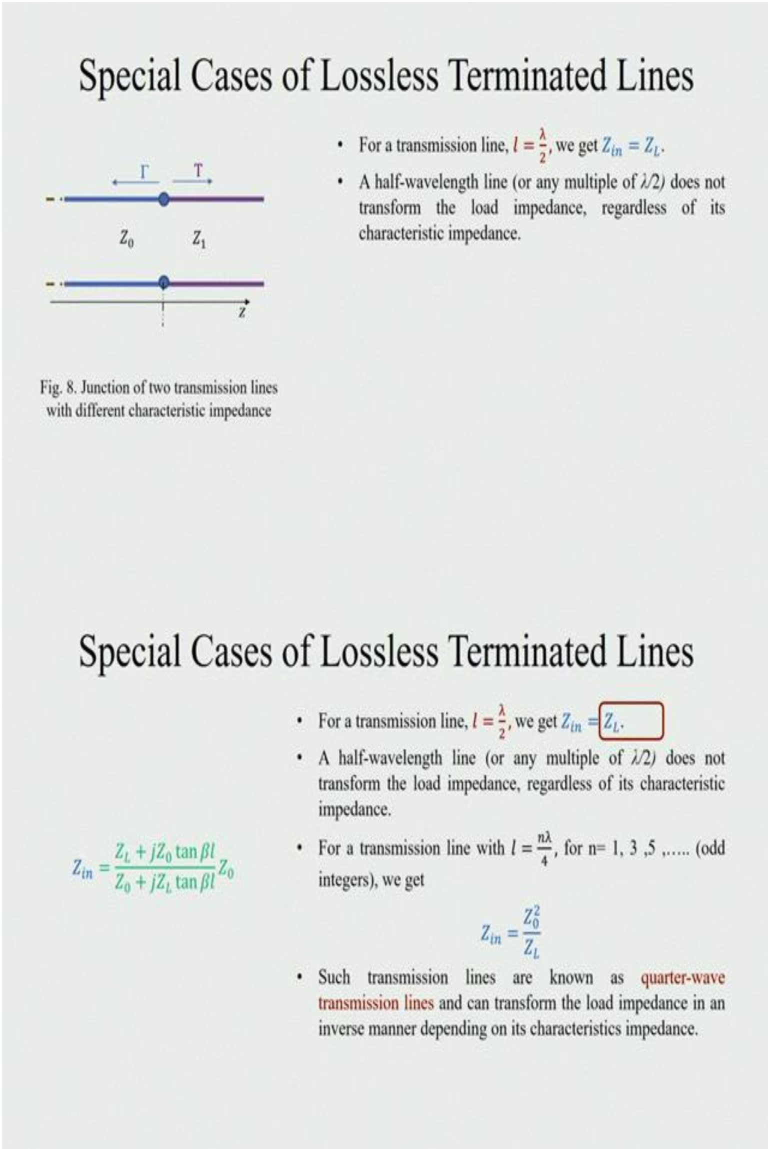

We continue with the junction of two transmission line with different characteristic impedances and we see that if we have a transmission line of length l is equal to $\frac{\lambda}{2}$, then Z_{in} becomes equal to Z_L. We have seen that the input impedance of a lossless line terminated to an impedance Z_L is given by $\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} Z_0$.

Now, depending upon the location where the input impedance is observed, we find that if l is equal to $\frac{\lambda}{2}$, then z in becomes equal to Z_L and a half wavelength or any multiple of $\frac{\lambda}{2}$ does not transform the load impedance, regardless of the characteristic impedance of the line. So this is a property that if we take a transmission line of length to $\frac{\lambda}{2}$, or multiples of $\frac{\lambda}{2}$, in that case, the input impedance seen is same as the load impedance.

We consider another case when the transmission line is an odd multiple of $\frac{\lambda}{4}$ in length. So in this case we get using this equation z in becomes equal to $\frac{Z_0^2}{Z_L}$. So for a transmission line, this type of transmission lines sections which are of quarter wavelength are known as quarter wave transmission line and can transfer the load impedance in an inverse manner depending upon the characteristic impedance value, the properties of quarter wave transformer will study in detail later.

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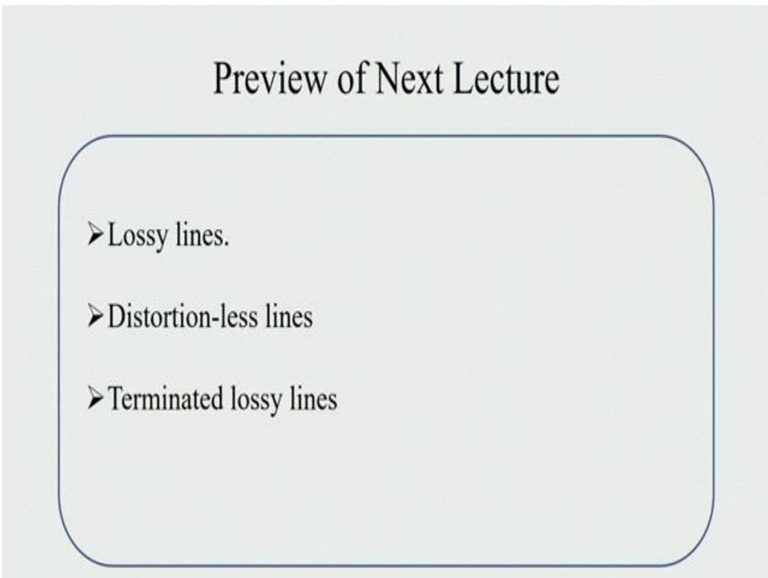

So we have discussed the various issues related to the wave propagation in a lossless transmission line. We have seen the special cases when the lossless line is terminated to a short-circuit or an open circuit. Similarly, we have seen the impedance transformation property of half wave transmission line, quarter wave transmission line. In the next lecture we will see transmission line having loss, in fact we will cover the lossy transmission line particularly when the loss is small. Then we will consider another special case which is called the distortion less line and finally we will conclude our discussion in that lecture by considering the terminated lossy lines.