Microwave Engineering
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Lecture 20 – Microwave Filters Part-2

We continue our discussion on the design of filters and in this lecture we discuss the design of prototype equal ripple or Chebyshev low pass filter. And then we will also discuss how the transformations are made from low pass to high pass, from low pass to bandpass and from low pass to bandstop. Let us now consider the Chebyshev low pass filters. So before we go into the design of the filters, prototype filters, let us first briefly discuss the Chebyshev polynomials.

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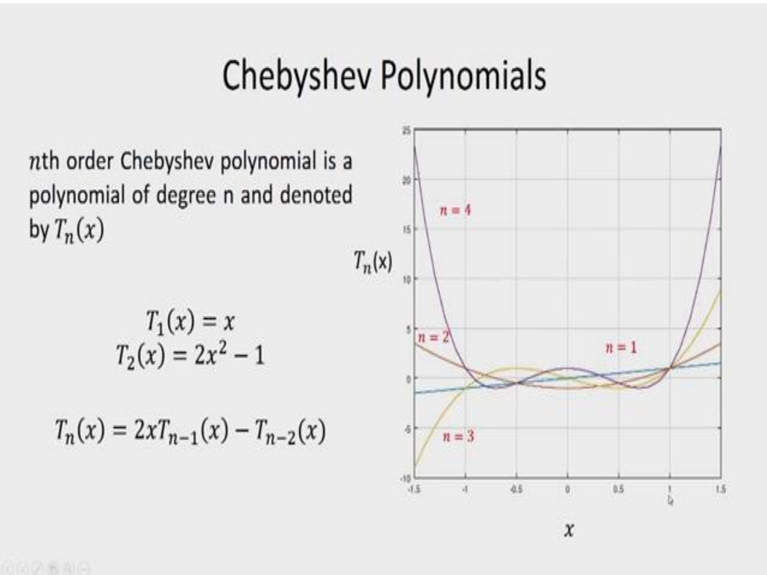
$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$



Nth order Chebyshev polynomial is a polynomial of degree n and denoted by Tn x. Here we have T_1 x is equal to x, T_2 x is equal to 2x square minus 1, and we can calculate Tn x from Tn minus 1x and T_n minus 2x, Tn x is equal to 2x Tn minus 1x minus T_n minus 2x. Now, the nature of variation of Chebyshev polynomials, some lower order Chebyshev polynomials are shown this figure. For example, for n is equal to 1 it is a straight line, for n equal to 2 we find that this is the variation, for n is equal to 3 we can see that there is some oscillation, this is for n equal to 4 and therefore we will find that the Chebyshev polynomial behavior is such that it will exhibit some oscillations in this region, x is equal minus 1 to x is equal to 1.

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Let the cutoff frequency be $\omega_c = 1$ rad/sec.

$$P_{LR} = 1 + k^2 T_N^2(\omega)$$

Since,

$$T_N(0) = \begin{cases} 0 & \text{for N odd} \\ \pm 1 & \text{for N even} \end{cases}$$

Therefore, at $\omega = 0$,

$$P_{LR} = \begin{cases} 1 & \text{for N odd} \\ 1 + k^2 & \text{for N even} \end{cases}$$

As

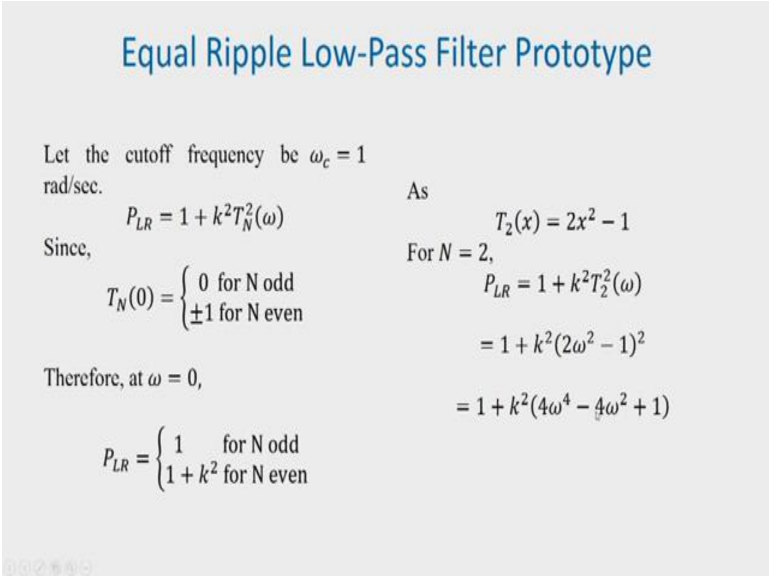
$$T_2(x) = 2x^2 - 1$$

For $N = 2$,

$$P_{LR} = 1 + k^2 T_2^2(\omega)$$

$$= 1 + k^2 (2\omega^2 - 1)^2$$

$$= 1 + k^2 (4\omega^4 - 4\omega^2 + 1)$$



And we can design an equal ripple low pass filter prototype using Chebyshev polynomials. Let us first consider the cut-off frequency omega c to be equal to 1 radian per second, and PLR is given by, PLR is equal to 1 plus k square Tn square omega. Now since Tn 0 is 0 for N odd and it is having a value of plus or minus 1 for N even, therefore at omega equal to 0 we can write, PLR is equal to 1 for N odd and 1 plus k square for N even.

We have seen that T_2 x is given by 2x square minus 1. Now, if we consider N equal to 2, then PLR will become 1 plus k square T_2 square omega, and if we substitute T_2 omega it will be 1 plus k square 2 omega square minus 1 whole square. When expanded this will become 1 plus k square 4 omega to the power 4 minus 4 omega square plus 1.

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For the two element network,

we have seen

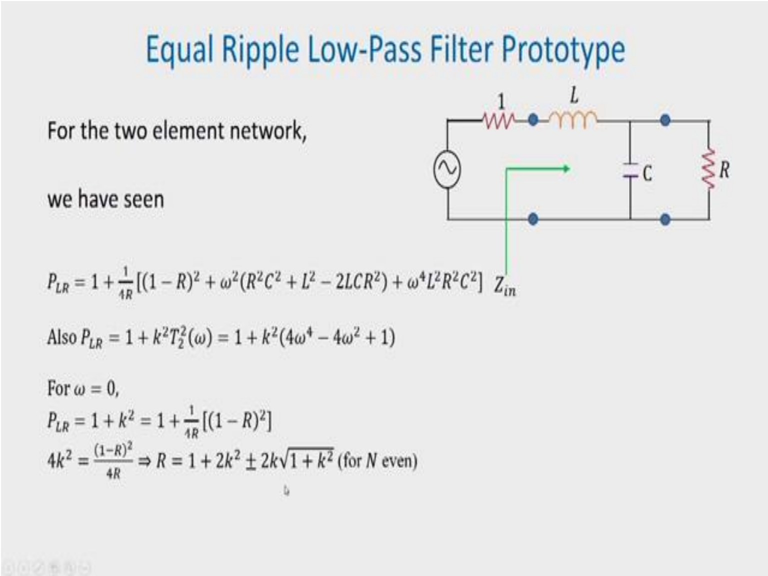
$P_{LR} = 1 + \frac{1}{4R}[(1-R)^2 + \omega^2(R^2C^2 + L^2 - 2LCR^2) + \omega^4 L^2 R^2 C^2]$

Also $P_{LR} = 1 + k^2 T_2^2(\omega) = 1 + k^2(4\omega^4 - 4\omega^2 + 1)$

For $\omega = 0$,

$P_{LR} = 1 + k^2 = 1 + \frac{1}{4R}[(1-R)^2]$

$4k^2 = \frac{(1-R)^2}{4R} \Rightarrow R = 1 + 2k^2 \pm 2k\sqrt{1+k^2}$ (for N even)



Now, when we have a prototype network containing these two elements L and C, for this two-element- network, we have already seen that PLR expression can be written out in this form and from the Chebyshev polynomial the PLR expression is given by this. So, if we equate these two equations for omega equal to 0, then we get PLR is equal to 1 plus k square, and from here we get 1 plus 1 by 4R 1 minus R whole square.

And therefore 4k square becomes 1 minus R divided by 4R, and therefore, R can be solved as R is equal to 1 plus 2k square plus minus 2k root 1 plus k square for even N, and here we have N equal to 2.

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Equating the two expressions for P_{LR}

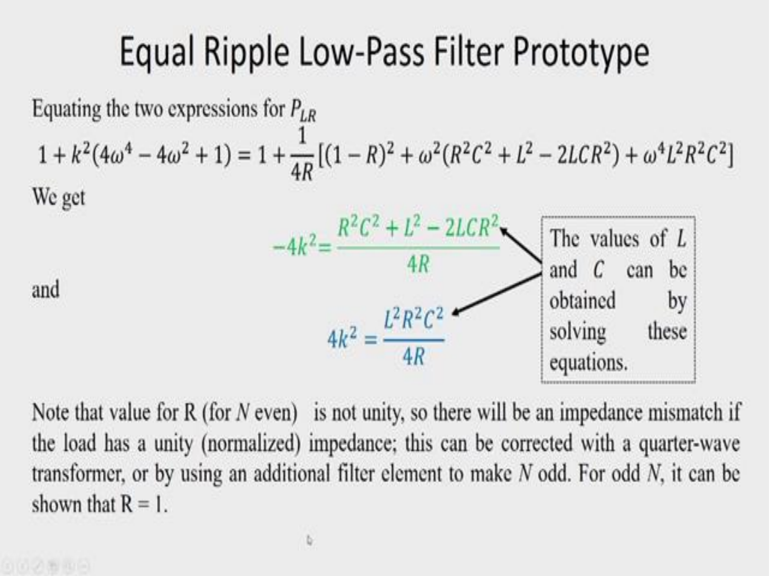
$$1 + k^2(4\omega^4 - 4\omega^2 + 1) = 1 + \frac{1}{4R}[(1-R)^2 + \omega^2(R^2C^2 + L^2 - 2LCR^2) + \omega^4 L^2R^2C^2]$$

We get

$$-4k^2 = \frac{R^2C^2 + L^2 - 2LCR^2}{4R}$$

and

$$4k^2 = \frac{L^2R^2C^2}{4R}$$



Equating the two expressions for PLR, we get this equation. The left-hand side is from the Chebyshev polynomial, and right-hand side comes by reducing the expression for PLR for the circuit shown in the previous slide. Now what we can do? We can equate the coefficients of omega square and omega to the power 4 from both sides. In that case we get two equations; one is minus 4k square is equal to R square C square plus L square minus 2LC R square divided by 4R. So this is one equation we get.

And another equation we get by equating the coefficients of omega to the power 4, so get 4k square equal to L square R square C square divided by 4R. Now, R-value has already been calculated. We have already found out the expression for R, and therefore, we can solve for L and C from this set of equations. We should note that the value of R for N even is not unity, so there will be an impedance mismatch if the load has unity impedance. This can be corrected with a quarter wavelength transformer or by using an additional filter element to make N odd. When N is odd, one can show that R is equal to 1.

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Element Values for Equi-ripple Low-Pass Filter Prototypes

$g_0 = 1,\ \omega_c = 1$, 0.5 dB ripple

N	g_1	g_2	g_3	g_4	g_5	g_6
1	0.6986	1.0000				
2	1.4029	0.7071	1.9841			
3	1.5963	1.0967	1.5963	1.0000		
4	1.6703	1.1926	2.3661	0.8491	1.9841	
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000

For practical filter, it will be necessary to determine the order of the filter. This is usually dictated by a specification on the insertion loss at some frequency in the stop-band of the filter.

So, from these considerations, we can get the element values for the equal ripple low-pass filter prototype. Prototypes we have already seen earlier, and we considered g not equal to 1, omega c equal to 1. Now here we also need to specify the passband ripple, so this table is shown for 0.5 dB passband ripple. And we have seen from the behavior of the Chebyshev polynomial that Chebyshev functions they exhibit ripple in the passband, and this calculation can be done for a specified value of the ripple.

Now, we note that when N is equal to 1, gN plus 1 is 1, when N is equal to 3 we have gN plus 1 is again 1, so for all odd N_s the load resistance normalized load becomes unity whereas for even values of N the normalized load is not unity. So, once we design the prototype by using the values of g_1, g_2, g_3 and we can map them to the prototype circuits which we discussed earlier and then we can carry out the impedance scaling as well as frequency scaling to get the actual cut-off frequency and also the source impedance may be changed to the desired value of the source impedance.

So, all this impedance and frequency scaling activities are to be performed to get a low pass filter with a desired cut-off frequency. The order of the filter once again will be determined by the specified value of the insertion loss at some frequency in the stopband of the filter.

As we said that in the filter design process we first need to find out the prototype filter and once the prototype filter is designed, prototype low pass filter is designed it can be transformed to the low pass filter with the actual cut-off frequency or we can transform this low pass filter into a high pass filter with desired cut-off frequency or to bandpass filter with desired passband or a bandstop filter with the specified stopband.

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Loss pass to high pass transformation is achieved by the frequency substitution:

$\frac{-\omega_c}{\omega}$ for ω

When this transformation is applied

$j\omega L_k$ becomes $-j\frac{\omega_c}{\omega}L_k = \frac{1}{j\omega C_k'}$, where $C_k' = \frac{1}{\omega_c L_k}$

and

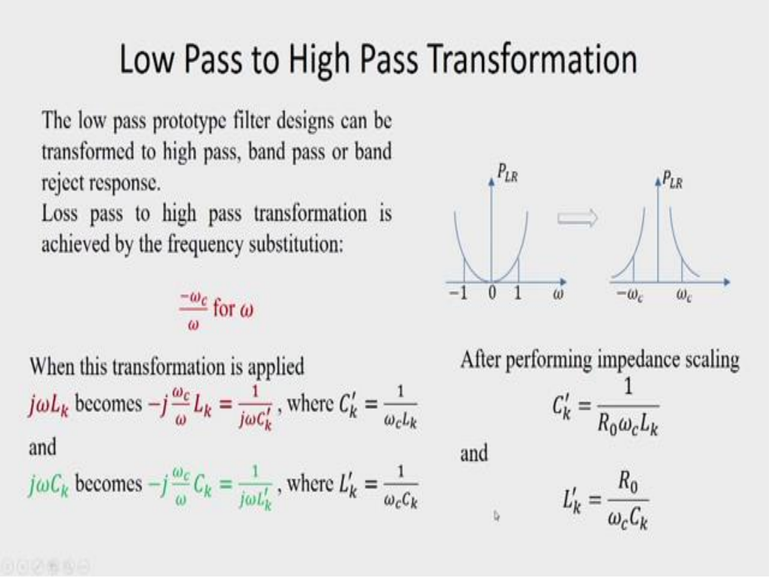
$j\omega C_k$ becomes $-j\frac{\omega_c}{\omega}C_k = \frac{1}{j\omega L_k'}$, where $L_k' = \frac{1}{\omega_c C_k}$

After performing impedance scaling

$$C_k' = \frac{1}{R_0\omega_c L_k}$$

and

$$L_k' = \frac{R_0}{\omega_c C_k}$$



So, let us see how this transformations work. The low pass prototype filter designs can be transformed to high pass, band pass or band reject response. Low pass to high pass transformation is achieved by frequency substitution. So, we have…this is the response of the prototype filter, we can see that the cut-off frequency is 1 and the PLR increases to the specified value at omega c equal to 1.

Now, this PLR where it has minimal cut at insertion loss at 0 is transformed to a high pass characteristic. So, here we see that insertion loss becomes very high below omega c and above omega c the insertion loss decreases, and this type of transformation can be achieved by substituting omega by minus omega c by omega. And when this transformation is applied, a series inductance j omega L_k becomes minus j omega c by L_k and which essentially becomes 1 by j omega C_k dash, where this C_k dash is 1 by omega C L_k.

And similarly, j omega C_k or L_k dash is j omega C_k is transformed to 1 by j omega L_k dash, and therefore, this L k dash becomes 1 by omega C_k. So, we can find that with this frequency substitution we have already taken omega c into account, the cut-off frequency will be shifted to omega c and then next we need to do the impedance scaling because the source impedance is still unity, and then this C_k dash becomes 1 by R not omega c L_k and L_k dash becomes R not by omega c C_k. So, this gives us the values of the L and C elements in the actual filter which will have a source impedance of R not and cut-off frequency omega c.

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For LPF to BPF

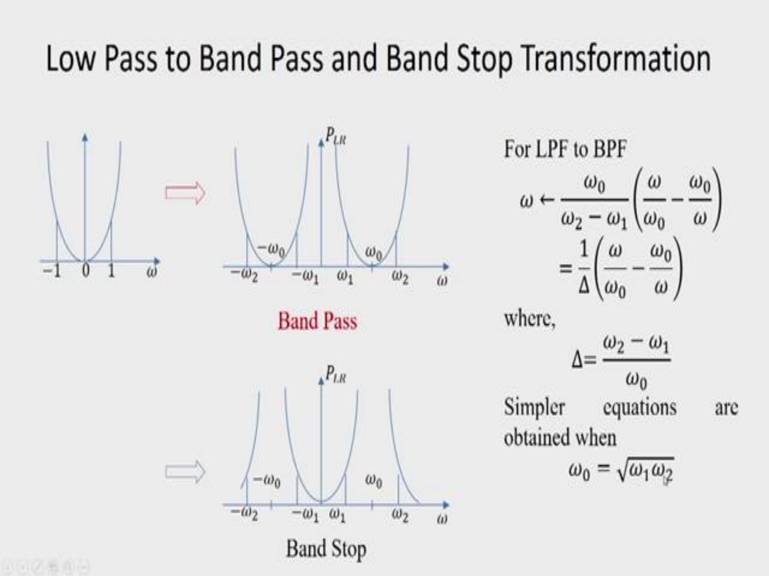
$$\omega \leftarrow \frac{\omega_0}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$
$$= \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

where,

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$$

Simpler equations are obtained when

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$



Let us now consider the low pass to bandpass and bandstop transformation. So the first transformation is a bandpass transformation where this low pass filter characteristic are mapped to a filter having a center frequency omega and passband edges having frequencies omega 1 and omega 2.

Similarly, if you go for bandstop transformation, we get a filter where the center frequency of the rejected band is omega naught, and the attenuation remains high for the bandages determined by omega 1 and omega 2. For performing low pass to bandpass transformation we need to substitute omega by omega naught divided by omega 2 minus omega 1 into omega by omega naught minus omega naught by omega. And this we can write 1 by delta omega by omega naught minus omega naught by omega where we have delta equal to omega 2 minus omega 1 by omega. And we can get simpler equation if omega naught is chosen as a geometric mean of omega 1 and omega 2. Therefore omega naught is root omega 1 omega 2.

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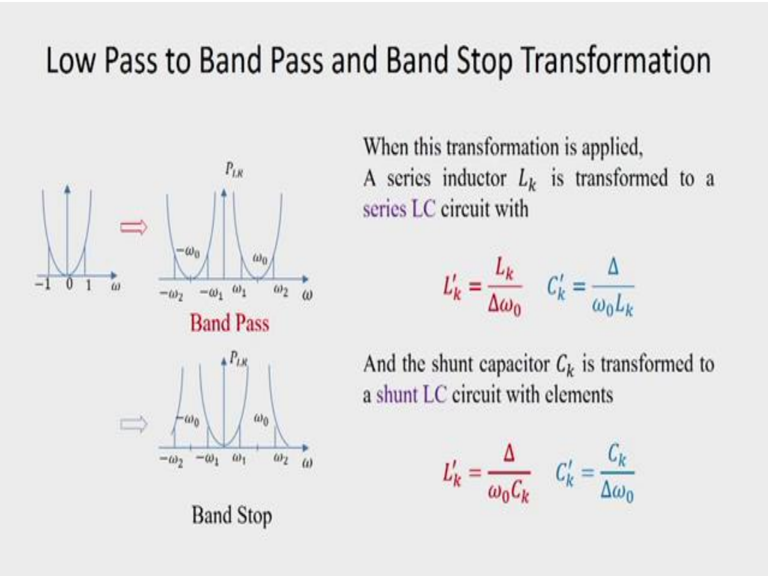
When this transformation is applied,

A series inductor L_k is transformed to a series LC circuit with

$$L_k' = \frac{L_k}{\Delta\omega_0} \quad C_k' = \frac{\Delta}{\omega_0 L_k}$$

And the shunt capacitor C_k is transformed to a shunt LC circuit with elements

$$L_k' = \frac{\Delta}{\omega_0 C_k} \quad C_k' = \frac{C_k}{\Delta\omega_0}$$



Similarly when we perform band stop transformation, so when this bandpass transformation is applied a series inductor L_k is transformed to a series LC circuit and the values of these L and C parameters are given by this, L_k dash is L_k by delta omega, and C_k dash becomes delta by omega naught L_k and a shunt capacitor C_k is transformed to a shunt LC circuit. And the element values are given by these expressions.

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For LPF to Band Stop

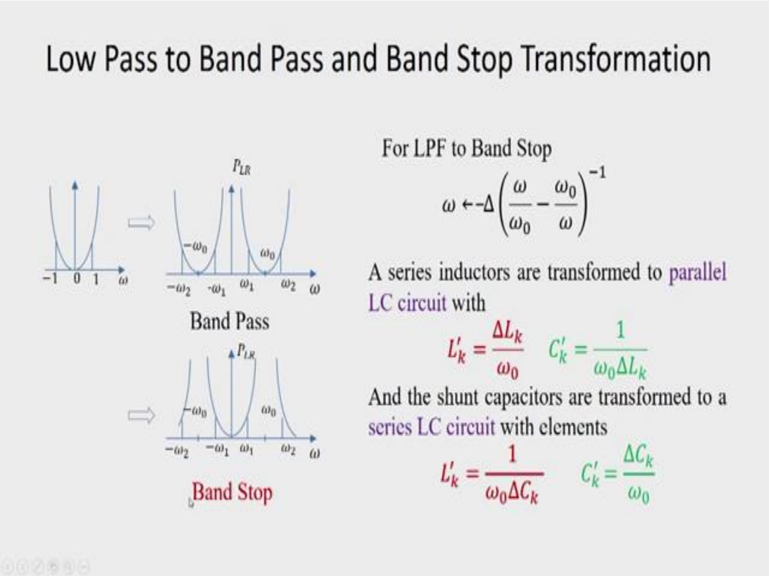
$$\omega \leftarrow \Delta \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^{-1}$$

A series inductors are transformed to parallel LC circuit with

$$L_k' = \frac{\Delta L_k}{\omega_0} \quad C_k' = \frac{1}{\omega_0 \Delta L_k}$$

And the shunt capacitors are transformed to a series LC circuit with elements

$$L'_k = \frac{1}{\omega_0 \Delta C_k} \qquad C'_k = \frac{\Delta C_k}{\omega_0}$$



When we go for a band stop transformation, so for a low pass to band stop transformation, we substitute omega by delta omega by omega naught minus omega naught by omega raised to the power minus 1. So, essentially it is the reciprocal of the bandpass transformation and when this frequency transformation is applied, a series inductor is transformed into a parallel LC circuit, and we can find out the values of this parallel LC circuit L and C from the given L_k delta and omega naught.

Similarly, we can find out C_k dash is equal to 1 by omega naught delta L_k. And also the shunt capacitors are transformed into series LC circuit, and the values of these series LC circuit elements L and C are determined from the original shunt capacitor C_k and the relation are as shown.

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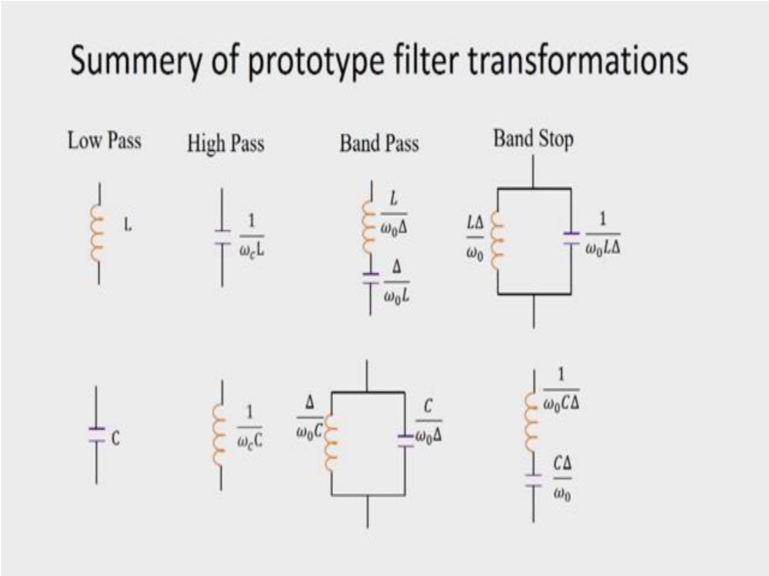

So in summary if we summarize the transformations, we have in the low pass circuit L and C, when we make low pass to high pass transformation this L is transformed to C and C is transformed to L. When we go for bandpass transformation, this element L is now transformed to a series LC circuit, and the C is transformed to parallel LC circuit and when we go for bandstop transformation this L is transformed to a parallel LC circuit whereas C is transformed to a series LC circuit.

You can see that the values of these series and parallel circuit elements can be found out in terms of the original circuit element. For example, here these L and C both are related to original L and Delta we have defined it to be omega 2 minus omega 1 divided by omega naught and omega naught we have defined root omega 2 into omega 1. So we can calculate the component values of these transformed circuits, and therefore when in the prototype we will have to substitute them by respective combination of elements.

We have already done the frequency transformation, the impedance scaling also needs to be done for transforming the source impedance. Impedance scaling also need to be done to complete the procedure.

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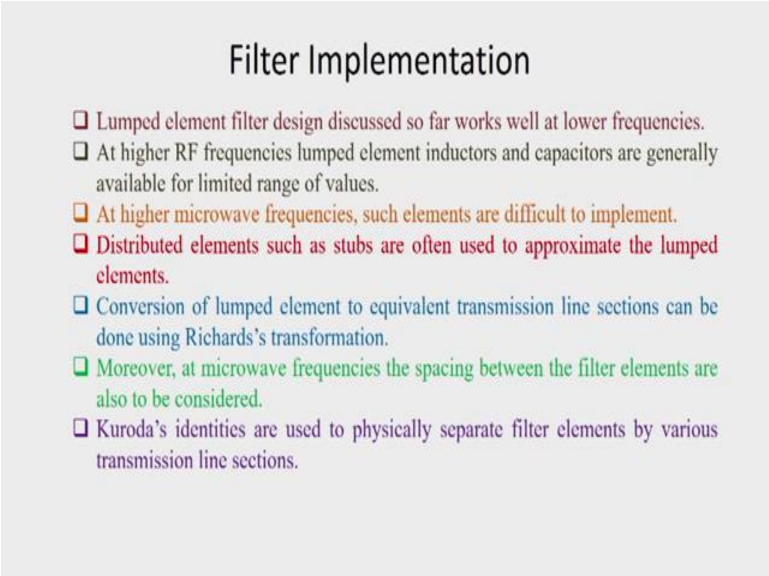

Let us now very briefly discuss some of the issues related to the filter implementation. Because so far in our discussion, we have considered lumped elements L and C, now these elements as lumped elements circuits can be used, so these elements as lumped components can be used only at the lower microwave frequencies. As we go to higher microwave frequencies it becomes very difficult to realize these capacitors and inductors in the lumped form, and often these are realized in the form of transmission line sections.

So lumped element design discussed so far works well at lower frequencies. At higher RF frequencies, lumped element inductors and capacitors are generally available for a limited range of values. At higher microwave frequencies, such elements are difficult to implement. So we need to use distributed elements such as stubs, which often used to approximate the lumped elements. And conversion of lumped element to equivalent transmission line sections can be done using Richard's transformation.

Moreover, when we connect these elements in a microwave circuit, the spacing between the filter elements is also to be considered at the microwave frequencies. And Kuroda's identities are used to physically separate filter elements by various transmission line sections. So, without affecting the response we can have these elements connected together by suitable transmission line sections.

In this module, we have discussed the various power dividers. We have also seen the design issues involved. We have also seen the different issues involved in the design of microwave frequency filters. We have discussed to somewhat detail the insertion loss method of filter design and how we can start with a prototype low pass filter design and later on we can transform it into a low pass filter of desired cut-off frequency or to a high pass or to a bandpass or to a band stop filter.

In the next module, we will start with the semiconductor devices. We will discuss the microwave transistors, the BJTs, and field-effect transistors. We will also discuss some of the important microwave frequency diodes, such as Schottky diodes, PIN diodes, and their application. We will also discuss the devices based on transferred electron effect, the gun devices.