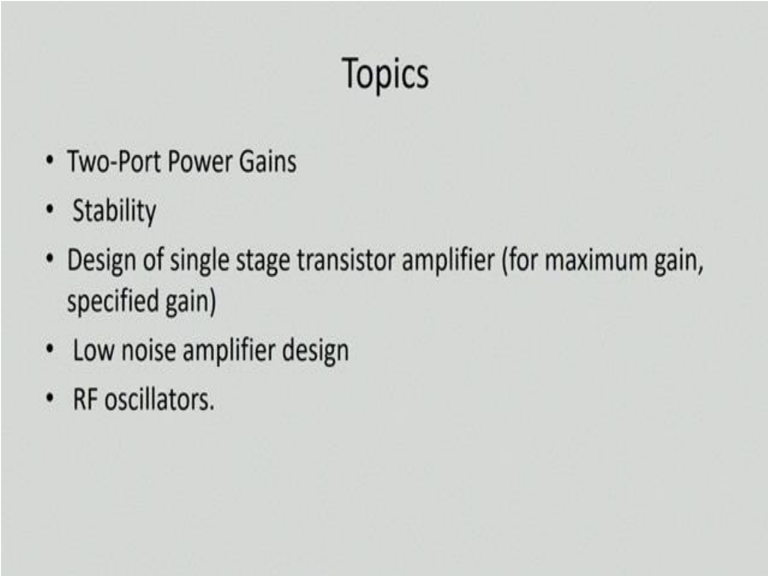
Microwave Engineering
Professor Ratnajit Bhattacharjee
Department of Electronics & Electrical Engineering
Indian Institute of Technology Guwahati
Lecture No. 25
Microwave Amplifiers and Oscillators

(Refer Slide Time: 0:30)



We start a new module microwave amplifiers and oscillators. In this module, we are going to cover the following topics, will have a discussion on two-port power gains as these amplifiers are essentially two-port circuits. So, we will have a discussion on two-port power gains. We will discuss the stability and then will discuss the design of single-stage transistor amplifier and will see the designs for the cases of maximum gain and also for some specified again. We will see some basics of low noise amplifier design, and finally we will discuss some basics of RF oscillators.

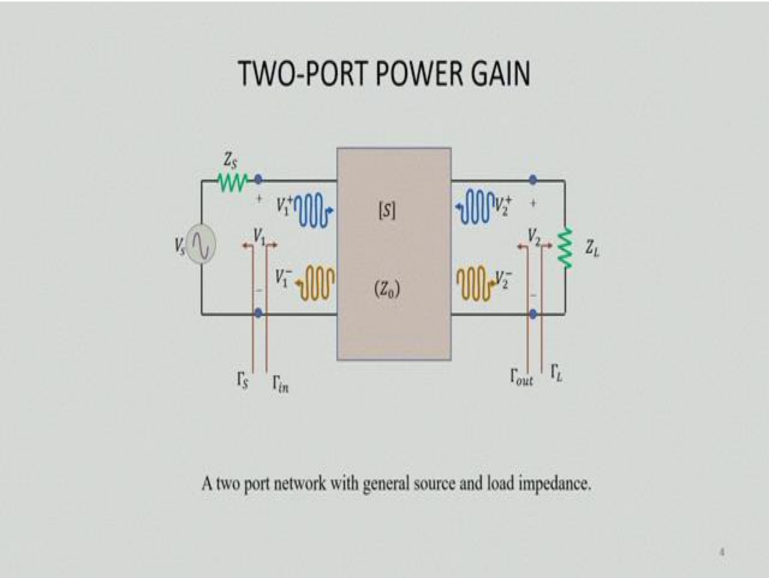
In our previous module, we briefly discussed the basics of microwave frequency transistors, both bipolar junction transistor as well as field-effect transistor or FET. We have seen that there are wide variety of same conductor devices, such as silicon BJT, gallium arsenide or silicon-germanium HBTs, silicon MOSFETs, gallium arsenide MESFETs, and other gallium arsenide or gallium nitrate-based HEMTs.

So, these are the variety of transistors which are available and can be used in the design of microwave amplifier and oscillators. Here, in this module, our discussion on the design of amplifiers and oscillators will remain limited to the terminal characteristics of the transistors.

That means will represent the transistors by their S parameters and then carry out the design, or in some cases we can also represent the transistors by their equivalent circuit model.

So, our discussion starts with some general discussion of two-port power gain, which is useful for amplifier design and then will discuss the stability criterions for such networks. And whatever results will drive and discussing the two-port networks, this will be applied two design of single-stage transistor amplifiers, including design for maximum gain, design for specified again, and also low noise figure design.

(Refer Slide Time: 4:24)

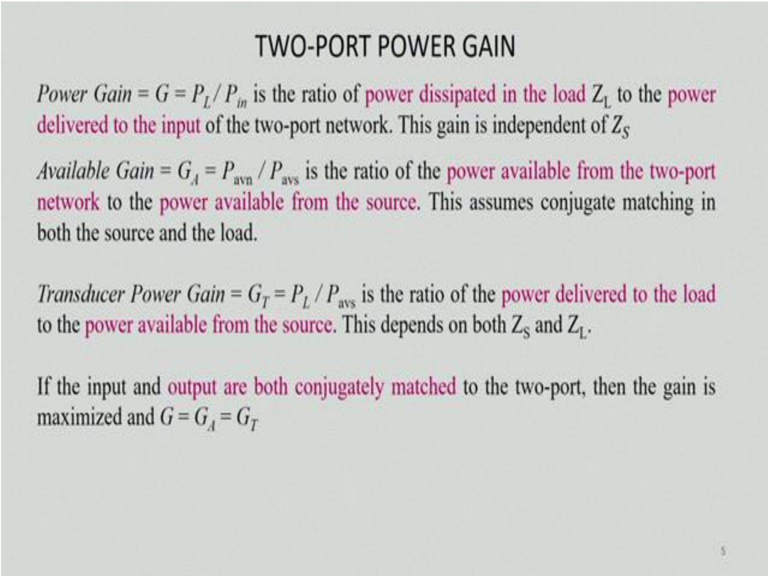


So, we start our discussion with the basics of a two-port network, and we consider the two-port power gain. This figure shows the two-port network where we have these two-port represented by its S parameter, and Z naught is the normalizing impedance of this network. We have a voltage source V_S with a source impedance Z_S, feeding the network. And also the two-port is terminated to a load impedance Z_L.

Here V_1 is the voltage in the input port, which essentially is a sum of incident voltage V_1 plus and be reflected voltage V_1 minus. Similarly, we have voltage V_2 at the output port, and V_2 plus is the voltage that is incident on port 2; V_2 minus is the voltage because of the signal going out of this port. If we look towards the source, we get a reflection coefficient gamma S.

Similarly, if we look towards the input of this two-port with the load connected, then we get an input reflection coefficient gamma in. Gamma out is the reflection coefficient when we look from the output port of the network, and gamma L is the load reflection coefficient.

(Refer Slide Time: 6:50)



So, for this type of two-port network, we can define different gains. The power gain G of such network is defined as the ratio of the load power and input power. So G is given by P_L by P_{in}. Now this gain is independent of Z_S. It is defined in terms of P_L and P_{in}. We define another gain, which is called available gain. Now G_A is defined as the ratio of power available at the output of the network and power that is available from the source.

So, when we talk of power available from the source. That means we talk of matching, conjugate matching at the source. Similarly, when we talk of available power from the network, we are referring to the matching at the load end, and therefore, this available power gain G_A, it assumes that source and load end, both are matched conjugately.

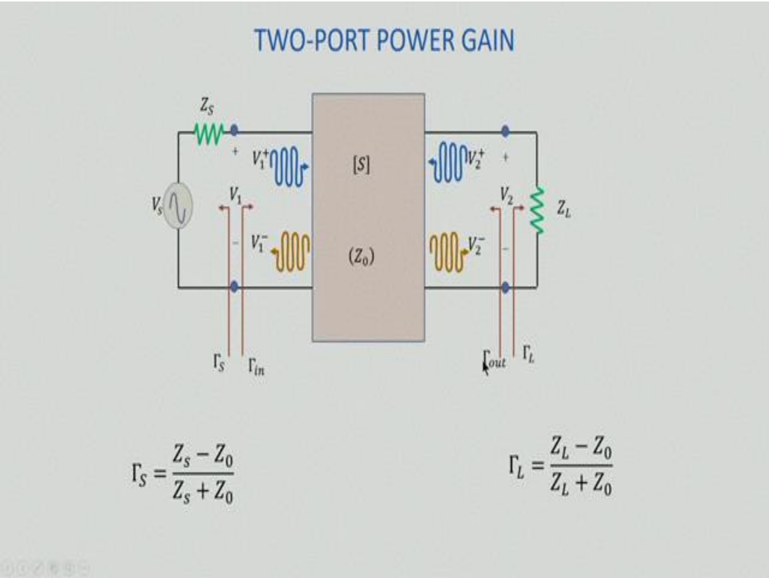
The power gain, which will be very, very useful in our discussion, is called transducer power gain, and it is represented by G_T. We define G_T the transducer power gain as P_L divided by Pavs. We have already mentioned that Pavs is the power that is available from the source under the matched condition that means the input port of the network is assumed to be conjugately matched to the impedance of the source.

And P_L is the load power that is delivered to the load this. And therefore, G_T is the ratio of the power delivered to the load to the power available from the source, and it depends on both Z_S and Z_L. Now, for a network, for which the input and output are conjugately matched to the two-port, then the gain is maximized, and we will have essentially G equal to G_A equal to G_T.

(Refer Slide Time: 10:46)

$$\Gamma_S = \frac{Z_s - Z_0}{Z_s + Z_0}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$



Next, we derived some of the useful relations which are necessary in order to derive the expression for these gains G, $G_{A,}$ and G_T. So, we start with the two-port network. Here we can see that the reflection coefficient gamma S can be defined as Z_S minus Z naught divided by Z_S plus Z naught. As we have already mentioned that Z naught is the reference impedance, we are considering the entire system.

Similarly, at the load end, we can define gamma L to be equal to Z_L minus Z naught divided by Z_L plus Z naught. So we have defined these two reflection coefficients, one looking toward the source, the other one looking toward the load. Now, we need to find out gamma in and gamma out.

(Refer Slide Time: 12:15)

$V_1^- = S_{11}\, V_1^+ + S_{12}\, V_2^+ = S_{11}\, V_1^+ + S_{12}\Gamma_L V_2^-$

$V_2^- = S_{21}\, V_1^+ + S_{22}\, V_2^+ = S_{21}\, V_1^+ + S_{22}\Gamma_L V_2^-$

$$\frac{V_1^-}{V_1^+} = S_{11} + S_{12}\Gamma_L \frac{V_2^-}{V_1^+}$$

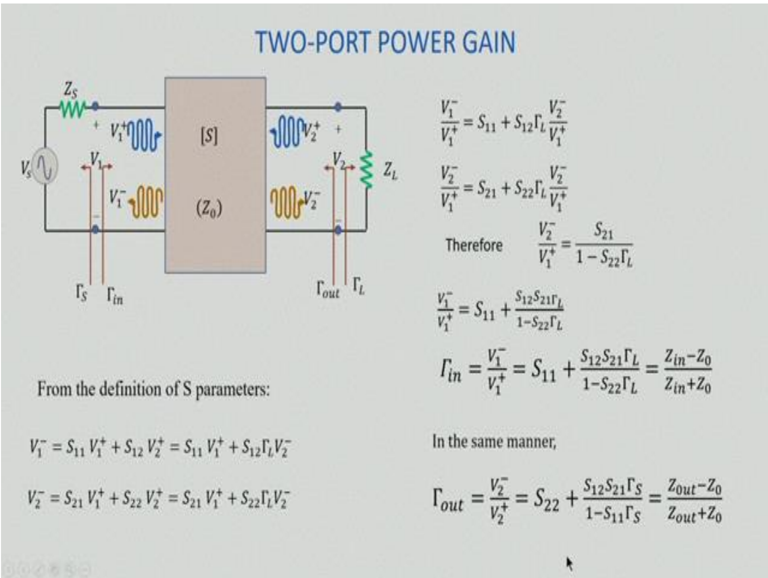
$$\frac{V_2^-}{V_1^+} = S_{21} + S_{22}\Gamma_L \frac{V_2^-}{V_1^+}$$

$$\frac{V_2^-}{V_1^+} = \frac{S_{21}}{1 - S_{22}\Gamma_L}$$

$\frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$

$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} = \frac{Z_{in}-Z_0}{Z_{in}+Z_0}$

$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S} = \frac{Z_{out}-Z_0}{Z_{out}+Z_0}$



So for this two-port network, we have the S parameters, they are related to this voltage S V_1 plus V_1 minus V_2 plus and V_2 minus as V_1 minus is equal to S_{11} V_1 plus, plus S_{12} V_2 plus. Now we have already defined gamma L, and if you look at this network, our V_2 plus is actually V_2 minus into the reflection coefficient at the load. So if you substitute V_2 plus, in terms of V2 minus, the signal that is incident on the load. We can write V_1 minus equal to S_{11} V_1 plus, plus S_{12} gamma L V_2 minus.

In the same manner, we can write V_2 minus is S_{21} V_1 plus, plus S_{22} V_2 plus, and we replace V_2 plus by gamma L V_2 minus. Now with these definitions, we need to find out gamma in and gamma in is the reflection coefficient looking into this two-port from the point as shown here. And therefore, gamma in will be V_1 minus divided by V_1 plus, from this expression, we can write V_1 minus divided by V_1 plus is equal to S_{11} plus S_{12} gamma L V_2 minus by V_1 plus.

Now, V_2 minus by V_1 plus can be calculated from the second expression. And we can write V_2 minus by V_1 plus is equal to S_{21} plus S_{22} gamma L V_2 minus by V_1 plus. And therefore, we can write V_2 minus by V_1 plus is equal to S_{21} divided by 1 minus S_{22} gamma L. Now. We can replace this V_2 minus by V_1 plus here. And therefore, we can write V_1 minus divided by V_1 plus is equal to S_{11} plus S_{12}, S_{21} gamma L divided by 1 minus S_{22} gamma L.

And this is V_1 minus by V_1 plus is nothing but our gamma in, and therefore, we can write gamma in to be equal to S_{11} plus S_{12}, S_{21} gamma L divided by 1 minus S_{22} gamma L. So, this is in terms of the S parameters. Also if Z_{in} is the impedance looking through this input port. In that case, we can write gamma in is Z_{in} minus Z_0 divided by Z_{in} the plus Z_0. We can carry out, in the same manner if we find V_2 minus by V_2 plus in this output port, then we can write gamma out is equal to V_2 minus divided by V_2 plus and that can be written as S_{22} plus S_{12}, S_{21} into gamma S divided by 1 minus S_{11} gamma S.

Now, looking from this port, if the impedance is Z_{out}, in that case, we can also write gamma out is equal to Z_{out} minus Z_0 divided by Z_{out} plus Z_0. So gamma out also can be expressed either in terms of the S parameters or in terms of the output impedance of the port Z_{out}.

(Refer Slide Time: 19:03)

$$V_1 = V_s \frac{Z_{in}}{Z_{in} + Z_S} = V_1^+ + V_1^- = V_1^+(1 + \Gamma_{in})$$

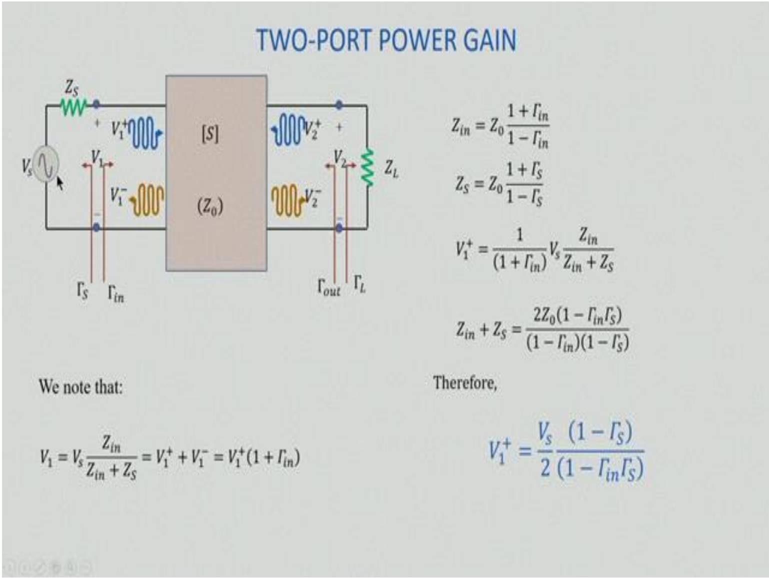
$$Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}}$$

$$Z_S = Z_0 \frac{1 + \Gamma_S}{1 - \Gamma_S}$$

$$V_1^+ = \frac{1}{(1 + \Gamma_{in})} V_s \frac{Z_{in}}{Z_{in} + Z_S}$$

$$Z_{in} + Z_S = \frac{2Z_0(1 - \Gamma_{in}\Gamma_S)}{(1 - \Gamma_{in})(1 - \Gamma_S)}$$

$$V_1^+ = \frac{V_S}{2}\frac{(1-\Gamma_S)}{(1-\Gamma_{in}\Gamma_S)}$$



Now, if we refer to this circuit once again, we find that V_1 by voltage division, it is Z in divided by Z_{in} plus Z_S into V_S. And V_1 is equal to V_1 plus, plus V_1 minus, and therefore, we can write V_1 to be equal to V_1 plus 1 plus V_1 minus divided by V_1 plus, which is gamma in. From this expression gamma in equal to Z_{in} minus Z_0 divided by Z in plus Z0, we can write Z_{in} equal to Z naught 1 plus gamma in divided by 1 minus gamma in.

And in the same manner, we can write Z_S equal to Z_0 1 plus gamma S divided by 1 minus gamma S., And now we can make the substitution, here we can write V_1 plus is equal to 1 by 1 plus gamma in into V_S Z_{in} divided by Z_{in} plus Z_S. And if you calculate Z_{in} plus Z_S, we get Z in plus ZS is equal to $2Z_0$1 minus gamma in into gamma S divided by 1 minus gamma in into 1 minus gamma S.

So, we have now the expression for Z in plus Z_S. We substitute Z_{in} here and also Z_{in} plus Z_S in the denominator and once we make this substitution, we get V_1 plus is equal to V_S by 2, 1 minus gamma S divided by 1 minus gamma in gamma S. So in this expression, we have actually expressed V_{in} plus, in terms of the source voltage V_S.

(Refer Slide Time: 22:47)

$$V_1^+ = \frac{V_S}{2}\frac{(1-\Gamma_S)}{(1-\Gamma_{in}\Gamma_S)}$$

$$P_{in} = \frac{1}{2Z_0}|V_1^+|^2\ (1 - |\Gamma_{in}|^2) = \frac{|V_S|^2}{8Z_0}\frac{|1 - \Gamma_S|^2}{|1 - \Gamma_{in}\Gamma_S|^2}(1 - |\Gamma_{in}|^2)$$

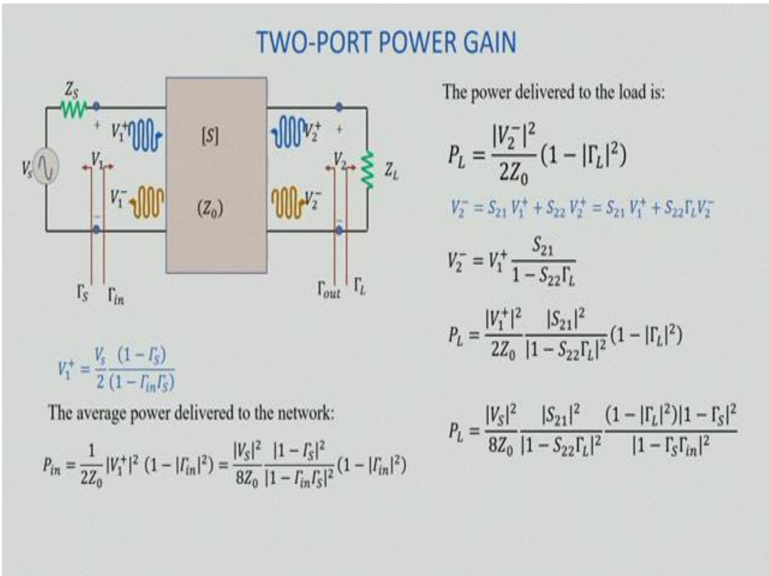
$$P_L = \frac{|V_2^-|^2}{2Z_0}(1 - |\Gamma_L|^2)$$

$V_2^- = S_{21}\, V_1^+ + S_{22}\, V_2^+ = S_{21}\, V_1^+ + S_{22}\Gamma_L V_2^-$

$$V_2^- = V_1^+ \frac{S_{21}}{1 - S_{22}\Gamma_L}$$

$$P_L = \frac{|V_1^+|^2}{2Z_0}\frac{|S_{21}|^2}{|1 - S_{22}\Gamma_L|^2}(1 - |\Gamma_L|^2)$$

$$P_L = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2}{|1 - S_{22}\Gamma_L|^2}\frac{(1 - |\Gamma_L|^2)|1 - \Gamma_S|^2}{|1 - \Gamma_S\Gamma_{in}|^2}$$



So we rewrite this expression here for V_1 plus. And now, we can find out the average power that is delivered to the network P in is equal to 1 by 2Z naught, V1 plus magnitudes square into 1 minus mod gamma in square. So the incident power is mod V1 plus square divided by 2Z naught. And when this term is multiplied by magnitude of gamma in square, it gives the reflected power. And the difference of these two incident and reflected power is the input power.

We have now expression for V_1 plus and when this expression is substituted here, we get P in is equal to mod of V_S square divided by 8Z naught into 1 minus mod of gamma S square divided by mod of 1 minus gamma in gamma S square into 1 minus mod of gamma in square. So, we have the expression for P in, now the power that goes into the network.

In the same manner, the power delivered to the load; it is V_2 minus magnitude square divided by 2Z naught. This is the incident power and minus the reflected power is incident power V_2 minus mod square divided by 2Z naught multiplied by gamma L square. So we get, this is the power which actually is delivered to the load. So, once we have expression for P_L, the power delivered to the load and P_{in} power delivered to the network. We can calculate G, the gain of the network as P_L by P_{in}.

But in the air expression of P_L we find that it is in terms of V_2 minus square whereas in P_{in} we have it in terms of source voltage V_S magnitude square. Now we see that V_2 minus is S_{21} V_1 plus, plus S_{22} V_2, plus which can be written as, S_{21} V_1 plus, plus S_{22} gamma L V_2 minus. And from this, we can write V_2 minus is equal to V_1 plus S_{21} divided by 1 minus S_{22} gamma L.

And therefore, P_L becomes V_1 plus magnitudes square divided by 2Z naught S_{21} magnitude square divided by 1 minus S_{22} gamma L magnitude square into 1 minus magnitude gamma L square. And we already have the relation between V_1 plus and V_S, and once we substitute magnitude of V_1 plus square, we now get the expression for the power P_L also, in terms of magnitude of V_S square. From these two expressions, we can find the ratio to calculate the gain G.

(Refer Slide Time: 29:00)

$$P_{in} = \frac{|V_S|^2}{8Z_0} \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_{in}\Gamma_S|^2} (1 - |\Gamma_{in}|^2)$$

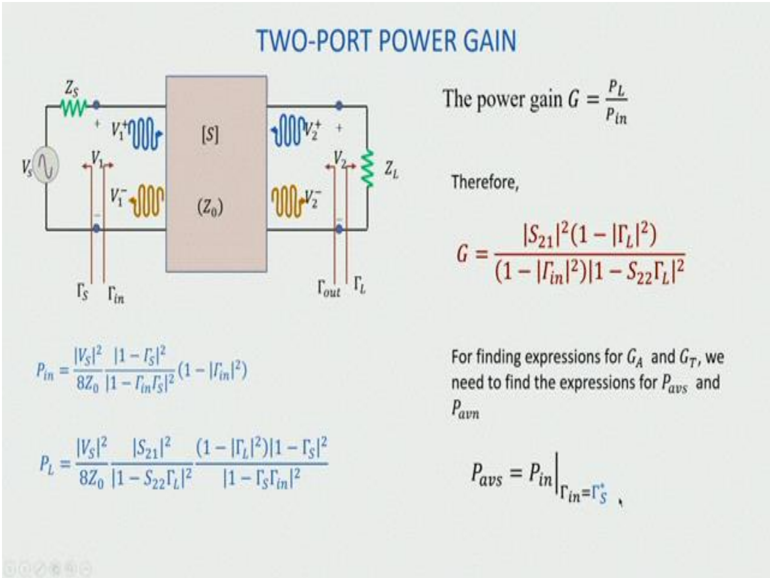
$$P_L = \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2}{|1 - S_{22}\Gamma_L|^2} \frac{(1 - |\Gamma_L|^2)|1 - \Gamma_S|^2}{|1 - \Gamma_S\Gamma_{in}|^2}$$

The power gain $G = \frac{P_L}{P_{in}}$

Therefore,

$$G = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{(1 - |\Gamma_{in}|^2)|1 - S_{22}\Gamma_L|^2}$$

$$P_{avs} = P_{in}|_{\Gamma_{in} = \Gamma_S^*}$$



So for the two-port shown, we have P_{in} and P_L given by these expressions. And once we have these expressions, we can find out the power gain G to be equal to P_L by P_{in}. So, once we substitute the expressions for P_L and P_{in}, we get G equal to mod of S_{21} square into 1 minus mod of gamma L square divided by 1 minus mod of gamma in square into 1 minus S_{22} gamma L mod square.

For finding expressions for the other two gains G_A the available gain and G_T transducer gain, we need to find the expressions for P_{avs}. That means power available from the source and P_{avn} power available from the network. Now P_{avs} can be found as P_{input}, P_{in} for the condition gamma in is equal to gamma S conjugate.

(Refer Slide Time: 30:44)

$$P_{in} = \frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|^2}(1-|\Gamma_{in}|^2)$$

$$P_L = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2}{|1-S_{22}\Gamma_L|^2}\frac{(1-|\Gamma_L|^2)|1-\Gamma_S|^2}{|1-\Gamma_S\Gamma_{in}|^2}$$

$$P_{avs} = P_{in}|_{\Gamma_{in}=\Gamma_S^*}$$

$$\text{Therefore,}\ \ P_{avs} = \frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2}{(1-|\Gamma_S|^2)}$$

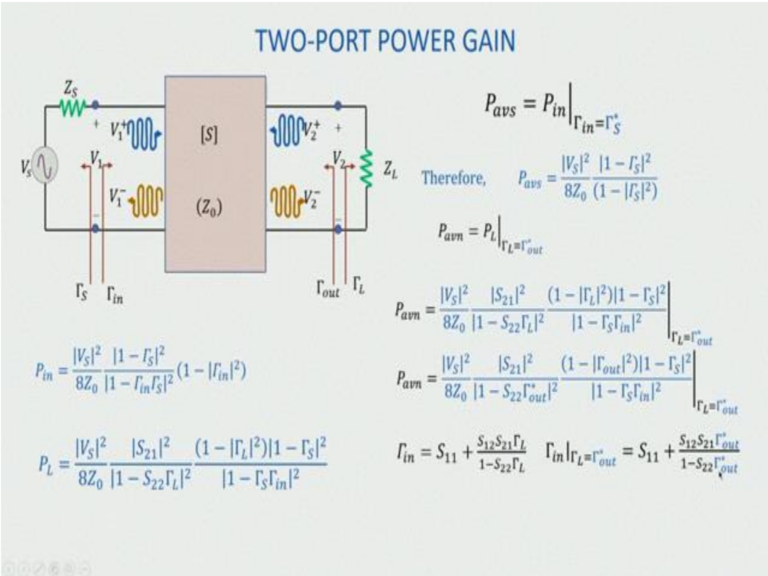
$$P_{avn} = P_L|_{\Gamma_L=\Gamma_{out}^*}$$

$$P_{avn} = \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2}{|1 - S_{22}\Gamma_L|^2} \frac{(1 - |\Gamma_L|^2)|1 - \Gamma_S|^2}{|1 - \Gamma_S\Gamma_{in}|^2}\Bigg|_{\Gamma_L = \Gamma_{out}^*}$$

$$P_{avn} = \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2}{|1 - S_{22}\Gamma_{out}^*|^2} \frac{(1 - |\Gamma_{out}|^2)|1 - \Gamma_S|^2}{|1 - \Gamma_S\Gamma_{in}|^2}\Bigg|_{\Gamma_L = \Gamma_{out}^*}$$

$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$

$\Gamma_{in}|_{\Gamma_L = \Gamma_{out}^*} = S_{11} + \frac{S_{12}S_{21}\Gamma_{out}^*}{1 - S_{22}\Gamma_{out}^*}$



Therefore, we can find out P_{avs} as mod of V_S square divided by 8Z naught 1 minus gamma S mod square divided by 1 minus mod of gamma S square and P_{avn} available power from the network, this can be calculated as PL evaluated as at gamma L is equal to gamma out conjugate. Now, when we substitute gamma L is equal to gamma out conjugate here, we get P_{avn} is equal to mod of V_S square by 8Z naught mod of S_{21} square divided by 1 minus S_{22} gamma out conjugate mod square, 1 minus mod of gamma out square into 1 minus gamma S mod square divided by one minus gamma S, gamma in square.

So, in this expression, we find that this parameter gamma in is also dependent upon gamma L. And therefore, gamma in 1 minus gamma S gamma in, this factor we need to calculate when gamma L is equal to gamma out conjugate. So, we can write gamma in, for gamma L is equal to gamma out conjugate is equal to S_{11} plus S_{12} S_{21} gamma out conjugate divided by 1 minus S_{22} gamma out conjugate.

(Refer Slide Time: 33:21)

$$P_{avs} = \frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2}{(1-|\Gamma_S|^2)}$$

$$P_{avn} = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2}{|1-S_{22}\Gamma_{out}^*|^2}\frac{(1-|\Gamma_{out}|^2)|1-\Gamma_S|^2}{|1-\Gamma_S\Gamma_{in}|^2}\Bigg|_{\Gamma_L=\Gamma_{out}^*}$$

$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$

$\Gamma_{in}\rfloor_{\Gamma_L=\Gamma_{out}^*} = S_{11} + \frac{S_{12}S_{21}\Gamma_{out}^*}{1-S_{22}\Gamma_{out}^*}$

$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$

$(1-S_{11}\Gamma_S)\,(\Gamma_{out}-S_{22}) = S_{12}S_{21}\Gamma_S$

$$1-\Gamma_S\Gamma_{in}\rfloor_{\Gamma_L=\Gamma_{out}^*} = \frac{(1-S_{11}\Gamma_S)(1-|\Gamma_{out}|^2)}{1-S_{22}\Gamma_{out}^*}$$

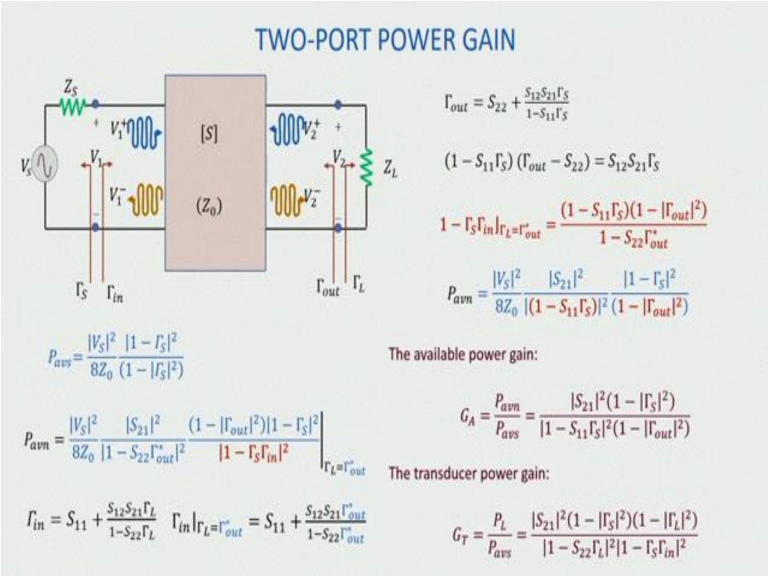
$$P_{avn} = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2}{|(1-S_{11}\Gamma_S)|^2}\frac{|1-\Gamma_S|^2}{(1-|\Gamma_{out}|^2)}$$

The available power gain:

$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2|1-\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)}$$

The transducer power gain:

$$G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2|1-\Gamma_S\Gamma_{in}|^2}$$



Now, we have this expressions already derived, gamma out is given by S_{22} plus S_{12} S_{21} gamma S divided by 1 minus S_{11} gamma S. Therefore, we can write gamma out minus S_{22} into 1 minus S_{11} gamma S is S_{12} S_{21} gamma S. And then if we find out 1 minus gamma S gamma in for gamma L is equal to gamma out conjugate. This comes out to be one minus S_{11} gamma S into one minus gamma out mod square divided by 1 minus S_{22} gamma out conjugate.

Now, please note that this expression, we are substituting. So, if you take modulus of this N square and then substitute here, then we will get Pavn because this will be 1 by mod of 1 minus gamma S gamma in square. So, some of the terms will get canceled, particularly when this will go at the numerator 1 minus S22 gamma out conjugate mod of square. It will get canceled with this term, like that, if we cancel out the corresponding terms, we get avn to be equal to mod of VS square by 8Z naught mod S21 square divided by 1 minus S_{11} gamma S mod square into 1 minus gamma S mod square divided by 1 minus mod of gamma out square.

So, we have now the expression for avs and avn, from this two expressions, we can now find out the expressions for GA which is avn divided by avs and this comes out to be mod of S_{21} square into 1 minus gamma S mod square divided by 1 minus S_{11} gamma S mod square into 1 minus mod gamma out square. Similarly, if we find P_L by P_{avs} then we can be expression for GT the transducer gain and which is shown here.

(Refer Slide Time: 36:49)

The power gain:

$$G = \frac{|S_{21}|^2(1 - |\Gamma_L|^2)}{(1 - |\Gamma_{in}|^2)|1 - S_{22}\Gamma_L|^2}$$

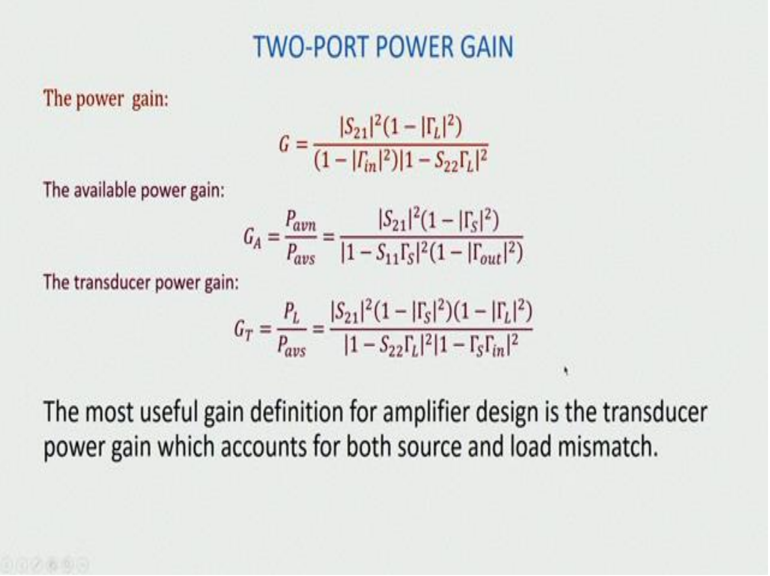
The available power gain:

$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2|1 - \Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2(1 - |\Gamma_{out}|^2)}$$

The transducer power gain:

$$G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|1 - S_{22}\Gamma_L|^2|1 - \Gamma_S\Gamma_{in}|^2}$$

The most useful gain definition for amplifier design is the transducer power gain which accounts for both source and load mismatch.



So, if we summarise our discussion, we find that we have now the expression for the power gain G, available power gain G_{A}, and transducer power gain G_T. Now, among these three, the most useful gain definition for amplifier design is the transducer power gain, which accounts for both source and load mismatch. Here we have both gamma S and gamma L, in the other two expressions, we get either gamma S or gamma L. And G_T has both gamma S and gamma L.

(Refer Slide Time: 37:42)

$$G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2|1-\Gamma_S\Gamma_{in}|^2}$$

$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$

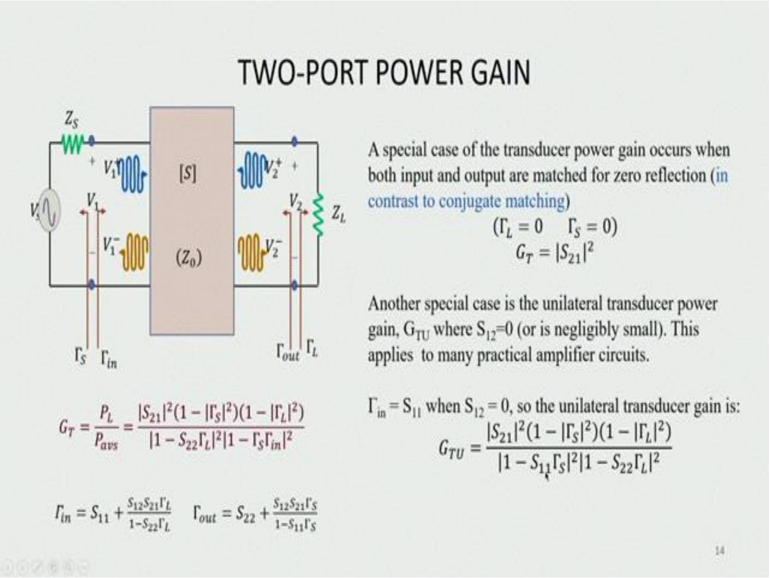
$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$

$$(\Gamma_L = 0 \qquad \Gamma_S = 0)$$

$$G_T = |S_{21}|^2$$

$\Gamma_{in} = S_{11}$ when $S_{12} = 0$, so the unilateral transducer gain is:

$$G_{TU} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-S_{11}\Gamma_S|^2|1-S_{22}\Gamma_L|^2}$$

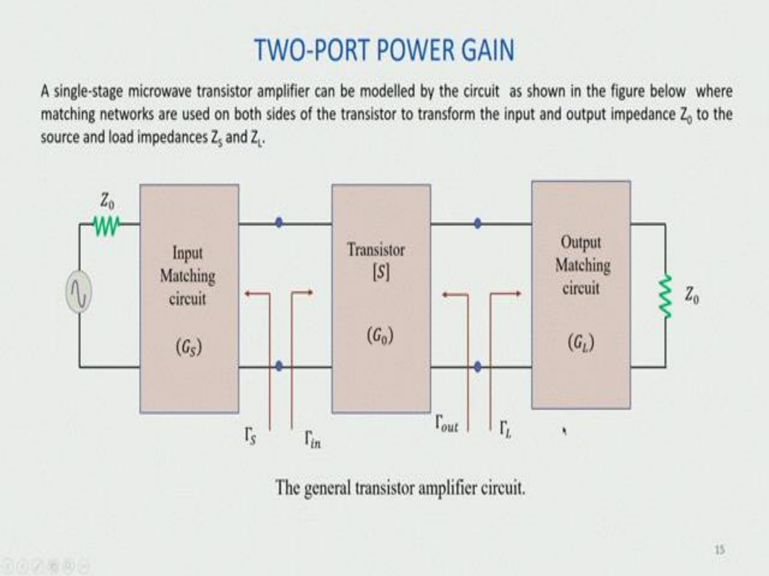


Now, let us go to some specific cases. Special case of the transducer power gain occurs when both input and output are matched for zero reflection. So, that means we have gamma S equal to 0, gamma L equal to 0. So this is in contrast to conjugate matching. Now, when gamma S and gamma L, they are 0. In that case, G_T becomes simply mod of S_{21} square.

Here we can see that if you put gamma S 0 here, gamma L 0 here, gamma L 0 here and gamma S 0 here. Then all these terms will become 1and we will be left with G_T equal to mod S_{21} square. Another special case is the unilateral transducer gain when we have S_{12} equal to 0. That means it is unilateral in the sense there is no signal flow from port to 21. So either S_{12} is 0 or it is negligibly small. In this condition, we get an expression for G_T which we did not by G_{TU} transducer gain unilateral.

And will see that this condition applies to many practical amplifier circuits, particularly when the transistor has S_{12} equal to 0. Now, when S_{12} equal to 0 from this expression, we find that gamma in becomes S_{11}. Similarly, gamma out becomes S_{22} and therefore, we get a modified expression for G_T where this gamma in has been now substituted by S_{11}.

(Refer Slide Time: 40:13)



So far, we were discussing the power gain for a general two-port network. A single-stage microwave amplifier can be modelled by a circuit which is shown in the figure below, where we can see that instead of Z_S and R_L, we have Z naught as the source impedance and Z naught as the load and we have actually to matching networks, input matching network, and output matching network.

Now, this matching network transforms on both sides of the transistor. The input and output impedance Z naught to Z_S and Z_L. So, our earlier discussion was with Z_S and Z_L. Now, the purpose of this matching network is to transform Z naught to Z_S and Z naught to ZL. And we have shown G_S, G naught and G_L are the three gains. So this G naught is the gain which you get from the transistor and G_S and G_L are the gains of the matching network.

(Refer Slide Time: 42:01)

$$G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2|1-\Gamma_S\Gamma_{in}|^2}$$

The separate effective gain factors:

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_{in}\Gamma_S|^2} \qquad G_0 = |S_{21}|^2 \qquad G_L = \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{22}\Gamma_L|^2}$$

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

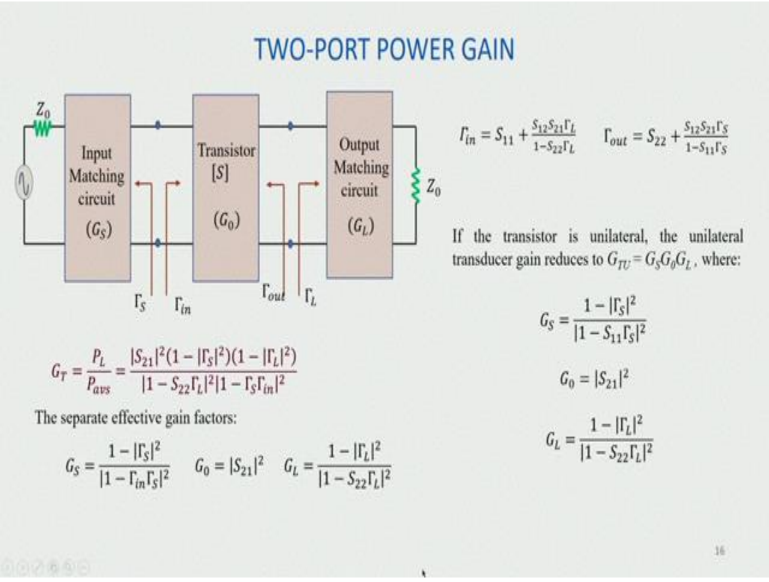
$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

If the transistor is unilateral, the unilateral transducer gain reduces to $G_{TU} = G_S G_0 G_L$, where:

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_{11}\Gamma_S|^2}$$

$$G_0 = |S_{21}|^2$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{22}\Gamma_L|^2}$$



So from this expression of G_T, we can now separate the effective gain factors G_S, G naught, and G_L. We find that G_S is equal to 1 minus gamma S mod square 1 minus mod gamma S square divided by mod of 1 minus gamma in gamma S square and G naught is equal to mod S_{21} square, G_L is equal to 1 minus mod gamma L square divided by mod of 1 minus S_{22} gamma L square.

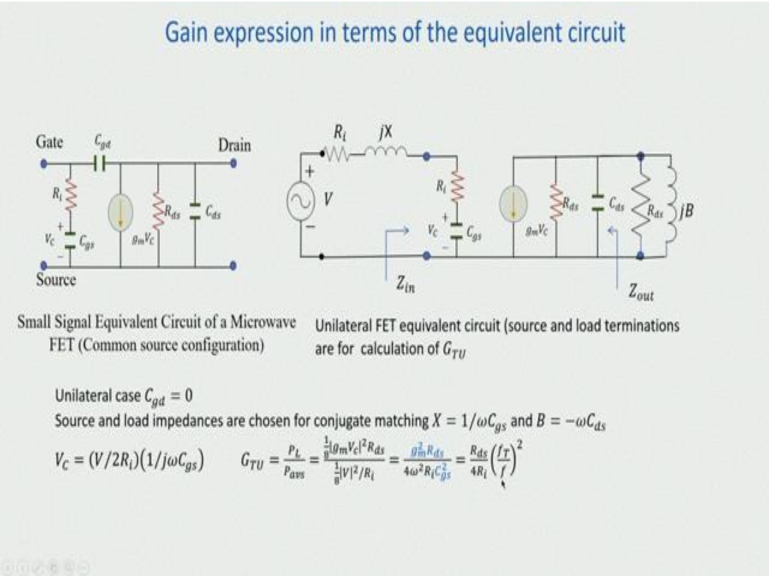
Now, these are the expressions for gamma in and gamma out. If the transistor is unilateral, the unilateral transducer gain reduces to G_{TU} equal to G_S G naught G_L, where we have G_S now when S_{12} is 0, gamma in becomes equal to S_{11}. So G_S we replace S_{11} for gamma in and gamma S becomes 1 minus mod gamma S square divided by mod of 1 minus S_{11} gamma S square. G naught remains the same mod of S_{21} square, and G_L also remains same.

(Refer Slide Time: 43:52)

Unilateral case $C_{gd} = 0$

Source and load impedances are chosen for conjugate matching $X = 1/\omega C_{gs}$ and $B = -\omega C_{ds}$

$V_C = (V/2R_i)(1/j\omega C_{gs})$ $\qquad G_{TU} = \frac{P_L}{P_{avs}} = \frac{\frac{1}{8}|g_m V_C|^2 R_{ds}}{\frac{1}{8}|V|^2/R_i} = \frac{g_m^2 R_{ds}}{4\omega^2 R_i C_{gs}^2} = \frac{R_{ds}}{4R_i}\left(\frac{f_T}{f}\right)^2$



So these are the gain expressions we have seen when we consider the S parameter representation of the transistor. Now, let us look into the gain expression in terms of the equivalent circuit. In our previous module we have seen that a small signal equivalent circuit of a microwave FET, operating in common source configuration is represented like this. Now, let us consider a unilateral case. That means we assume that C_{gd} equal to 0, and also we have conjugate matching.

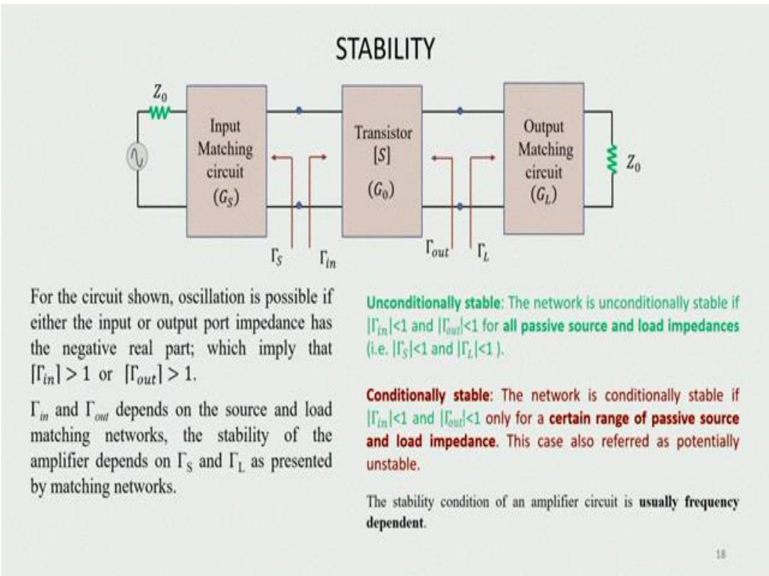
So here we can see the source resistance real part is R_i and source reactance jX is so chosen that it cancels out the reactance provided by C_{gs}. That means minus J 1 by omega Cgs and jX, they cancel out each other. Similarly, the load is chosen in such a way that in conjugate matching is obtained. And therefore, will have for conjugate matching the load susceptance is chosen here to be equal to minus omega C_{ds}.

So, essentially these two will cancel out each other. So, under this condition since, we have unilateral case C_{gd} equal to 0, we can find out G_{TU}. So, the voltage V_C is actually V by $2R_i$, this will give the current multiplied by 1 by J omega C_{gs}. This is the voltage V_C and the output current gmVC, now can be calculated for this current source. And G_{TU} is half of this current will go through Rds, this load Rds. And therefore, we can write P_L is equal to 1 by 8 mod of gmV_C square into R_{ds}.

And the power available from the source is 1 by 8 V square by R_i, this is because we have R_i plus R_i, two R_i and therefore, the current is essentially V by $2R_i$ and V square half V_I square will give us this expression. Now, when the expression for V_C is substituted, we get G_{TU} to be equal to gm square Rds divided by 4 omega square R_i C_{gs} square. Now, this can be written in this form R_ds by $4R_i$ f_T by f whole square. Where f_T is the unity gain frequency in which we define our previous module.

Therefore, we see that the gain of this unilateral transistor reduces as 1 by f square. So, this is how we can find out the gain, unilateral gain of the transistor device, provided these values are given under match condition. So in this manner we can find out the unilateral gain G_{TU} from the equivalent circuit, as shown in the figure under the matching condition. And we can determine the value of G_{TU}, once we have the values of f_T, R_{ds}, R_i. We can calculate G_{TU} at a given frequency.

(Refer Slide Time: 50:01)



Let us now turn our attention to another very important issue in the design of the transistor amplifier circuits. This is stability. If you consider the circuit as shown above the oscillation in this circuit is possible if either input or output port impedances negative real part. So, if we have a negative real part, we can show that magnitude of gamma in will become greater than 1 or magnitude of gamma out will become greater than 1, depending on, which port impedance has negative real part.

Gamma in and gamma out depends on the source and load matching network. The stability of the amplifier depends on gamma S and gamma L S presented by the matching networks. So, we talk of a particular situation, which is unconditionally stable: The network is unconditionally stable if mod gamma in is less than 1 and mod gamma out is less than 1 for all passive source and load impedances. That is, whenever mod gamma S is less than 1 and mod gamma L is less than 1, mod gamma in if it is less than 1 and mod gamma out is also less than 1. Then we have an unconditionally stable case.

Conditionally stable: The network is conditionally stable if mod gamma in is less than 1, and mod gamma out is less than 1 only for a certain range of passive source and load resistances. That means this condition of mod gamma in less than 1 and mod gamma out less than 1, it is not valid, these conditions are not valid for all gamma S and gamma L. And this case is also referred as potentially unstable. And also the stability condition of an amplifier circuit is usually frequency dependent, an amplifier may be stable over a given range of frequencies.

(Refer Slide Time: 53:15)

The condition that must be satisfied by Γ_S and Γ_L if the amplifier is to be unconditionally stable:

$$|\Gamma_{in}| = \left|S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}\right| < 1$$

$$|\Gamma_{out}| = \left|S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}\right| < 1$$

The determinant of the scattering matrix:

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

From,

$$|\Gamma_{in}| = 1,$$

$$\left|S_{11}+\frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}\right|=1$$

and

$$\Delta = S_{11}S_{22}-S_{12}S_{21}$$

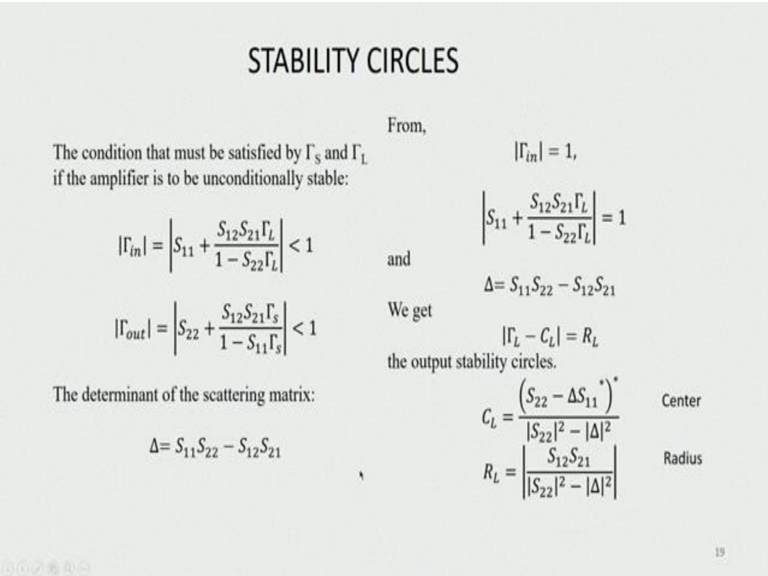
We get

$$|\Gamma_L - C_L| = R_L$$

the output stability circles.

$$C_L = \frac{\left(S_{22}-\Delta S_{11}^{\ *}\right)^*}{|S_{22}|^2-|\Delta|^2}$$

$$R_L = \left|\frac{S_{12}S_{21}}{|S_{22}|^2-|\Delta|^2}\right|$$



The condition that must be satisfied by gamma S and gamma L, if the amplifier is to be unconditionally stable: we have already stated mod gamma in is less than 1 and mod gamma out is less than 1. We represent the determinant of the scattering matrix S by delta, which is equal to S_{11} S_{22} minus S_{12} S_{21}. Now, if we equate mod gamma in to 1, then will get the condition that mod of S_{11} plus S_{12} S_{21} gamma L divided by 1 minus S_{22} gamma L is 1.

And starting from this condition, we can derive mod of gamma L minus C_L is equal to R_L. Now C_L is given by conjugate of S_{22} minus delta S_{11} conjugate divided by mod of S_{22} square minus mod of delta square. Similarly, the value R_L, which actually represents a radius, C_L represents the center because gamma L minus C_L is equal to R_L gives a circle.

And RL is given by mod of S_{12} S_{21} divided by mod of S_{22} square minus mod delta square. So we see that we can have a circle with center C_L and radius R_L, and it demarcates the value of gamma L. For which mod gamma in is greater than 1 or less than 1.

(Refer Slide Time: 55:39)

Similarly from,

$$|\Gamma_{out}| = 1,$$

$$\left|S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}\right| = 1$$

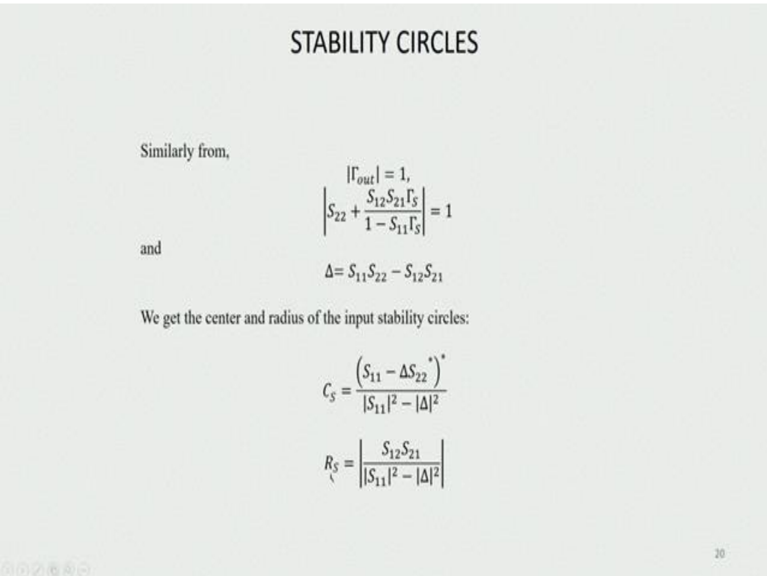
and

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

We get the center and radius of the input stability circles:

$$C_S = \frac{\left(S_{11} - \Delta S_{22}^*\right)^*}{|S_{11}|^2 - |\Delta|^2}$$

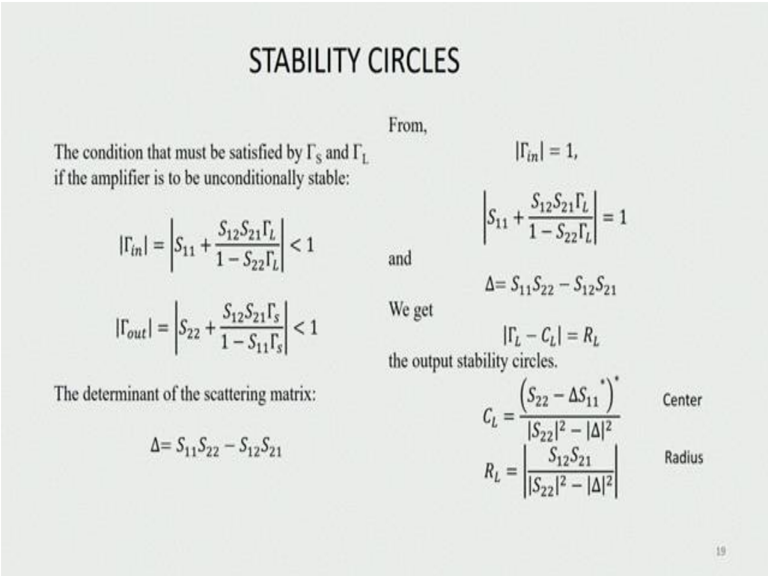
$$R_S = \left|\frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2}\right|$$

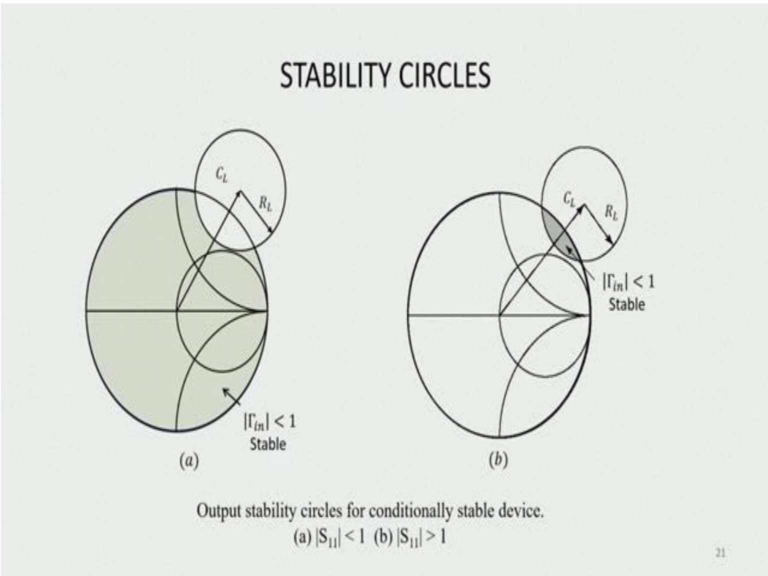


In the same manner from mod gamma out equal to 1, we can get another circle with center CS and radius R_S. And the corresponding expressions are shown.

(Refer Slide Time: 55:59)

If the device is unconditionally stable, the stability circles must be completely outside (or totally enclose) the Smith chart.





Now, from our previous discussions, we find that if the devices is unilateral. That means S_{12} is equal to 0, then mod gamma in is less than 1, if mod of S_{11} is less than 1. Similarly, mod of gamma out is less than 1, if mod of S_{22} is less than 1. So for a unilateral device, it is the values of the S parameters, S_{11}, and S_{22} that directly decides the stability.

Otherwise mod gamma in less than 1 and mod gamma out less than 1, essentially defines a range of values for gamma S and gamma L where the amplifier will be stable. And we find this range of values for gamma L and gamma S by plotting the stability circles, which we have just discussed. For example, if we consider the output stability circle, here we have plotted it for two cases, one mod of S_{11} is less than 1. So this is the Smith chart, and this is the circle with radius R_L and C_L.

Now, we find that we determine the stable region by the location of gamma L is equal to 0. If you have Z_L equal to Z naught, then gamma L will be 0. And here we find that gamma L equal to 0 is the center of the Smith chart, which is outside this circle and the intersection of the Smith chart, the shaded region it gives the values of gamma in for which the amplifier will be stable, which means mod gamma in is less than 1.

Now, if you consider the other case where mod of S_{11} is greater than 1. Then by the same argument, we find that in this region the center of the Smith chart which represents gamma L is equal to 0 is in the unstable region. And the stable region is given by this shaded region. So given the S parameters of the transistors give, we can find out the values of gamma L that we can use to have mod gamma in less than 1. That means to make the transistor amplifier a stable. Similarly, we can do it for gamma S by plotting the input stability circles.

(Refer Slide Time: 60:08)

If the device is unconditionally stable, the stability circles must be completely outside (or totally enclose) the Smith chart.

$$\big||C_L| - R_L\big| > 1 \rightarrow |S_{11}| < 1$$

$$\big||C_S| - R_S\big| > 1 \rightarrow |S_{22}| < 1$$

Rollet's condition*:*

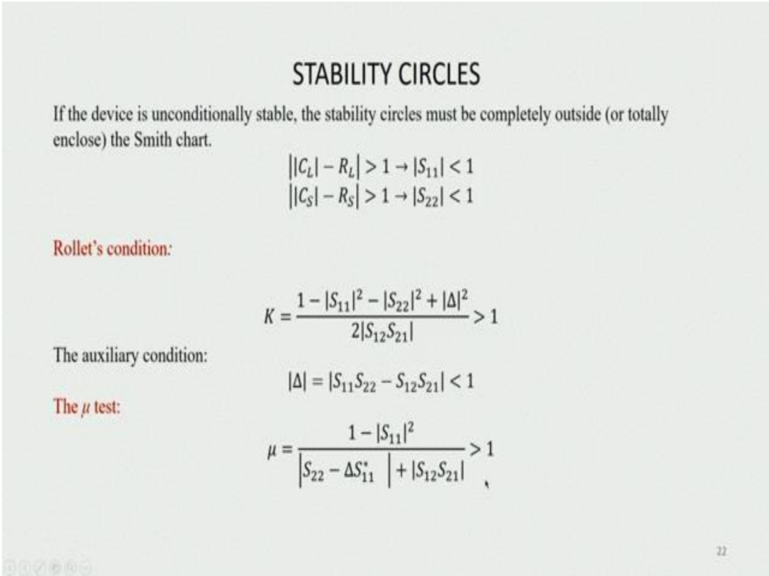
$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1$$

The auxiliary condition:

$$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| < 1$$

The μ test:

$$\mu = \frac{1 - |S_{11}|^2}{\left|S_{22} - \Delta S_{11}^*\right| + |S_{12}S_{21}|} > 1$$



So, we see that if the devices unconditionally stable, the stability circles must be completely outside the Smith chart, or the stability circle must totally enclose the Smith chart. Now, mod of C_L minus R_L greater than 1, this implies mod of S_{11} less than 1 and similarly mod of C_S minus R_S modulars greater than 1, it implies mod of S_{22} less than 1. Given the transistor S parameters from the discussion we had so far, we get a set of conditions known as Rollets conditions.

And we calculate a parameter K which is 1 minus mod S_{11} square minus mod S_{22} square plus delta square plus mod delta square divided by 2 mod, S_{12} S_{21} and if K greater than 1. And the auxiliary condition delta is less than 1, where delta is S_{11} S_{22} minus S_{12} S_{21}. So if the conditions are satisfied. Then the transistor or the device is unconditionally stable. There is another test, which is known as mu test, and for unconditional stability, we must have mu defined by 1 minus mod S_{11} square divided by mod of S_{22} minus delta S_{11} conjugate plus S_{12} S_{21} mod. This parameter mu must be greater than 1.