Microwave Engineering
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Lecture 26
Design of single stage transistor amplifier

So, we have discussed the gain of general two-port circuits, and we have seen the different gain like power gain, available gain, transducer gain. And, we have discussed the conditions related to the stability of the two-port. We have also discussed the conditions related to the stability of the two-port device. Let us now move on to discussion of the issues related to the design of a single-stage transistor amplifier.

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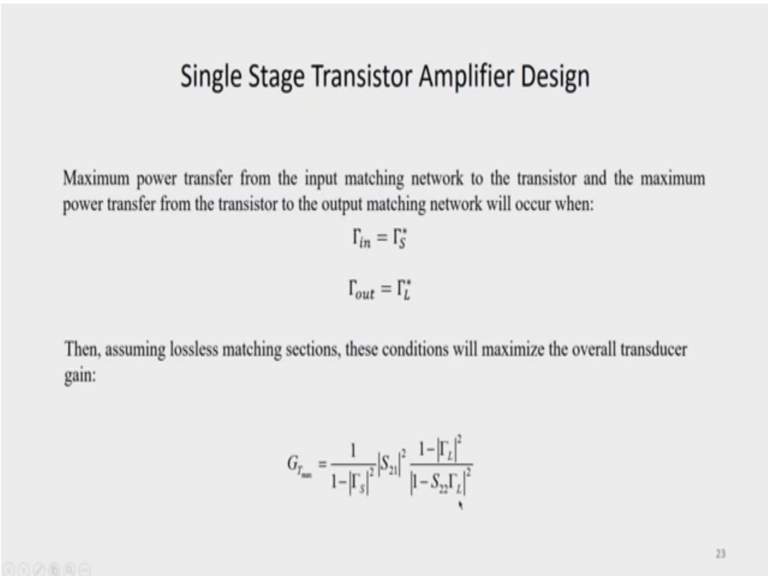
$$\Gamma_{in} = \Gamma_S^*$$

$$\Gamma_{out} = \Gamma_L^*$$

$$G_{T_{\max}} = \frac{1}{1-\left|\Gamma_S\right|^2}\left|S_{21}\right|^2 \frac{1-\left|\Gamma_L\right|^2}{\left|1-S_{22}\Gamma_L\right|^2}$$



So, the maximum power transfer from the input matching network to the transistor and maximum power transfer from the transistor to the output matching network will occur when both input and output, they are conjugately matched. That is, gamma is equal to gamma s conjugate, and gamma out is equal to gamma l conjugate. We assumed that the matching network that we are designing, these are lossless matching network and, with the assumptions of lossless matching sections.

These conditions will maximize the overall transducer gain, and we can write G_t max, the maximum of the transducer gain as 1 by 1 minus mod of gamma s square, mod of S_{21} square into 1 minus mod gamma l square divided by 1 minus S_{22} gamma l mod square.

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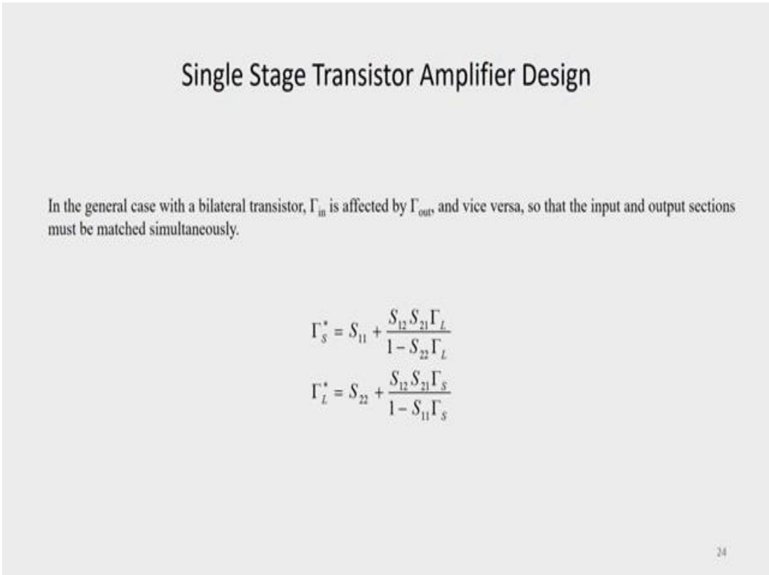
$$\Gamma_S^* = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_L^* = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$



So, in the general case with a bilateral transistor, gamma is affected by gamma out and vice versa. So, the input and output sections are required to be matched simultaneously. That means gamma will be affected by gamma out; gamma out will affected by any changes that we make in gamma in. So, we can write from our earlier discussion that gamma is equal to gamma S conjugate.

Therefore, gamma S conjugate can be written as S_{11} plus S_{12} S_{21} gamma l divided by 1 minus S_{22} gamma l. This is essentially the expression of gamma. And, similarly, we can write gamma l conjugate is equal to S_{22} plus S_{12} S_{21} gamma S divided by 1 minus S_{11} gamma S. So, this right-hand side is essentially the expression for gamma out.

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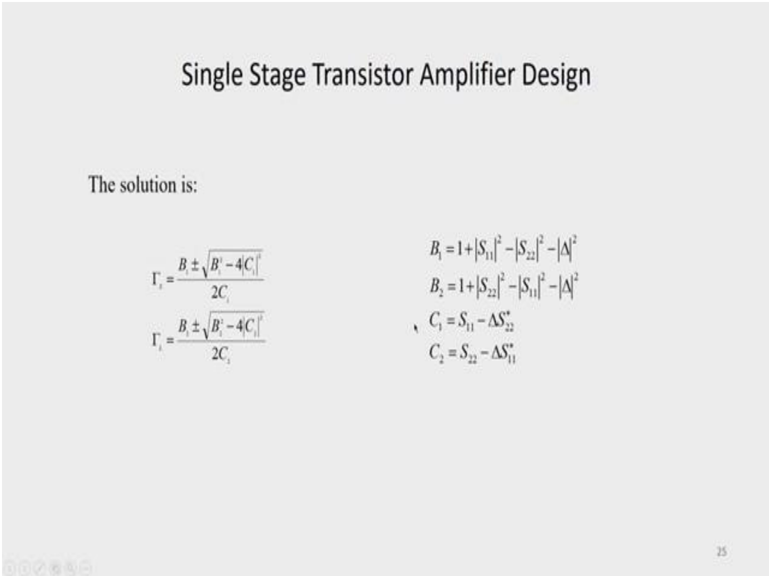
$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \qquad B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} \qquad C_1 = S_{11} - \Delta S_{22}^*$$

$$C_2 = S_{22} - \Delta S_{11}^*$$



From these conditions we can solve for gamma s and gamma l. The solution for gamma s is B_1 plus-minus root B_1 square minus 4 mod C_1 square divided by 2 C_1. And, gamma l is equal to B_2 plus-minus root B_2 square minus 4 mod C_2 square divided by 2 C_2. Here this B_1 B_2, C_1, and C_2 these are given by B_1 is equal to 1 plus mod S_{11} square minus mod S_{22} square minus mod of delta square. B_2 is 1 plus mod S_{22} square minus mod S_{11} square minus mod of delta square. And, C_1 is equal to S_{11} minus delta S_{22} conjugate C_2 is S_{22} minus delta S_{11} conjugate.

So, given the s parameters of the transistor, we can see that we can get the values of gamma s and gamma l. Things become very much simplified if we have a unilateral transistor that means. S_{12} is equal to 0. In that case we get gamma S equal to S_{11} conjugate and gamma l is equal to S_{22} conjugate. Because gamma in becomes is equal to S_{11} in case of a unilateral device. S_{12} is equal to 0 and gamma out becomes S_{22}. Now, once we have this gamma s and gamma l computed.

Now, this at the reflection coefficients which had to be seen looking into the matching networks at both ends. And, therefore, we can start with Z naught and design a matching network to transform it into an impudence which will give us the required gamma s and gamma l.

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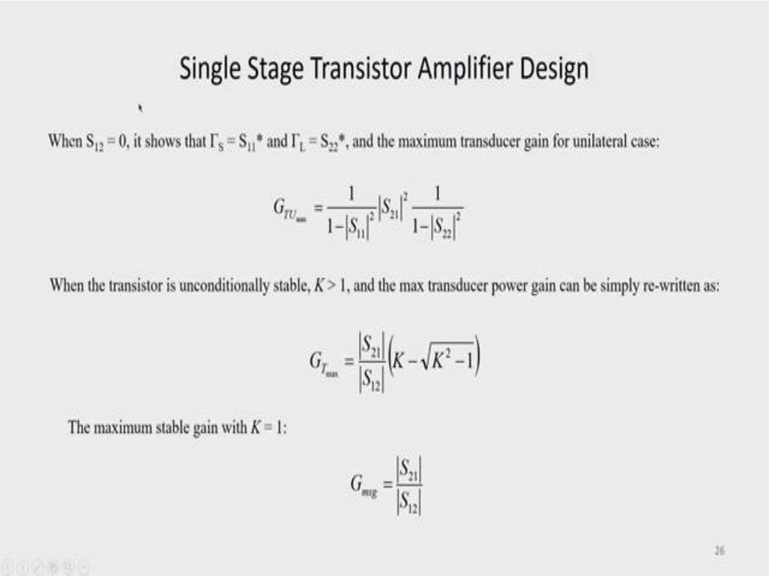
$$G_{TU_{\max}} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2}$$

$$G_{T_{\max}} = \frac{|S_{21}|}{|S_{12}|}\left(K-\sqrt{K^2-1}\right)$$

$$G_{msg} = \frac{|S_{21}|}{|S_{12}|}$$



For a unilateral transistor, G_{tu} max is 1 by 1 minus mod S_{11} square into mod S_{21} square into 1 by 1 minus mod S_{22} square. So, when the transistor is unconditionally stable K is greater than 1. And, the maximum transducer power gain can be simply rewritten as G_t max is mod of S_{21} divided by mod of S_{12} into K minus root K square minus 1. And, we define maximum stable gain that we can get is for K is equal to 1 and this is G maximum stable gain G_{ms} g is mod S_{21} divided by mod S_{12}. Please note that when S_{12} is equal to 0 we have a unilateral case, and that is written separately here.

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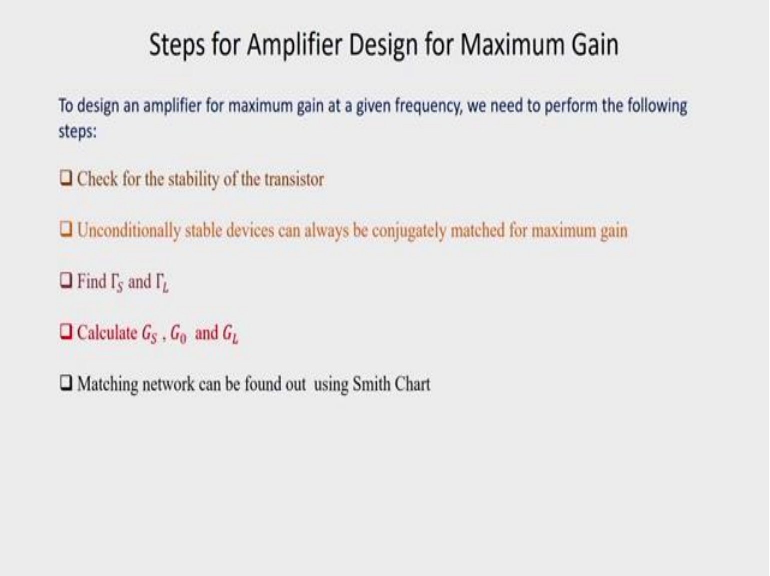

So, we can flow the steps for an amplifier design for maximum gain. We check for the stability of the transistor. We have seen that unconditionally stable devices can always be conjugately matched for maximum gain. So, we find out from the given s-parameters of the transistors at the specified frequency. We find out gamma s and gamma l. And, then we can calculate G_s, G_0, and G_l, and product of these three will give us the overall gain.

And, since we know gamma s and gamma l we can design the matching network using Smith chart by transforming Z naught by the appropriate matching network. Now, matching networks can be designed in different ways; we can use section of transmission lines and staff, we can use l section design that we have discussed. So, design of the matching network is left to the choice of the designer and also the form in which the fabrication will be simpler. Now, we have discussed how we can design an amplifier for maximum gain.

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$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}$$

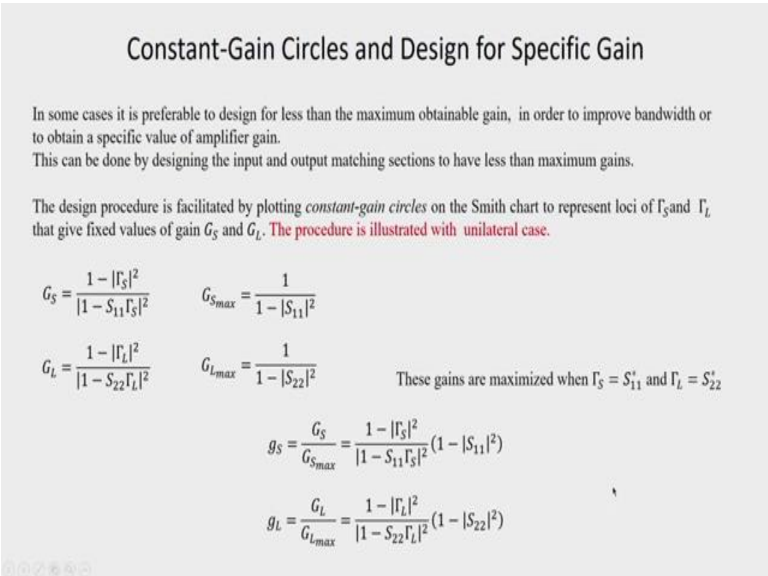
$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

$$G_{S_{max}} = \frac{1}{1 - |S_{11}|^2}$$

$$G_{L_{max}} = \frac{1}{1 - |S_{22}|^2}$$

$$g_S = \frac{G_S}{G_{S_{max}}} = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}(1 - |S_{11}|^2)$$

$$g_L = \frac{G_L}{G_{L_{max}}} = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}(1 - |S_{22}|^2)$$



But, in certain applications, it may be preferable to design less than the maximum obtainable gain. For example, if we, we know that the higher the gain, the bandwidth will lower. So, in order to improve the bandwidth, we may sacrifice for gain. Or this amplifier may be working as a part of system where we require the amplifier to provide some specific values of gain. So, we have seen that G_0 is essentially mod of S_{21} square. However we can achieve gains lower than the maximum achievable gain.

By designing and input and output matching sections to have gained less than there maximum values. That means instead of designing G_s and G_l for the maximum values we can design them for lower values and achieve the desired gain. The design procedure is facilitated by plotting constant gain circles on the Smith chart to represent loci of gamma s and gamma l. That gives fixed values of gains G_s and G_l. So, we illustrate this procedure with a unilateral case.

So, we have G_s given by this expression, G_l given by this; G_s is equal to 1 minus mod of gamma l square divided by mod of 1 minus S_{11} gamma S_{11} square. Similarly, G_l is 1 minus mod of gamma l square divided by mod of 1 minus S_{22} gamma l square. And, therefore, we have G_s max to be equal to 1 by 1 minus mod S_{11} square. That means when we have since we have unilateral transistor, gamma in is now equal to S_{11}, and gamma in conjugate becomes equal to gamma s, and therefore, this gamma s can be replaced by S_{11} conjugate.

In the same manner, gamma out becomes S_{22} when S1 to S0. Therefore, Gl max becomes 1 by 1 minus mod of S_{22} square. Now, we define small G_s is equal to G_s by G_s max, and this can be written as 1 minus mod gamma s square divided and by 1 minus S_{11} gamma S mod S square into 1 minus mod S_{11} square. And, similarly, we define the normalized G_l, capital G_l divided G_l max is equal to small G_l and its corresponding expression.

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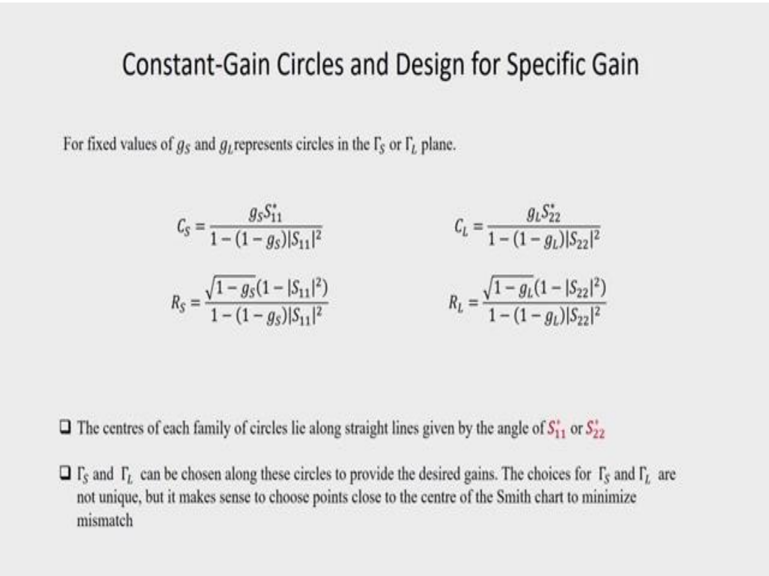
$$C_S = \frac{g_S S_{11}^*}{1-(1-g_S)|S_{11}|^2}$$

$$R_S = \frac{\sqrt{1-g_S}(1-|S_{11}|^2)}{1-(1-g_S)|S_{11}|^2}$$

$$C_L = \frac{g_L S_{22}^*}{1-(1-g_L)|S_{22}|^2}$$

$$R_L = \frac{\sqrt{1-g_L}(1-|S_{22}|^2)}{1-(1-g_L)|S_{22}|^2}$$



Now, if we fix values of G_s and G_l, they represent circles in gamma s or gamma l plane. And their centers and radius are given by C_s R_s for the G_s constant G_s and C_l R_l for constant G_l. Now, we can see that the center of each family of circles, so for different values of G_s and G_l we can have a family of constant gain circles. And, the centers of each family of circles lie along a straight line given by angels of S_{11} conjugate or S_{22} conjugate. Gamma s and gamma l can be chosen along these circles to provide the desired gains.

So, we can plot this circle's family of circles in the Smith chart. And, for a given G_s or G_l we can find out the corresponding gamma s, directly from the gamma l or directly from the Smith chart. The choices for gamma s and gamma l are not unique because we are representing constant gain circles. So, any point on that circle is a candidate but, it makes sense to choose funds closer to the center of the Smith chart to minimize mismatch.

So, these gamma s and gamma l are chosen in these circles. So, that it is closest to the center of the smith chart, and therefore, the mismatch is reduced. We explain here with some numerical values.

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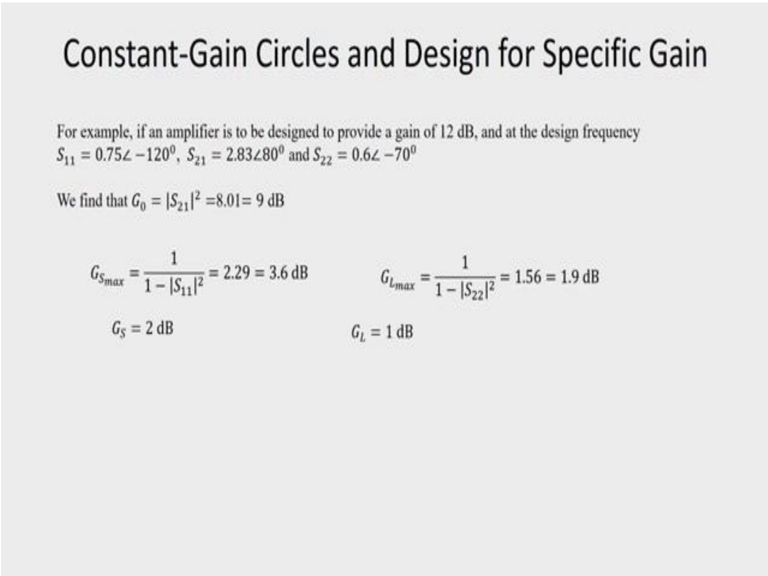
$$G_{S_{max}} = \frac{1}{1 - |S_{11}|^2} = 2.29 = 3.6 \text{ dB}$$

$$G_{L_{max}} = \frac{1}{1 - |S_{22}|^2} = 1.56 = 1.9 \text{ dB}$$



For example, if an amplifier is designed to provide a gain of 12 dB and at the design frequency. Suppose we have S_{11} is equal to 0.75, angel is minus 120 degrees S_{21} is 2.83 with angel 80 degrees, and S_{22} is 0.6 with angel 70 degrees. And, we have already mentioned that S_{12} is 0. In that case we find that G naught is roughly 9 dB, and Gs max is 3.6 dB, and G_l max is 1.9 dB. So, what we can do in order to realize this12 dB gain, we can choose G_s to be 2 dB and Gl to be 1 dB. And, also we know that this constant G_l circle their center will now lie at 120 degrees. Because it lies along the line defining the angle of S_{11} conjugate. Similarly, the other circle will lie at 70 degrees.

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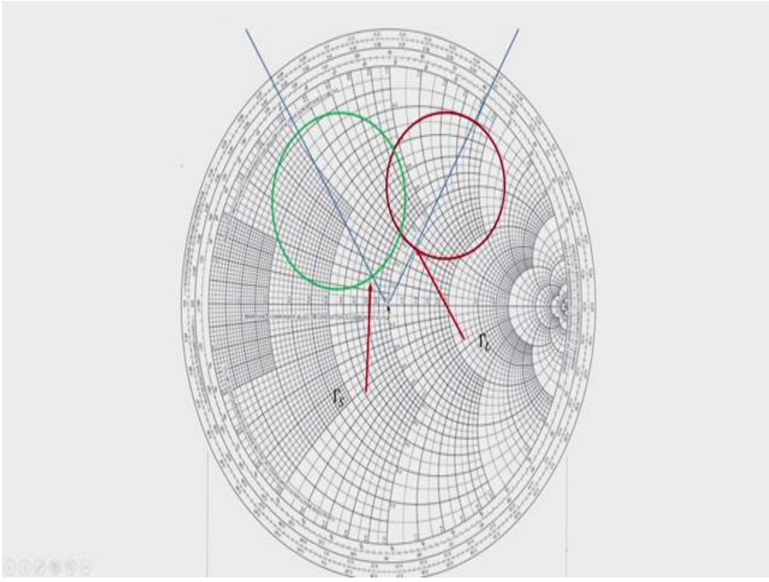

So, these are not exact values just to show the representative values of how we can use this information. For, example this will be G_s circle this maybe G_l circle. And, the points that are closest to the center of the Smith chart and intersecting with the circle is this. Similarly, this is the closest point intersecting with constant G_l circle. And nearest to the Smith chart, so we will choose this value of gamma l and this value as gamma s. Once, again it should be noted that this circle has not been drone to the scale this is just for representation purposes only.

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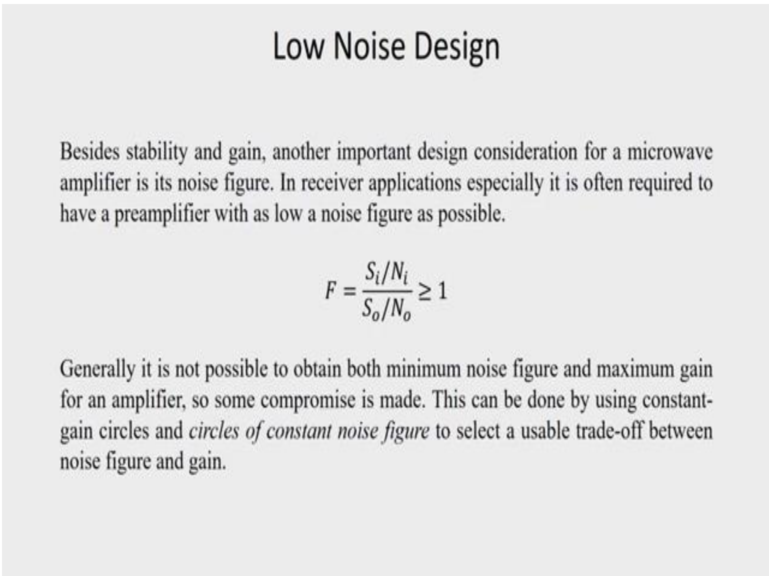
$$F = \frac{S_i/N_i}{S_o/N_o} \geq 1$$

$$F = F_{min} + \frac{R_N}{G_S}\left|Y_S - Y_{opt}\right|^2$$



Next, we move on to low noise design. So, we have seen the two cases of single-stage transistor amplifier design. Designing for maximum gain and designing for specified gain. Let us now consider another issue the low wise design. So, besides stability and gain another important design consideration for a microwave amplifier is its noise figure. Particularly in receiver applications it is often required to have a preamplifier with a noise figure as low as possible.

Now, we know that the amplifiers add noise, and the noise figure is defined as the signal to noise ratio at the input divided by signal to noise ratio at the output of the system. And, usually it is greater than 1 because of S naught by N naught. At the output of the amplifier becomes lower than Si by Ni at the input of the amplifier.

So, while designing microwave amplifiers, it is not possible to obtain both minimum noise figure and maximum gain. Some compromise is made where we operate the amplifier at slightly reduces gain to achieve a specified noise figure. And, this can be done by using constant gain circles and circles of constant noise figure to select a suitable rate of between noise figure and gain.

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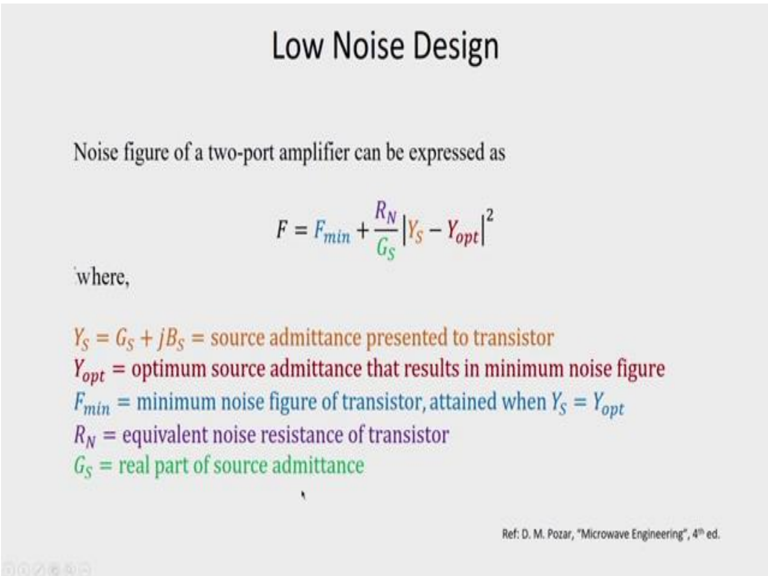
$$F = F_{min} + \frac{R_N}{G_S}\left|Y_S - Y_{opt}\right|^2$$



A noise figure of two-port amplifier can be expressed as F equal to F min, minimum noise figure plus Rn by Gs into mod of Ys minus Y opt square. Now, this Y_s G_s plus jBs is a source admittance. Y opts the optimum source admittance, which results in a minimum noise figure. So, we can see that when Y_s is equal to Y opt F becomes equal to F_{min}. And, F_{min} minimum noise of the transistor, R_n is equivalent noise resistance of the transistor. And, G_s is the real part of the source admittance.

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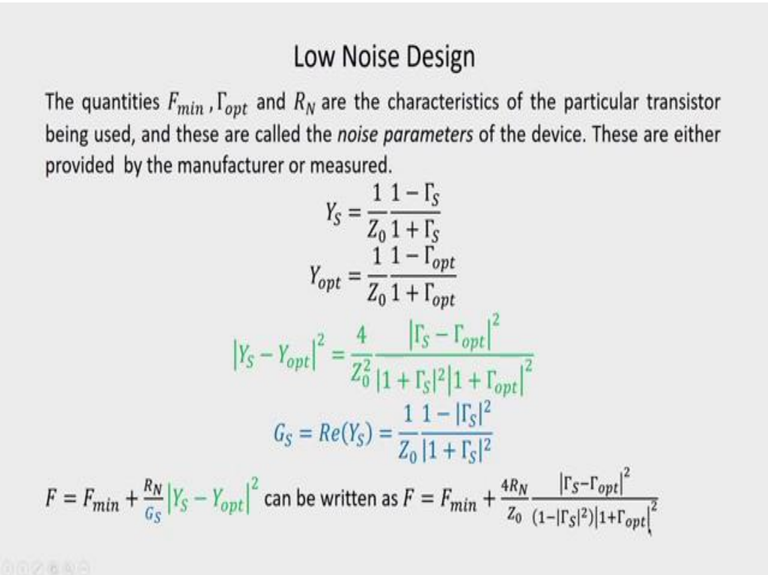
$$Y_S = \frac{1}{Z_0}\frac{1-\Gamma_S}{1+\Gamma_S}$$

$$Y_{opt} = \frac{1}{Z_0}\frac{1-\Gamma_{opt}}{1+\Gamma_{opt}}$$

$$\left|Y_S - Y_{opt}\right|^2 = \frac{4}{Z_0^2}\frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{|1+\Gamma_S|^2\left|1+\Gamma_{opt}\right|^2}$$

$$G_S = Re(Y_S) = \frac{1}{Z_0}\frac{1-|\Gamma_S|^2}{|1+\Gamma_S|^2}$$

$F = F_{min} + \frac{R_N}{G_S}\left|Y_S - Y_{opt}\right|^2$ can be written as $F = F_{min} + \frac{4R_N}{Z_0}\frac{\left|\Gamma_S-\Gamma_{opt}\right|^2}{(1-|\Gamma_S|^2)\left|1+\Gamma_{opt}\right|^2}$



The quantities F min gamma opt as you will see it is related to Y opt, and R_n are the characteristics of the particular transistor been used. And, these are called the noise parameters of the device. Now, these parameters can be either provided by the manufacturer in the data sheet or this can be measured. So, we have Y_s related to gamma s as 1 by Z naught 1 minus gamma s divided by 1 plus gamma s. In the same manner, Y_{opt} related to 1 by Z naught is related to gamma opts 1 by Z naught 1 minus gamma_{opt} divided by 1 plus gamma opt.

Now, in the expression, for we have mod of Ys minus Y opt square, so this term can be calculated from Y_s and Y_{opt}. And, also G_s which is the real part of Y_s, can be calculated. And, then in the expression for F we substitute G_s and mod of Y_s minus Y opt square from here. And, then we get the expression for F as F_{min} plus 4 R_n divided by Z naught into gamma s minus gamma opt magnitude square divided by 1 minus magnitude of gamma s square. Into 1 plus gamma opt magnitude square.

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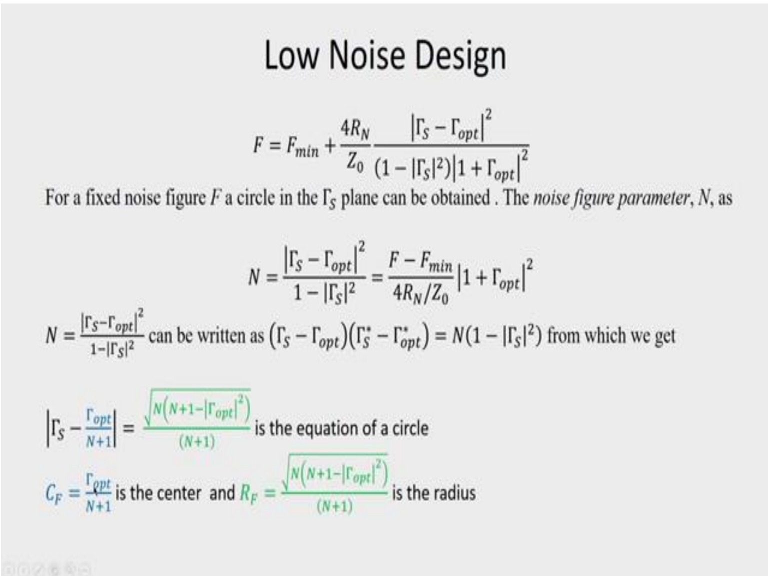
$$F = F_{min} + \frac{4R_N}{Z_0}\frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{(1-|\Gamma_S|^2)\left|1+\Gamma_{opt}\right|^2}$$

$$N = \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{1-|\Gamma_S|^2} = \frac{F - F_{min}}{4R_N/Z_0}\left|1+\Gamma_{opt}\right|^2$$

$N = \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{1-|\Gamma_S|^2}$ can be written as $\left(\Gamma_S - \Gamma_{opt}\right)\left(\Gamma_S^* - \Gamma_{opt}^*\right) = N(1-|\Gamma_S|^2)$ from which we get

$\left|\Gamma_S - \frac{\Gamma_{opt}}{N+1}\right| = \frac{\sqrt{N\left(N+1-\left|\Gamma_{opt}\right|^2\right)}}{(N+1)}$ is the equation of a circle

$C_F = \frac{\Gamma_{opt}}{N+1}$ is the center and $R_F = \frac{\sqrt{N\left(N+1-\left|\Gamma_{opt}\right|^2\right)}}{(N+1)}$ is the radius



Now, for a fixed noise figure F, a circle in the gamma s plane can be obtained from this expression. So, a parameter n, which is the noise figure parameter, is introduced. And, it is defined as mod gamma s minus gamma opt square divided by 1 minus mod of gamma s square. So, essentially the ratio of these two terms and then we get this equal to F minus F min divided by 4 R_n divided by Z naught. Mod of 1 plus gamma opt square. Now, this expression N is equal to gamma s minus gamma opt magnitude square divided by 1 minus mod gamma s square.

It can be written in this form gamma s minus gamma opt into gamma s conjugate minus gamma opt conjugate is equal to N into 1 minus magnitude of gamma S square. And, from this expression, we can actually get the equation of the circle. And, the center of this circle C_f is gamma opt divided by N plus 1, and radius of the circle R_f is given by square root of N into N plus 1 minus gamma opt square divided by N plus 1. Now, we see that gamma opt a complex quantity, and this center C_f will lie along a line defined by the angel of gamma opt in the Smith chart.

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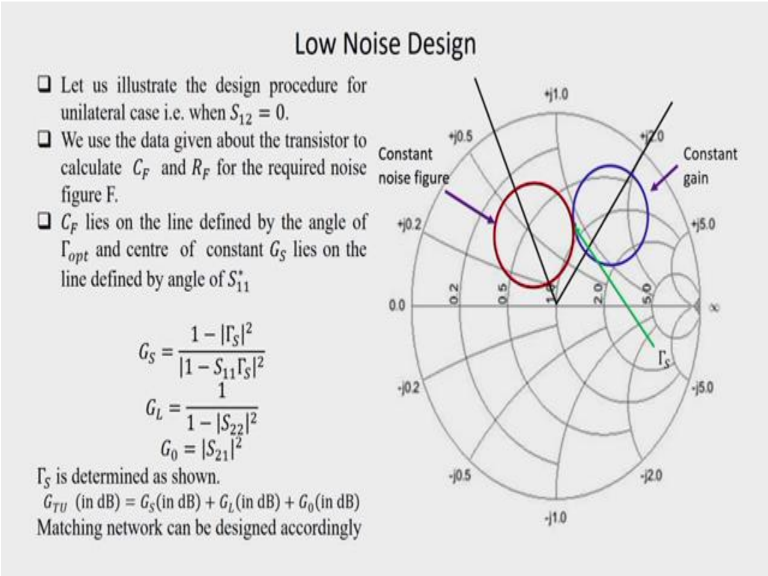

So, let us illustrate the design procedure for the unilateral case that is when S1 to equal to 0. We use the data given about the transistor to calculate C_f and R_f for the required noise figure F. C_f lies on the line defined by the angle of gamma opt and center of constant G_s circle lies on the line defined by the angel of S_{11} conjugate. So, these are the two circles. This is the G_s circle just touching the constant the circle corresponding to the required noise figure F. And, this point of intersection is the gamma s, which actually gives the value of G_s for the required F.

And, once we have this gamma s we can find out G_s G_l we can find out to be 1 by 1 minus mod of S_{22} square. Because S_{12} is 0 and G naught is given by mod of S_{21} square, and we can calculate Gtu in dB as sum of G_s G_l and G naught in dB. So, now we have the gain of G_s but designed in such a way it satisfies the requirement of the noise figure F. Once, we have gamma s and gamma L, we can see that gamma s value we have already calculated. And, once we have this gamma s and gamma l, we can design a matching network following the procedure.

That is used to transform Z naught to either gamma s or gamma l using stub section of transmission lines or plumbed elements. So, this is in brief how we can design an amplifier with a specified noise figure. And, that design is achieved by adjusting the gain G_s, and we cannot have maximum gain from such design. So, we have discussed different issues related to the design of amplifiers. So, in the next lecture we will discuss R_f oscillators and will see some of the steps involved in design of such oscillators.