**Microwave Engineering**

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# Lecture 29 - Electronic transit time effect

We have seen two limitations of the conventional vacuum tubes when we try to operate them at the microwave frequencies. One limitation comes from the effect of parasitic elements, the parasitic capacitance and inductance, and another limitation comes from the transit time effect, electron transit time from cathode to grid.

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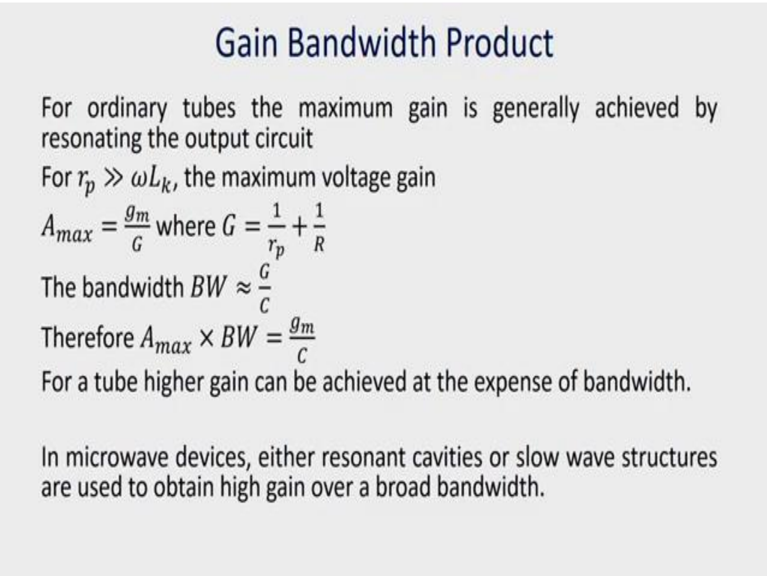
For $r_p \gg \omega L_k$, the maximum voltage gain

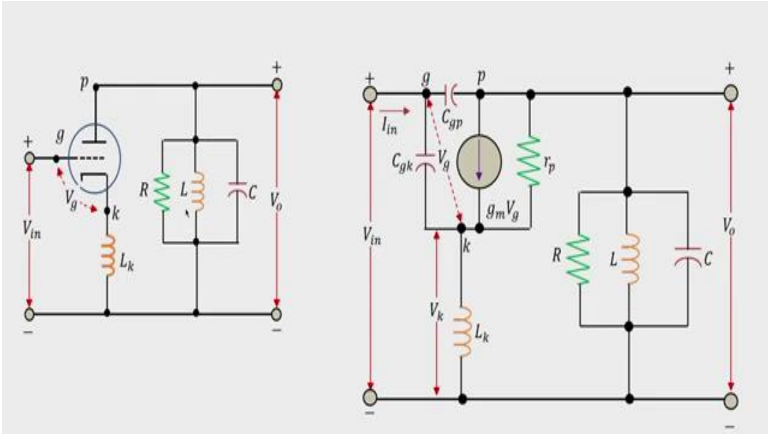
$A_{max} = \frac{g_m}{G}$ where $G = \frac{1}{r_p} + \frac{1}{R}$

The bandwidth $BW \approx \frac{G}{C}$

Therefore $A_{max} \times BW = \frac{g_m}{C}$

For a tube higher gain can be






Triode equivalent circuit taking into account the inter electrode capacitances and cathode inductance as the parasitic elements

Ref: S. M. Liao, Microwave devices and Circuits; 3/e, Prentice Hall of India, 2004.

Conventional tubes have another limitation in the form of a gain-bandwidth product. For ordinary tubes the maximum gain is generally achieved by resonating the output circuit. So, if we consider the circuit here we see that there is a resonated circuit RLC resonated circuit present and what we assume now that omega L k is very very less as compared to the plate resistance r P.

For r P very very large compared to omega L k the maximum voltage gain $A_{max}$ which becomes V output by $V_g$ and this becomes equals to gm by G, where G is equal to 1 by r P plus 1 by R. Now this particular condition occurs at resonance. Now we find out the bandwidth around this resonant frequency where the gain, maximum gain becomes 1 by root 2. And if we find out the bandwidth then it will be approximately G by C.

And therefore, $A_{max}$ product of $A_{max}$ and bandwidth is equal to gm by C. Now, we find that for a given tube and a resonant circuit we have the right-hand side independent of frequency. And therefore, if we want to maximize the gain we will have to do it at the expense of bandwidth. So for this type of tubes higher gain can be achieved at the expense of bandwidth.

In microwave devices, either resonant cavities or slow-wave structures are used to obtain high gain over a broad bandwidth. So we have discussed the major limitations which prevent the use of conventional tubes like triodes, tetrodes or pentodes at the microwave frequency range. And next we will see how specially designed tubes operate at the microwave frequency, and they provide high gain. They can be operated at, operated over broadband of frequencies. So we start our discussion on microwave tubes with a two-cavity klystron. So this type of klystron tubes is used as amplifiers.

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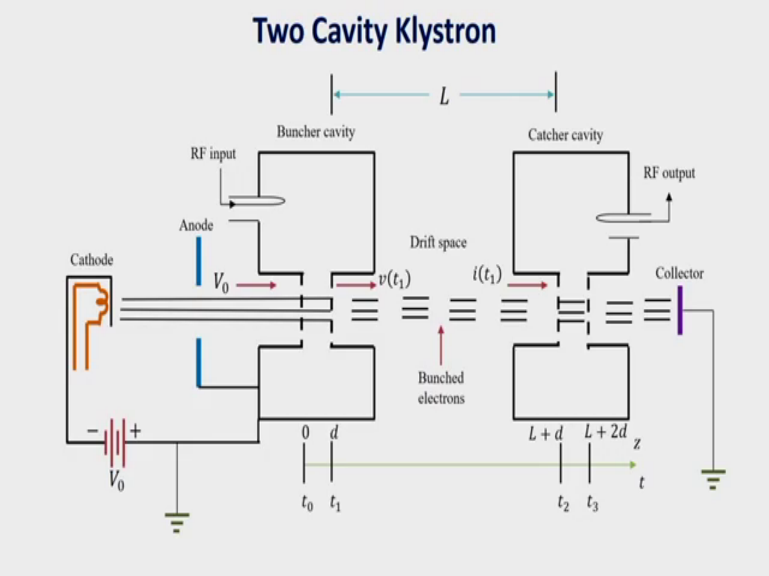


Here, we have a cathode which is heated, and it emits electrons which are accelerated by the anode and this accelerated electrons pass through the anode, we have two specially designed cavities, and here the cavity walls are made narrow and also the electrons can pass through with, RF input is applied at one cavity which is called the buncher cavity, this is buncher cavity we will see that the RF input applied here will help to form bunches of electrons.

Separated from the buncher cavity by a distance L is the catcher cavity, and this bunched electrons while passing through this catcher cavity will deliver energy and the oscillations in the catcher cavity will build up. And finally RF output is taken out from this cavity, so as a result we will get an amplified version of the signal at this RF output. The electrons, after delivering their energy they slow down, and they are collected by the collector.

Here we have two scales, one showing the distance z, and another showing time t, so at the instant $t_0$ the electrons will enter the buncher cavity. At the instant $t_1$ they will leave the buncher cavity. It may be noted that in this region there is no electric field and this is the drift space, so the emitted electrons from the cathode they come to the buncher cavity and RF input applied at the buncher cavity, we will see that they will produce velocity modulation resulting in bunching of electrons and then this bunched electrons they will move in the drift space.

And finally, when they pass through the catcher cavity they will deliver energy to the catcher cavity. And finally they will be collected by the collector. So these are the bunched electrons. So let us first see the operation of this type of tubes in brief and how the transit time effect is utilized here for producing velocity modulation of the electrons.

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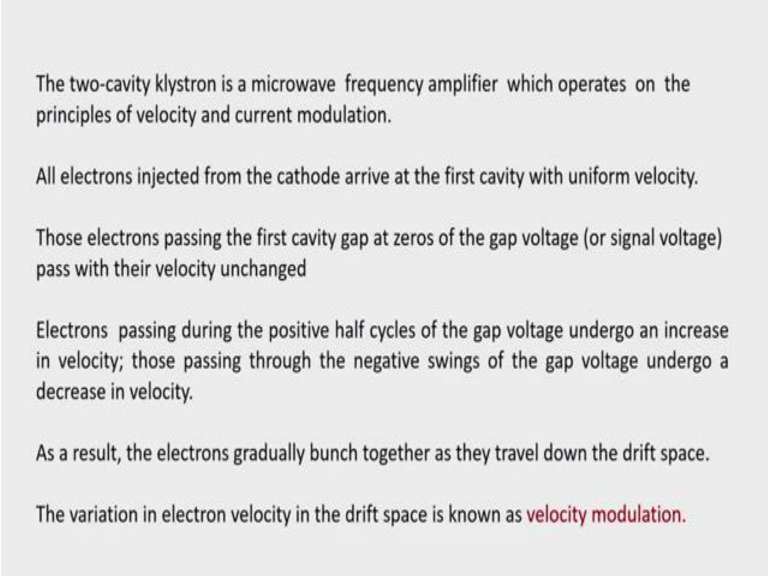


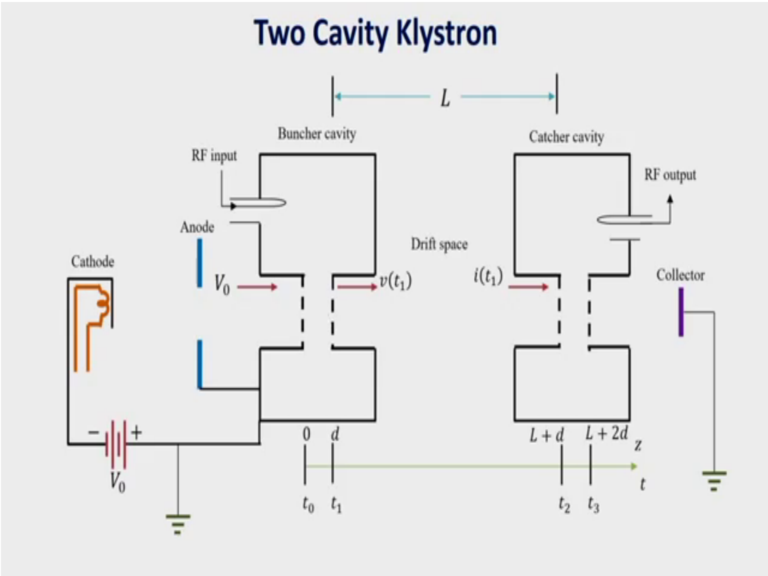


The two-cavity klystron is a microwave frequency amplifier which operates on the principle of velocity and current modulation. All electrons injected from the cathode arrive at the first cavity with uniform velocity. These electrons passing the first cavity gap at zeroes of the gap voltage pass with their velocity unchanged. So, if it happens that the electrons appear at the buncher cavity at an instant when the RF signal is going through its 0 at that time the electrons velocity will remain unchanged.

Electrons passing during the positive half cycle of the gap voltage undergo an increase in velocity, and those passing through the negative swings of the gap voltage undergo a decrease in velocity. So depending upon the voltage present at the gap, here we can see that when the

RF voltage is present, the electric field will either accelerate the electrons, or it will retard the electrons, and when this electric field here is 0, the velocity will remain unaffected.

Now, when we have the electrons, they are now moving with different velocities as they leave the buncher cavity. If it happens that the electrons which entered the buncher cavity in some earlier instant of time they have been slowed down and the electrons which entered the buncher cavity at a later instant of time and they got accelerated then these electrons, the first moving electrons after some distance will catch up this slow-moving electrons which entered earlier and we will have a bunch of electrons formed.

So, as a result, the electrons gradually bunched together as they travel down the drift space. Now, the variation in electron velocity in the drift space is known as velocity modulation. Now, as a result, since there are bunching of electrons at some instant of time the density of electrons will be higher. At some instant of time, the density of electrons will be lower.

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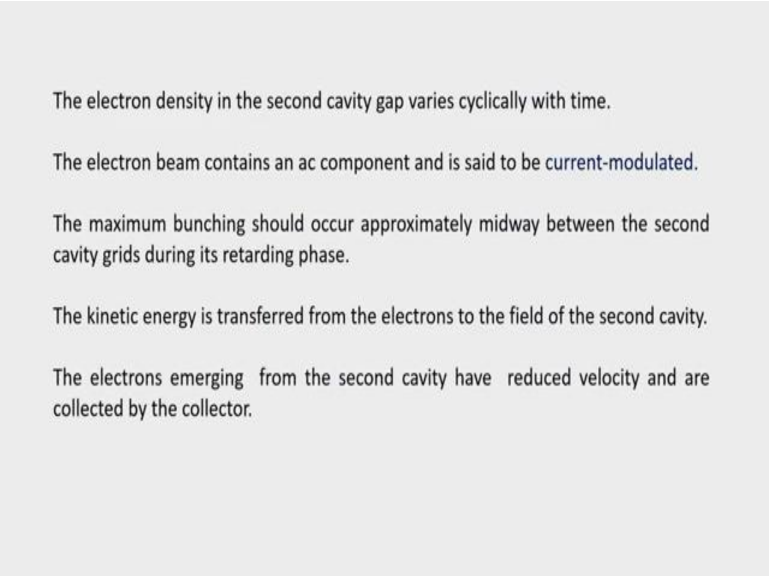


And the electron density in the second cavity gap varies cyclically with time. Therefore, the electron beam that passes through the catcher cavity, the second cavity contains an AC component and is said to be current modulated. Now, if we want the power transfer from the beam to the field in the catcher cavity, the maximum bunching should occur approximately midway between the second cavity grids and during the retarding phase.

So when the second cavity field has a retarding phase, it will decelerate the electrons, and the field will grow. So energy will get transferred from the electrons to the field. The kinetic energy is thus transferred from electrons to the field of the second cavity. The electrons emerging from the second cavity have reduced velocity and are collected by the collector. So because of this energy transfer from the electrons to the field in the second cavity, the field in second cavity grows and we have amplification of signal possible.

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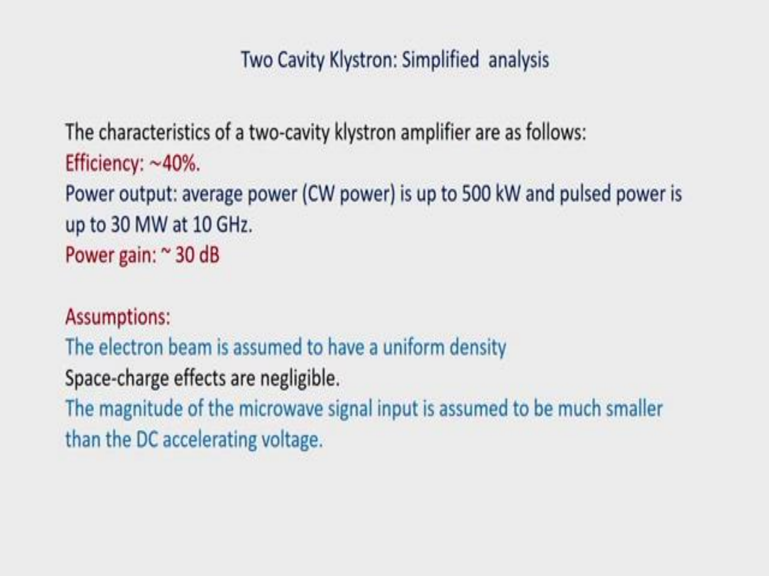


Let us now discuss some features of two-cavity klystron and also let us do some simplified analysis of this type of klystron device. The main characteristics of a two-cavity klystron amplifier are as follows: Its efficiency is of the order of 40 percent. The power output we generally talk of two types of power output, one is the CW power or continuous wave power, it can be up to 500 kilowatts and if it is a pulsed power that means the klystron is delivering power over a brief period and then it goes off, then again deliver power.

So, in that case, the peak power may go as high as 30 megawatts at around 10 Gigahertz of frequency. So the pulse power is the power delivered over a very small period of time followed by another period when the device essentially does not give any output power. So it is the duty cycle which we will decide the average power output. And power gain that can be achieved with this type of amplifier is again of the order of 30 dB. So 30 dB power gain means a power gain of 1000.

While carrying out the simplified analysis of this type of klystron devices, we make the following assumptions: The electron beam is assumed to have uniform density across its cross-section. The space charge effects are negligible, so when we have a beam of electron there will be mutual repulsion, and those effects are not considered here. The magnitude of the microwave signal input is assumed to be much smaller than the DC accelerating voltage.

So the input signal that we put in the buncher cavity its magnitude is considered to be very very less as compared to the DC accelerating voltage between the anode and the cathode. So assume that the electrons had just emitted from the cathode because of the cathode being thermally heated we have after being accelerated through a voltage V naught the kinetic energy half MV square is equal to eV naught.

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$$\frac{1}{2}mv^2 = eV_0$$

$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6\sqrt{V_0}$ m/s

Applied microwave voltage $V_s = V_1 \sin(\omega t)$

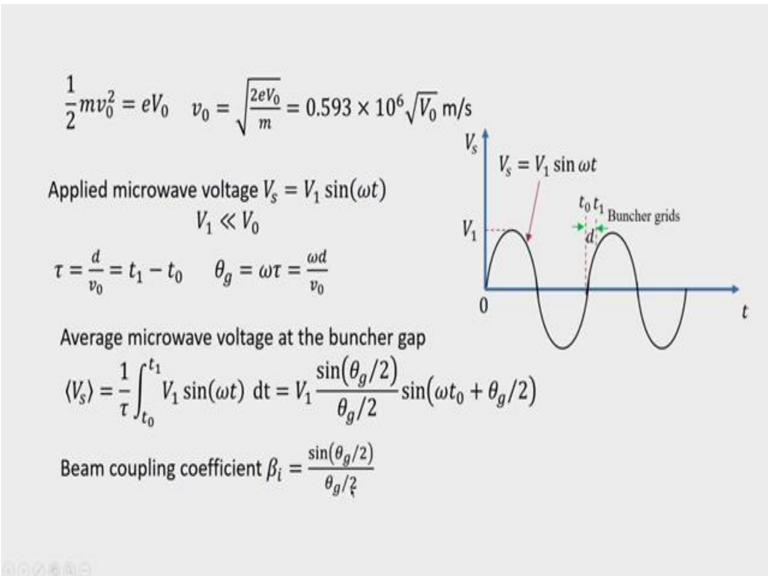
$$V_1 \ll V_0$$

$\tau = \frac{d}{v_0} = t_1 - t_0 \qquad \theta_g = \omega\tau = \frac{\omega d}{v_0}$

Average microwave voltage at the buncher gap

$$\langle V_s \rangle = \frac{1}{\tau}\int_{t_0}^{t_1} V_1 \sin(\omega t)\ \mathrm{dt} = V_1 \frac{\sin(\theta_g/2)}{\theta_g/2}\sin(\omega t_0 + \theta_g/2)$$

Beam coupling coefficient $\beta_i = \frac{\sin(\theta_g/2)}{\theta_g/2}$




And therefore, we can write the velocity of the electron after being accelerated by a voltage capital V naught is given by under root 2 eV naught by m is the electronic charge, m is the mass of the electron, so when these typical values are substituted we get the velocity of the electron to be 0.593 into 10 to the power 6 under root capital V naught meter per second.

Let the applied microwave signal be $V_s$ is equal to $V_1$ sin omega t, and as we have already mentioned that $V_1$ is very very less compared to V naught. Now this voltage $V_s$ will appear across the buncher cavity, and since the spacing between the buncher grids is kept small we can see that as the electron passes through the buncher cavity it will experience an average voltage that can be calculated by integrating this signal from the entry time t naught to exit time t1.

And therefore, we can write tau, which is d by v naught is equal to t1 minus t naught, so this is the transit time and theta g this is the transit angle is omega tau is omega d by v naught. And average microwave voltage that it experiences at the buncher gap is 1 by tau $t_0$ to $t_1$ $V_1$ sin omega t dt and when this integral is evaluated, we can write average voltage to be equal to $V_1$ sine theta g by 2 divided by theta g by 2 sine omega t naught plus theta g by 2. We define a parameter beta i which is the beam coupling coefficient, and its value is given by sine theta g by 2 divided by theta g by 2.

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$$\mathrm{v}(t_1) = \sqrt{\frac{2e}{m}\left(V_0 + \beta_i V_1 \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right)} = \sqrt{\frac{2eV_0}{m}\left(1 + \frac{\beta_i V_1}{V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right)}$$

$\frac{\beta_i V_1}{V_0}$ is the depth of modulation

Since $\frac{\beta_i V_1}{V_0} \ll 1$ , $\mathrm{v}(t_1) = v_0\left(1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right)$

Alternately,

$$\mathrm{v}(t_1) = v_0\left(1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right)$$

$$v(t_1) = \sqrt{\frac{2e}{m}\left(V_0 + \beta_i V_1 \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right)} = \sqrt{\frac{2eV_0}{m}\left(1 + \frac{\beta_i V_1}{V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right)}$$

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Now this voltage, the average voltage is the extra voltage that the electrons they experience during their passage through the buncher cavity, and depending upon at which instant the electrons enter the buncher cavity this average voltage will vary. And therefore, now we can write velocity of the electron at the time of exit t1 is 2e by m, earlier it was only V naught, now it is V naught plus beta i $V_1$ sine Omega t naught plus theta g by 2.

And if we take out V naught then it becomes 2 eV naught by m, 1 plus beta i $V_1$ by V naught sine omega t naught plus theta g by 2. Now this parameter, beta i $V_1$ by V naught it is the depth of modulation because depending upon this we now find that apart from this velocity 2 eV naught by m under root which is the DC component of the velocity or v naught small v naught this will now change because of this factor.

And since we have assumed that $V_1$ by V naught is very, very small compared to 1 and beta i is sine theta G by 2 divided by theta G by 2, so it will also be less than equal to 1. Therefore, this entire term will be very very less compared to 1, and then we can write root 2 eV naught by m is equal to small v naught, the velocity of the electron only under the applied DC field and here we can do a Taylor series expansion, so this is approximately V naught 1 plus beta i $V_1$ by 2 V naught this half is coming from this square root sine omega t naught plus theta g by 2.

So now, we find that the velocity v at $t_1$ it has modulation, and if we write everything in terms of $t_1$ then we can write v $t_1$ is equal to v naught 1 plus beta i $V_1$ divided by 2 V naught sine omega t1 minus theta g by 2.

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$$v(t_1) = v_0\left(1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right)$$

$$v_{max} = v_0\left(1 + \frac{\beta_i V_1}{2V_0}\right)$$

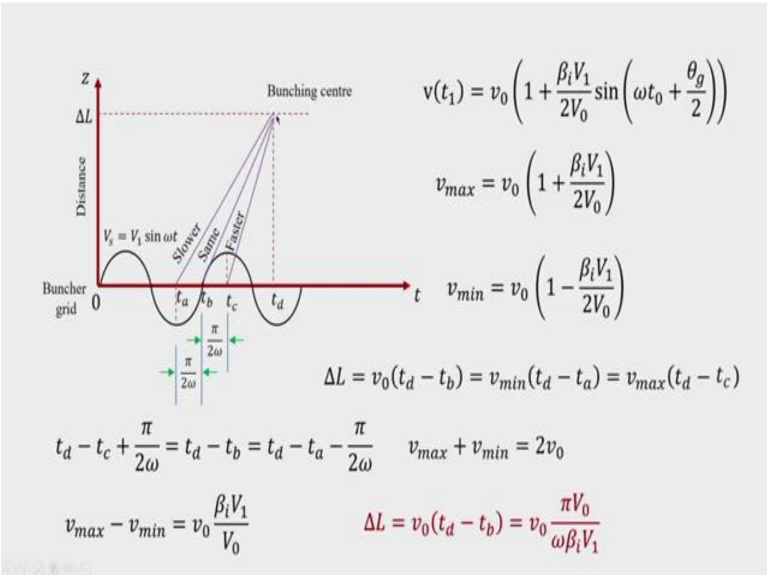
$$v_{min} = v_0\left(1 - \frac{\beta_i V_1}{2V_0}\right)$$

$$\Delta L = v_0(t_d - t_b) = v_{min}(t_d - t_a) = v_{max}(t_d - t_a)$$

$$t_d - t_c + \frac{\pi}{2\omega} = t_d - t_b = t_d - t_a - \frac{\pi}{2\omega}$$

$$v_{max} - v_{min} = v_0\frac{\beta_i V_1}{V_0}$$

$$\Delta L = v_0(t_d - t_b) = v_0\frac{\pi V_0}{\omega\beta_i V_1}$$




Now, this is the diagram where we explain the effect of this velocity modulation, and we have already mentioned that when this velocity modulation is present, the slower moving electrons which entered earlier they can meet faster-moving electrons which enter later. So here we find that at time t the electron entered, it experienced a retarding voltage and it slowed down. At this instant the buncher cavity grids did not apply any field, so the electrons move with the same velocity V naught. Here at $t_c$ the electrons experience an accelerating field, and this electron moves faster.

Now when we have this type of a scenario where we have the earlier entering electron is slowed down and later entering electron it is, its velocity is increased after travelling some distance which we are denoting by delta L here, they will meet each other. Now this time is marked as t d here, and this is representing one-fourth of a cycle, so this time is capital T by 4 and which is equal to pi by twice omega.

Now we have seen that v $t_1$ is equal to v naught 1 plus beta i $V_1$ by 2 V naught sine omega t naught plus theta g by 2. Therefore, the maximum velocity that an electron can attain is when sine omega t plus theta g by 2 becomes 1, and therefore $v_{max}$ has been denoted as v naught 1 plus beta i $V_1$ by 2 V naught. Similarly, when this sign becomes negative the $v_{min}$ minimum velocity is v naught 1 minus beta i $V_1$ by 2 V naught.

And we can write delta L, this is v naught into $t_d$ minus t b, or it may be written as v min into $t_d$ minus $t_a$, or it can also be written as v max into $t_d$ minus $t_c$. Now we can write $t_d$ minus $t_c$ plus pi by 2 omega, so this is actually $t_d$ minus $t_b$, and this is same as $t_d$ minus $t_a$ minus pi by 2 omega. We also find that v max plus v min is equal to 2 v naught, and v max minus v min is equal to v naught beta i $V_1$ by V naught.

So using these relations, for example, we can multiply here by $v_{min}$, we can multiply here by v max, and then v max $t_d$ minus $t_c$ can be replaced by v naught $t_d$ minus $t_b$. Similarly, this side we can multiply it by v min, so with this type of rearrangement of terms we can find out that delta L which is equal to v naught t d minus t b, v naught into $t_d$ minus $t_b$ and this can be written as v naught pi capital V naught divided by omega beta i $V_1$.

So this gives us the expression for the distance where the bunching occurs, and we mentioned that we want the bunching to occur in the middle of the catcher grid and at an instant when the phase is a retarding phase. Delta L gives us an expression for the distance where bunching occurs.

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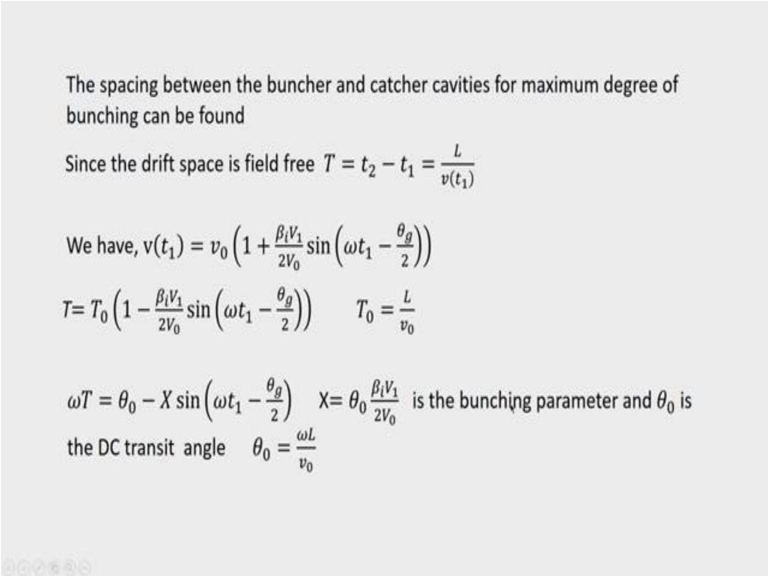
Since the drift space is field free $T = t_2 - t_1 = \frac{L}{v(t_1)}$

We have, $v(t_1) = v_0\left(1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right)$

$T = T_0\left(1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right)$ $\quad T_0 = \frac{L}{v_0}$

$\omega T = \theta_0 - X\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)$ $\quad X = \theta_0 \frac{\beta_i V_1}{2V_0}$ is the bunching parameter and $\theta_0$ is the DC transit angle $\quad \theta_0 = \frac{\omega L}{v_0}$




Let us see how the spacing between the buncher and catcher cavities can be adjusted for the maximum degree of bunching. Since the drift space is field-free we can write capital T is equal to $t_2$ minus $t_1$ where $t_2$ is the time when the electron entered the catcher cavity, and this can be written as L by v $t_1$ this is because the disk space is field free and the electron moves with the velocity v $t_1$.

Now, we have already seen V $t_1$ is v naught into 1 plus beta i $V_1$ divided by 2 V naught sine omega t1 minus theta g by 2. And therefore, we can write capital T is equal to L by v naught we can write T naught and 1 by this term can be written as 1 minus beta i $V_1$ by 2 V naught sine omega t1 minus theta g by 2. Please note that this minus sign comes from the fact that beta i $V_1$ by 2 V naught it is very, very less compared to 1. And therefore, 1 plus beta i $V_1$ by 2 V naught sine omega t1 minus theta g by 2 raised to the power minus 1 can be approximated by this expression.

And if you multiply both sides by omega, we get omega capital T is equal to theta naught, theta naught is omega T naught, this is the DC transit angle and then we have omega t naught beta i $V_1$ by 2V naught that means theta naught into beta i $V_1$ by 2 V naught which we represent by X and this X is the bunching parameter of the klystron.

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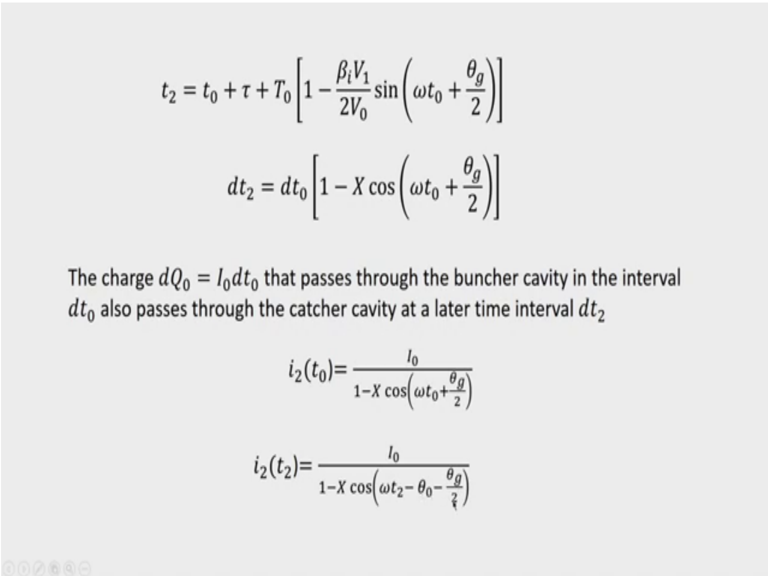
$$t_2 = t_0 + \tau + T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

$$dt_2 = dt_0\left[1 - X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

The charge $dQ_0 = I_0 dt_0$ that passes through the buncher cavity in the interval $dt_0$ also passes through the catcher cavity at a later time interval $dt_2$

$$i_2(t_0) = \frac{I_0}{1 - X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)}$$

$$i_2(t_2) = \frac{I_0}{1 - X\cos\left(\omega t_2 - \theta_0 - \frac{\theta_g}{2}\right)}$$




Now $t_2$ is equal to t naught the time of entry of the electrons in the buncher cavity plus tau the transit time of the electrons between the buncher grids plus this term t naught 1 minus beta i $V_1$ by 2 V naught sine omega t naught plus theta g by 2, this is the transit time of the electrons in the drift space. So sum of this three times gives the time $t_2$ the entry time of electron in the catcher cavity.

Therefore, we can write $dt_2$ is equal to if we differentiate with respect to t naught, then we can write $dt_2$ is equal to dt naught into 1 minus X cos omega t naught plus theta g by 2, this is, this we obtain by differentiating this $t_2$ expression with respect to t naught. Now, in the buncher cavity a charge amounting to dQ naught equal to I naught dt naught this passes, and this amount of charge passes in the interval dt naught in the buncher cavity.

The same amount of charge will appear at the catcher cavity at a later time interval $dt_2$. And therefore, $i_2$ into $dt_2$ will give the same amount of charge that has passed through the catcher cavity, and we can write $i_2$ as a function of t naught is equal to I naught divided by 1 minus X cos omega t naught plus theta g by 2. So this expression we get by equating $i_2$ $dt_2$ is equal to I naught dt naught. And in terms of $t_2$ we can write $i_2$ $t_2$ is equal to I naught divided by 1 minus X cos omega $t_2$ minus theta naught minus theta g by 2.

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Beam current in the catcher cavity is periodic and it can be expanded in Fourier series

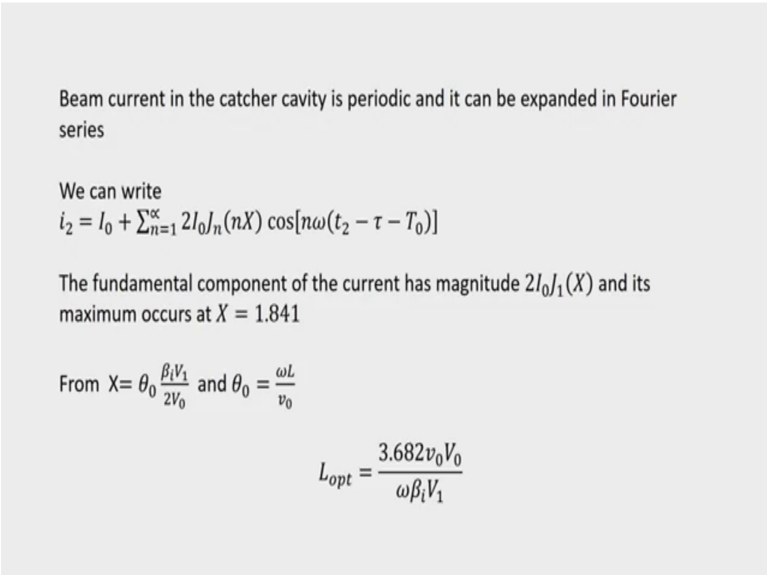
We can write

$i_2 = I_0 + \sum_{n=1}^{\propto} 2I_0 J_n(nX)\cos[n\omega(t_2 - \tau - T_0)]$

The fundamental component of the current has magnitude $2I_0J_1(X)$ and its maximum occurs at $X = 1.841$

From X= $\theta_0 \frac{\beta_i V_1}{2V_0}$ and $\theta_0 = \frac{\omega L}{v_0}$

$$L_{opt} = \frac{3.682 v_0 V_0}{\omega \beta_i V_1}$$



$$t_2 = t_0 + \tau + T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

$$dt_2 = dt_0\left[1 - X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

The charge $dQ_0 = I_0 dt_0$ that passes through the buncher cavity in the interval $dt_0$ also passes through the catcher cavity at a later time interval $dt_2$

$$i_2(t_0) = \frac{I_0}{1 - X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)}$$

$$i_2(t_2) = \frac{I_0}{1 - X\cos\left(\omega t_2 - \theta_0 - \frac{\theta_g}{2}\right)}$$

The beam current in the catcher cavity $i_2$ as we have seen it is periodic, we have just seen that $i_2$ is periodic. And therefore, it can be expanded in a Fourier series. So once we expand this in terms of Fourier series and find out the Fourier coefficients, $i_2$ can be written as $i_2$ equal to I naught plus summation of n equal to 1 to infinity 2 I naught J n $n^{th}$ order Bessel function J n nX cos n omega $t_2$ minus tau minus T naught.

Now this $i_2$ has several harmonic components because of the catcher cavity, because of the resonant behavior of the catcher cavity this cavity will actually pick up the fundamental component and the fundamental component of the current has a magnitude of 2 I naught J 1x and the maximum of this fundamental component occurs at X is equal to 1.841. So J 1x the first-order Bessel function it attains its peak value for X equal to 1.841.

Suppose the arrangement of the cavities are made in such a way that we are able to achieve the peak of this fundamental component, then from this relation X is equal to theta naught beta i V 1 by 2 V naught and theta naught is equal to omega L by V naught. And we also know that X is equal to 1.841. We get the optimum value of L, this L what should be the spacing between the cavities, so as to have a peak of the fundamental current. This comes out to be 3.682 small v naught capital V naught divided by omega beta i $V_1$. So at this spacing we get the maximum bunching to occur at the center of the catcher cavity.