Microwave Engineering
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Lecture 03
Introduction to Microwave Engineering and Transmission line theory

In the previous lecture, we discussed about the wave propagation in a transmission line and particularly focused on the characteristics of lossless line.

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- Terminated lossy lines

In this lecture we discussed about the lossy transmission line, we discussed about a special form of lossy line which is called Distortionless line and we also discuss the Terminated lossy lines.

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Lossy lines

We have seen that for a transmission line, the complex propagation constant is given by:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

where, α is the attenuation constant and β is the phase constant.

$$\therefore\ (\alpha + j\beta)^2 = (R + j\omega L)(G + j\omega C)$$

$$\Rightarrow (\alpha^2 - \beta^2) + j2\alpha\beta = (RG - \omega^2 LC) + j(\omega LG + R\omega C)$$

Equating real and imaginary parts, we get

$$(\alpha^2 - \beta^2) = (RG - \omega^2 LC) \text{ and } 2\alpha\beta = (\omega LG + R\omega C).$$

So, we have already seen that for a transmission line, the complex propagation constant is given by, γ is equal to $\alpha + j\beta$, here this component α actually attenuates the wave as it propagates and β is the phase constant. So, that is why we called α is the attenuation constant and β is the phase constant. Now, if we square this equation on both sides we can write, $(\alpha + j\beta)^2$ is equal to $(R + j\omega L)(G + j\omega C)$.

Now, if we expand these equations and write both the left and right hand side in the form of real and imaginary parts and then if we equate these real and imaginary parts we get $(\alpha^2 - \beta^2)$ is equal to $(RG - \omega^2 LC)$ and $2\alpha\beta$ is equal to $(\omega LG + R\omega C)$.

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Lossy lines

$$(\alpha^2 - \beta^2) = (RG - \omega^2 LC)$$
$$2\alpha\beta = (\omega LG + R\omega C)$$

The above set of equation can be solved for α and β.

$$\alpha = \sqrt{\frac{1}{2}\left[(RG - \omega^2 LC) + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}\right]}$$

$$\beta = \sqrt{\frac{1}{2}\left[(\omega^2 LC - RG) + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}\right]}$$

So, this is the real and imaginary part separated. Once we have this form, we have two unknowns alpha and beta to be calculated in terms of the line parameters that is R, G, L, and C and also frequency ω. So, if I solve these equations we get the solutions for α and β, and these are the general solution for α and β in a lossy line.

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Lossy lines: Low loss case

- For practical transmission lines, the loss is usually small and some approximations can be made to simplify the expressions for the parameters γ and Z_0 .
- Propagation constant γ can be written as

$$\gamma = \sqrt{(j\omega L)(j\omega C)\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)} = j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \frac{RG}{\omega^2 LC}}$$

- For low loss case, we can assume $R \ll \omega L$ and $G \ll \omega C$ at the operating frequency, ω.

$\therefore \frac{RG}{\omega^2 LC} \ll 1$, can be neglected. Using Taylor series approximation and retaining only the significant terms, we get

$$\gamma \cong j\omega\sqrt{LC}\left(1 - \frac{j}{2}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right)$$

$$\therefore \alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) \text{ and } \beta = \omega\sqrt{LC}$$

Now what happens in practical cases, the transmission line losses are usually small and when we are using short section of transmission line essentially the signal attenuation will not be much and some approximation can be made to simplify the expression for the parameters γ and Z_0, for the case of no loss line. So, we can rewrite the propagation constant in this form and which can be re-written in the form shown where we have $j\omega\sqrt{LC}$ that is taken out and we have written in the form $\frac{R}{\omega L}, \frac{G}{\omega C}$.

For the low loss case we assume that at the operating frequency R is very very small compared to ωL and G is very small compared to ωC. And therefore this product term $\frac{RG}{\omega^2 LC}$ will be even smaller and as compared to one it can be neglected. Then once we neglect this term and for the remaining term we applied Taylor series approximation and retain only the significant terms, then we can write γ, the propagation constant, approximately $j\omega\sqrt{LC}\sqrt{1-\frac{j}{2}\left(\frac{R}{\omega L}+\frac{G}{\omega C}\right)}$.

So, now what we can do, we can find out the real and imaginary parts of this equation and from there we find that, α is $\frac{1}{2}\left(R\sqrt{\frac{C}{L}}+G\sqrt{\frac{L}{C}}\right)$ and β is $\omega\sqrt{LC}$.

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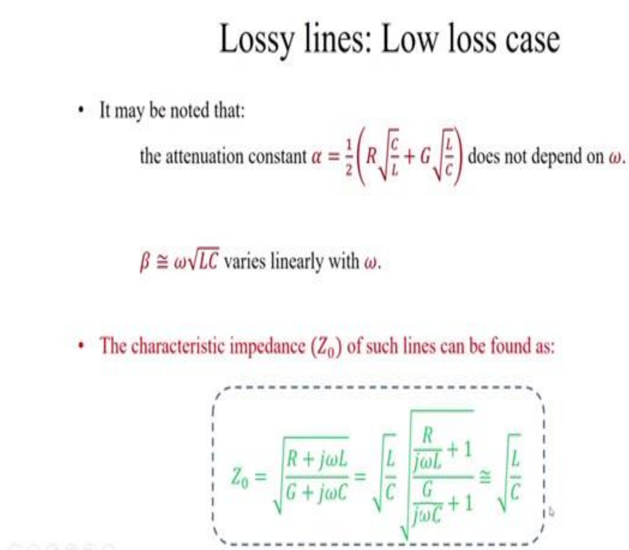

So, if we look at the expression for α and β, we find that attenuation constant α now does not depend upon ω, it depends only on the transmission line parameters and the phase constant β varies linearly with ω. And characteristic impedance Z_0 for such line can also be found out and we write Z_0 is equal to $\sqrt{\frac{L}{C}}\sqrt{\frac{\frac{R}{j\omega L}+1}{\frac{G}{j\omega C}+1}}$ and this $\frac{R}{j\omega L}$ with respect to 1 will be very small similarly, $\frac{G}{j\omega C}$ that term also will be very small compared to 1 and Z_0 can be approximated as $\sqrt{\frac{L}{C}}$.

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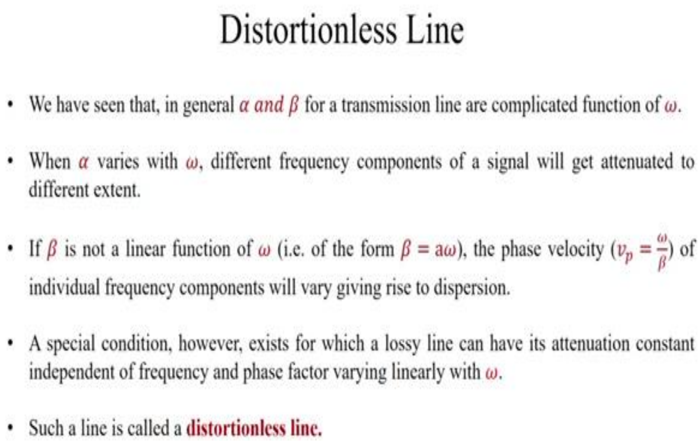

Now we come to a very special case of transmission line which is known as Distortionless line. So we have seen in general α and β for a transmission line are complicated function of ω the angular frequency. When α varies with ω different frequency components of a signal if it is present will get attenuated to different extent. Similarly, if β is not a linear function of ω that is of this form β is equal to some aω.

The phase velocity v_p which is given by $\frac{\omega}{\beta}$ of the individual frequency components will vary giving rise to dispersion. So, we find that, if alpha is varying with ω and β is not a linear function of ω, in that case both the attenuation and the phase velocity for different frequency component will be different. However, a special condition exists for a lossy line that can have attenuation constant independent of frequency and phase constant varying with ω linearly. And these type of lines, if we can realize, they will not distort the signal and therefore this type of lines are called Distortionless line.

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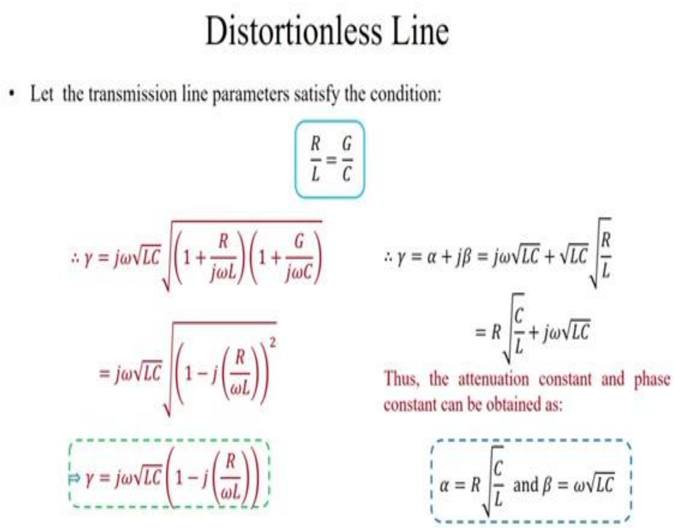

So, let us assume that the transmission line parameters we have designed in such a way that they satisfy the condition $\frac{R}{L}$ is equal to $\frac{G}{C}$. So, we have designed the transmission line to satisfy this requirement. In that case we once again write γ the propagation constant in this form where we have the terms $\frac{R}{j\omega L}$ and $\frac{G}{j\omega C}$. Then since, $\frac{R}{L}$ is $\frac{G}{C}$ we can club these two terms and we get $\left(1 - j\left(\frac{R}{\omega L}\right)\right)^2$.

And therefore, γ expression now simplifies to $j\omega\sqrt{LC}\left(1 - j\left(\frac{R}{\omega L}\right)\right)$. So if we compute the α and β the real and imaginary part of γ, we get α to be equal to R,C by L $R\sqrt{\frac{C}{L}}$ and β equal to $\omega\sqrt{LC}$.

So, we find that β is now varies linearly with ω and α does not contain any ω term. So this is a specific condition in a transmission line and if this condition can be achieved then there would not be any distortion of the signal.

Please note that although, we are considering in terms of the single frequency ω, when a signal is transmitted this signal will have multiple frequency, more than one frequencies, practical signal which are used for communication and if this condition is satisfied then all frequency components of that signal will be attenuated, there strength will be reduced but they will be attenuated equally.

Similarly, the phase shift beta being a linear function of ω will result in to phase velocity which same for all the frequency components of the signal. So, the signal waveform will not get distorted. And that is why we call this type of transmission line having this property of $\frac{R}{L}$ is equal to $\frac{G}{C}$ as Distortionless line.

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Distortionless Line

- With this condition $\frac{R}{L} = \frac{G}{C}$ is satisfied

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$= \sqrt{\frac{L}{C}} \sqrt{\frac{1 + j\frac{R}{\omega L}}{1 + j\frac{G}{\omega C}}}$$

$$\Rightarrow Z_0 = \sqrt{\frac{L}{C}}$$

Thus, Z_0 becomes real quantity.

With this condition $\frac{R}{L}$ is equal to $\frac{G}{C}$ is satisfied, let us see what happens to the characteristic impedance Z_0, of the line. The characteristic impedance Z_0 is defined as $\sqrt{\frac{R+j\omega L}{G+j\omega C}}$. Now we can write this expression in to the form shown where we find that in the numerator under the root $R + j\omega L$ is same as $G + j\omega C$ in the denominator. And these two terms will get cancel, finally we get Z_0 equal to $\sqrt{\frac{L}{C}}$. And characteristic impedance becomes real and is very same as that of the Lossyless line. So, this is an interesting property of the characteristic impedance of the Distortionless line.

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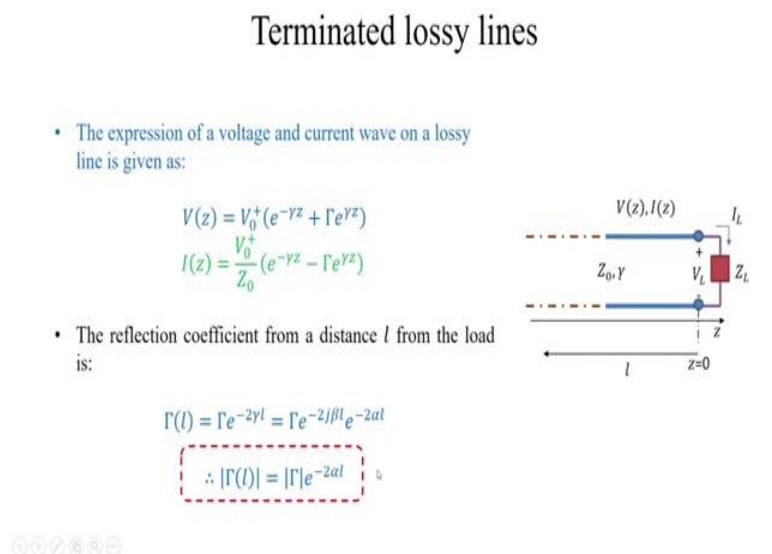

So, let us now consider another case when a transmission line having a propagation constant γ and characteristic impedance Z_0, is terminated to a load impedance Z_L. We set that reference distance z is equal to 0 on the load itself and we see what happens at a distance z is equal to $-l$.

So, the expression for voltage and current wave on this lossy line can be given as, here, Γ is the reflection co-efficient at the load and we have seen that Γ is equal to $\frac{Z_L - Z_0}{Z_L + Z_0}$. Now, if you are interested to know the reflection coefficient at a distance l from the load going towards the negative z direction, then we can write $\Gamma(l)$ is equal to Γ the load reflection coefficient multiplied by $e^{-2\gamma l}$ and this can be written in this form and therefore the $|\Gamma(l)|$ will be $|\Gamma| e^{-2\alpha l}$.

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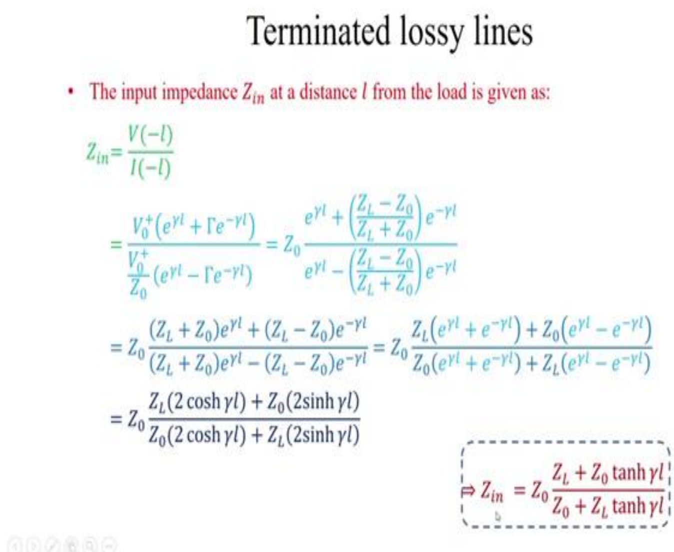

We can compute the input impedance of a lossy transmission line as follows, we consider a distance l from the load and we can write Z_{in} to be equal to $V(z=-l)$ divided by $I(z=-l)$ and then we substitute the equations for the voltage and current and then replacing the reflection coefficient γ expression by $\frac{Z_L-Z_0}{Z_L+Z_0}$ and reorganizing the terms we get in this form.

And finally we find that the input impedance expression comes in terms of hyperbolic functions and from this stage if we divide by $2\cosh\gamma l$ we get a more compact form of the input impedance of a lossy transmission line which is given by Z_{in} is equal to $Z_0\frac{Z_L+Z_0\tanh\gamma l}{Z_0+Z_L\tanh\gamma l}$. Please note that when α equal to 0 then this $\tanh\gamma l$ will be $\tanh j\beta l$ and which can be finally written as $j\tan\beta l$, from this expression we will get the expression for the input impedance of the line that we derive for the Lossless case.

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Terminated lossy lines

- The power delivered to the input of the terminated line at $z = -l$ is given as:

$$P_{in} = \frac{1}{2} Re\{V(-l) I^*(-l)\}$$

$$= \frac{1}{2} Re\left\{\left[V_0^+\left(e^{\gamma l} + \Gamma e^{-\gamma l}\right)\right]\left[\frac{V_0^+}{Z_0}\left(e^{\gamma l} - \Gamma e^{-\gamma l}\right)\right]^*\right\}$$

$$= \frac{|V_0^+|^2}{2Z_0}\left(e^{2\alpha l} - |\Gamma|^2 e^{-2\alpha l}\right)$$

$$= \frac{|V_0^+|^2}{2Z_0}\left(1 - |\Gamma|^2 e^{-4\alpha l}\right) e^{2\alpha l}$$

$$\boxed{\Rightarrow P_{in} = \frac{|V_0^+|^2}{2Z_0}\left(1 - |\Gamma(l)|^2\right) e^{2\alpha l}} \qquad \because |\Gamma(l)| = |\Gamma| e^{-2\alpha l}$$

Let us now consider the power in a terminated lossy line. The power that is delivered to the input of the terminated line at the position $z = -l$ can be written as P_{in} equal to half real part of $V(z = -l)\, I^*(z = -l)$. And when we substitute once again the expression for the voltage and the current then we can get and only retain the real part, we get the expression for the input power at Z is equal to minus L.

And this is been written in this particular form. Now if we use our earlier relation of $\Gamma(l) = |\Gamma| e^{-2\alpha l}$, we can represent, here please note in the previous equation the power is in terms of reflection coefficient gamma at the load, whereas in this expression the power is in terms of the reflection coefficient at $(z = -l)$.

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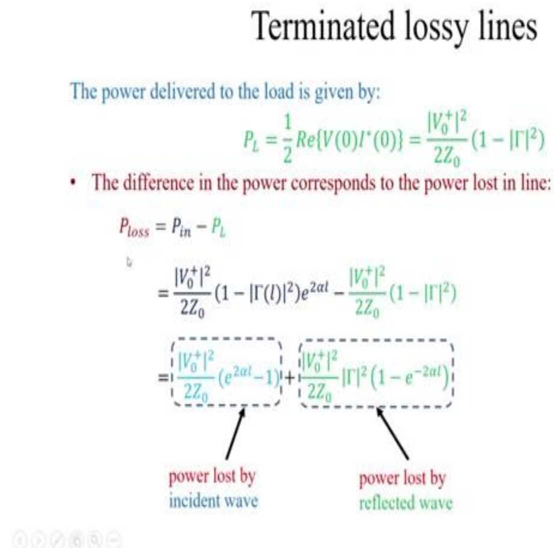

So, once we get the input power, we proceed to calculate the power that is delivered to the load, this is given by $\frac{1}{2}Re\{V(z=0)I^*(z=0)\}$ and we can write this as $\frac{|V_0^+|^2}{2}(1-|\Gamma|^2)$, where have the Γ is the load reflection coefficient.

The difference in the power corresponds to the power lost in the line. Please note that in Lossless case we have power only delivered the load, but in case of lossy transmission line as the wave propagates because of this attenuation factor alpha, some power will be dissipated throughout the line and whatever power we compute at $(z=-l)$ and the power that we compute at the load position. So, if we take the difference of these two power, that will be giving as the power lost in the line. So, P_{loss} is p in expression which we have already computed minus P_L and this can be found out by substituting the expressions for P_{in} and P_L. And after rearranging these terms we can write the P_{loss} into two components. The first component it gives the power lost by the incident wave, and the second component is the power lost by the reflected wave.

So, in a transmission line we will have both the waves present and both incident and the reflected wave, because of the attenuation in the line we will lose power and these two expressions account for the power lost by the incident and the reflected component of the wave.

P_L is the load power which is finally delivered and P_{loss} is the total power loss because of the incident and reflected wave. So this brings us to the end of our discussion about the lossy line.

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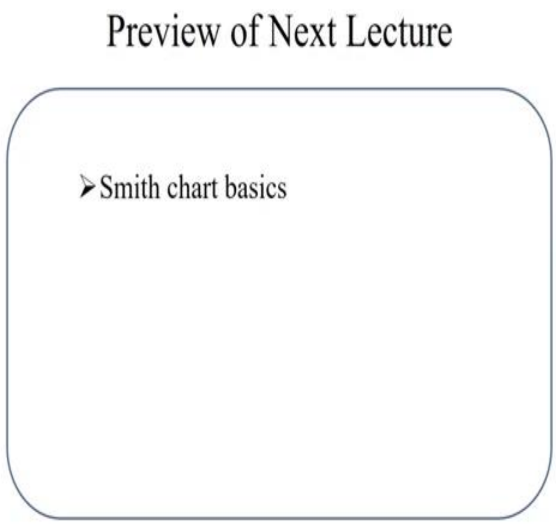

In the next lecture we are going to cover, what is known as Smith chart? The basics of smith chart. Smith chart is a widely used graphical tool for solving transmission line problems and in the next class we will discuss about the Smith chart.