

Microwave Engineering
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Lecture 30 - Reflex Klystron, Magnetron and TWT

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$$i_{2\text{ind}} = \beta_0 i_2 = \beta_0 2I_0 J_1(X) \cos[\omega(t_2 - \tau - T_0)]$$

Output Power and Efficiency

- ❑ As already mentioned, The maximum bunching should occur approximately midway between the catcher cavity grids.
- ❑ The phase of the catcher gap voltage must be maintained in such a way that the bunched electrons while passing through the grids, encounter a retarding phase so that they can transfer kinetic energy to the field of the catcher cavity.
- ❑ The fundamental component of the induced microwave current in the catcher is given by :

$$i_{2\text{ind}} = \beta_0 i_2 = \beta_0 2I_0 J_1(X) \cos[\omega(t_2 - \tau - T_0)]$$
- ❑ β_0 is the beam coupling coefficient of the catcher gap

So, we have found out the optimal separation, l_{opt} between the buncher and catcher cavity. Let us now discuss the output power and efficiency that can be achieved in this type of 2 cavity klystron. As already mentioned, the maximum bunching should occur approximately midway between the catcher cavity grids. The phase of the catcher gap voltage must be maintained in such a way that the bunched electrons, while passing through the grids encounter a retarding phase so that they can transfer kinetic energy to the field of the catcher cavity.

The fundamental component of the induced microwave current in the catcher cavity is given by I_2 induced is equal to $\beta_0 I_2$, which is $\beta_0 2 I_0 J_1(X) \cos[\omega(t_2 - \tau - T_0)]$. Here β_0 is the beam coupling coefficient of the catcher gap.

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$$\beta_i = \beta_0$$

$$I_{2\text{ind}} = \beta_0 I_2 = \beta_0 2I_0 J_1(X)$$

If the buncher and catcher cavities are identical, then

$$\beta_i = \beta_0$$

The magnitude of the fundamental component of current induced in the catcher cavity is

$$I_{2\text{ind}} = \beta_0 I_2 = \beta_0 2I_0 J_1(X)$$

If the buncher and catcher cavities are identical, then we will have beta i is equal to beta naught. And the magnitude of the fundamental component of current induced in the catcher cavity is $i_{2\text{ind}}$ is equal to beta naught I_2 which can be written as beta naught $2 i_{\text{naught}} J_1 X$. Now, this beta naught I_2 , this current actually flows through the parallel combination of these resistances R_{sh0} representing the conductivity of the catcher cavity walls.

R_B is the beam loading resistance usually very high, and R_L is the load resistance that is connected to the output of the cavity. And this can be written as the combination of these three resistances can be represented by R_{sh} , and V_2 is the output voltage, and beta naught I_2 is the current that flows through R_{sh} .

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$$P_{\text{out}} = \frac{(\beta_0 I_2)^2}{2} R_{\text{sh}} = \frac{\beta_0 I_2 V_2}{2}$$

$$\text{Efficiency} \equiv \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{\beta_0 I_2 V_2}{2 I_0 V_0}$$

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If $\beta_0 = 1$ (perfect coupling), the beam current I_2 approaches its maximum value $2I_0 \times 0.582$, and $V_2 = V_0$, the maximum efficiency becomes $\sim 58\%$.

Now output power beta naught I_2 square by 2 R_{sh} and which can also be written as beta naught I_2 by 2 V_2 and the electronic efficiency or the efficiency can be written as P_{out} by P_{in} , P_{out} is beta naught $I_2 V_2$ by 2 and input power is I_0 into V_0 , and therefore, if we consider that beta naught equal to 1 perfect coupling and the beam current I_2 is such that it approaches its maximum value which is given by $2 I_0$ into 0.582, and also V_2 equal to V_0 , then we can achieve the maximum efficiency. It should be noted that this is the maximum efficiency that we can achieve, which is approximately 58 percent.

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Reflex Klystron

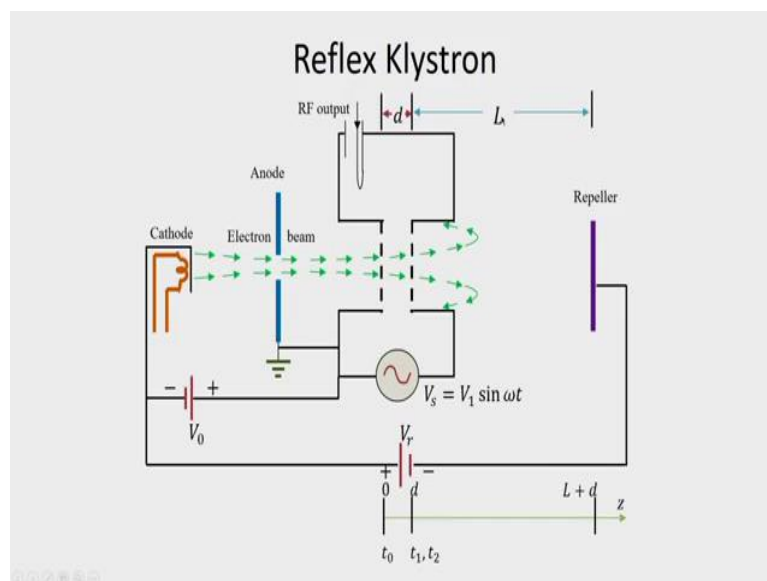
- ❑ The reflex klystron is a single-cavity klystron oscillator.
- ❑ It is a low-power generator of 10 to 500-mW output at a frequency range of 1 to 25 GHz.
- ❑ The electronic efficiency is about 20 to 30%.

Usage:

- ❑ As laboratory source for microwave measurements, in microwave receivers, as local oscillators in commercial, military, and airborne Doppler radars.

So, we have seen two-cavity klystron which is essentially an amplifier; we can have more than two cavities also. Let us now start a new topic, which is reflex klystron. A two-cavity klystron amplifier with proper feedback arrangement can be operated as oscillator, but generally the reflex klystron is a single-cavity klystron oscillator. It is a low power generator, the output of the order of 10 to 500 milliwatt in the frequency range of 1 to 25 gigahertz, and its electronics efficiency is of the order of 20 to 30 percent. This device is extensively used as a laboratory source for microwave measurement, in microwave receivers, as well as local oscillators in commercial, military, and airborne Doppler radars.

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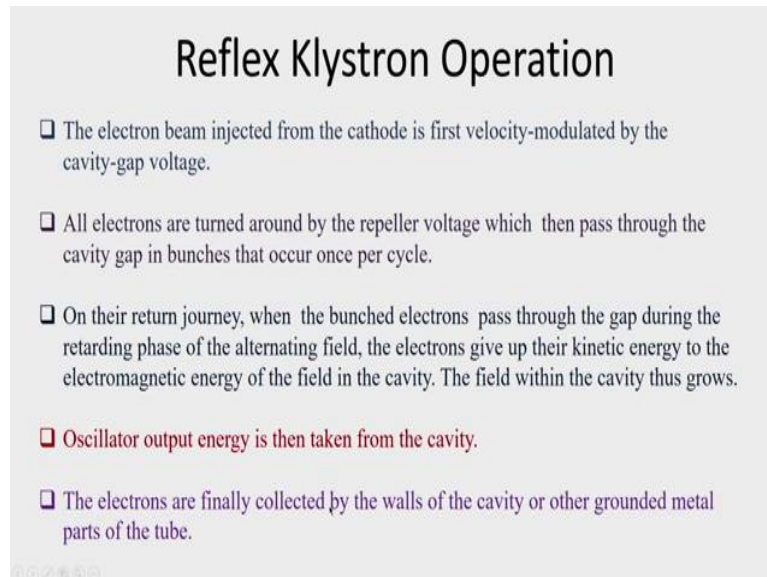
The schematic of the reflex klystron oscillator is shown here. We have a cathode which is heated, and the anode is maintained at a higher potential than the cathode, and we can see that

here the anode is at the ground potential; cathode is at lower potential as compared to anode. And there is only one cavity, so this cavity does the function of bunching the electrons as well as this is the cavity where the electrons also transfer their energy.

In reflex klystron, we have a repeller which is maintained at a lower potential, and its job is to repel the electrons back to the cavity. Since it is an oscillator, it has only one output, but for the sake of explanation we are assuming a small AC signal already present here. So when the electron beams, when the electrons are emitted by the cathode, they travel, they are first accelerated by the anode, and then when they pass through the cavity grids the RF signal present in the cavity produces velocity modulation of the electrons.

These electrons then travel first to the cavity, and finally, the repeller repels them back, and they come to the cavity after travelling some distance. Now, here d is the distance between the grids, t_0 is the time when an electron enters, t_1 is the time when it leaves this cavity after being velocity modulated, and t_2 is the time when the electrons come back to the same cavity again after traveling certain distance. The separation between the cavity and the repeller is l .

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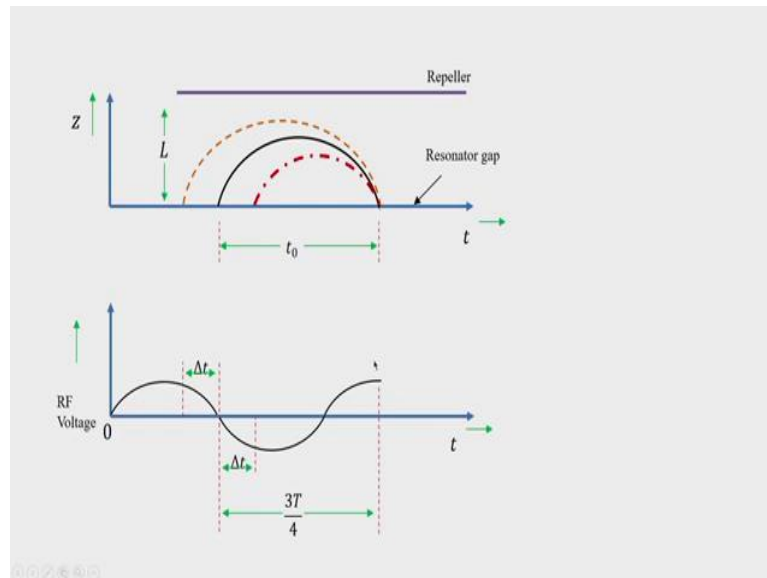
Reflex Klystron Operation

- ❑ The electron beam injected from the cathode is first velocity-modulated by the cavity-gap voltage.
- ❑ All electrons are turned around by the repeller voltage which then pass through the cavity gap in bunches that occur once per cycle.
- ❑ On their return journey, when the bunched electrons pass through the gap during the retarding phase of the alternating field, the electrons give up their kinetic energy to the electromagnetic energy of the field in the cavity. The field within the cavity thus grows.
- ❑ Oscillator output energy is then taken from the cavity.
- ❑ The electrons are finally collected by the walls of the cavity or other grounded metal parts of the tube.

So, the electron beam injected from the cathode is the first velocity modulated by the cavity gap voltage. All electrons are turned around by the repeller voltage, which then passes through the cavity gap in bunches and that occurs once per cycle. On their return journey, when the bunched electrons pass through the gap during the retarding phase of the alternating field, the electrons give up their kinetic energy to the electromagnetic energy of the field in the cavity. And the field within the cavity thus grows. The oscillator output energy is taken from the cavity.

The electrons are finally collected by the walls of the cavity or other grounded metal parts of the tube.

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So, here we are explaining, so an electron, here we are explaining how bunching occurs, so an electron which was accelerated, which entered the cavity earlier and got accelerated, it travels a longer distance towards the repeller. Similarly, an electron which entered the cavity when the voltage, AC voltage is 0, its velocity remains unchanged, and it travels a lesser distance and comes back to the cavity again.

Similarly, the electron which was slowed down by the field in the cavity, this electron travels the least distance and finally, if all the parameters are adjusted properly after traveling certain distance, the electrons they bunch at this point in the resonator gap, and at this instant if the voltage RF voltage in the cavity it is such that it retards this electron, then they will deliver their kinetic energy to the field and the field will grow.

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$$T' = t_2 - t_1 = \frac{2mL}{e(V_r + V_0)} v(t_1) = T'_0 \left[1 + \frac{\beta_i V_1}{2V_0} \sin \left(\omega t_1 - \frac{\theta_g}{2} \right) \right]$$

$$T'_0 = \frac{2mLv_0}{e(V_r + V_0)}$$

$$\omega(t_2 - t_1) = \theta'_0 + X' \sin \left(\omega t_1 - \frac{\theta_g}{2} \right)$$

where,

$$\theta'_0 = \omega T'_0 \quad \text{round-trip DC transit angle}$$

$$X' \equiv \frac{\beta_i V_1}{2V_0} \theta'_0 \quad \text{bunching parameter of the reflex klystron oscillator}$$

The round-trip transit time in the repeller region is given by

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$$X' \equiv \frac{\beta_i V_1}{2V_0} \theta'_0 \quad \text{bunching parameter of the reflex klystron oscillator.}$$

Now the round-trip transit time in the repeller region can be given as capital T dash equal to t2 minus t1, and this can be found as 2mL by e Vr plus V naught. m is the electronic mass, is the charge of the electron and Vr plus V naught is the total voltage multiplied by the velocity of the electron V t1, and therefore, it can be written in this form. T dash is equal to T naught dash into 1 plus beta i V1 by 2 V naught sin omega t1 minus theta g by 2. We follow the same type of analysis as we have done in the case of 2 cavity klystron.

And here t0 dash is 2mL v naught divided by eVr plus capital V naught. Now, the transit angle can be found out by multiplying t2 minus t1 by omega, and this can be written in the form of DC transit angle theta naught dash. It is the round-trip DC transit angle plus X dash sin omega t1 minus theta g by 2. Here, X Dash is the bunching parameter of the reflex klystron oscillator. And it is given by beta i V1 divided by 2 V naught theta naught dash.

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$$\omega(t_2 - t_1) = \omega T'_0 = \left(n - \frac{1}{4}\right) 2\pi = N2\pi = 2\pi n - \frac{\pi}{2}$$

$$\text{Efficiency} = \frac{P_{ac}}{P_{dc}} = \frac{2X'J_1(X')}{2\pi n - \pi/2}$$

For a maximum energy transfer, the round-trip transit angle, referring to the centre of the bunch, is given by

$$\omega(t_2 - t_1) = \omega T'_0 = \left(n - \frac{1}{4}\right) 2\pi = N2\pi = 2\pi n - \frac{\pi}{2}$$

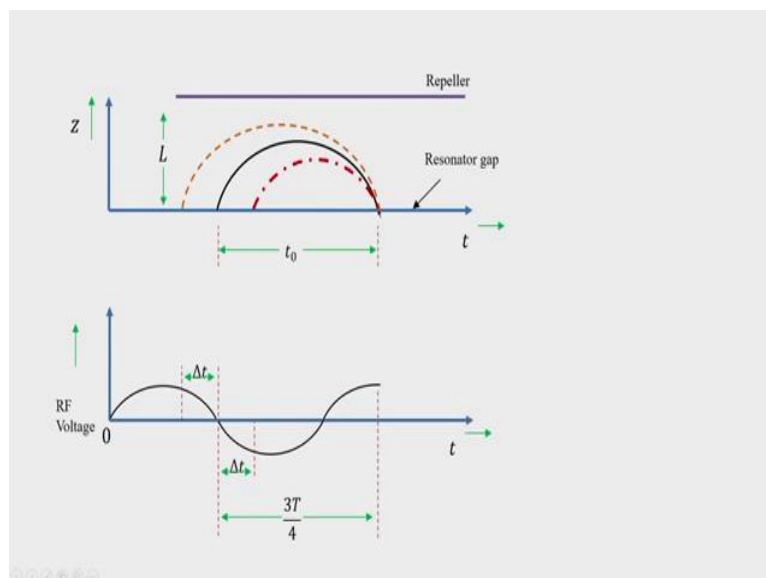
The electronic efficiency of a reflex klystron oscillator can be found as

$$\text{Efficiency} = \frac{P_{ac}}{P_{dc}} = \frac{2X'J_1(X')}{2\pi n - \pi/2}$$

In practice, the mode of $n = 2$ (or $1\frac{3}{4}$ mode) has the most output power

$X'J_1(X')$ reaches a maximum value of 1.25 at $X' = 2.408$.

Therefore maximum efficiency is 22.7%

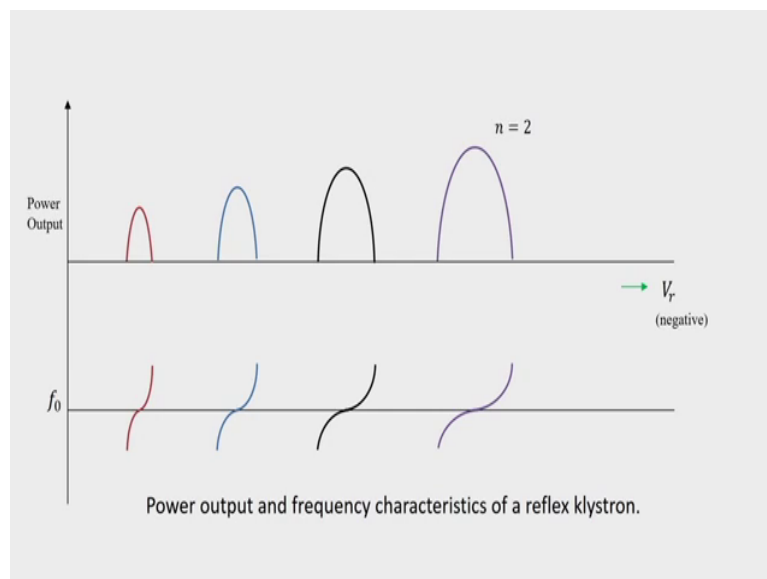


So, having defined these parameters, we find that for a maximum energy transfer, the round-trip transit angle referring to the center of the bunch is given by $\omega(t_2 - t_1)$. It must be equal to $\omega T'_0 = \left(n - \frac{1}{4}\right) 2\pi$. We can see that here we showed the first bunching, which occurs as $3T/4$, that means which is this case is when $n = 1$. When $n = 2$, we will have $1\frac{3}{4}T$.

So, in general, we can write $n - \frac{1}{4} \cdot 2\pi$, and this can be written as $2\pi n - \pi$ by 2, and capital N is equal to $n - \frac{1}{4}$ is the mode of operation. The electronic efficiency of the reflex klystron, it can be calculated by finding out the AC power, and it is given by $2 X_{J1} X_{dash}$ divided by $2\pi n - \pi$ by 2. Now we are not going into the detailed derivation of this for reflex klystron.

In practice, the mode of n equal to 2 or $1 \frac{3}{4}$ mode, it has the most output power. And we also find that this quantity $X_{dash} J 1 X_{dash}$, it reaches its maximum value of 1.25 at X_{dash} equal to 2.408. And once we substitute n equal to 2 here, $X_{dash} J 1, X_{dash}$ equal to 1.25, then we get the maximum efficiency to be 22.7 percent. At the beginning of this discussion, we have mentioned that reflex klystron has efficiency ranging between 20 to 30 percent, so we get a value of 22.7 percent.

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Depending upon the adjustment of the repeller voltage, we can see that if the repeller voltage is very high, the electrons will be sent back to the cathode after traveling short distance, whereas when the repeller voltage is low the electrons can move for further distance towards the repeller, and therefore based on the setting of the repeller voltage we can have different modes.

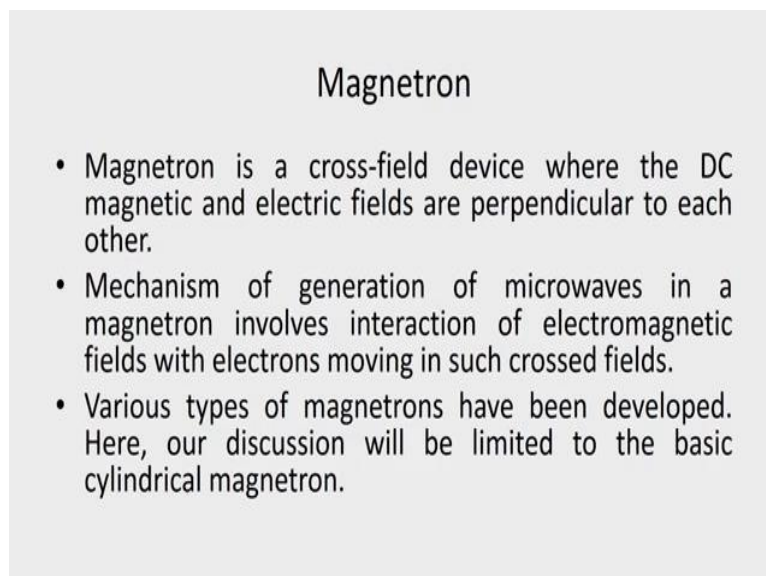
The mode one with the highest power we have already mentioned is $1 \frac{3}{4}$ mode, and this mode will have different amount of power, usually as the value of n increases, we get lower power. Also the frequency around the resonant frequency f_0 where this power is available is indicated, and these are only some representative plots showing the behavior of the device.

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So, we have discussed the basics of the reflex klystron, how it operates the efficiency that is achieved? Next we move onto another topic, Magnetron.

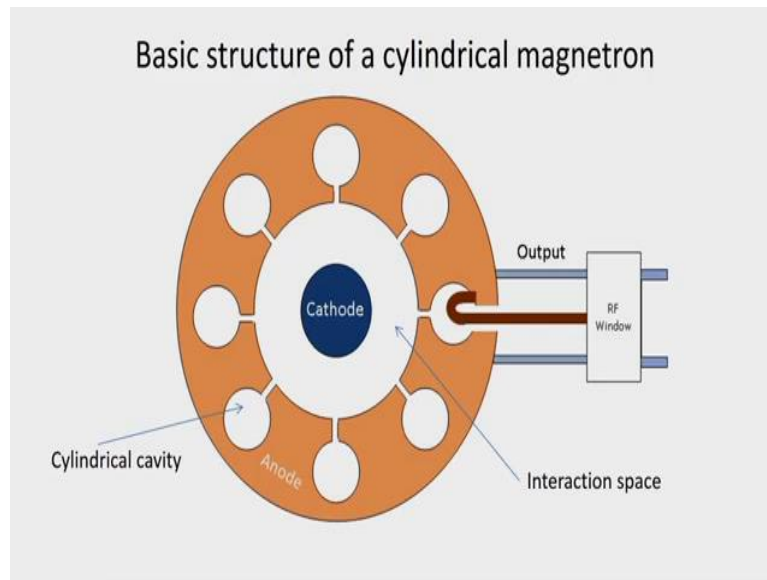
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A magnetron is a cross-field device, and it has both DC magnetic and electric fields, which are applied to the device perpendicular to each other. The mechanism of generation of microwaves in a magnetron involves interaction of electromagnetic fields with electrons moving in such crossed fields. The mechanism of generation of microwaves in a magnetron involves interaction of electromagnetic fields with electrons moving in such cross fields.

Various types of magnets have been developed. Here, our discussion will be limited to the basic cylindrical magnetron. Now a magnetron is an oscillator, and it can generate both continuous wave or pulse power. Generally, the peak power that can be generated by these magnetrons is very high, may go several megahertz, but that power will exist only for a short duration. In a pulsed magnetron, the average power will be of the order of few hundred kilowatts.

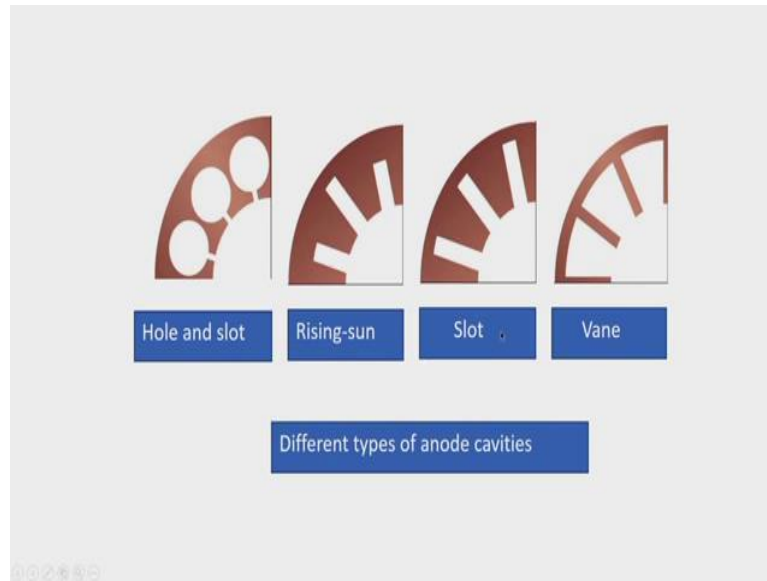
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As we have said, we will discuss the basic magnetron, a cylindrical magnetron, and the cross-section of such magnetron is shown. Here, we have a cathode surrounded by an anode block where we have cylindrical cavities. Now between anode and cathode is the interaction space. These cavities, as we can see has opening to the interaction space, so the electrons emitted by the cathode will be accelerated by the field present between cathode and anode.

And also, a magnetic field perpendicular to this electric field is applied along the axis of the magnetron. Finally, the RF output is taken out. Here, a loop coupling has been shown, and since this type of device, these are evacuated devices we need to connect this device to rest of the circuitry operating under normal atmospheric condition through an RF window, which acts as a barrier between this vacuum and the normal atmosphere outside the window.

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We have shown the cross-section of a hole and slot magnetron. The anode cavities, different types of anode cavities, are used like rising-sun, simple slot, vane. Let us now, discuss some basics about the motion in cylindrical coordinates the governing equations, because we are dealing with the cylindrical magnetron, and then electron is subjected to electric and magnetic force because of the field present, so how we describe the motion in terms of the mathematical equations these basics let us discuss.

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$$\mathbf{v} = \frac{dr}{dt} \mathbf{u}_r + r \frac{d\phi}{dt} \mathbf{u}_\phi + \frac{dz}{dt} \mathbf{u}_z$$

$$\mathbf{F} = -e\mathbf{E} = m\mathbf{a}$$

$$\frac{d^2r}{dt^2} - r \left(\frac{d\phi}{dt} \right)^2 = -\frac{e}{m} E_r$$

$$\frac{1}{r} \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right) = -\frac{e}{m} E_\phi$$

$$\frac{d^2z}{dt^2} = -\frac{e}{m} E_z$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \left[\frac{d^2r}{dt^2} - r \left(\frac{d\phi}{dt} \right)^2 \right] \mathbf{u}_r + \frac{1}{r} \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right) \mathbf{u}_\phi + \frac{d^2z}{dt^2} \mathbf{u}_z$$

$$\mathbf{F} = -e\mathbf{v} \times \mathbf{B} = m\mathbf{a}$$

$$\frac{d^2r}{dt^2} - r \left(\frac{d\phi}{dt} \right)^2 = -\frac{e}{m} \left(B_z r \frac{d\phi}{dt} - B_\phi \frac{dz}{dt} \right)$$

$$\frac{1}{r} \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right) = -\frac{e}{m} \left(B_r \frac{dz}{dt} - B_z \frac{dr}{dt} \right)$$

$$\frac{d^2 z}{dt^2} = -\frac{e}{m} \left(B_\phi \frac{dr}{dt} - B_r r \frac{d\phi}{dt} \right)$$

$$\omega = \frac{v}{R} = \frac{eB}{m} \text{ rad/sec}$$

$$\text{Here R is obtained from } \frac{mv^2}{R} = evB$$

Equations of motion in cylindrical coordinates

$$\mathbf{v} = \frac{dr}{dt} \mathbf{u}_r + r \frac{d\phi}{dt} \mathbf{u}_\phi + \frac{dz}{dt} \mathbf{u}_z \quad \mathbf{a} = \frac{d\mathbf{v}}{dt} = \left[\frac{d^2 r}{dt^2} - r \left(\frac{d\phi}{dt} \right)^2 \right] \mathbf{u}_r + \frac{1}{r} \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right) \mathbf{u}_\phi + \frac{d^2 z}{dt^2} \mathbf{u}_z$$

$F = -eE = ma$ $F = -e\mathbf{v} \times \mathbf{B} = ma$

$$\frac{d^2 r}{dt^2} - r \left(\frac{d\phi}{dt} \right)^2 = -\frac{e}{m} E_r$$

$$\frac{1}{r} \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right) = -\frac{e}{m} E_\phi$$

$$\frac{d^2 z}{dt^2} = -\frac{e}{m} E_z$$

$$\frac{d^2 r}{dt^2} - r \left(\frac{d\phi}{dt} \right)^2 = -\frac{e}{m} \left(B_z r \frac{d\phi}{dt} - B_\phi \frac{dz}{dt} \right)$$

$$\frac{1}{r} \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right) = -\frac{e}{m} \left(B_r \frac{dz}{dt} - B_z \frac{dr}{dt} \right)$$

$$\frac{d^2 z}{dt^2} = -\frac{e}{m} \left(B_\phi \frac{dr}{dt} - B_r r \frac{d\phi}{dt} \right)$$

Cyclotron angular frequency of the circular motion of the electron is

$$\omega = \frac{v}{R} = \frac{eB}{m} \text{ rad/sec} \quad \text{Here R is obtained from } \frac{mv^2}{R} = evB$$

The velocity in cylindrical coordinate, we can write $\frac{dr}{dt} \mathbf{u}_r$ is the unit vector along r , then $r \frac{d\phi}{dt} \mathbf{u}_\phi$ is the unit vector along ϕ and $\frac{dz}{dt} \mathbf{u}_z$ is the unit vector along z . We are using a cylindrical coordinate r , ϕ and z . Now, the acceleration, which is time derivative of velocity, it can be written in this form. Again, it is shown as its r , ϕ and z components. Now, if an electron is placed in an electric field, then it will experience a force of minus e , electric field E , and this can be written as mass into acceleration.

So, from the expressions we have given for acceleration, we can now write, we can equate component-wise, so the r component of the field and r component of acceleration, we can write $\frac{d^2 r}{dt^2} - r \left(\frac{d\phi}{dt} \right)^2$ is equal to, from here, minus e by m and E_r component. Similarly, we can write for E_ϕ and E_z component. When an electron, it moves with a velocity V in a magnetic field, the force that acts on it is given by minus $e \mathbf{v} \times \mathbf{B}$, and that can be written as now ma .

And once again using the relation given for a , we can write. So, we have these sets of equations, please note that in our case in the cylindrical magnetron we are applying only E_r , and also we

are applying only B_z , so these two-component, the equations which contain E_r and B_z , those two equations will be useful to us, and we also define cyclotron angular frequency of circular motion of the electron.

Now, because v and F , these two are perpendicular to each other, this force is perpendicular to both v and B , so this force being a perpendicular force will try to move the electron in a circular path. So, the electron trajectory, it will change into a circular path, and cyclotron angular frequency of the circular motion of the electron, it is given by ω is equal to v by R is equal to $e B$ by m radian per second.

Now, this R we obtain from the fact that mv^2 by R must be the centrifugal force of the electrons must be balanced by $e v B$, the force produced by the magnetic field. And from this we find that v by R is equal to $e B$ by m , and this v by R is the cyclotron angular frequency ω . Now, having discussed these basics we can now move onto study the electron motion under combined magnetic field.

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$$\mathbf{F} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = m \frac{d\mathbf{v}}{dt}$$

$$\frac{d^2 r}{dt^2} - r \left(\frac{d\phi}{dt} \right)^2 = \frac{e}{m} \left(E_r - B_z r \frac{d\phi}{dt} \right)$$

$$\frac{1}{r} \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right) = \frac{e}{m} B_z \frac{dr}{dt}$$

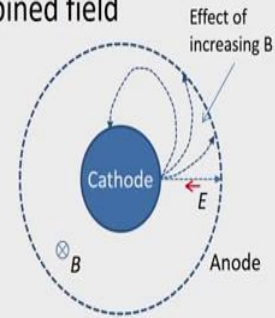
Electron motion under combined field

When the DC voltage and the magnetic flux are adjusted properly, the electrons will follow cycloidal paths in the cathode-anode space under the combined force of both electric and magnetic fields as shown.

Electron Motion under combined electric and magnetic field is given by

$$F = -e(E + v \times B) = m \frac{dv}{dt}$$

$$\frac{d^2 r}{dt^2} - r \left(\frac{d\phi}{dt} \right)^2 = \frac{e}{m} \left(E_r - B_z r \frac{d\phi}{dt} \right)$$



Cathode radius: a Anode radius: b

$$\frac{1}{r} \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right) = \frac{e}{m} B_z \frac{dr}{dt}$$

So, in our magnetron, we have both electric field, applied the electric field, which is radial and applied a magnetic field, which is along the axis. So, an electron will move under the influence of both the fields. When the DC voltage and magnetic flux are adjusted properly, the electrons will follow cycloidal path in the cathode-anode space under the combined force of both electric and magnetic fields, as shown.

So, when no magnetic field is there since we have an anode is at higher potential, we have the electric field E , so the electrons will be emitted by the cathode, will be accelerated by the anode, and it will move towards the anode. As we apply this magnetic field B , now the electrons will experience a sideways force, and its path will change from this linear towards circular, as we keep on increasing B .

And if B is increased beyond certain value, the curvature of this path will be such that the electron, instead of reaching anode it will come back to the cathode. So, we see that for a given electric field we will have a magnetic field for which the electrons will be coming back to the cathode instead of going to anode. Now, the motion under combined electric and magnetic field it is given by F is equal to sum of the two forces. So, minus $e E$ plus v cross B , and this can now be written as $m \frac{dv}{dt}$.

So, from our previous discussion, we can write $\frac{d^2 r}{dt^2} - r \left(\frac{d\phi}{dt} \right)^2$, this now becomes $\frac{e}{m} E_r$, please note that, our e is from anode to cathode, so this minus sign is not there as we saw in the earlier case, minus $B_z r \frac{d\phi}{dt}$, and $\frac{1}{r} \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right)$ is equal to $\frac{e}{m} B_z \frac{dr}{dt}$. So, these two equations actually come from the discussion

we did earlier separately for the electric field and magnetic field. Now, we need to combine these terms, so that so as to take into account the combined force that an electron experiences.

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$$\frac{d^2r}{dt^2} - r \left(\frac{d\phi}{dt} \right)^2 = \frac{e}{m} E_r - \frac{e}{m} B_z r \frac{d\phi}{dt} \qquad \frac{1}{r} \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right) = \frac{e}{m} B_z \frac{dr}{dt}$$

$$\Rightarrow \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right) = \frac{e}{m} B_z r \frac{dr}{dt} = \frac{1}{2} \omega_c \frac{d}{dt} (r^2)$$

where $\omega_c = \frac{e}{m} B_z$ is cyclotron angular frequency

Integrating,

$$r^2 \frac{d\phi}{dt} = \frac{1}{2} \omega_c r^2 + \text{constant}$$

At $r = a$, where a is cathode radius, $\frac{d\phi}{dt} = 0$,

$$\frac{d\phi}{dt} = \frac{1}{2} \omega_c \left(1 - \frac{a^2}{r^2} \right)$$

Equations of Motion for Electrons in a Cylindrical Magnetron

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Integrating,

$$r^2 \frac{d\phi}{dt} = \frac{1}{2} \omega_c r^2 + \text{constant}$$

At $r = a$, where a is cathode radius, $\frac{d\phi}{dt} = 0$,

$$\frac{d\phi}{dt} = \frac{1}{2} \omega_c \left(1 - \frac{a^2}{r^2} \right)$$

So, this is the same set of equations. So, from these two equations, we can write $\frac{d}{dt}$ of r^2 $\frac{d\phi}{dt}$ is equal to $\frac{e}{m} B_z r \frac{dr}{dt}$. Now, $r \frac{dr}{dt}$ we can write half $\frac{d}{dt}$ of r^2 and $\frac{e}{m} B_z$ by m we can write ω_c the cyclotron frequency. Integrating the above equation, we get $r^2 \frac{d\phi}{dt}$ is equal to half $\omega_c r^2$ plus some constant of integration. Now, at r is equal to a , where a is the cathode radius, we should have $\frac{d\phi}{dt}$ is equal to 0.

The Phi component of the velocity is zero, and therefore, if we put $d\phi/dt$ equal to 0, the constant evaluates as minus half omega a square. So, when this constant is substituted, and then we divide by r^2 we get $d\phi/dt$ is equal to half omega c $1 - a^2$ by r^2 .

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Since the magnetic field does not work on the electrons, the kinetic energy of electron is given by,

$$\frac{1}{2} m v^2 = eV$$

However, electron velocity has r and ϕ components,

$$v^2 = \frac{2e}{m} V = v_r^2 + v_\phi^2 = \left(\frac{dr}{dt}\right)^2 + \left(r \frac{d\phi}{dt}\right)^2$$

At $r = b$, where b is anode radius, $V = V_0$, and $\frac{dr}{dt} = 0$, electrons just graze the anode,

$$b^2 \left(\frac{d\phi}{dt}\right)^2 = \frac{2e}{m} V_0$$

$$\frac{d\phi}{dt} = \frac{1}{2} \omega_c \left(1 - \frac{a^2}{b^2}\right)$$

Since the magnetic field does not work on the electrons, it because the force is always perpendicular to the motion of the electron. The kinetic energy of the electron is given by half mv^2 , and that can be equated to eV . However, the electron velocity has r and ϕ components, and therefore, we can write v^2 is equal to $2e/mV$, and that can be written as $V r^2$, r component of the velocity square plus ϕ component of the velocity square.

And r component of the velocity is given by dr/dt , and ϕ component of the velocity is given by $r d\phi/dt$. Now, at r equal to b , where b represents the anode radius. We have potential V equal to V_0 , and also, if we have dr/dt equal to 0, then the electrons will just graze the anode. So if we put this conditions, then we get $b^2 (d\phi/dt)^2$ is equal to, because dr/dt we are putting 0, $2e/m$ and this V becomes V_0 , and therefore, we get $d\phi/dt$ is equal to half omega c.

We have also seen $d\phi/dt$ is equal to half omega c $1 - a^2$ by b^2 , and therefore, now we can substitute this $d\phi/dt$ here by this expression.

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$$b^2 \left[\frac{1}{2} \omega_c \left(1 - \frac{a^2}{b^2} \right) \right]^2 = \frac{2e}{m} V_0$$

Hull cutoff magnetic equation,

$$B_{0c} = \frac{\left(8V_0 \frac{m}{e} \right)^{\frac{1}{2}}}{b \left(1 - \frac{a^2}{b^2} \right)}$$

If $B_0 > B_{0c}$ for a given V_0 , electrons will not reach the anode.

Conversely, the cutoff voltage is given by,

$$V_{0c} = \frac{e}{8m} B_0^2 b^2 \left(1 - \frac{a^2}{b^2} \right)^2$$

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This equation is also known as **Hull cutoff voltage equation**

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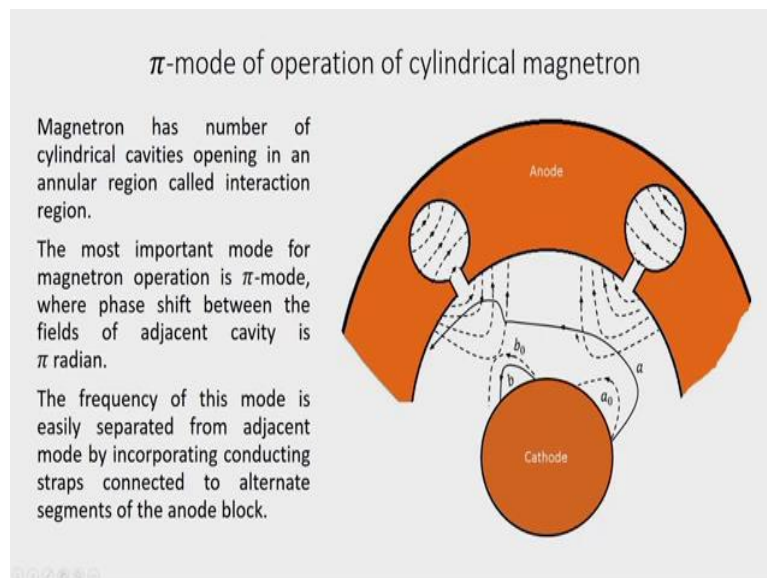
If $V_0 < V_{0c}$ for a given B_0 , electrons will not reach the anode.
This equation is also known as **Hull cutoff voltage equation**

So, once we make this substitution, we get $b^2 \left[\frac{1}{2} \omega_c \left(1 - \frac{a^2}{b^2} \right) \right]^2 = \frac{2e}{m} V_0$ whose square is equal to $\frac{2e}{m} V_0$. So, this equation is the Hull cut-off magnetic equation, and from here we can, for a given V_0 we can find out the cut-off magnetic field B_{0c} , so this B_{0c} we can get by substituting ω_c is equal to $\frac{e B_0}{m}$, and then we rearrange the terms, we get the Hull cut-off magnetic field.

And if V_{naught} is greater than B_{oc} , the applied magnetic field is greater than this cut-off magnetic field for the given V_{naught} , the electrons will not reach the anode, because we have derived this condition considering that dr/dt becomes 0 at the anode. Alternatively, we can give B , we can find a cut-off voltage. So, V_{oc} is the cut-off voltage for a given B_{naught} .

And if applied voltage V_{naught} is less than V_{oc} for a given V_{naught} , electrons will not reach the anode, and this is called Hull cut-off voltage equation. So, these two voltage equations, the cut-off magnetic field equation and cut-off voltage equations, they are very important because they determine what should be the applied magnetic field so that the electrons do not reach the anode or what should be the applied V_{naught} for a given B_{naught} so that the electrons will not reach the anode.

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Now, we qualitatively discuss the very important mode of operation of magnetron, which is called pi mode. So, a magnetron has number of cylindrical cavities opening in the annular region between cathode and anode, which is called the interaction region, and pi mode is the mode where the phase shift between the fields in the adjacent cavities is pi radian. We will see why it is important.

And for practical magnetron, the frequency of this mode is easily separated from adjacent mode by incorporating, conducting straps connected to alternate segments of the anode block. So, it is very easy to separate out this particular mode from the other adjacent modes. Now,

qualitatively let us see how the electrons transfer energy to the cavities when operating under π mode.

Suppose, as we have seen that the voltage when it is below the voltage and the fields, magnetic fields, they can be set up in such a way so as to decide whether an electron will reach anode or it will come back. Suppose, for the time being let us assume that there is no RF field present in these cavities and the DC fields have been set in such a way that this electron, which is denoted by anode, it is emitted and it is repelled back to the cathode.

Now, in the presence of this RF field, and if the field configuration is favorable, that means here, we can see that the electric field component, they are actually in the same direction as the velocity of the electron here. So, what will happen, here the electron will get retarded, and as a result its radius will increase, and this electron in the presence of this RF field instead of following this path, its trajectory will be what is shown as a, with larger radius of curvature.

And this field being a retarding field, it will actually extract energy from the electron, and its radius of curvature will increase further. Now if the adjustments are made in such a way by the time this electron marked a, traveling along this trajectory comes to the second cavity, when this field now changes because we are talking of π mode, so there is a phase shift of π , you can see in the field, now by that the electron travels to the second cavity, its field polarity changes.

In that case, it will further retard the electron, and this field also will extract energy from the electron, and eventually the electron will get collected at the anode. So, these are the electrons which will contribute to the building up of field in the cavity.

Let us now consider another case, when this electron was emitted here correspondingly another electron be B naught whose trajectory is shown in the absence of RF field but because of this RF field present and this field is now opposite to the direction of movement of the electron, it will reduce the radius of curvature and instead of following B naught path it will follow this trajectory B and come back to the cathode.

So, we see that, but in that process, this electron following this trajectory B, instead of delivering energy to the field will extract energy from the field. So we see that we have both type of electrons, the electrons which deliver energy to the field and electrons which extract

energy from the field. Now, this type of electrons following this trajectory B, they are quickly removed from the interaction space.

Whereas the electrons following the trajectory a, they remain for a much longer time in this interaction space, and they visit multiple cavities, and therefore, they deliver more energy than what is extracted by these electrons shown by the trajectory B, and as a result, overall the oscillation grows. So, qualitatively this is how in a magnetron the oscillation can grow and finally a steady state is reached, and power can be extracted from the magnetron using appropriate arrangement as discussed earlier. So, with this we come to the end of our discussion on magnetron.(Refer Slide Time: 56:21)

TRAVELING WAVE TUBE (TWT)

Next, we will discuss traveling wave tube. We have seen the working of different microwave tubes such as klystron, reflex klystron, and magnetron. Let us now consider another microwave tube, which is known as traveling wave tube or in short TWT. We will discuss only the basic operation of such tubes without going into many mathematical details.

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- ❑ In TWT, the microwave circuit is non-resonant and the wave propagates with the same speed as the electrons in the beam.
- ❑ A slow-wave structure is used to slow down the wave
- ❑ A small amount of velocity modulation is caused by the weak electric fields associated with the traveling wave.
- ❑ This velocity modulation later translates to current modulation, which then induces an RF current in the circuit, causing amplification.

In TWT, or traveling wave tube, the microwave circuit is non-resonant, so we have seen in the case of klystron or magnetron, we had resonators. Here, the microwave circuit is non-resonant, and the wave propagates with the same speed as the electrons in the beam. A slow-wave structure is used to slow down the wave. A small amount of velocity modulation is caused by the weak electric fields associated with the travelling wave.

This velocity modulation later translates to current modulation, which then induces an RF current in the circuit, causing amplification. So, we have this slow-wave structure, where we have a signal is given at the input, and then as this wave propagates down the slow-wave structure, it interacts with the electron beam, and that results into amplification of the signal, provided certain conditions are met.

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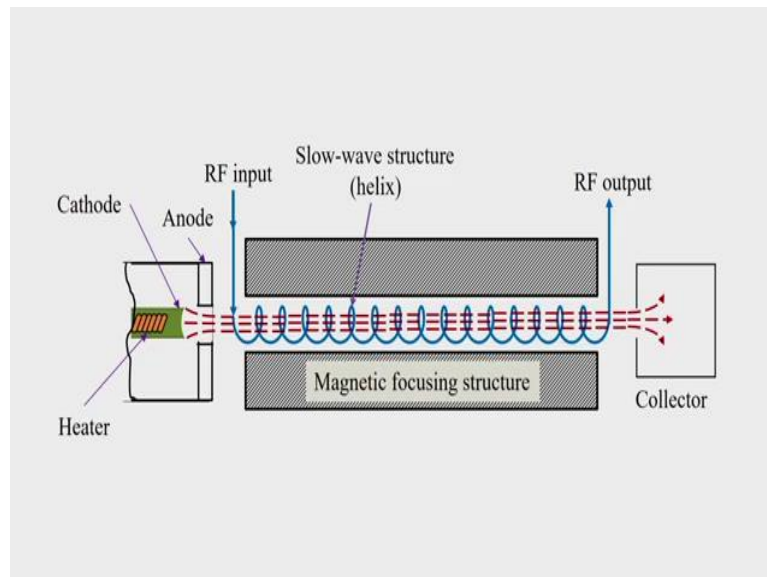
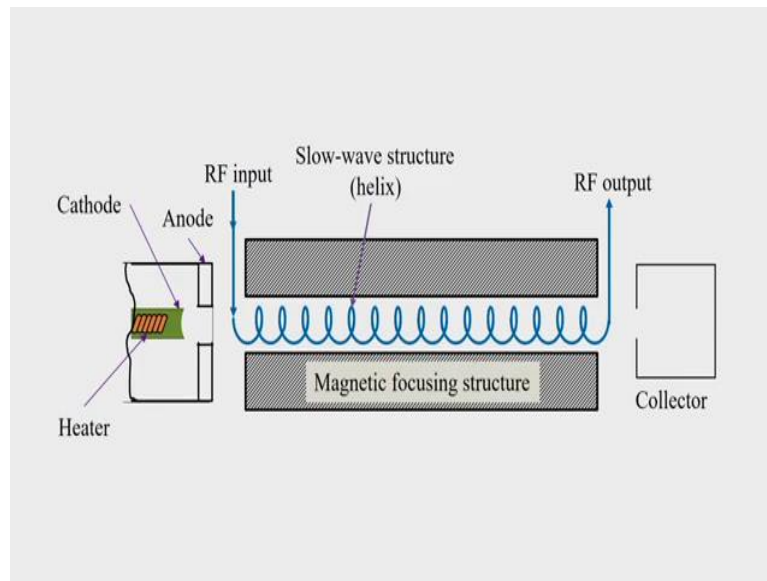
- ❑ The interaction of electron beam and RF field in the TWT is continuous over the entire length of the circuit
- ❑ The wave in the TWT is a propagating wave

A helix travelling wave tube consists of an electron beam and a slow-wave structure. The electron beam is kept focused by a constant magnetic field along the electron beam and the slow-wave structure.

The interaction of the electron beam and the RF field in the TWT is continuous over the entire length of the circuit. This is in contrast to say klystron, where we had the interaction of the field and the electrons for a very brief period when the electrons pass through the buncher or catcher cavity. So, in a TWT this interaction is continuous, and as we have mentioned, the wave in a TWT is a propagating wave, whereas, in klystron or magnetron, the wave remains confined within the cavities.

A helix traveling wave tube consists of an electron beam and the slow-wave structure. Now, TWT has also come in different forms. We will discuss the basic TWT configuration, which is a helix TWT. The electron beam is kept focused by a constant magnetic field along the electron beam and the slow-wave structure. So, we have a slow-wave structure, which is a helical structure, and an axial magnetic field is applied to keep the electron beam focused.

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So, the very basic structure of a helix TWT is shown. So, it has a cathode which emits electrons. We are not showing the biasing arrangements here. The anode accelerates the electron, and then, this beam of electron they enter the slow-wave structure, and RF input is provided here which sets up the wave in this slow-wave structure, and these electrons when traveling from, traveling within the interaction space, they interact with the wave in the slow-wave structure and this interaction is continuous.

The bunching of electron takes place due to velocity modulation, and when bunching of electron takes place, it results into current modulation, and finally, the RF input is amplified by exchange of energy from the electron beam, and the electrons after leaving the interaction

space are collected by the collector. Here, we see the magnetic focusing structure, which applies a field so that the beam remains focused.

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- ❑ The applied signal propagates around the turns of the helix and produces an electric field at the centre of the helix, directed along the helix axis.
- ❑ When the electrons enter the helix tube, an interaction takes place between the moving axial electric field and the moving electrons.
- ❑ On the average, the electrons transfer energy to the wave on the helix. This interaction causes the signal wave on the helix to become larger.

The applied signal propagates around the turns of the helix and produces an electric field at the center of the helix, which is directed along the helix axis. When the electrons entered the helix tube, an interaction takes place between the moving axial electric field and the moving electrons. On average, the electrons transfer energy to the wave on the helix. This interaction causes the signal wave on the helix to become larger. And this is how we get amplification of the signal.

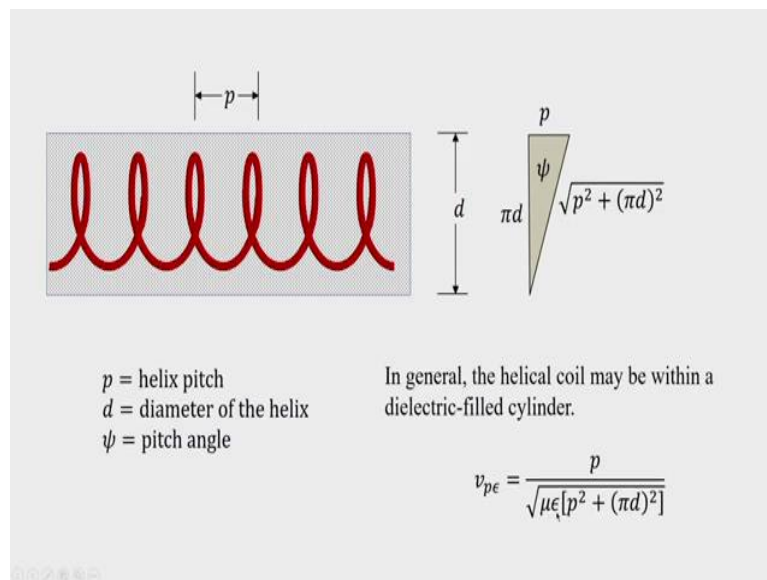
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- ❑ Those electrons which enter the helix at zero field are not affected by the signal wave; those electrons entering the helix at the accelerating field are accelerated, and those entering at the retarding field are decelerated.
- ❑ As the electrons travel further along the helix, they bunch at the collector end. The bunching shifts the phase by $\pi/2$. The electrons in the bunch encounter a stronger retarding field.
- ❑ Microwave energy of the electrons is delivered by the electron bunch to the wave on the helix.

Those electrons which enter the helix at zero fields are not affected by the signal wave; those electrons entering the helix at the accelerating field are accelerated, and those entering the retarding field are deaccelerated. As the electrons travel further along the helix, they bunch at the collector end, and bunching shifts the phase by π by 2. The electrons in the bunch encounter a stronger retarding field. The microwave energy of the electrons is delivered by the electron bunch to the wave on the helix.

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$$v_{p\epsilon} = \frac{p}{\sqrt{\mu\epsilon[p^2 + (\pi d)^2]}}$$



So, here, we briefly discuss the slow-wave structure. We have shown a helical structure. So, this distance between the strands, it is the pitch p of the helix and d is the diameter of the helix, and we can see that πd , we can find out, actually πd is the distance that is travelled from this end to this end, and p is the pitch. So, we can write that the actual distance is under root p square plus πd square. Here, this angle ψ is the pitch angle. Now, this helix structure may also be in a dielectric field cylinder. In that case, we get the phase velocity $V_{p\epsilon}$ is equal to p by root $\mu\epsilon$ p square plus πd square.

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$$v_p \approx \frac{pc}{\pi d} = \frac{\omega}{\beta}$$

□ For a very small pitch angle, the phase velocity along the coil in free space is approximately represented by

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For a very small pitch angle, the phase velocity along the coil in free space is approximately represented by v_p is equal to pc divided by πd and this is ω by β .

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p = helix pitch
 d = diameter of the helix
 ψ = pitch angle

In general, the helical coil may be within a dielectric-filled cylinder.

$$v_{p\epsilon} = \frac{p}{\sqrt{\mu\epsilon[p^2 + (\pi d)^2]}}$$

Here, we can say that when μ is equal to μ_0 and ϵ equal to ϵ_0 and p^2 is neglected with respect to $(\pi d)^2$, then we have $v_p = \frac{pc}{\pi d}$, so it becomes cp by πd , so this is the velocity in the axial direction.

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- ❑ For a very small pitch angle, the phase velocity along the coil in **free space** is approximately represented by

$$v_p \approx \frac{pc}{\pi d} = \frac{\omega}{\beta}$$

- ❑ In the helical slow-wave structure, a translation back or forth through a distance of one pitch length results in an identical structure. Thus the period of a helical slow-wave structure is its pitch.

In the helical slow-wave structure, a translation back or forth through a distance of one pitch length results in an identical structure, and thus the period of a helical slow-wave structure is its pitch.

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The characteristics of the travelling wave tube are as follows:

Frequency range: 3 GHz and higher

Bandwidth: about 0.8 GHz

Efficiency: 20 to 40%

Power output: up to 10 kW average

The characteristics of the traveling wave tubes are – It is usually operated in the frequency range 3 gigahertz and higher. Bandwidth is about 0.8 gigahertz, so this is a huge bandwidth. Efficiency 20 to 40 percent and power output up to 10-kilowatt average. And power gain up to 60 dB. So TWT is often used as amplifiers in different systems, such as radar, communication satellites, and many other broadband applications.

So, in this module, we have seen some basics of some of the commonly used microwave tubes. We discussed the limitations of conventional tubes. In the microwave frequency range, and then we have seen how these limitations are overcome in specially designed tubes, such as klystron, magnetron and finally in TWT.

It may be mentioned that when it comes to a very high power microwave application, the tubes are extensively used as microwave signal sources or for amplification purposes. In the next module, we will introduce devices based on ferrites. We will first discuss the basic properties of ferrites, briefly discuss the wave propagation on ferrites, and then we will discuss two very important devices, isolator, and circulator which are designed using ferrites.