Microwave Engineering
Professor Ratnajit Bhattacharjee
Department of Electronics & Electrical Engineering
Indian Institute of Technology, Guwahati
Lecture 04
Introduction to Microwave Engineering and Transmission Line Theory

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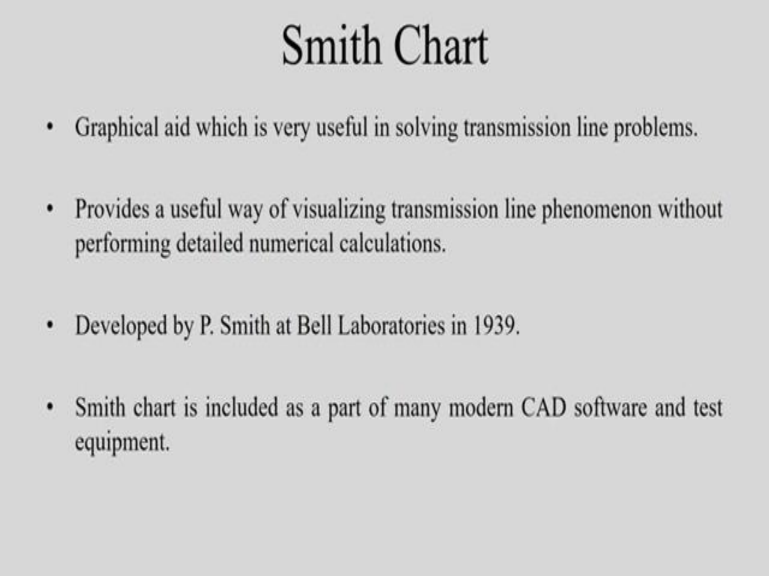

In the earlier lectures, we have seen the different characteristics of the transmission line. In this lecture we discuss the basics of Smith chart. A Smith chart is a graphical aid which is very useful in solving transmission line problems, and it provides a useful way of visualizing transmission line phenomena without performing detailed numerical calculations. This chart was developed by P.Smith at Bell Laboratories in 1939.

But even though this chart was developed long back, the Smith chart is included as a part of many modern CAD software and test equipment, so smith chart is being extensively used till date for solving various transmission line programs.

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We know that, $\varGamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

$$\therefore \frac{Z_L}{Z_0} = \frac{1+\varGamma}{1-\varGamma}$$

Denoting the normalized load impedance

$\frac{Z_L}{Z_0} = r_L + jx_L$ and $\varGamma = \varGamma_r + j\varGamma_i$

we can write

$$r_L + jx_L = \frac{(1+\varGamma_r) + j\varGamma_i}{(1-\varGamma_r) - j\varGamma_i}$$

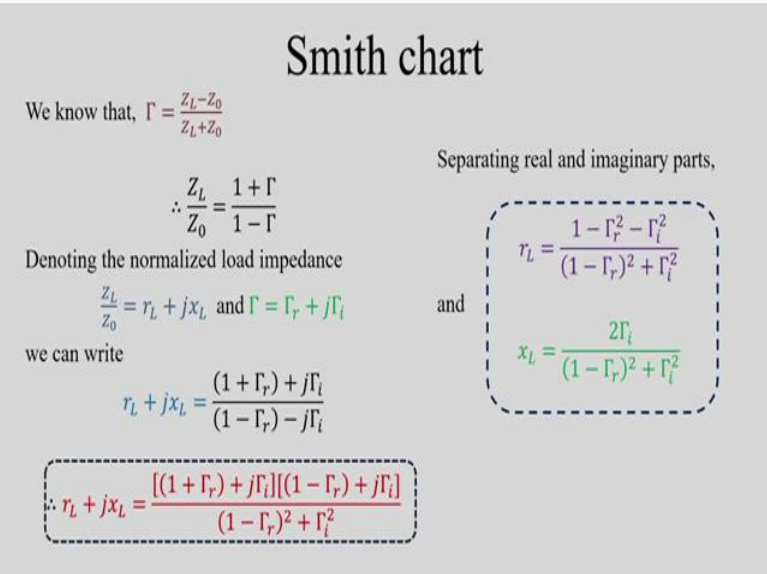
$$\therefore r_L + jx_L = \frac{[(1+\varGamma_r) + j\varGamma_i][(1-\varGamma_r) + j\varGamma_i]}{(1-\varGamma_r)^2 + \varGamma_i^2}$$

Separating real and imaginary parts,

$$r_L = \frac{1 - \varGamma_r^2 - \varGamma_i^2}{(1-\varGamma_r)^2 + \varGamma_i^2}$$

and

$$x_L = \frac{2\varGamma_i}{(1-\varGamma_r)^2 + \varGamma_i^2}$$



Now let's see how this Smith chart is organized. We know that the reflection coefficient gamma at the load is given by zl minus z not divided by zl plus z not. If we normalize the load impedance zl by z not, then we can write, normalized load impedance as one plus gamma divided by one minus gamma.

Now let's denote this normalized load impedance by its real and imaginary parts rl and xl. So, zl by z not is rl plus j xl, and gamma is equal to gamma r plus j gamma I, where gamma r and gamma I jr the real and imaginary parts of the reflection coefficient gamma. Now, if we substitute this gamma here, then we can write rl plus j xl in this form one plus gamma r plus j gamma I divided by one minus gamma r minus j gamma.

Now what we can do, we can multiply the right-hand side by conjugate of this term both at the numerator and at the denominator. So once we do that and we simplify, so this becomes 1 minus gamma r whole square plus gamma I square and this term we can expand and once we separate the real and imaginary parts we get,

Normalized resistance rl is equal to one minus gamma r square minus gamma I square divided by one minus gamma r square plus gamma I square, and similarly, xl is two gamma I divided by one minus gamma r square plus gamma i square. Now these two equations are very important in developing the Smith chart.

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From $r_L = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}$, we can write:

$$\Gamma_r^2(1+r_L)+\Gamma_i^2(1+r_L)-2\Gamma_r r_L = 1-r_L$$

$$\Gamma_r^2 - \frac{2\Gamma_r r_L}{(1+r_L)} + \Gamma_i^2 = \frac{(1-r_L)}{(1+r_L)}$$

$$\Gamma_r^2 - \frac{2\Gamma_r r_L}{(1+r_L)} + \left(\frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \frac{(1-r_L)}{(1+r_L)} + \left(\frac{r_L}{1+r_L}\right)^2$$

$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2$

Smith Chart

From $r_L = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}$, we can write:

$$\Gamma_r^2(1+r_L) + \Gamma_i^2(1+r_L) - 2\Gamma_r r_L = 1 - r_L$$

$$\Gamma_r^2 - \frac{2\Gamma_r r_L}{(1+r_L)} + \Gamma_i^2 = \frac{(1-r_L)}{(1+r_L)}$$

$$\Gamma_r^2 - \frac{2\Gamma_r r_L}{(1+r_L)} + \left(\frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \frac{(1-r_L)}{(1+r_L)} + \left(\frac{r_L}{1+r_L}\right)^2$$

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2$$

So we take the expression for the normalized resistance, and this equation can be re-arranged in this form, then if you divide by one plus gamma rl throughout, then we can put this equation in the function. Now by adding rl divided by one plus rl whole square on both sides we get this equation and from this, we get gamma r minus rl by one plus rl whole square plus gamma I square one by one plus rl whole square.

So we can see that this gives us a circle if we plot it in the complex gamma plain, we find that the center of the circle is given by rl divided by one plus rl and zero. So let us see how these circles look in the complex gamma plain.

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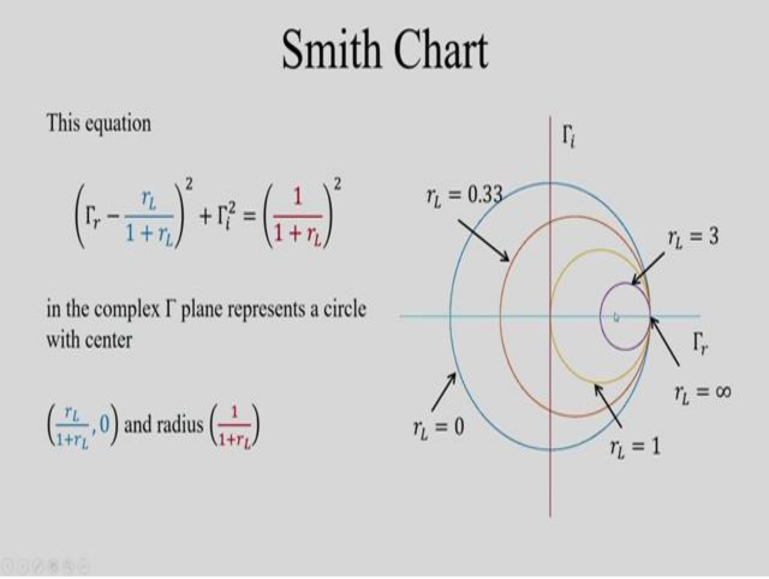
This equation

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2$$

in the complex Γ plane represents a circle with center

$\left(\frac{r_L}{1+r_L},\ 0\right)$ and radius $\left(\frac{1}{1+r_L}\right)$



As I have said, the center is rl divided by one plus rl zero, and the radius is one by one plus rl. So for different values of rl if we plot these circles in the gamma plane this is gamma r, and this is gamma I axis, so we can say that,

For rl equal to zero we will get this circle, so this is the rl equal to zero circles and if we will take rl is equal to point three, then we get this particular circle, so in fact one can rl is equal to one gives this circle and rl is equal to three finally we observe that as the value of rl increases the radius decreases and finally when rl tens to infinity the circle becomes a point here.

And for rl equal to zero, we get a circle in the gamma plane with a radius equal to unity. So please note that this circle has the outer most circle has a radius of unity, and all others circles for different values of rl they pass through this point gamma r is equal to one and the radii keep on decreasing.

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In the same manner, we have

$$x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

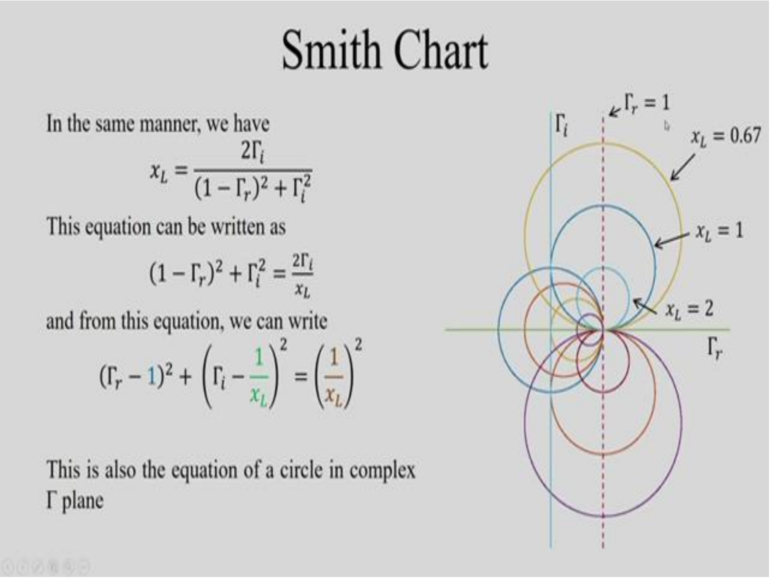
This equation can be written as

$(1-\Gamma_r)^2 + \Gamma_i^2 = \frac{2\Gamma_i}{x_L}$

and from this equation, we can write

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

This is also the equation of a circle in complex Γ plane



In the same manner, if we consider the expression for normalized reactance xl then we get, one minus n gamma r square plus gamma I square is equal to two gamma I by xl. Now what we can do we can add one by xl square on both sides take this turn to the left and rearrange then we get an equation of this form gamma r minus one square plus gamma I minus one by xl square is equal to one by xl square.

Now this equation again gives rise to a set of circles as we take before the example of xl, so let us see how these circles look. So here once again this is the gamma r axis, and this is the gamma I axis, please note that the centers of all these circles will now lie on gamma r equal to one, so this is shown by this dotted line gamma r is equal to one and then,

For example, please note this radius is one by xl, so smaller the value of xl larger will be the radius. So for xl is equal to point six seven we have this circle, for xl is equal to one we get this circle, for xl is equal to two we get this circle, so we see that as xl becomes larger and larger we are approaching towards this point, in fact xl is equal to infinity will bring you here.

And as xl becomes smaller, this circle it becomes larger and the radius becomes larger and the center moves over this line. So finally when xl tends to zero this circle will pass through this line. We notice that we have two sets of circles we find that xl can be positive or negative, so based on that we will get these two sets of circles and here we see that for negative xl the centers move in this part of the gamma r equal to one line.

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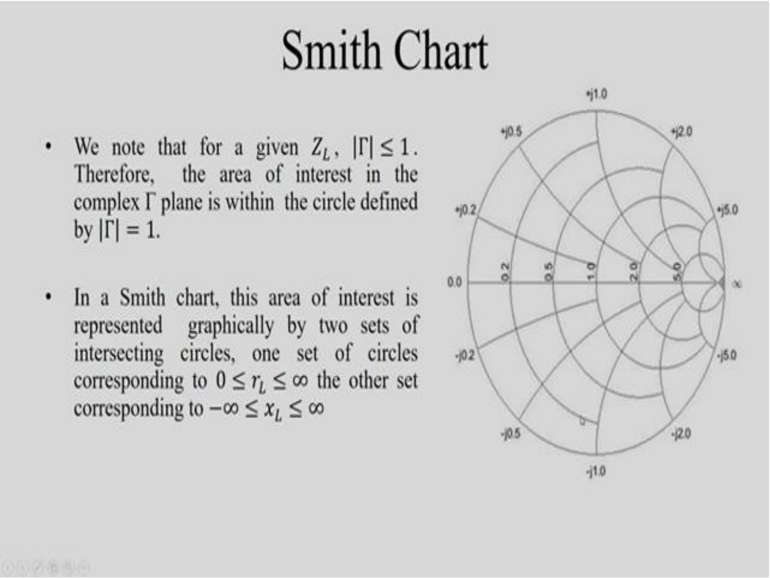
- We note that for a given Z_L, $|\Gamma| \leq 1$. Therefore, the area of interest in the complex Γ plane is within the circle defined by $|\Gamma| = 1$.
- In a Smith chart, this area of interest is represented graphically by two sets of intersecting circles, one set of circles corresponding to $0 \leq r_L \leq \infty$ the other set corresponding to $-\infty \leq x_L \leq \infty$



Now for a given zl, for a passive load we know that mod gamma is less than one. Therefore, our area of interest in the complex gamma plane is within the circle which is defined by mod gamma equal to one. So if you revisit here, you can see that this region is the region of interest. So if we retain this part only.

In a Smith chart this area of interest is represented graphically by two sets of intersecting circles, one set of circles corresponding to rl in the range of zero to infinity, other set corresponding to xl in the range of minus infinity to infinity. So if we draw this circle the Smith chart will look as shown in the figure, and here you can see that these circles are constant are circles, whereas these arcs are part of the constant x circle, and we have the negative values represented here for the reactance part.

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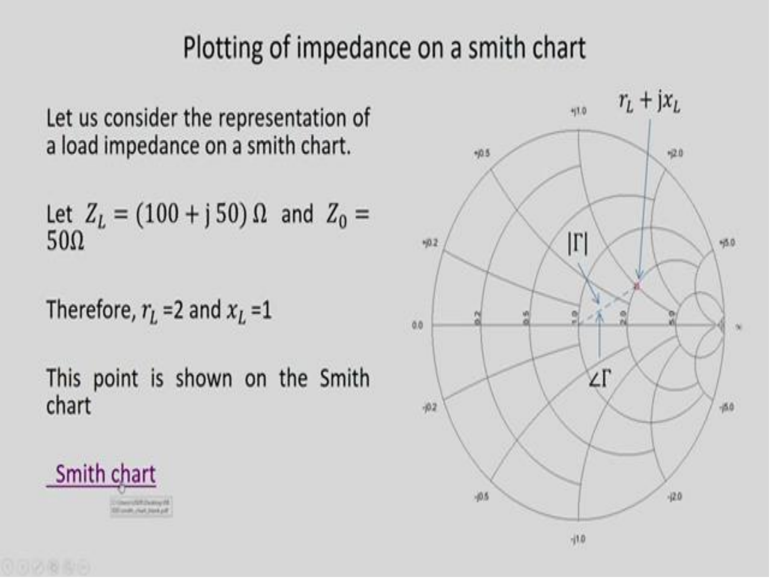
Let us consider the representation of a load impedance on a smith chart.

Let $Z_L = (100 + \text{j } 50)\ \Omega$ and $Z_0 = 50\Omega$

Therefore, r_L =2 and x_L =1

This point is shown on the Smith chart

 Smith chart



Now once we have this chart, we can now plot impedance on this chart. Please note that we can plot the normalized impedance, and the actual impedance can be obtained by multiplying the impedance shown on the chart by characteristic impedance by which the impedance has been normalized.

So let us start with an example, let zl is equal to hundred plus j fifty ohm and normalizing impedance z not is fifty ohm therefore, our rl normalized resistance part is two and xl normalized reactive part is one, and this point is shown on the Smith chart, so this is the point, which is determined by the intersection of r is equal to two circles and x is equal to one circle.

So this is the point denoted by the normalized impedance, and if you put a line from the center of the Smith chart to this impedance point, then the length of this line represents the magnitude of the reflection coefficient gamma, also this angle represents the angle of gamma, angle with respect to this axis, that is subtended by this line. So either from this point now we find that we get a direct measure the length of this line as the magnitude of the reflection coefficient, and this is theta or angle of the reflection coefficient.

Now here we have represented Smith chart by only a few circles the actual Smith chart that is used in practice for doing transmission line based calculations contains a large number of values for both r and x, and there are certain additional parameters which are provided in that chart which helps in doing transmission line calculation.

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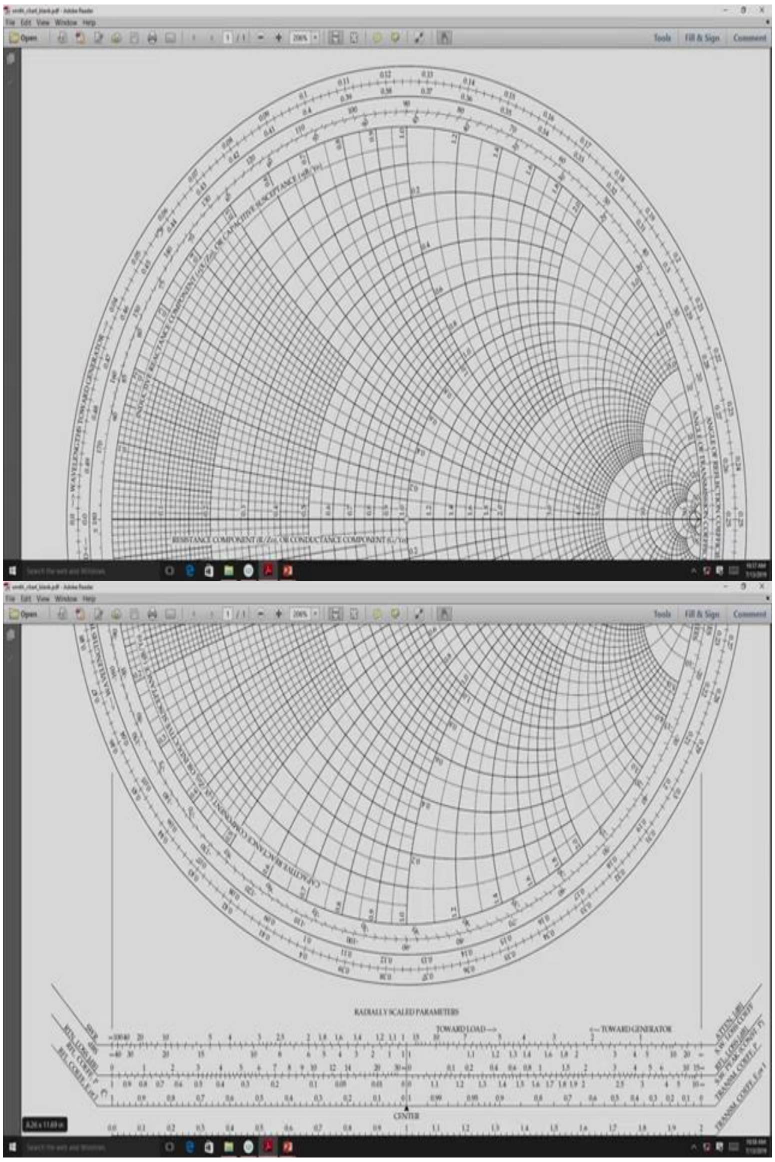

So let's see a practical Smith chart. So here you can see that the resistance circles and the reactance circles they are very closely spaced and their values are indicated, for example, this is r is equal to point one r is equal to point two r is equal to one circle r is equal to two circle r is equal to five. So all these circles these values are provided. Similarly, here x is equal to one, x is equal to one point six, x is equal to point five, so you can see that all these circles are very closely spaced now.

And this part of the Smith chart it represents the inductive reactive components that means j x by z not, when it is used as an impedance chart, Smith chart can also be used for calculation of advertences, so this part will also represent jb by y not. Now we can see that here that the peripheral of the chart we have the wavelengths marking, please note that in a Smith chart, if you move around the center of the chart make a complete circle, then essentially a lambda by two distance is covered on the transmission line.

Now here you can see the markings are wavelength towards the generator. So for any clockwise movement essentially it is wavelength towards generator, we move from the load towards the generator, and if you draw a line from the center of the Smith chart intersecting these circles then the wavelength that is being covered can be read directly. Similarly we have if you move counter-clockwise it is wavelength towards load.

Please note that is zero starts from here, and then it increases this way. Whereas, wavelengths towards generator here is the zero and wavelength increases in this way. And finally this full Smith chart covers lambda by two, there are some additional scales provided which help directly computing several other parameters like swr, return loss, the magnitude of reflection coefficient. So these parameters can be directly computed from the Smith chart.

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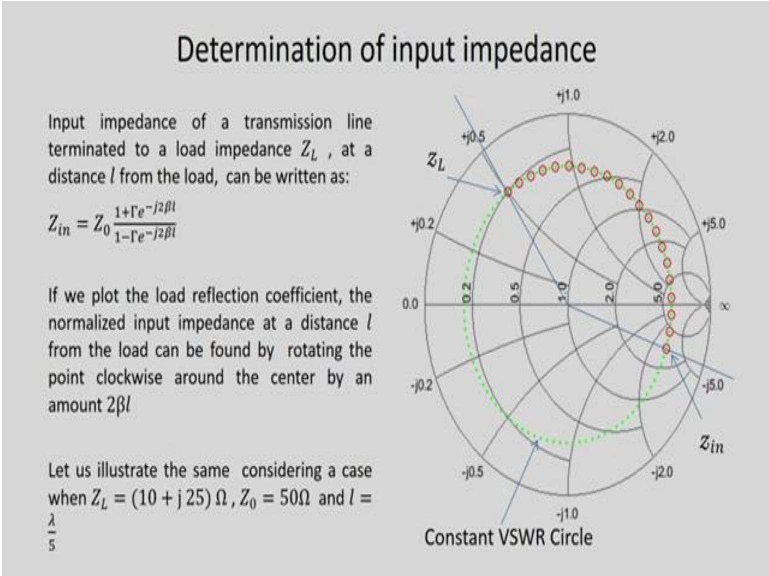
Input impedance of a transmission line terminated to a load impedance Z_L , at a distance l from the load, can be written as:

$$Z_{in} = Z_0 \frac{1+\Gamma e^{-j2\beta l}}{1-\Gamma e^{-j2\beta l}}$$

If we plot the load reflection coefficient, the normalized input impedance at a distance l from the load can be found by rotating the point clockwise around the center by an amount $2\beta l$

Let us illustrate the same considering a case when $Z_L = (10 + \text{j } 25)\ \Omega$, $Z_0 = 50\Omega$ and $l = \frac{\lambda}{5}$



Let's now see how we can utilize this Smith chart for determining the input impedance of a transmission line which is terminated by some load. So we know that in terms of the reflection coefficient, the input impedance z in can be written as z not one plus gamma e to the power minus j two beta l, one minus gamma e to the power minus j two beta l.

Now, if you plot the load reflection co-efficient this gamma and then normalized input impedance at a distance l from the load can be formed by rotating the point clockwise around the center by an amount two beta l. So please note beta is two pi by lambda and when l is lambda by two we get a phase shift of two pi that means a complete rotation.

Now let us illustrate how we find the input impedance by taking an example, and we consider the case when zl is ten plus j twenty-five ohm and z not is fifty ohms, and l is lambda by five. So here in this chart this is the starting point zl, which is point two j point five if you divide ten plus j twenty-five by fifty you will get, point two point five and from this point now if you draw a circle this circle will be called the constant VSWR circle.

So any point on this circle has the same magnitude of reflection coefficient and, therefore the same value of the VSWR. Now starting from this point as he moves along this constant VSWR circle, the movement is indicated by these small circles, and we reach this point. Now you know that from the wavelength marking provided on the chart you can find out this to be equal to lambda by five or point two lambda starting from this point and you stop here so,

You can now read out the impedance at this point, so you can see that it is closer to a circle, which will be around r equal to three, and j is equal to approximately in actual Smith chart we can read out the value. So in that way you can find out the normalized input impedance, and then you multiply it by z not that is fifty ohm, and then you get the actual impedance seen at this point.

The impedance at intermediate points can be also seen in, for example, if we stop at this length then we get an input impedance of one plus j two that means here it is fifty plus j hundred ohm, now we can see that starting from the load as we move down the line, we cross this axis that means here the input resistance becomes purely resistive.

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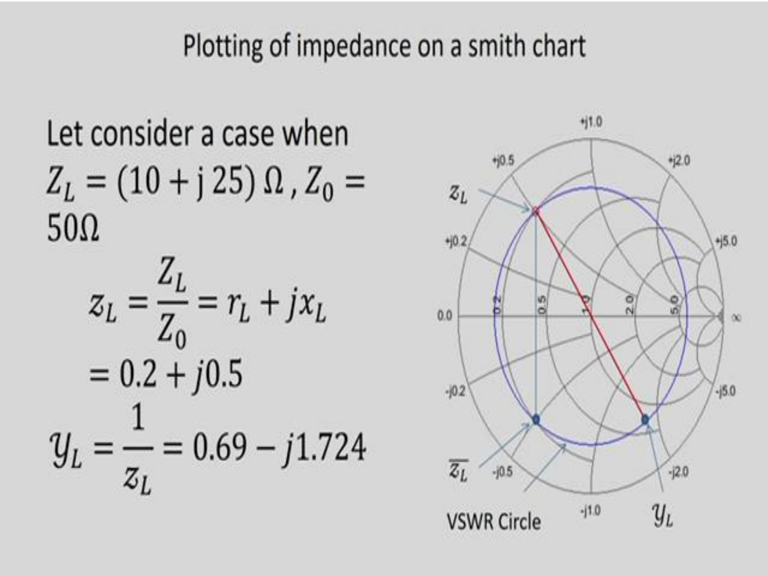
Let consider a case when $Z_L = (10 + \text{j } 25)\ \Omega$, $Z_0 = 50\Omega$

$$z_L = \frac{Z_L}{Z_0} = r_L + jx_L = 0.2 + j0.5$$

$$\mathcal{Y}_L = \frac{1}{z_L} = 0.69 - j1.724$$



So let us consider another case when zl is ten plus j twenty-five and z not is fifty ohm and we have seen the normalized impedance is point two plus j point five, now yl one by zl is point six nine minus j one point seven two four. Now how we can mark this yl directly on the Smith chart.

So we start with marking zl and then we draw the constant VSWR circle then if we pass a line, through the center of the Smith chart where it touches this circle this point gives us yl which is equal to point six nine minus j one point seven two four and here you can see the values are close and from the actual Smith chart it can be readout.

Similarly, if we drop a line from here, intersecting this axis then the point where we touch the constant VSWR circle will give conjugate of zl. So this Smith chart starting from a load point we can plot the yl we can plot zl bar, please note that when we are using this chart as an impedance chart this point on the chart will represent the short circuit point because here zl becomes zero and this point will represent the open circuit point where both rl and xl becomes infinity.

We have seen some basics of the Smith chart. We will use the Smith chart in our later lectures extensively particularly when we will be solving the impedance matching problem.

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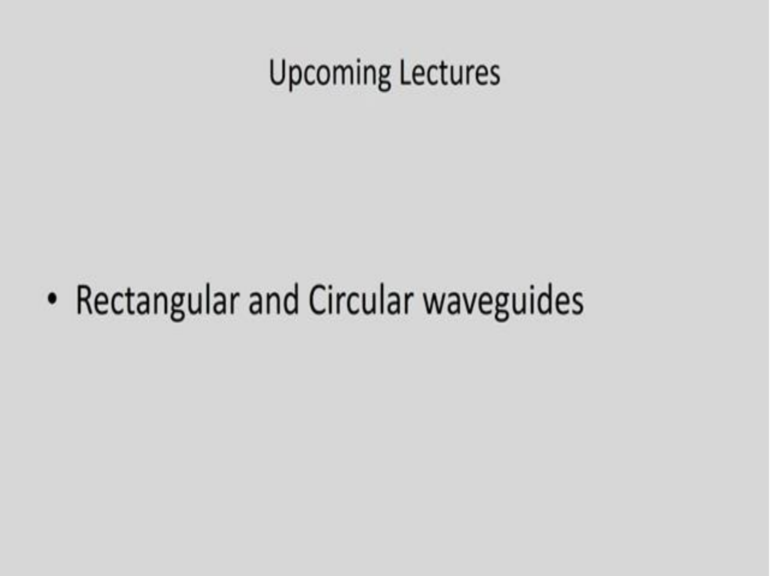

So far we can find our discussion to transmission lines, we consider another type of waveguiding structure which are in the form of hollo metallic pipes of rectangular or circular cross-section these are called rectangular and circular waveguides. So our next lecture will start with the discussion of this type of waveguides.