Microwave Engineering

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Lecture 7

Rectangular and Circular Waveguides

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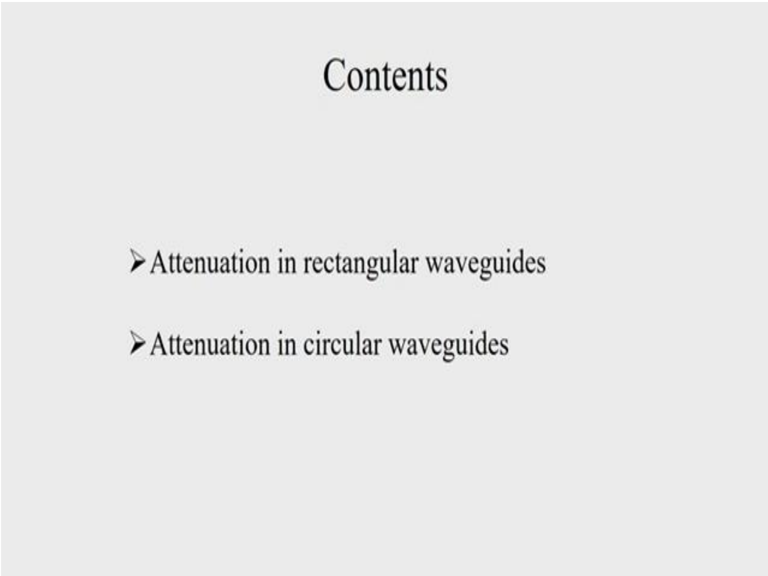

We now cover these two topics, attenuation in rectangular waveguides and attenuation in circular waveguides. We start with formulating the attenuation in rectangular waveguides.

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- We have seen that for dominant TE_{10} mode

$$H_z = A_{10}\, cos\frac{\pi x}{a}e^{-j\beta z}$$

$$E_y = \frac{-j\omega\mu a}{\pi}A_{10}\, sin\frac{\pi x}{a}e^{-j\beta z}$$

$$H_x = \frac{j\beta a}{\pi}A_{10}\, sin\frac{\pi x}{a}e^{-j\beta z}$$

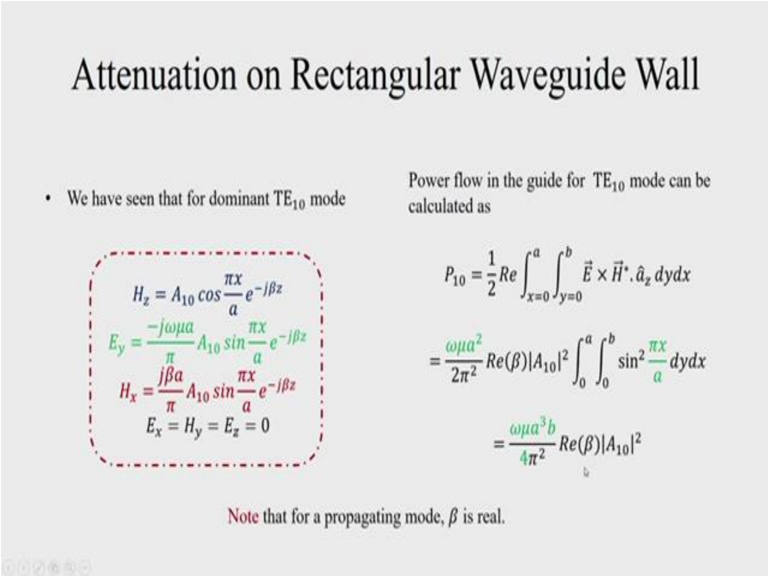
$$E_x = H_y = E_z = 0$$

Power flow in the guide for TE_{10} mode can be calculated as

$$P_{10} = \frac{1}{2}Re\int_{x=0}^{a}\int_{y=0}^{b}\vec{E}\times\vec{H}^{*}.\hat{a}_z dydx$$

$$= \frac{\omega\mu a^2}{2\pi^2} Re(\beta)|A_{10}|^2 \int_0^a \int_0^b \sin^2 \frac{\pi x}{a} dydx$$

$$= \frac{\omega\mu a^3 b}{4\pi^2} Re(\beta)|A_{10}|^2$$



We will restrict our discussion for the dominant TE$_{10}$ mode only, and for TE$_{10}$ mode we have seen the longitudinal H z is given in this form, and we have the transverse field component E y and H x and E x, H y and E z equal to 0. Now, we can calculate the power flow inside the waveguide as half real part of E cross H star is the pointing vector and then dot the area dx dy and a z is the unit vector for this elemental area.

So if we evacuate this expression we get an expression for the power that flows inside the waveguide because of the dominant TE$_{10}$ mode, and this can be written as omega mu a cube b by 4 pi r square real part of beta and A 10 magnitude square. Please note that when the wave will propagate inside the waveguide beta will be a real quantity.

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- Power loss per unit length due to finite wall conductivity is

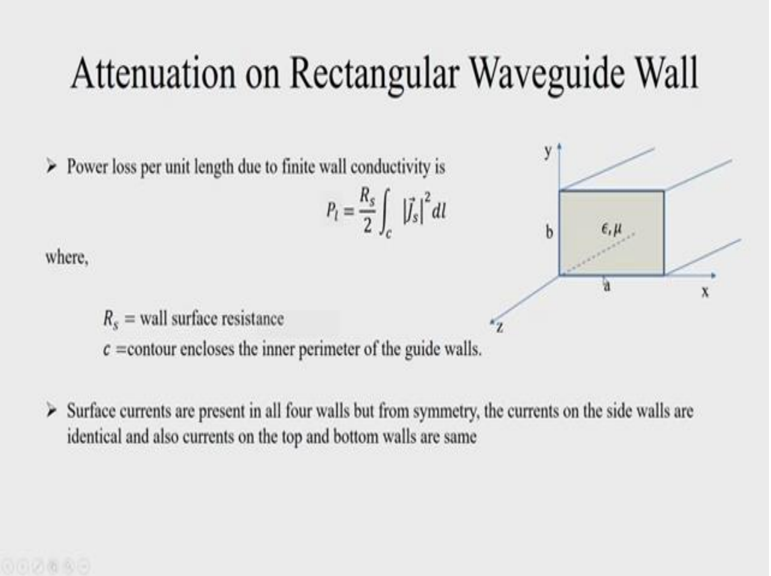
$$P_e = \frac{R_s}{2} \int_c |\vec{J}_s|^2 dl$$

where,

R_s = wall surface resistor

c =contour encloses the inner perimeter of the guide walls.

- Surface currents are present in all four walls, but from symmetry, the currents on the side walls are identical and also currents on the top and bottom walls are same



Now, let us see what causes attenuation inside the waveguide. We have calculated the field components inside the rectangular waveguide considering the walls to be a perfect conductor. In practice this conductors will have finite conductivity, and a small amount of power will be lost on the metallic walls of the waveguide. Further, another source of attenuation is because of the loss in the dielectric media feeling this waveguide, for the timing we are assuming this dielectric inside the waveguide to be a perfect dielectric it does not dissipate any power.

Now, how we calculate the power that is lost on the waveguide walls, this power loss per unit length due to finite conductivity is given by P l is equal to R s by 2 c is the contour and J magnitude of surface current squared into dl. Now wherefrom this surface current comes? We know from the boundary condition that the magnetic fields for TE_{10} mode that means H z and H x they will be discontinuous on this metallic boundary, and we can calculate the surface current that is induced on the walls of the waveguide by n cross H.

R s is the wall surface resistance, and c is the contour that encloses the inner perimeter of the waveguide walls. Now, once we are able to find out J s we can evaluate this integral, and this will give us the power loss per unit length, and from there we can find out the attenuation constant.

Now, the surface currents will be present in all the four walls, but from the symmetry, the currents on the sidewalls will be identical, and similarly the currents on the top and bottom walls their magnitude will be the same and therefore, what we can do? Since we are evaluating mod J square we can evaluate only on two walls and multiply this by 2.

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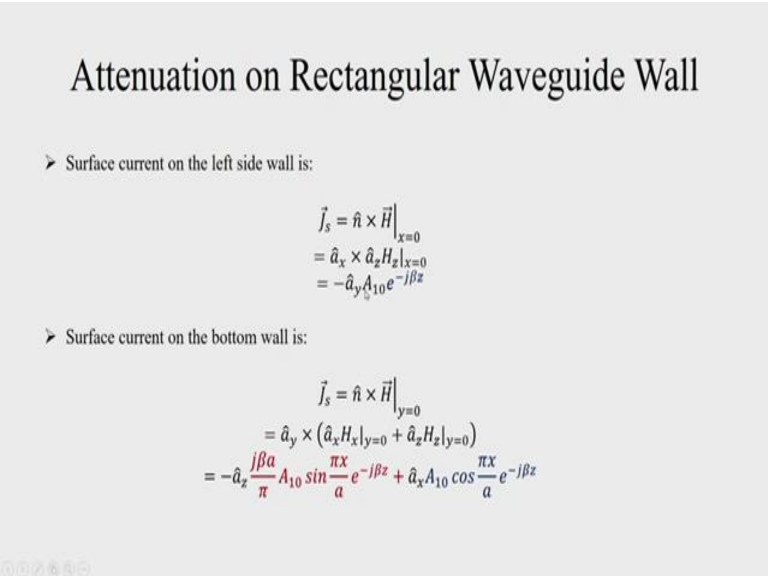
- Surface current on the left side wall is:

$$\vec{J}_s = \hat{n} \times \vec{H}\Big|_{x=0}$$
$$= \hat{a}_x \times \hat{a}_z H_z|_{x=0}$$
$$= -\hat{a}_y A_{10} e^{-j\beta z}$$

- Surface current on the bottom wall is:

$$\vec{J}_s = \hat{n} \times \vec{H}\Big|_{y=0}$$
$$= \hat{a}_y \times \left(\hat{a}_x H_x|_{y=0} + \hat{a}_z H_z|_{y=0}\right)$$
$$= -\hat{a}_z \frac{j\beta a}{\pi} A_{10}\, sin\frac{\pi x}{a} e^{-j\beta z} + \hat{a}_x A_{10}\, cos\frac{\pi x}{a} e^{-j\beta z}$$



The surface current on the left side wall is given by J s is equal to n cross H evaluated at x is equal to 0. Now, if we look at the expression of the field component then H x becomes 0 at x is equal to 0 since the H x dependence with x is given by sign pi x by A, and therefore x is equal to 0 it becomes 0. So, from the x is equal to 0 walls we have only H z, and the vector that is perpendicular vector to this wall is a x and therefore we can write the induced surface current to be equal to minus a y A 10 e to the power minus j beta z.

Similarly, surface current on the bottom wall is contributed by both H x and H z component and for the bottom wall the unit normal vector is a y and therefore once we evaluate this expression a y cross a x H x at y equal to 0 plus a z H z at y equal to 0 and substitute the expression for H x and H z then we get this as the surface current on the bottom wall. So once you have the surface current densities calculated on the top and the bottom wall, and then we can utilize this expression in calculating the power loss per unit length.

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Thus, the rectangular components of the surface current can expressed as:

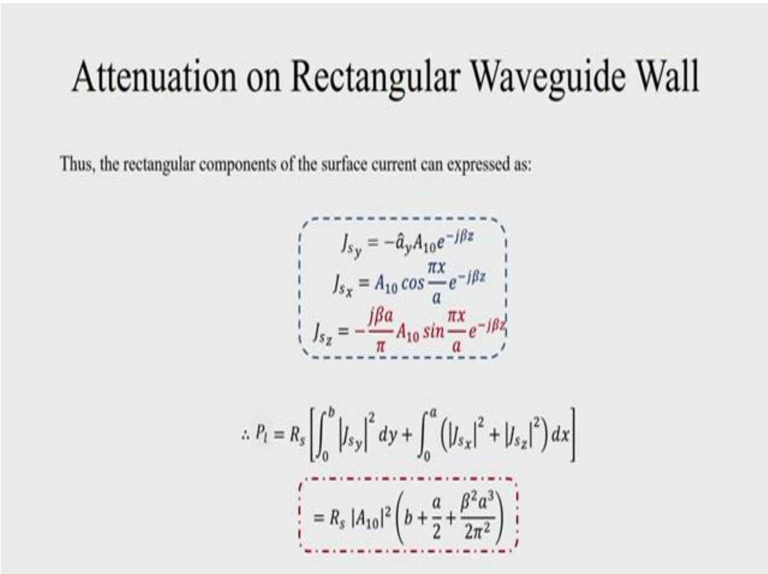
$$J_{s_y} = -\hat{a}_y A_{10} e^{-j\beta z}$$

$$J_{s_x} = A_{10} \cos\frac{\pi x}{a} e^{-j\beta z}$$

$$J_{s_z} = -\frac{j\beta a}{\pi} A_{10} \sin\frac{\pi x}{a} e^{-j\beta z}$$

$$\therefore P_e = R_s \left[\int_0^b \left|J_{s_y}\right|^2 dy + \int_0^a \left(\left|J_{s_x}\right|^2 + \left|J_{s_z}\right|^2\right) dx\right]$$

$$= R_s |A_{10}|^2 \left(b + \frac{a}{2} + \frac{\beta^2 a^3}{2\pi^2}\right)$$



So now we have the J s y, J s x and J s z these current densities are as given and therefore if we substitute these quantities then we get and evaluate this integral we get the expression for power loss per unit length on the waveguide walls and this is given by R_s mod A 10 square b plus a by 2 plus beta square a cube by 2 pi square.

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The attenuation constant is given by

$$\alpha = \frac{P_e(z=0)}{2P_0}$$

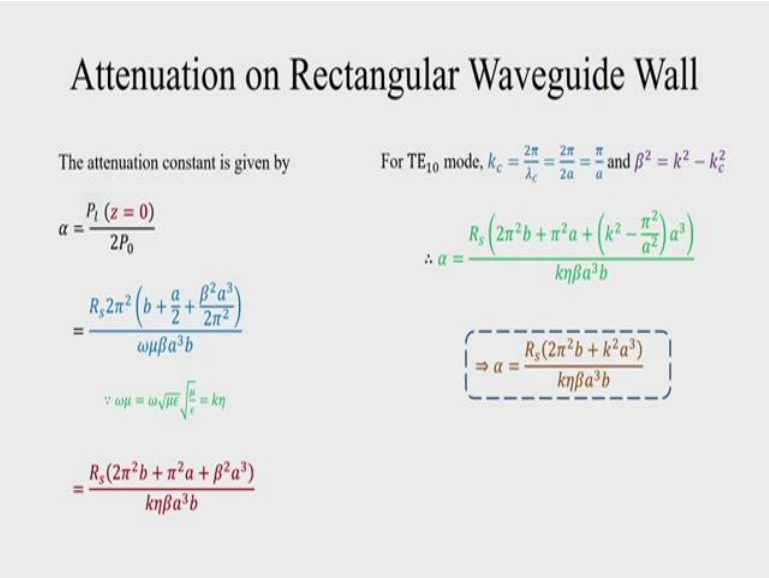
$$= \frac{R_s 2\pi^2 \left(b + \frac{a}{2} + \frac{\beta^2 a^3}{2\pi^2}\right)}{\omega\mu\beta a^3 b}$$

$$= \frac{R_s(2\pi^2 b + \pi^2 a + \beta^2 a^3)}{k\eta\beta a^3 b}$$

For TE_{10} mode, $k_c = \frac{2\pi}{\lambda_c} = \frac{2\pi}{2a} = \frac{\pi}{a}$ and $\beta^2 = k^2 - k_c^2$

$$\therefore \alpha = \frac{R_s\left(2\pi^2 b + \pi^2 a + \left(k^2 - \frac{\pi^2}{a^2}\right)a^3\right)}{k\eta\beta a^3 b}$$

$$\Rightarrow \alpha = \frac{R_s(2\pi^2 b + k^2 a^3)}{k\eta\beta a^3 b}$$



Now, this is the power loss per unit length and the attenuation constant is given by power loss at z is equal to 0 divided by 2P naught. Now, this P naught here we have already computed this P naught here will be the power of the dominant TE_{10} mode which we have calculated, and therefore if you substitute these expressions then we get an expression for the attenuation constant alpha.

Now, further simplification is possible by considering that omega mu can be written as omega root mu epsilon into root mu by epsilon, which is equal to k eta, eta is the intrinsic impedance of the dielectric and once you substitute that we get the equation of this form. Now, for TE 10 mode the cut-off wavelength is lambda c is related to cut-off wave number k c as 2pi by lambda c and lambda c is 2a.

So, k c can be written as pi by a, and we know that beta square is equal to k square minus k c square. So, if we now substitute this beta square by k square minus lambda by a whole square then expression for alpha becomes as shown here, and on simplification, it becomes alpha equal to R s 2pi square b plus k square a cube divided by k eta beta a cube b. So, this is the expression for the attenuation constant in a rectangular waveguide.

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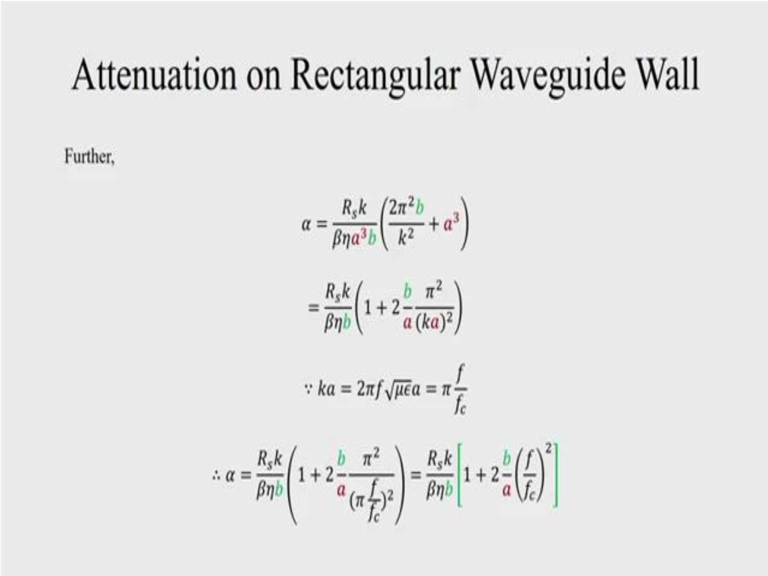
Further,

$$\alpha = \frac{R_s k}{\beta\eta a^3 b}\left(\frac{2\pi^2 b}{k^2} + a^3\right)$$

$$= \frac{R_s k}{\beta\eta b}\left(1 + 2\frac{b}{a}\frac{\pi^2}{(ka)^2}\right)$$

$$\because ka = 2\pi f\sqrt{\mu\epsilon}a = \pi\frac{f}{f_c}$$

$$\therefore \alpha = \frac{R_s k}{\beta\eta b}\left(1 + 2\frac{b}{a}\frac{\pi^2}{(\pi\frac{f}{f_c})^2}\right) = \frac{R_s k}{\beta\eta b}\left[1 + 2\frac{b}{a}\left(\frac{f}{f_c}\right)^2\right]$$



We can put this attenuation constant expression, starting with this expression. We can if you take this a cube outside we can put the equation for alpha in this particular form and we find that for a TE_{10}mode type propagation this factor ka can be written as pi f by f c and therefore alpha can be written as R s k divided by beta eta b 1 plus 2 b by a f by f c whole square.

So, here in this particular expression, we have expressed alpha in terms of the operating frequency and cut-off frequency. Please note that attenuation constant is a function of frequency and f is greater than f c here, of course, we will have another frequency-dependent term, which is beta. So, in general as the frequency of operation in a waveguide is varied the attenuation constant changes.

Please note that our entire derivation is based on the fact that the attenuation on the waveguide walls is very very small, and we have computed the surface current density in terms of the field components H x and H z which were evaluated considering that there is no attenuation on the waveguide walls. We have seen how we can calculate the attenuation constant for a rectangular waveguide, which is propagating TE_{10} mode and when we consider only the losses in the waveguide walls.

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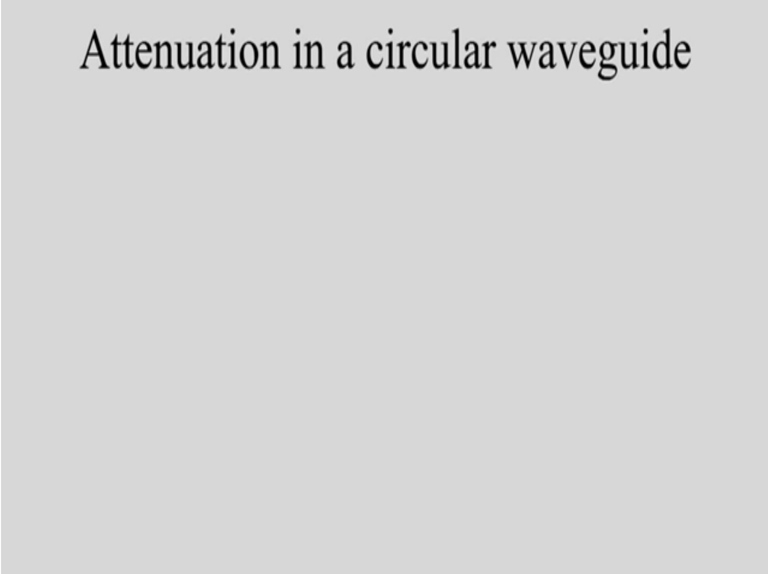

Now, let us try to find out the attenuation constant in a circular waveguide in a similar manner. So, we will restrict or discussion to the dominant TE_{11} mode in the circular waveguide, and as before, we will assume that the dielectric region within the waveguide is lossless whatever attenuation takes place that is because of the power loss in the waveguide wall.

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The attenuation due to lossy waveguide conductor for TE_{11} mode in a circular waveguide can be found as follows:

For $B = 0$, the TE_{11} field equations can be written as

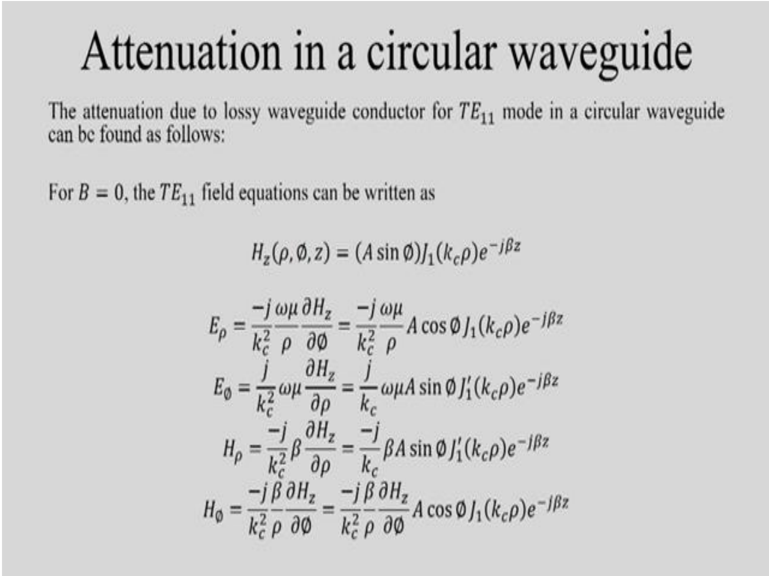
$$H_z(\rho, \emptyset, z) = (A \sin \emptyset) J_1(k_c \rho) e^{-j\beta z}$$

$$E_\rho = \frac{-j}{k_c^2} \frac{\omega\mu}{\rho} \frac{\partial H_z}{\partial \emptyset} = \frac{-j}{k_c^2} \frac{\omega\mu}{\rho} A \cos \emptyset \, J_1(k_c \rho) e^{-j\beta z}$$

$$E_\emptyset = \frac{j}{k_c^2} \omega\mu \frac{\partial H_z}{\partial \rho} = \frac{j}{k_c} \omega\mu A \sin \emptyset \, J_1'(k_c \rho) e^{-j\beta z}$$

$$H_\rho = \frac{-j}{k_c^2} \beta \frac{\partial H_z}{\partial \rho} = \frac{-j}{k_c} \beta A \sin \emptyset \, J_1'(k_c \rho) e^{-j\beta z}$$

$$H_\emptyset = \frac{-j}{k_c^2} \frac{\beta}{\rho} \frac{\partial H_z}{\partial \emptyset} = \frac{-j}{k_c^2} \frac{\beta}{\rho} \frac{\partial H_z}{\partial \emptyset} A \cos \emptyset \, J_1(k_c \rho) e^{-j\beta z}$$



So, while calculating the different field components for a circular waveguide, we have seen the phi dependence of the field can be written as either sin n phi or cos n phi and in general it is given by A sin n phi plus B cos n phi. Now, let us consider that B is equal to 0 and we write the field components for the TE_{11} mode that means n is equal to 1.

So, we can write H z as a function of rho phi and z as A sin phi J 1 k c rho e to the power minus j beta z and once we have written H z in this manner we can find out the e rho component, it is only a substitution of H z here and finding out the derivative with respect to phi.

Similarly, we can find out E phi component by substituting H z and finding the derivative with respect to rho and similarly we can find out the magnetic field components H rho and H phi. So, once we have all these field components computed, now we can find out the power that propagates along the circular waveguide for the dominant TE_{11} mode.

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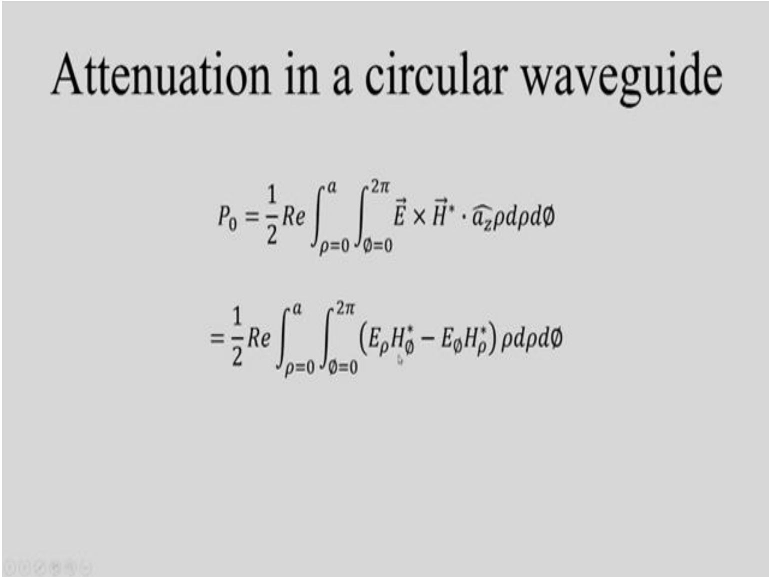
$$P_0 = \frac{1}{2} Re \int_{\rho=0}^{a} \int_{\emptyset=0}^{2\pi} \vec{E} \times \vec{H}^* \cdot \widehat{a_z} \rho d\rho d\emptyset$$

$$= \frac{1}{2} Re \int_{\rho=0}^{a} \int_{\emptyset=0}^{2\pi} \left(E_\rho H_\emptyset^* - E_\emptyset H_\rho^*\right) \rho d\rho d\emptyset$$

$$= \frac{\pi\omega\mu |A|^2 Re(\beta)}{4 {k_c}^4} \left({P'}_{11}^2 - 1\right) J_1^2(k_c a)$$



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$$E_\emptyset = \frac{j}{k_c^2} \omega\mu \frac{\partial H_z}{\partial \rho} = \frac{j}{k_c} \omega\mu A \sin \emptyset \, J_1'(k_c \rho) e^{-j\beta z}$$

$$H_\rho = \frac{-j}{k_c^2} \beta \frac{\partial H_z}{\partial \rho} = \frac{-j}{k_c} \beta A \sin \emptyset \, J_1'(k_c \rho) e^{-j\beta z}$$

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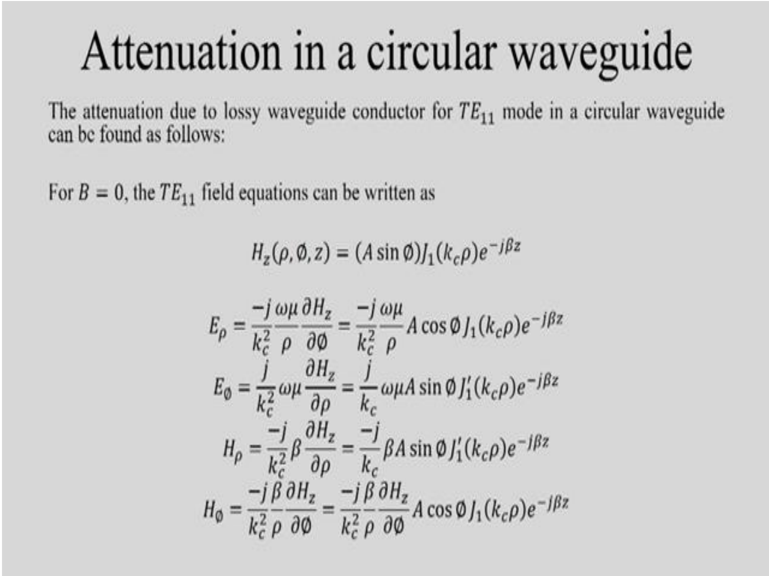

So, the power P naught can be written as half real part of E cross H star is the pointing vector and then integrating this pointing vector from rho equal to 0 to rho equal to a and phi is equal to 0 to phi equal to 2pi and this is the unit area a z rho d rho d phi and once this is expanded we find that the power can be actually computed in terms of E rho, H phi, E phi, and H rho and we can substitute the expressions for E rho, H phi, E phi and H rho from the already found out expressions.

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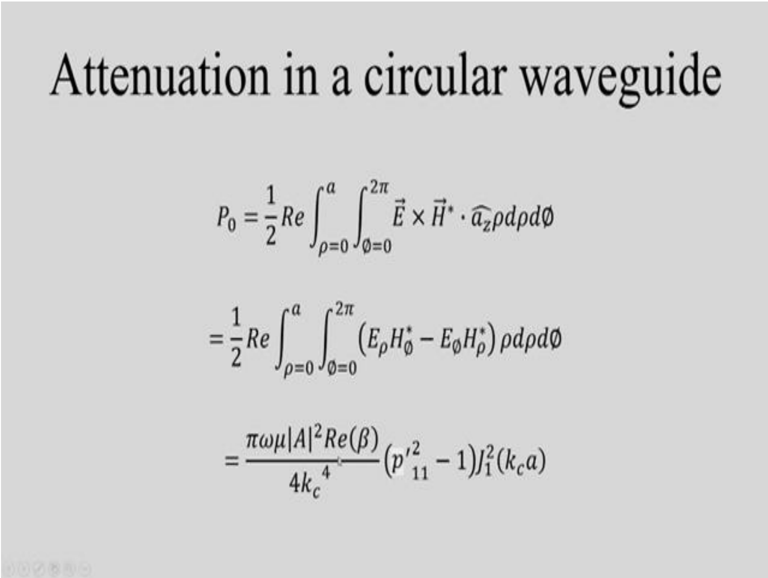
$$P_0 = \frac{1}{2} Re \int_{\rho=0}^{a} \int_{\emptyset=0}^{2\pi} \vec{E} \times \vec{H}^* \cdot \widehat{a_z} \rho d\rho d\emptyset$$

$$= \frac{1}{2} Re \int_{\rho=0}^{a} \int_{\emptyset=0}^{2\pi} \left(E_\rho H_\emptyset^* - E_\emptyset H_\rho^* \right) \rho d\rho d\emptyset$$

$$= \frac{\pi \omega \mu |A|^2 Re(\beta)}{4{k_c}^4} \left({P'}_{11}^2 - 1 \right) J_1^2(k_c a)$$



And once we do this substitution and do some simplification then we get the power P naught for that TE_{11} mode is given by this expression.

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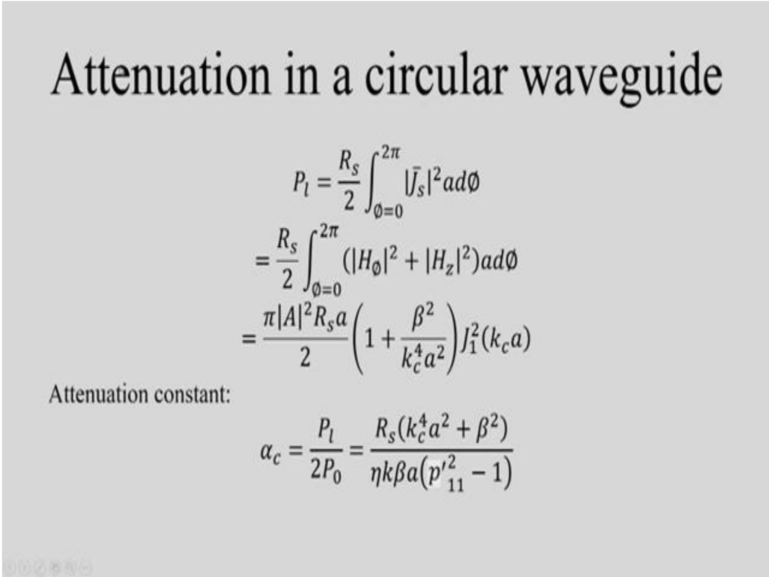
$$P_l = \frac{R_s}{2} \int_{\emptyset=0}^{2\pi} |\bar{J_s}|^2 a d\emptyset$$

$$= \frac{R_s}{2} \int_{\emptyset=0}^{2\pi} (|H_\emptyset|^2 + |H_z|^2) a d\emptyset$$

$$= \frac{\pi |A|^2 R_s a}{2} \left(1 + \frac{\beta^2}{k_c^4 a^2} \right) J_1^2(k_c a)$$

Attenuation constant:

$$\alpha_c = \frac{P_l}{2P_0} = \frac{R_s (k_c^4 a^2 + \beta^2)}{\eta k \beta a \left({P'}_{11}^2 - 1 \right)}$$



So, as before once you have found out the power for the TE_{11} mode. Now, we proceed to calculate the power loss on the conductor walls per unit length and that can be found out as R s by 2 phi is 0 to 2pi mod of J square where J is the current induced on the circular waveguide wall because of the tangential magnetic field components, so mod J s square ad phi.

And this can be found out in terms of H phi and H z square and once you substitute H phi and H z then we get this expression for the power loss per unit length of the circular waveguide. And once you have P l and P j rho evaluated we know that attenuation constant alpha c it is given by P l by 2P naught.

So, we substitute P l and P naught from these two expressions we have just derived and after doing some simplification we get alpha c as shown in this expression R s k c to the power 4 a square plus beta square divided by eta k beta a and then this is p dash 11 square which is the first root of the derivative of first-order Bessel function minus 1.

So, once we have this expression we can for a given frequency and waveguide dimensions we can find out the attenuation constant alpha c, and for a given length of the waveguide we can then calculate by how much the power will get attenuated as it travels down the line. So, in summary in this module we have covered the waveguides rectangular and circular waveguides.

We have seen the wave propagation in this type of waveguides, and we have studied what are the different modes which occur and we have seen which one is the dominant mode when it comes to a rectangular waveguide and a dominant mode in a circular waveguide. We have also seen how we can calculate the attenuation in the waveguide considering the finite conductivity of the waveguide walls.

So, this gives a comprehensive statement on different aspects of wave propagation in the waveguide. In the next module we will start our discussion on microwave networks and particularly scattering parameters, which are very much useful in designing microwave circuitry.