Micro Wave Engineering
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Lecture 08:
Microwave Networks and Scattering Matrix

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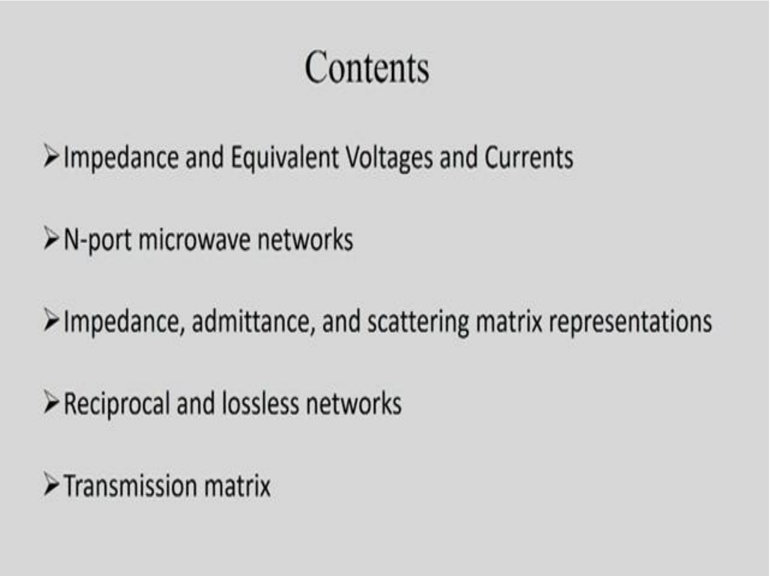

So, we start a new module, microwave networks, and scattering matrix. So, in this, we will cover the following contents, impedance and equivalent voltages, and currents, N-port microwave networks, impedance, admittance, and scattering matrix representations. And reciprocal and lossless networks, also we will discuss the transmission matrix.

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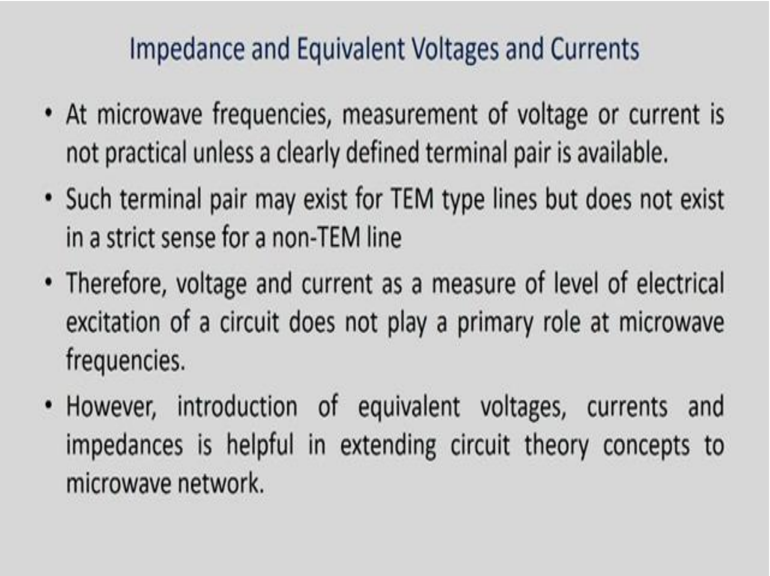


So, let us start over a discussion with impedance and equivalent voltages and currents, what happens at microwave frequencies, measurement of voltage or current is not practical unless a clearly defined terminal pair is available. So, this terminal pair we call it a port and such terminal pair may exist for TEM type lines that means transverse, electromagnetic wave propagation only but does not exist in a strict sense for not TEM lines.

And therefore, the voltage and current is a measure of level of electrical excitation of a circuit does not play a primary role at microwave frequencies. However we can introduce the concept of equivalent voltages, equivalent currents, and impedances and which is helpful in extending the circuit theory concepts to microwave network.

(Refer Slide Time 02:21)

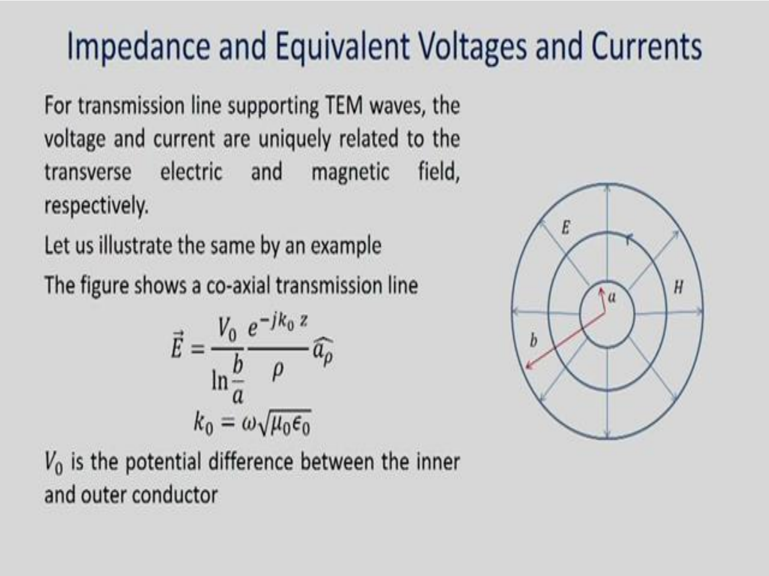
Let us illustrate the same by an example

The figure shows a co-axial transmission line

$$\vec{E} = \frac{V_0}{\ln\frac{b}{a}} \frac{e^{-jk_0 z}}{\rho} \widehat{a_\rho}$$

$$k_0 = \omega\sqrt{\mu_0 \epsilon_0}$$

V_0 is the potential difference between the inner and outer conductor



For transmission lines supporting TEM waves, the voltage and current are uniquely related to the transverse electric and magnetic fields, respectively. And let us illustrate this by an example. So, we show a section of a coaxial transmission line, so we have a coaxial transmission line within a radius a and the outer radius b, now this radial lines these are the electric field and the magnetic field lines are shown by this circle.

Now for such line, we can find out the electric field to be v naught by log of b by a into e to the power minus Jkz by rho. So this electric field is directed along rho, and here k naught is omega root Mue Naught epsilon Naught. Voltage V Naught is the potential difference between the inner and the outer conductor.

(Refer Slide Time 03:42)

Voltage wave associated with the electric field is

$$V = V_0 e^{-jk_0 z}$$

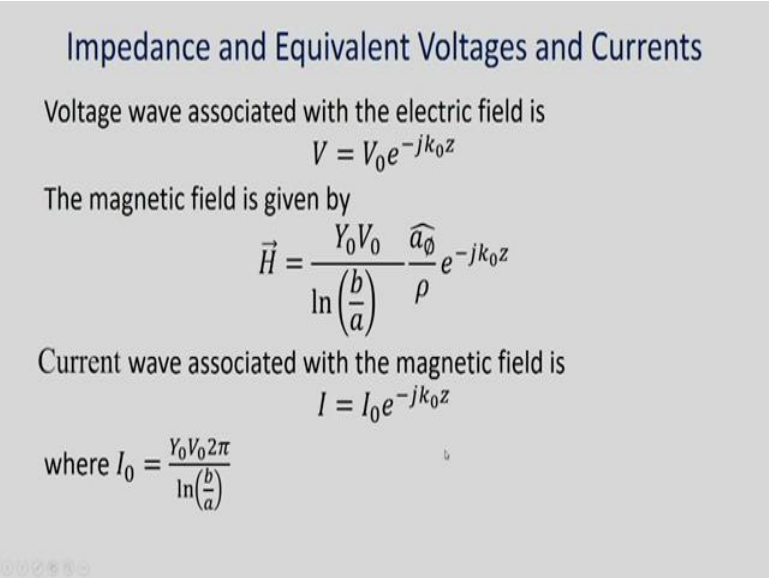
The magnetic field is given by

$$\vec{H} = \frac{Y_0 V_0}{\ln\left(\frac{b}{a}\right)} \frac{\widehat{a_\emptyset}}{\rho} e^{-jk_0 z}$$

Current wave associated with the magnetic field is

$$I = I_0 e^{-jk_0 z}$$

where $I_0 = \frac{Y_0 V_0 2\pi}{\ln\left(\frac{b}{a}\right)}$



So, for the same transmission line, we can associate a voltage wave with the electric field, and this is given by V is equal to V Naught every to the power minus j k Naught z. similarly for this type of coaxial conductor the magnetic field is given by, H is equal to Y Naught V Naught divided by log of b by a, e to the power minus jk z by rho by a phi. So, the magnetic field the current associated with the magnetic field can be written as I is equal to I Naught e to the power minus j k Naught z. please not that Y not is the characteristic admittance and I Naught is given by Y V Naught 2 Pi log of b by a. so, once we define these equivalent voltages and currents and relate them to the electric and magnetic fields.

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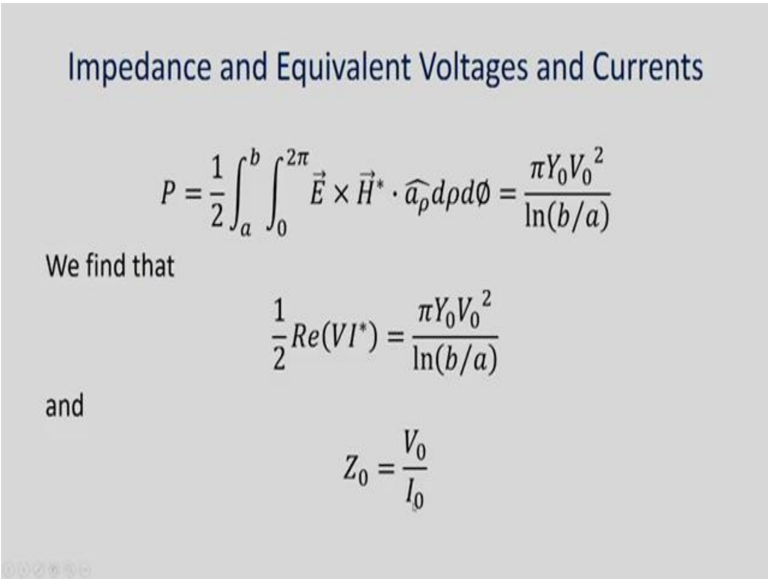
$$P = \frac{1}{2}\int_{a}^{b}\int_{0}^{2\pi}\vec{E}\times\vec{H}^{*}\cdot\widehat{a_\rho}\,d\rho d\emptyset = \frac{\pi Y_0 {V_0}^2}{\ln(b/a)}$$

We find that

$$\frac{1}{2}Re(VI^{*}) = \frac{\pi Y_0 {V_0}^2}{\ln(b/a)}$$

and

$$Z_0 = \frac{V_0}{I_0}$$



We can calculate also the power that is going through such coaxial transmission line and power is given by integration of the pointing vector over the cross-section of the transmission line. And once it is evaluated we get, Pi Y Naught V Naught square log of b by a. if we find the power from the current and voltage wave that means I and V which we have just defined, then half Re VI complex conjugate this also gives the same quantity Pi Y Naught V Naught square by log b by a. so, we can see that whatever power we calculate using this field equations, if we consider the equivalent voltage or current equation, the same power flow is evaluated on the coaxial lines.

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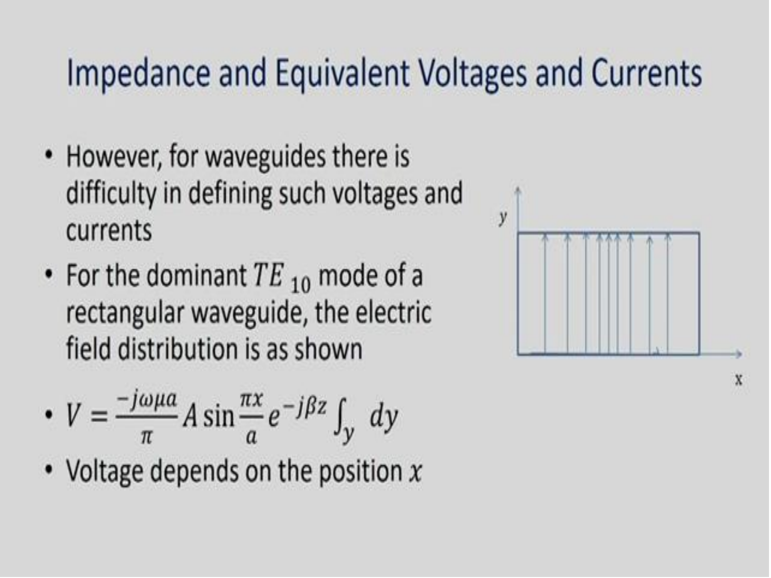
- $V = \frac{-j\omega\mu a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z} \int_y \, dy$
- Voltage depends on the position x



So, these voltages and currents can essentially represent in an equivalent manner transfers electric and magnetic fields. And also we find that Z Naught is equal to V Naught by I Naught, which is the characteristic impedance. So, when we have a TEM transmission line and we have a clearly defined pair of terminals. It is relatively easy to introduce the equivalent voltage and currents. Because these voltages and currents, these are well defined. When we go to a single conductor system like waveguide where propagation mode is non- TEM. We face difficulty in defining such voltages and currents, so this we illustrate.

Let us consider dominant TE_{10} mode of a rectangular waveguide and this electrical field distribution as we show in the figure, nowhere if you consider, if we want to calculate the voltage what do we will do, we will integrate this electric field from y equal to 0 to y equal to b, that means along y and once we substitute the expression for the UI and integrate we find that, the same integration. Now has a term sin Pi x by a, so for x is equal to 0 the voltage is evaluated to 0. Whereas x is equal to a, we will get again the voltage to be 0.

We will have a peak voltage in the middle so, what we find that, if we consider this type of single conductor transmission line like waveguide, we cannot uniquely define the voltage and the voltage will vary depending upon what this voltage is evaluated. So, voltage depends on the position x for this type of waveguide.

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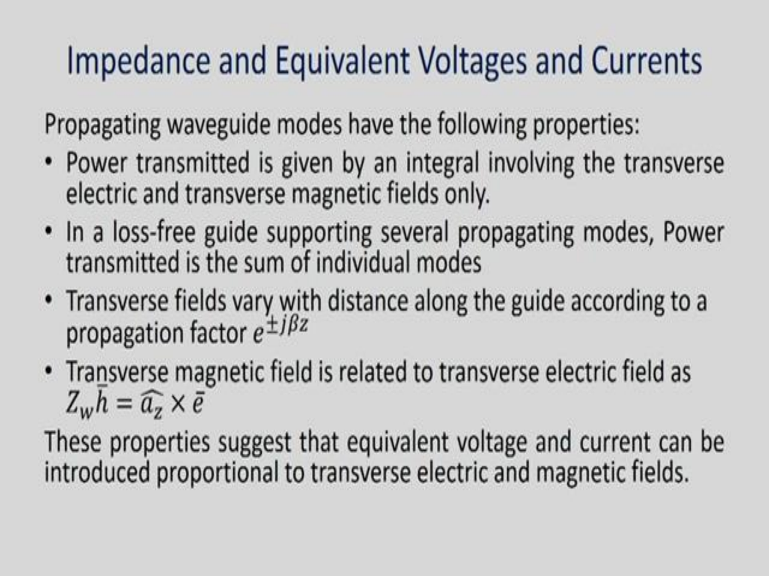
Transverse magnetic field is related to transverse electric field as $Z_w\bar{h} = \widehat{a_z} \times \bar{e}$



Now, next is how we can introduce actually the equivalent voltages and currents, in this type of waveguiding systems. So we note the following properties of the propagating modes in waveguides. The power transmitted is given by integral involving the transverse electric and magnetic fields only. So, when it comes to computation of power of the inside the waveguide mode it essentially involves the transverse electric and magnetic fields. In a loss-free waveguide supporting several propagating modes.

Power transmitted is the sum of individual modes, then the transverse field varies with distance along the guide according to a propagation factor e to the power plus minus j beta z. and the transverse magnetic fields are related to the transverse electric field as Z_w, which is the waveguide impedance, and h is the transverse magnetic field az cross e bar. So, these properties of the waveguide propagation will make use of in defining equivalent voltages and currents. So these properties essentially suggest that equivalent and voltage and current can be introduced proportional to the transverse electric and magnetic field.

So, whenever we define equivalent voltage and current, we try to relate those voltages and current to the transverse electric and magnetic field components.

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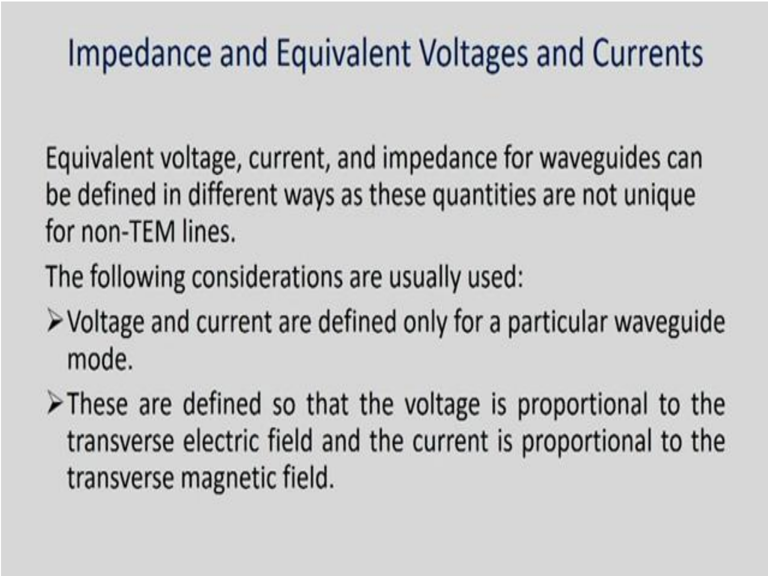

Now equivalent voltage, current, and impedance for waveguides can be defined in different ways as these quantities are not unique for non TEM lines. however, if we use the following consideration then, sum uniformity can be maintained, the voltage and current are defined only for particular waveguide mode. So, every waveguide mode is essentially represented similar to a transmission line. These are defined so that voltage is proportional to the transverse electric field and current is proportional to the transverse magnetic field.

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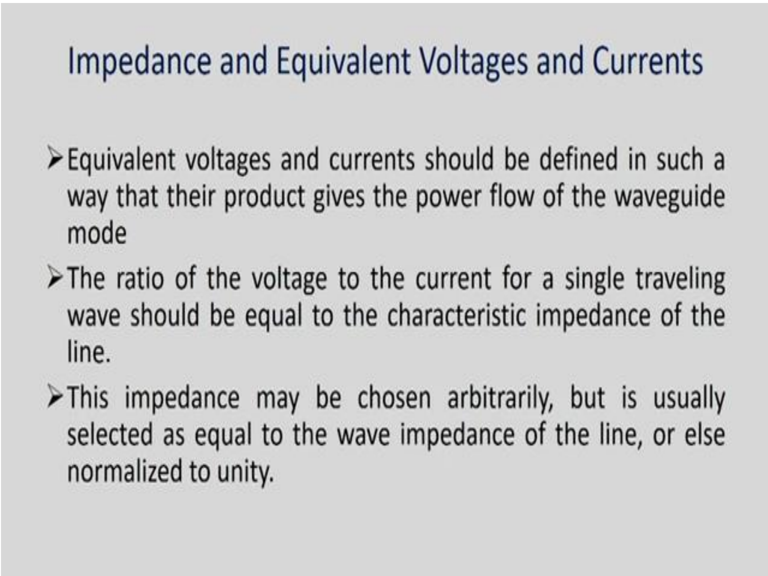


Equivalent voltages and currents should be defined in such a way their product gives the power flow of the waveguide mode. So, as we are defining these voltages and currents, equivalent voltages and currents for individual modes. Now that to be introduced in such a manner the voltage and current, when they are taken computation of power they will actually correspond to the power flow of that particular waveguide mode.

The ratio of voltage to current for a single traveling wave should be equal to the characteristic impedance of the line. Now impedance may be chosen arbitrarily, but usually selected as equal to the wave impedance of the line. Sometimes z Naught is said equal to Zw, or else it is normalized to unity.

(Refer Slide Time 12:32)

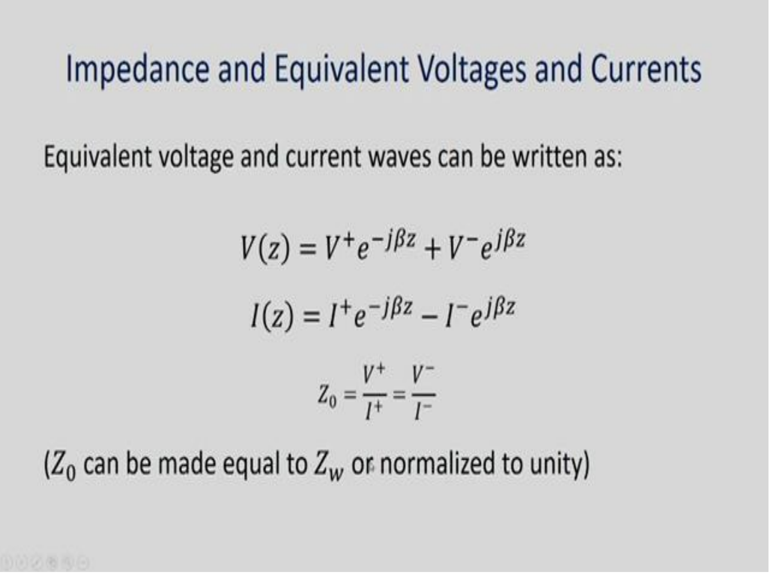
Equivalent voltage and current waves can be written as:

$$V(z) = V^{+}e^{-j\beta z} + V^{-}e^{j\beta z}$$

$$I(z) = I^{+}e^{-j\beta z} - I^{-}e^{j\beta z}$$

$$Z_0 = \frac{V^{+}}{I^{+}} = \frac{V^{-}}{I^{-}}$$

(Z_0 can be made equal to Z_w or normalized to unity)



So, the equivalent voltage and current waves can be written as V z is equal to V plus e to the power minus j beta z plus V minus e to the power j beta z. and I z can be written as I plus e to the power minus j beta z. minus I minus e to the power j beta z. and we define the characteristic impedance z Naught to be equal to V plus by I plus or V minus by I minus. and as we have said, z Naught can we made equal to Zw the wave impedance of the waveguide mode or it is normalized to unity.

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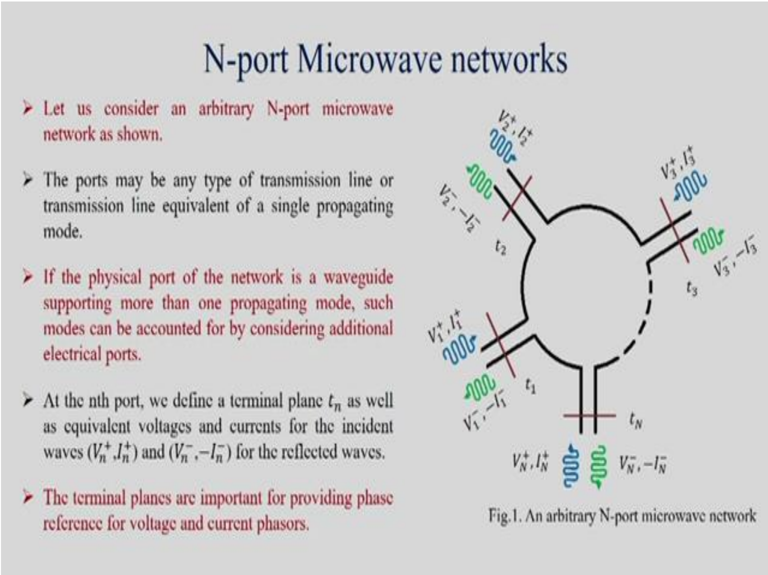

Now let us consider an N port microwave network, so this is the schematic of an N port microwave network, the port may be any type of transmission line or transmission line equivalent of a single propagating mode. So we can see that if it is a TEM transmission line we can define this port directly or if it is a waveguide type port then, we have a transmission line equivalent of a single propagating mode. If the physical port of a network is a waveguide supporting more than propagating mode.

So in any case, it might happen that the single physical port is supporting more than one propagating mode. In such cases the modes can be accounted by considering additional electrical ports. So, we can increase the number of ports accordingly. Now once we have the equivalent voltages and currents defined at the Nth port we define a terminal plain TEM.

For example, this is the t1 terminal plain and also the equivalent voltages and currents for both incident and reflected waves. Now what we do, this terminal plain essentially serves as the phase reference for these waves. So, if we shift the terminal plain there will be a change of phase, and we can define these terminal plains for each of the ports as shown, and we have this V1 plus I1 plus these are the waves that is going into the network through this port and V1 minus and minus I1 minus. These are the waves which are coming out of the port.

And we have already mentioned that these terminal plains are important for providing phase reference for voltage and current phases. So, this is how we have defined an N port microwave networks, let us now see some practical circuits where this type of multiport microwave network actually occur.

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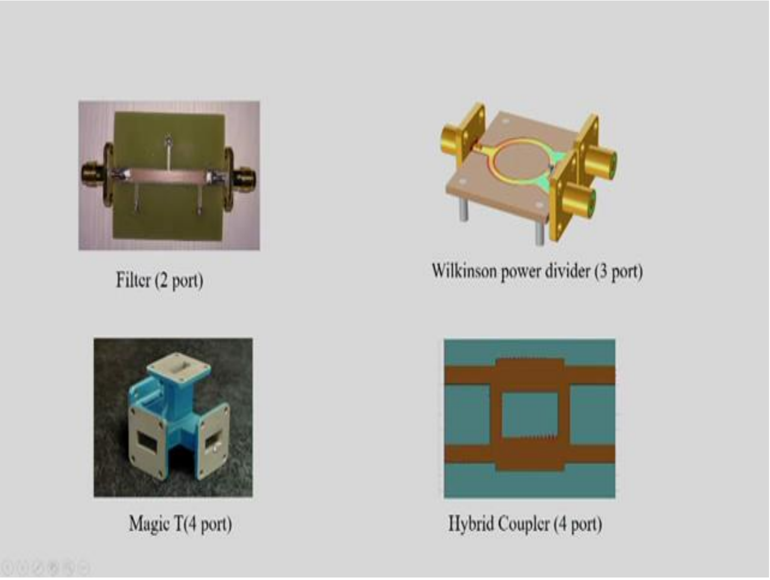

For example, here we show a 2 port network which is essentially a filter and this towards the ports, similarly we can consider a case of a power divider, here the input signal incident on this port is split between these 2 ports, so that it divides the power and it is a 3 port device. Similarly, we consider a waveguide version of a magic T which is a 4 port device and if we consider the microstrip or plainer version, this is called a hybrid coupler so you can see it also has 4 ports.

So, you can see that physical microwave circuits or networks will have one or more number of ports and how we define the voltages and currents In these ports? These are through if there TEM lines we directly define or if they are non TEM lines we can always have an equivalent voltage which is related to the transverse electric field and equivalent current which is related to the transverse magnetic field components.

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At the nth terminal plane,

$$V_n = V_n^+ + V_n^-$$

$$I_n = I_n^+ - I_n^-$$

The impedance matrix $[Z]$ of a microwave network relates this voltage and currents

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

$$[V] = [Z][I]$$

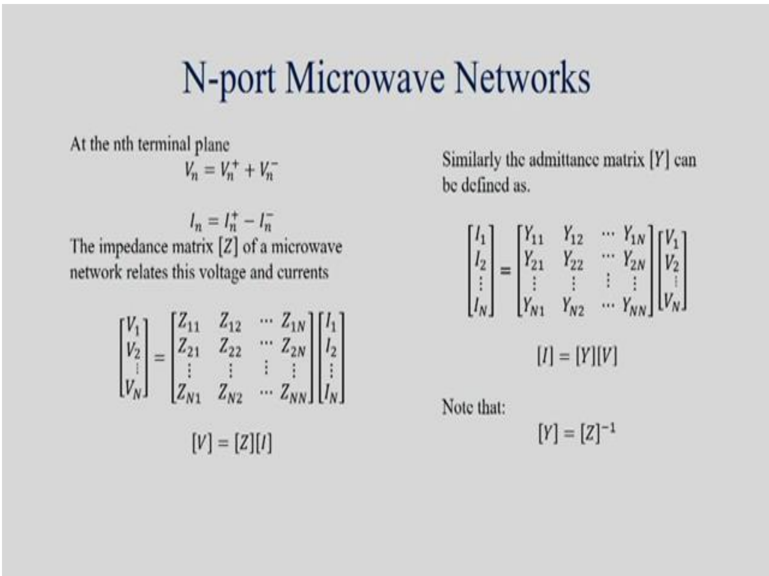
Similarly the admittance matrix $[Y]$ can be defined as.

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots & Y_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

$$[I] = [Y][V]$$

Note that:

$$[Y] = [Z]^{-1}$$



So, we have at the Nth terminal plain Vn is equal to Vn plus Vn minus so, this is essentially obtained by putting Z is equal to 0. And In becomes equal to In plus minus In minus. And the impedance matrix Z of a microwave network relates these voltages and currents. So, for an N port network, we can define V_1, V_2, Vn these are the voltages at pots 1, 2 and N and I_1, I_2, In these are the currents at pots 1,2 and up to n. and these 2 vectors, they are related by a matrix which is called the impedance matrix Z. Z_{11}, Z_{12}, Z_{1n}, Z_{21} like that Zij are entries for this matrix.

So, we have in the compact form we can write V is equal to Zi where V is a vector Z is a matrix, and i is the vector. In the same manner we can define another matrix, which is called the admittance matrix y here, I_1, I_2, In they are defined as a matrix y multiplied by the vector voltage vector V., and we can see that Z and Y they are inverse of each other.

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We note that

$$Z_{ii} = \left.\frac{V_i}{I_i}\right|_{I_k=0,k\neq i}$$

Therefore, Z_{ii} is the input impedance seen at port i when all other ports are open circuited.

Similarly,

$$Z_{ij} = \left.\frac{V_i}{I_j}\right|_{I_k=0,k\neq j}$$

is the transfer impedance between port i and j when all other ports are open circuited.

In the same manner,

$$Y_{ij} = \left.\frac{I_i}{V_j}\right|_{I_k=0,k\neq j}$$

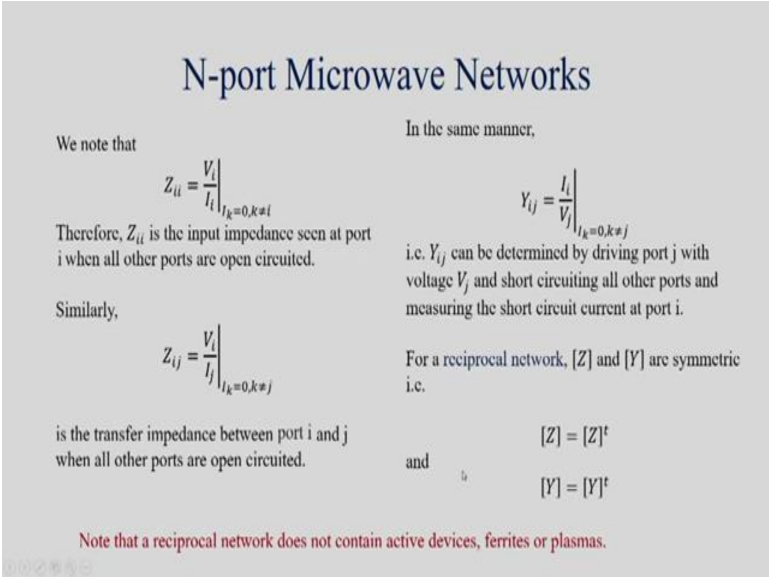
i.e. Y_{ij} can be determined by driving port j with voltage V_j and short circuiting all other ports and measuring the short circuit current at port i.

For a reciprocal network, $[Z]$ and $[Y]$ are symmetric i.e.

$$[Z] = [Z]^t$$

and

$$[Y] = [Y]^t$$



Now how we define these parameters? For example if we consider Z_{ii} like Z_{11} diagonal elements, Z_{11}, Z_{22}, Z_{33} so it is V_i by I_i and when Ik is equal to 0k is not equal to I. so if I go to the definition, for example, Z_{11} will be V_1 by I_1 when I_2, I_3 up to In they are all zero. And therefore Z_{ii} is the input impedance seen at the port I when all other ports are kept open-circuited.

Similarly, Z_{ij} is V_i by ij, and all Ik equal to zero for k not equal to j, and this will give the transfer impedance between port i and j even all other ports are open-circuited. So these z parameters, are open circuit parameters that means if we have to measure Z11 we will have to keep all other ports other that port1 open-circuited. In the same manner we can find Yij and you can determine by drawing port j with voltage Vj and short-circuiting all other ports, and measuring the short circuit port at port I.

Now for a reciprocal network, the impedance and admittance matrix is symmetric that means we can write Z is equal to Z transpose and Y equals to Y transpose. By reciprocal network we mean that it will not contain any active devices ferrites or plasmas and if a network contains any of these elements it cannot be modeled as a reciprocal network.So we have seen that for a reciprocal network, the Z and y matrices are symmetric. Let us now consider a lossless network.

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If the network is lossless

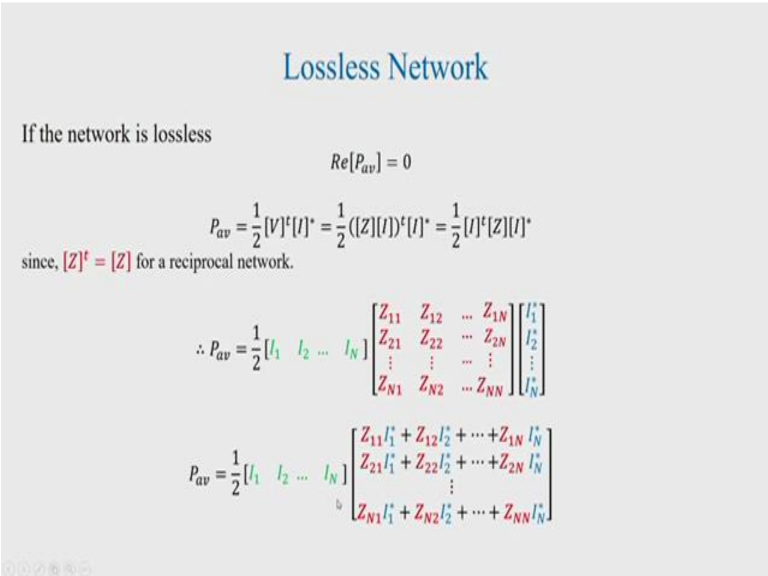
$$Re[P_{av}] = 0$$

$$P_{av} = \frac{1}{2}[V]^t[I]^* = \frac{1}{2}([Z][I])^t[I]^* = \frac{1}{2}[I]^t[Z][I]^*$$

since, $[Z]^t = [Z]$ for a reciprocal network.

$$\therefore P_{av} = \frac{1}{2}[I_1 \quad I_2 \quad \cdots \quad I_N]\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix}\begin{bmatrix} I_1^* \\ I_2^* \\ \vdots \\ I_N^* \end{bmatrix}$$

$$P_{av} = \frac{1}{2}[I_1 \quad I_2 \quad \cdots \quad I_N]\begin{bmatrix} Z_{11}I_1^* + Z_{12}I_2^* + \cdots + Z_{1N}I_N^* \\ Z_{21}I_1^* + Z_{22}I_2^* + \cdots + Z_{2N}I_N^* \\ \vdots \\ Z_{N1}I_1^* + Z_{N2}I_2^* + \cdots + Z_{NN}I_N^* \end{bmatrix}$$



So, if a network is lossless then, what we can write, we can write that the real part of the average power to be equal to 0. Now how we can calculate this average power, we can actually find the product of V, and I conjugate at each port, which in the matrix from we can write Vt transpose of V into I complex conjugate. And then once we substitute V is equal to Zi and then using the property of transverse we can write I transpose, it should be z transpose, but since z is symmetric, therefore, we can write I transpose Z, I complex conjugate.

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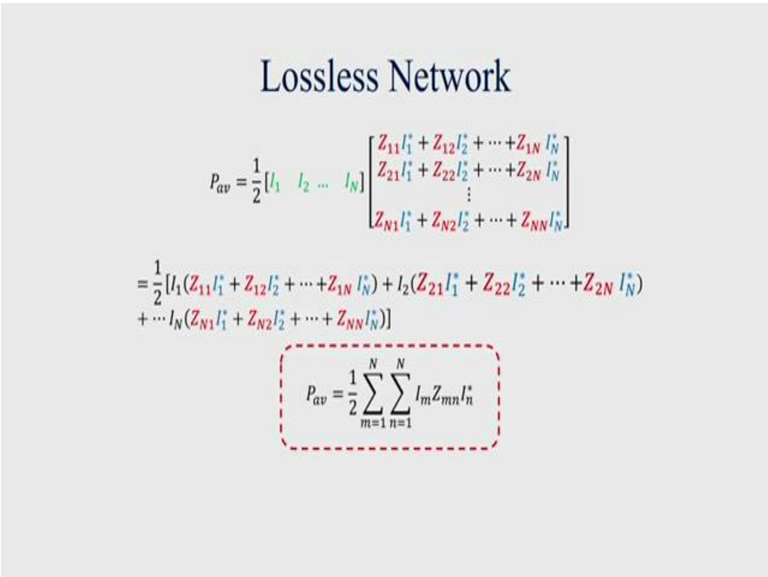
$$P_{av} = \frac{1}{2}[I_1 \quad I_2 \quad \dots \quad I_N]\begin{bmatrix} Z_{11}I_1^* + Z_{12}I_2^* + \cdots + Z_{1N}I_N^* \\ Z_{21}I_1^* + Z_{22}I_2^* + \cdots + Z_{2N}I_N^* \\ \vdots \\ Z_{N1}I_1^* + Z_{N2}I_2^* + \cdots + Z_{NN}I_N^* \end{bmatrix}$$

$$= \frac{1}{2}[I_1(Z_{11}I_1^* + Z_{12}I_2^* + \cdots + Z_{1N}I_N^*) + I_2(Z_{21}I_1^* + Z_{22}I_2^* + \cdots + Z_{2N}I_N^*) + \cdots I_N(Z_{N1}I_1^* + Z_{N2}I_2^* + \cdots + Z_{NN}I_N^*)]$$

$$P_{av} = \frac{1}{2}\sum_{m=1}^{N}\sum_{n=1}^{N} I_m Z_{mn} I_n^*$$



Now, this can be expanded, as shown. We can have I transpose as a rho vector, then the Z matrix and I conjugate as the column vector. Now once we find out this product, matrix product we can put the average power expression in this form and starting from this form, if we now find out the product of this rho with this column we can write the expression for average power in the form soon. And we can see that each term within this bracket first bracket, it is once summation and then it is been multiplied by I_1, I_2 so, we can write p over each as a summation of m to n, as a summation of m is equal to 1 to n,, n equal to 1 to n ,I_m Z_{mn} In conjugate.

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Since I_ns are independent, let us set all port currents to zero except for n^{th} port current.

$$\therefore Re[P_{av}] = 0$$

$$\Rightarrow Re(I_m Z_{nn} I_n^*) = 0$$

$$\therefore |I_n|^2 . Re(Z_{nn}) = 0$$

$$\because I_n \neq 0$$

$$Re(Z_{nn}) = 0$$

Let us now consider that all port currents except I_m and I_n are zero. Then

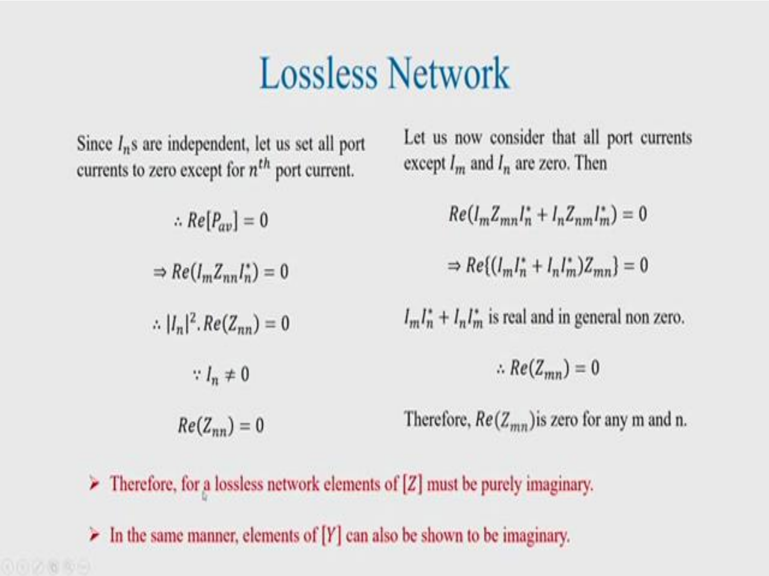
$$Re(I_m Z_{mn} I_n^* + I_n Z_{nm} I_m^*) = 0$$

$$\Rightarrow Re\{(I_m I_n^* + I_n I_m^*) Z_{mn}\} = 0$$

$I_m I_n^* + I_n I_m^*$ is real and in general non zero.

$$\therefore Re(Z_{mn}) = 0$$

Therefore, $Re(Z_{mn})$ is zero for any m and n.



Since, In's are independent, let us set all port currents to be zero, except for the Nth port. Therefore, this essentially when only 1 port is exited this double summation term results into these terms I_m Z_{nn} In s term, and this can be written as mode of In square real part of Z_{nn} is equal to zero. Since we have I_n not equal to 0. So, we find that real part of Z_{nn} is equal to zero. That means the diagonal elements, the real part of the diagonal elements of the Z matrix they are zero.

Let us now consider the off-diagonal elements, and to calculate this, let us now consider that all port currents except I_m and In are zero. Then in the same manner we can write I_m Z_{mn} In conjugate, plus In Z_{nm} I_m conjugate real part of this is equal to zero. Now this can be written as real part of I_m In conjugate plus In I_m conjugate Z_{mn} is equal to zero since we have the matrix which is symmetric. Now this quantity I_m In conjugate plus In I_m conjugate this quantity is always real.

If you expand I_m and I_n in terms of real and imaginary part and find out the expression this will a real quantity. And in general, it will be none-zero, so we can say that real part Z_{mn} is equal to zero. And therefore, real part of Z_{mn} is zero for any m and n, therefore if we consider a lossless network, the elements of Z must be purely imaginary, and in the same manner elements of Y can also be shown to be imaginary. So, this property that, the elements of a lossless reciprocal Z or Y matrix they are purely imaginary. We will see some examples later on.

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$$[Z] = \begin{bmatrix} j2 & 0.2 & j3 \\ 4 & -2 & 0.5 \\ j & 1 & j4 \end{bmatrix}$$

Neither lossless nor reciprocal

$$[Z] = \begin{bmatrix} j2 & 4 & j3 \\ 4 & -j2 & j0.5 \\ j3 & j0.5 & 4 \end{bmatrix}$$

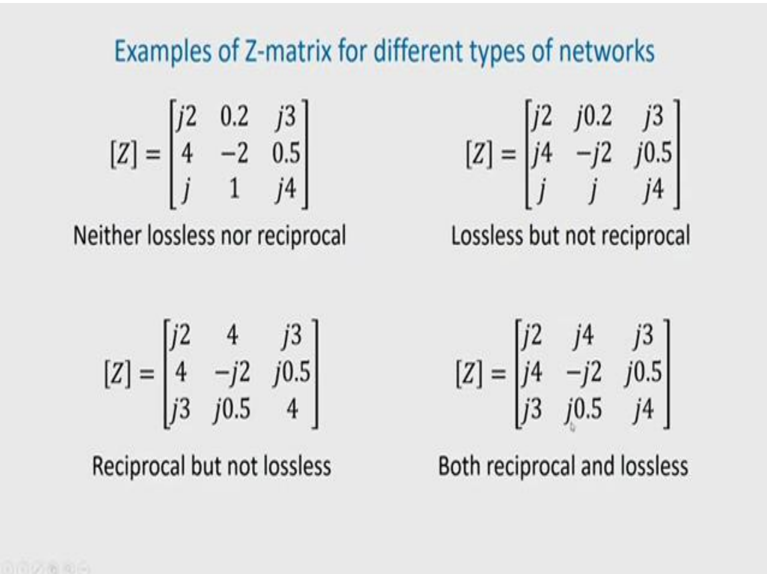
Reciprocal but not lossless

$$[Z] = \begin{bmatrix} j2 & j0.2 & j3 \\ j4 & -j2 & j0.5 \\ j & j & j4 \end{bmatrix}$$

Lossless but not reciprocal

$$[Z] = \begin{bmatrix} j2 & j4 & j3 \\ j4 & -j2 & j0.5 \\ j3 & j0.5 & j4 \end{bmatrix}$$

Both reciprocal and lossless



Now, let us see some Z matrices for different types of networks, for example if you take this network, here we find that it is neither lossless nor reciprocal. Because it has entries which are real, and it is not symmetric. So, this Z matrix is neither lossless nor reciprocal. Similarly, if you take this matrix it is lossless but not reciprocal because we have all the elements to be imaginary. This gives an example of a Z matrix, which is reciprocal but not lossless, and this is the case where the Z matrix is both reciprocal and lossless.

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Let us illustrate how we can find equivalent voltages and currents for TE_{10} mode in a rectangular waveguide.

For TE_{10} mode in a rectangular waveguide, when waves travelling in both +z and –z direction are present, the transverse field components can be written as:

$$E_y = A^+ \sin\frac{\pi x}{a} e^{-j\beta z} + A^- \sin\frac{\pi x}{a} e^{j\beta z}$$

$$= \left(A^+ e^{-j\beta z} + A^- e^{j\beta z}\right) \sin\frac{\pi x}{a}$$

We have,

$$\bar{h}(x, y) = \hat{a}_z \times \frac{\bar{e}(x, y)}{Z_w}$$

where, $\bar{h}$ and $\bar{e}$ are transverse field components and Z_w is the wave impedance.

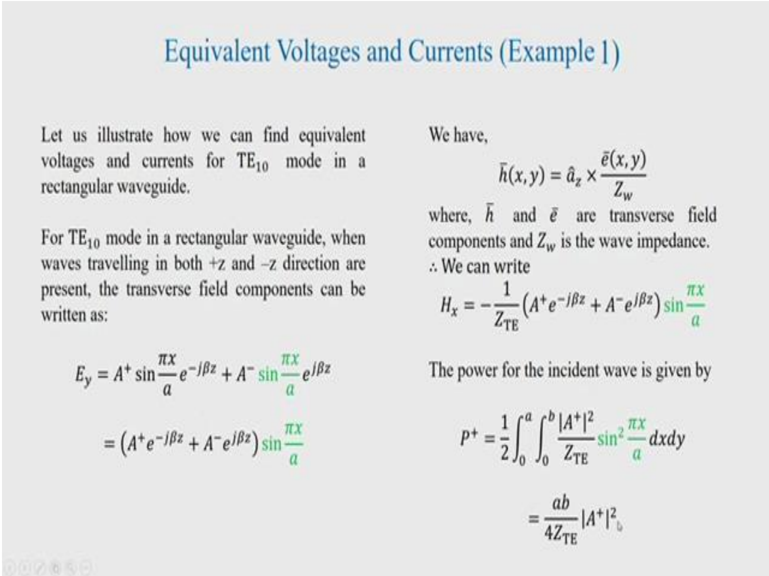
∴ We can write

$$H_x = -\frac{1}{Z_{\mathrm{TE}}}\left(A^+ e^{-j\beta z} + A^- e^{j\beta z}\right) \sin\frac{\pi x}{a}$$

The power for the incident wave is given by

$$P^+ = \frac{1}{2}\int_0^a \int_0^b \frac{|A^+|^2}{Z_{\mathrm{TE}}} \sin^2\frac{\pi x}{a}\, dxdy$$

$$= \frac{ab}{4Z_{\mathrm{TE}}}|A^+|^2$$



We discussed equivalent voltages and currents. so, let us consider an example to illustrate how we can find those equivalent voltages and currents. A particular, for a propagating mode in a waveguide. So, to illustrate this we consider TE_{10} mode propagation in a rectangular waveguide, and for a TE_{10} mode in a rectangular waveguide, when the waves traveling in both plus Z and minus Z directions are present the transverse field components we can write in this form a plus sin Pi x by a this is the variation of the field component ui, e to the power minus j beta z. and this is the wave traveling in the plus Z directions and a minus sin Pi x by a, e to the power j beta z. this gives the wave traveling in the minus z-direction.

So, we can put in this form, and we know that the transverse magnetic and electric field components, they are related by this equation, h transverse is az cross e transverse by Zw where Zw is the waveguide impedance, and these relations holds when we consider the wave is propagating along z. so, from this relation, once we have written ui we can write the expression for Hx to be equal to minus 1 by Z TE, a plus e to the power minus j beta z. plus a minus e to the power j beta z, sin Pi x by a, so, the power for the incident wave.

Let us denoted by p plus and from this ui and Hx component we can find this power to half 0 to a, 0 to b, if you integrate this a plus square Z_t sin square Pi x by a, that means over the cross-section of the waveguide then this will give us the power in the incident wave, and once we evaluate this integral, we get p plus to be equal to ab divided by 4 Z_{TE} a plus mode square

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Writing in the form of equivalent voltages and currents.

$$V(z) = V^{+}e^{-j\beta z} + V^{-}e^{j\beta z}$$

$$I(z) = I^{+}e^{-j\beta z} - I^{-}e^{j\beta z}$$

$$= \frac{1}{Z_0}\left(V^{+}e^{-j\beta z} - V^{-}e^{j\beta z}\right)$$

$$\therefore P^{+} = \frac{1}{2}V^{+}I^{+*}$$

Let C_1 and C_2 be two constants such that

$V^{+} = C_1A^{+} \qquad I^{+} = C_2A^{+}$

$$V^{-} = C_1A^{-} \qquad I^{-} = C_2A^{-}$$

We have seen that

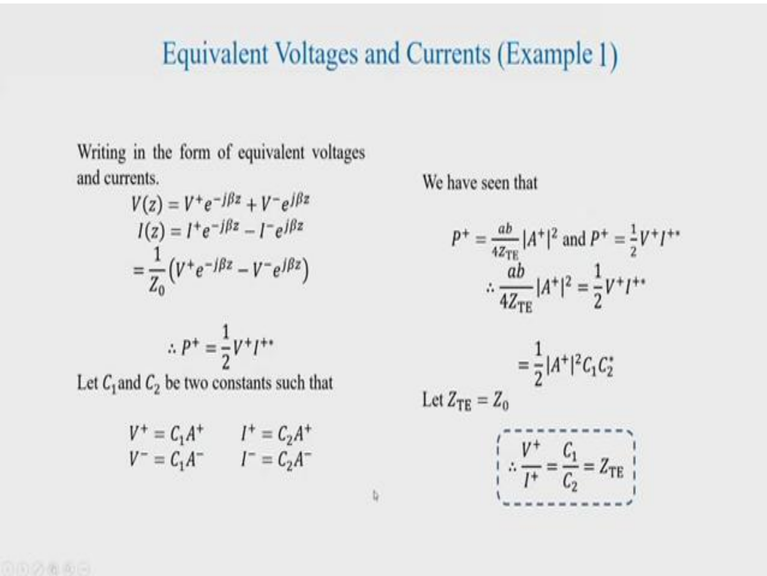
$P^{+} = \frac{ab}{4Z_{\mathrm{TE}}}|A^{+}|^2$ and $P^{+} = \frac{1}{2}V^{+}I^{+*}$

$$\therefore \frac{ab}{4Z_{\mathrm{TE}}}|A^{+}|^2 = \frac{1}{2}V^{+}I^{+*}$$

$$= \frac{1}{2}|A^{+}|^2C_1C_2^*$$

Let $Z_{\mathrm{TE}} = Z_0$

$$\therefore \frac{V^{+}}{I^{+}} = \frac{C_1}{C_2} = Z_{\mathrm{TE}}$$



Now we can also write equivalent voltages and currents. so, Vz can be written as V plus e to the power minus j beta z. plus V minus e to the power j beta z. and Iz can be written as I plus e to the power minus j beta z, minus I minus e to the power j beta z, and this Iz can also be written in terms of the voltages V plus and V minus. So, from these expressions if we consider the incident power P plus then, it is given by half V plus I plus conjugate, and now let us consider that to constant C1 and C2 which relates this V plus with a plus and V minus, with a minus, I plus with a plus and I minus with a minus, a plus a minus, these are the as we have seen, these are the magnitude of the transverse field components.

Now, we have 2 expressions for P plus 1 evaluated from the fields. Another evaluated from the voltage and current. And if we equate these 2 then, we can write ab by 4 Z_{TE} a plus mod square is equal to half V plus I plus conjugate, nowhere what do we can look, we can replace V plus I plus in terms of C1 A plus and C2 A plus. So, if we do that then this becomes half A plus modular square C1, C2 conjugate.

And let us as we said during over discussion of equivalent voltages and currents. Z Naught can be either set to be equal to Z TE or it can be normalized to unity. Let us set Z Naught equal to Z TE. In that case, we have V plus divided by I plus is equal to C1 by C2 and equal to Z_{TE}.

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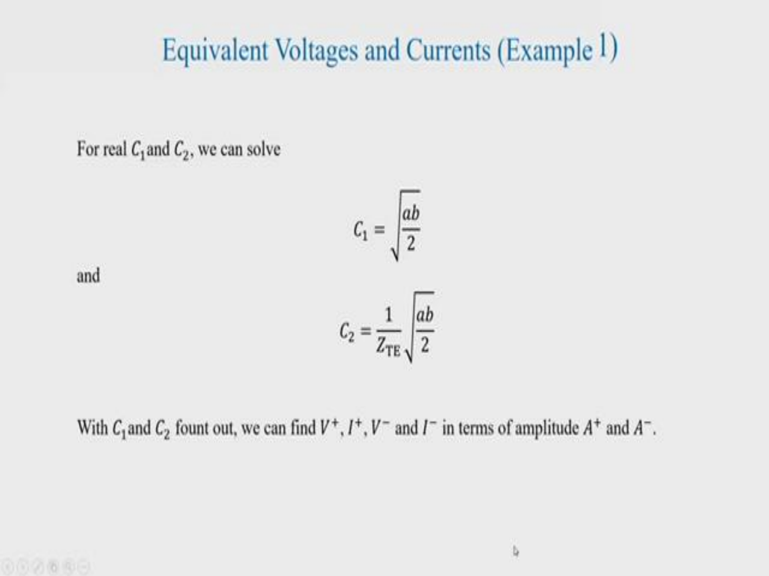
For real $C_1$and C_2, we can solve

$$C_1 = \sqrt{\frac{ab}{2}}$$

and

$$C_2 = \frac{1}{Z_{\text{TE}}}\sqrt{\frac{ab}{2}}$$

With $C_1$and C_2 fount out, we can find V^+, I^+, V^- and I^- in terms of amplitude A^+ and A^-.



So, if we consider C_1, C_2 to be real, from the previous 2 equations, that means ab by 4 Z_{TE} is equal to half C_1, C_2 conjugate, and C_1 by C_2 equal to Z_{TE}. From this 2, we can find out C_1 to be equal to under root ab by 2, and C_2 will be 1 by Z_{TE}, under root ab by 2. So, with C_1, C_2 known what we can do, we can find V plus I plus, V minus and I minus in terms of amplitude A plus and A minus.

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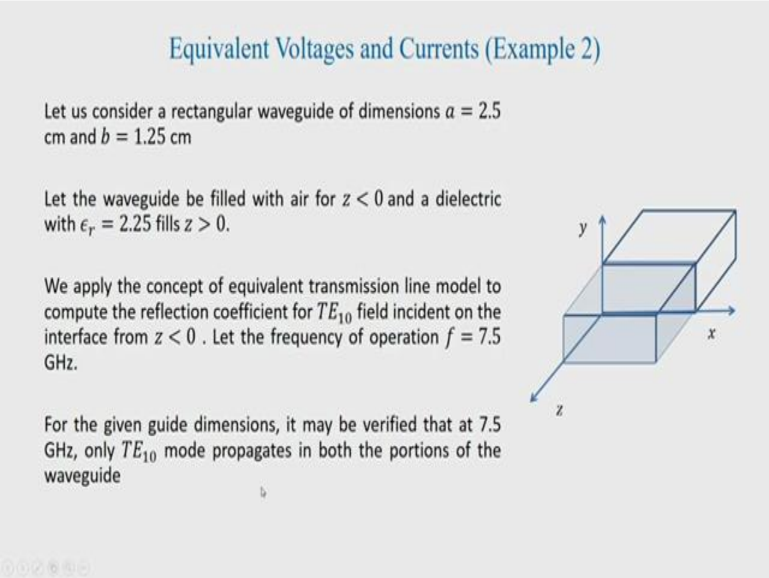

Let us now consider another example, let us consider a rectangular waveguide of dimensions, a is equal 2.5 centimeter and b is equal to 1.25 centimeter and, let the waveguide be a field with air for z less than 0, and the dielectric material with epsilon r is equal to 2.25 for z greater than 0. So, this is shown here, so in this part it is airfield, in this part of the waveguide it is dielectric field, now let us apply the concept of equivalent transmission line model to computed reflection coefficient for the TE_{10} field incident on the interface from z less than zero, so that means what we are assuming, a wave is incident from this side from the waveguide air field waveguide and then part of this incident wave we will get reflected, because there will be a change in impedance here. And partly it will get transmitted.

So, let us try to find out the reflection coefficient for this system assuming that the propagating mode is TE_{10} and the frequency of operation is 7.5 giga hertz. Now for the given guide dimensions, one may verify that at 7.5 giga hertz only TE10 mode propagates, in both the portions of the waveguide and that can be verified by calculating the cut off frequencies for different modes in both the waveguide. So, we will find that at 7.5 gigahertz only TE10 mode will get excited and higher-order modes will have cut off frequencies higher than 7.5 giga hertz in both the guides.

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Cutoff wavelength for TE_{10} mode in a rectangular waveguide is given by $2a$.

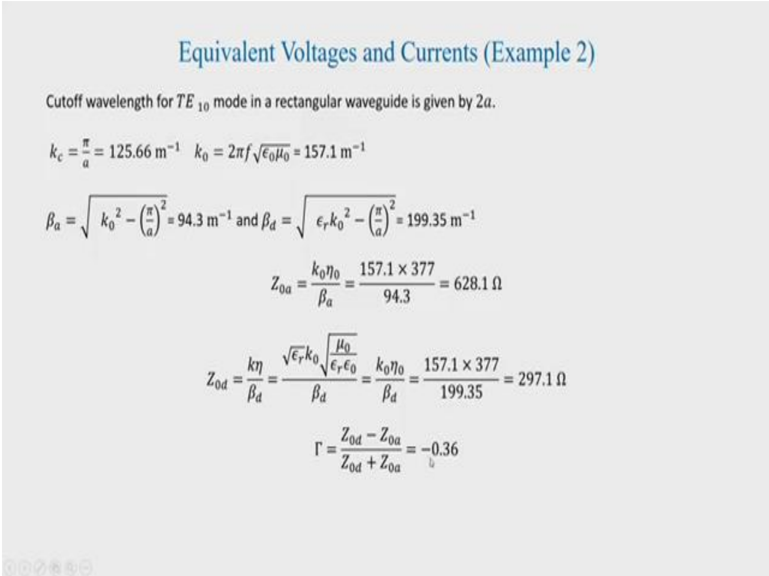
$k_c = \frac{\pi}{a} = 125.66\ \mathrm{m}^{-1}$ $\quad k_0 = 2\pi f\sqrt{\epsilon_0\mu_0} = 157.1\ \mathrm{m}^{-1}$

$\beta_a = \sqrt{k_0{}^2 - \left(\frac{\pi}{a}\right)^2} = 94.3\ \mathrm{m}^{-1}$ and $\beta_d = \sqrt{\epsilon_r k_0{}^2 - \left(\frac{\pi}{a}\right)^2} = 199.35\ \mathrm{m}^{-1}$

$$Z_{0a} = \frac{k_0\eta_0}{\beta_a} = \frac{157.1 \times 377}{94.3} = 628.1\ \Omega$$

$$Z_{0d} = \frac{k\eta}{\beta_d} = \frac{\sqrt{\epsilon_r}k_0\sqrt{\frac{\mu_0}{\epsilon_r\epsilon_0}}}{\beta_d} = \frac{k_0\eta_0}{\beta_d} = \frac{157.1 \times 377}{199.35} = 297.1\ \Omega$$

$$\Gamma = \frac{Z_{0d} - Z_{0a}}{Z_{0d} + Z_{0a}} = -0.36$$



Now, the cut off wavelength for TE$_{10}$ mode in a rectangular waveguide, it is given by 2a and, therefore we can find out the cut off wave number kc is 2 Pi by lambda c which becomes equal to Pi by a, and for the given dimension a it can be calculated to be 125.66 per meter. Similarly, k Naught is 2 Pi f root Mue Naught epsilon Naught and that can be calculated for the given frequency 7.5 giga hertz to be 157.1 per meter. So once we know k Naught we can find out beta a, the propagation constant in the airfield waveguide and that will be given by k Naught square minus Pi by a whole square, and if we substitute the values we will get the value of beta a to be 94.3.

Similarly, when beta e the inside the dielectric waveguide is calculated, there it is instead of k Naught it is k square, and therefore, it will be epsilon r into k Naught square, and it comes out to be 199.35 per meter. Now the wave impedance z Naught a for the TE$_{10}$ mode in the air part of the waveguide. It can be found out to be k Naught Eta Naught Eta Naught is the intrinsic impedance of the free space divided by beta a, and once these values are substituted, Eta Naught is 377 ohm.

So, it comes to be 628.1 ohm. Similarly z Naught d inside the dielectric portion of the guide can be found out as, k Eta by beta d, k we can write root epsilon rk Naught and Eta can be written as Mue Naught by epsilon r epsilon Naught. So this root epsilon r cancels and z Naught d is given by k Naught Eta Naught y beta d, so once again once, we substitute these values, we get this to be equal to 297.1 ohm. We know that the reflection coefficient gamma will be, z Naught d minus z Naught a divided by z Naught d plus z Naught a and once we substitute the values for z Naught a and z Naught d. the reflection coefficient comes out to be minus 0.36.

So, this is how we can apply the concept of equivalent transmission line voltages and currents. For a single propagating mode and we can calculate the reflection coefficient at the interface of this ARN dielectric.