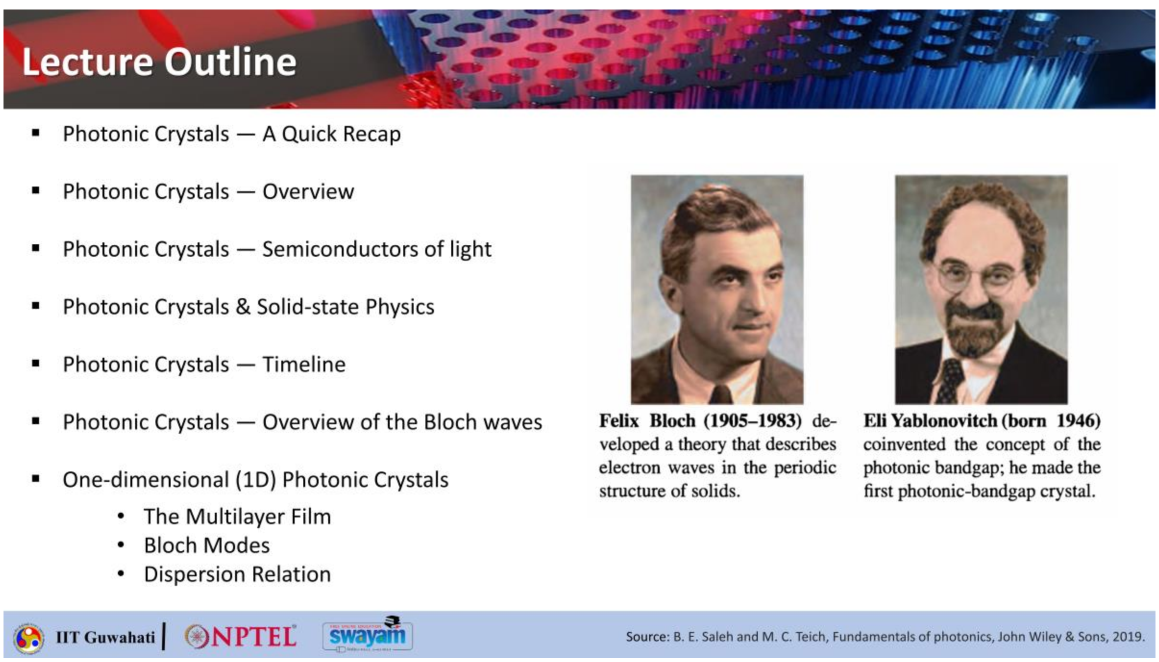
Lec 10: Fundamentals of 1D Photonic Crystal

Hello students, welcome to lecture 10 of the online course on Photonic Crystals Fundamentals and Applications.



Today's lecture, we will cover the fundamentals of 1D photonic crystals. So, here is the lecture outline. We will have a quick recap of the photonic crystals, the overview. We will discuss photonic crystals as semiconductors of light. We will also draw the analogy between photonic crystals and solid state physics.

We will discuss about the timeline of photonic crystals. We will provide overview of block waves and then go into details of 1D photonic crystals and take some examples like the multilayer film. their block modes and dispersion relation. So, in the context of this lecture these two gentlemen have made significant contribution that is the picture of Felix Bloch.

So, he developed a theory that describes electron waves in the periodic structure of solids and the same theory has also been applied for photonic crystals where we study about light propagation in periodic structure of dielectrics. And this is a picture of Eli Avelinovich. So he co-invented the concept of photonic bandgap along with S. John and he made the first photonic bandgap crystal. So his contribution to the field of photonic crystal is really enormous.

Photonic Crystals — A Quick Recap

- A ***photonic crystal (PhC)*** is a material that has been structured to possess a periodic modulation of the refractive index so that the structure influences the propagation and confinement of light within it.

- *Photonic Crystals are periodic optical structures that are designed to affect the motion of photons in a similar way that periodicity of a semiconductor crystal affects the motion of electrons.*

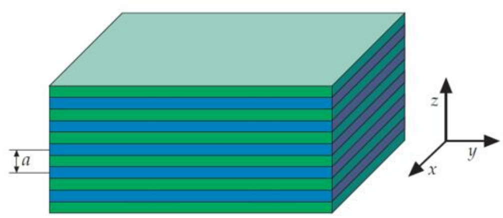


Figure: The multilayer film, a one-dimensional photonic crystal.

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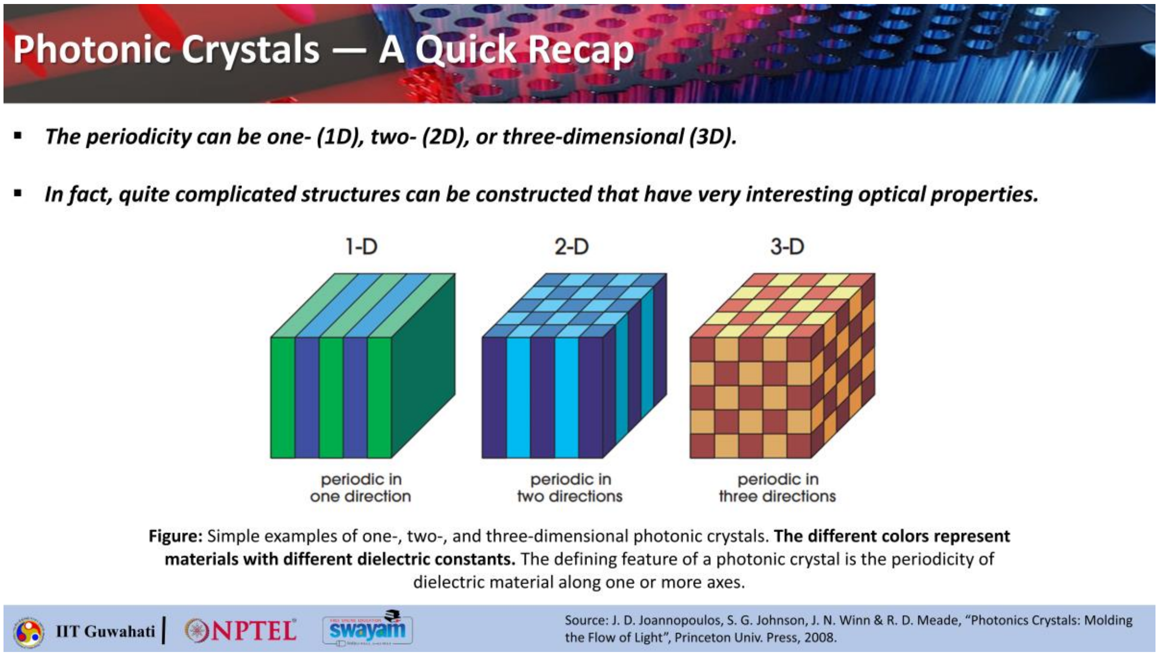
Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So look into the first topic which is a quick recap. So a photonic crystal is basically a material that has been structured to process a periodic modulation of the refractive index so that the structure influences the propagation as well as confinement of light within it. So the picture here shows a multilayer film which is basically a one-dimensional photonic crystal.



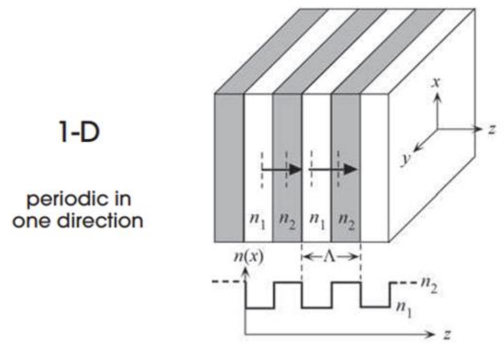
So you can see the green and the blue material they are basically two different types of material and they have been repeated periodically along the z direction okay and here A marks the period of this periodicity of the structure right. So, there is a direct analogy between you know semiconductor physics and the photonic crystals. So, photonic crystals are also sometimes called you know semiconductor crystals for light. So, as you understand the periodicity can be in one dimension, two dimension or three dimension and you know that way the photonic crystals are also called one dimensional, two dimensional or three dimensional photonic crystals. So, in fact you know you can actually make quite complicated structures like these okay and they possess very interesting optical properties. So, this we have already seen.

So, photonic crystals are periodic optical structures that are designed to affect the motion of photons in a similar way that periodicity of a semiconductor crystal affects the motion of electrons.

Okay always remember the different colors they represent materials with different dielectric constant. So, 1D it is very clear that you just have the periodicity in one dimension okay the material is uniform in other two dimension. When you go for now 2D photonic crystals what you see you basically have you know columns okay of two different materials which are repeated along the two dimension and here you can think of you have small cubes of different materials which are periodically repeated in all three axis or all three dimensions. So now let us have a quick overview of the photronic crystal.

Photonic Crystals — Overview

- **One-dimensional periodic structures** include stacks of identical parallel planar multi-layer segments.
- These are often used as gratings that reflect optical waves incident at certain angles, or as filters that selectively reflect waves of certain frequencies.



IIT Guwahati | NPTEL swayam Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition, 2013.

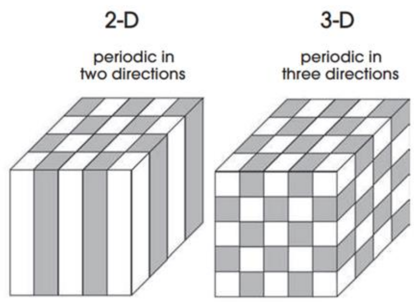
So let us start with 1D you know periodic structure that include stacks of identical parallel planar multilayer segments, something like this. So, these are often used as gratings that could reflect optical waves at certain angles and also as filters that can selectively reflect certain frequencies. So, you can see that if you consider the bright region to have refractive index n_1 and the dark region to have refractive index n_2, the profile refractive index profile shows you that n_1 is lower than n_2 and that is typically the convention. So, the target material has higher dielectric constant. And you can measure the periodicity like this, you know, from this point to this point.

So that is given as capital lambda. So that is the periodicity along z direction, along x and y direction the material is homogeneous.

Photonic Crystals — Overview

- **Two-dimensional periodic structures** include sets of parallel rods as well as sets of parallel cylindrical holes, such as those used to modify the characteristics of optical fibers known as holey fibers.

- **Three-dimensional periodic structures** comprise arrays of cubes, spheres, or holes of various shapes, organized in lattice structures much like those found in natural crystals.



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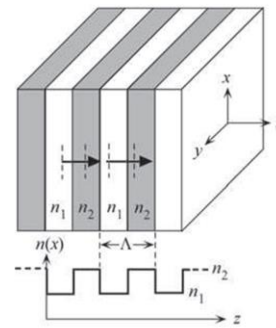
You can go for 2D 2D photonic crystals. They look like this as you have seen. So they basically include parallel rods as I have already discussed.
So you can actually have parallel rods or you can have parallel cylindrical holes. And this kind of cylindrical holes are important to modify the characteristics of optical fibers as we have discussed in the initial lectures and those are called holy fibers right. You can also think of you know three-dimensional structures where you can have arrays of cubes, spheres or even holes of different shapes which are organized in lattice structures which are very similar to those found in the natural crystals. So, when we go into more details, we will see those exact orientation, but you can think of all the possibilities of different unit cell structures that could repeat in 3 dimension to form a 3D photonic crystal. You can think of a renuance in 2 dimension to form 2D photonic crystals.

Photonic Crystals — Overview

- The periodic variation in n (shown in Figure) is normally assumed to extend indefinitely, whereas in practice, the PhCs have a finite size, for example, a certain number of layers.
- As in normal crystals, the periodic structures in Figure have a unit cell, which repeats itself to generate the whole lattice—that is, the whole crystal structure.
- For the 1D PhC in, for example, two adjacent layers form the unit cell. We can move this unit cell along z by a distance Λ, **the period (or periodicity),** many times to generate the whole 1D photonic crystal.



Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition, 2013.

So, we come back to our study and our objective today. So, we are mainly focusing on the 1D photonic crystal in this particular lecture. So, here you can notice that we are having periodic variation in the refractive index ok and it is assumed that this kind of you know variation will extend indefinitely like so infinitely long periodic array. But when you think in practice practice you want to make this device you will see that these devices of this photonic crystals they have finite size it means you only can have you know certain fixed number of layers so as in normal crystals the periodic structure okay this all in this figure also have a unit cell here here you can identify this dark and the bright region or layer as one unit cell and that is basically getting repeated periodically right so this unit cell once repeated can give you the whole crystal structure. So as discussed in 1D photonic crystal these two adjacent layer give you the photonic crystal and you can move along that direction that is the grating vector you can say, okay? And here the periodicity is given by this capital Lambda, okay? So it is called the period of the periodicity, okay? And you can generate the whole 1D photonic crystal once you repeat this unit cell using this period in this particular Z dimension, fine? Now let's look into the concept where, you know, photonic crystals are considered to be semiconductors of light.

Photonic Crystals — Semiconductors of light

- Optical waves, which are inherently periodic, interact with periodic media in a unique way, particularly when the scale of the periodicity is of the same order as that of the wavelength.
- For example, spectral bands emerge in which light waves cannot propagate through the medium without severe attenuation.
- Waves with frequencies lying within these forbidden bands, called **photonic bandgaps**, behave in a manner akin to total internal reflection, but are applicable for all directions.
- The dissolution of the transmitted wave is a result of destructive interference among the waves scattered by elements of the periodic structure in the forward direction.
- Remarkably, this effect extends over finite spectral bands, rather than occurring for just single frequencies.

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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

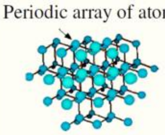
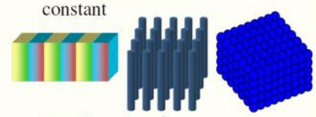
So optical waves which are inherently periodic interact with periodic media in a unique way, particularly when the scale of the periodicity becomes comparable to that of the wavelength of light. And if you remember the initial lecture where we discussed about the differences between optics, photonics and nanophotonics, we discussed that photonics is that particular science branch of optics, you can say. where we are talking about materials which are having dimensions of the order of the wavelength of light and this is where exactly photonic crystal comes into the picture. The photonic crystal has got periodicity which is comparable to the wavelength of light. So, here what happens you know some spectral bands will emerge where light waves cannot propagate through this medium at all ok.

That means the propagation has got severe attenuation ok. And the waves with frequencies lying within those forbidden bands are called photonic bandgap. They behave in a manner which is very similar to that of total internal reflection but difference is that photonic band gap is applicable for all directions. Means the light can have any incident angle but still it will be completely reflected. But when you compare photonic crystals it is when you compare this with total internal reflection where the reflection only takes place for a particular set of incident angle for you know the light.

So, the dissolution of the transmitted wave is a result of the destructive interference among the waves scattered by the elements of the periodic structure in the forward direction. So, this effect basically extends over finite spectral bands rather than just occurring for single frequencies. Now, this phenomena is analogous to the electronic properties of crystalline solids such as semiconductors. So, in that case the periodic wave associated with an electron travels in a periodic crystal lattice and energy band gap often materialize. So, if you put semiconductors and photonic crystals side by side.

Photonic Crystals — Semiconductors of light

- This phenomenon is analogous to the electronic properties of crystalline solids such as semiconductors.
- In that case, the periodic wave associated with an electron travels in a periodic crystal lattice, and energy bandgaps often materialize.

Semiconductors	Photonic Crystals
Periodic array of atoms	Periodic variation of dielectric constant
	
Atomic length scales	Length scale ~ λ
Natural structures	Artificial structures
Control electron flow	Control e.m. wave propagation
1950's electronic revolution	New frontier in modern optics

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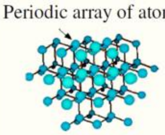
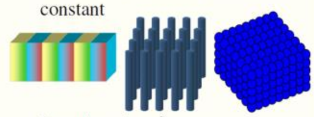
Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

You can say the semiconductors are basically periodic array of atoms whereas photonic crystals are basically periodic variation of dielectric constant. So you can have this variation along 1 dimension, 2 dimension or 3 dimension. The length scale for semiconductors are basically atomic length scale that means you know you are talking in terms of Armstrong okay. When you come to photonic crystal you are basically having length scale which is comparable to lambda and lambda for visible light is typically from 400 nanometer to you know 780 nanometer. So you are typically in those micrometer kind of length scale.

Photonic Crystals — Semiconductors of light

- **Because of this analogy**, the photonic periodic structures have come to be called **photonic crystals**.
- Photonic crystals enjoy a whole raft of applications, including use as waveguides, fibers, resonators, lasers, filters, routers, switches, gates, and sensors; etc

Semiconductors	Photonic Crystals
Periodic array of atoms	Periodic variation of dielectric constant
	
Atomic length scales	Length scale ~ λ
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Semiconductors are natural structures whereas photonic crystals are artificial structures. Semiconductors control the flow of electron whereas photonic crystal controls the propagation of electromagnetic wave or light. and semiconductors have caused this 1950s electronic revolution whereas photonic crystals are bringing up lot new more frontiers in modern optics and as i mentioned in the initial lectures that even in the emergence of 6G technologies photonic crystal is going to play a very very important role in terms of you know terahertz topological photonic insulators based devices So, because of this analogy you can say that the photonic periodic structures have come to be called as photonic crystals. Right. And photonic crystals enjoy a whole raft of applications.

They can be used as web guides, filters, resonators, lasers, you know, fibers, routers, switches, gates, sensors, etc. So you can think of, you know, making any kind of, you know, optical communication system using photonic crystals. So that way, photonic crystal is a very, very handy tool. concept and it is a very handy tool for the optical engineers. Now, let us put a comparison between the photonic crystals and the solid state physics, okay.

Photonic Crystals & Solid-State Physics

- A similarity between the physics of PhCs and solidstate physics gives the possibility to draw the analogy between some properties and computation methods applied to solid-state and PhCs physics.
- The most important similarities between PhC and solid-state physics are as follows:
  - Periodic modulation of the refractive index in a PhC forms a lattice similar to atomic lattice of solid-state;
  - Behavior of photons in a PhC is similar to electron and hole behavior in an atomic lattice;
  - Due to the lattice periodicity both PhC and solid-state provide band gap, the range of energies which particle cannot have inside the structure.
- From theoretical viewpoint, determination of the eigen functions in a PhC is very similar to calculation of the particle wave functions in the solid-state.
- This similarity is used to obtain photonic band structure.

Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

The similarity between the physics of photonic crystals and solid state physics gives us the possibility to draw the analogy between some properties and computation method which are applied in you know solid state and photonic crystal physics. The most important similarities are something like you know periodic modulation of refractive index in photonic crystal forms a lattice which is similar to the atomic lattice of solid state. The behavior of photons in a photonic crystal is similar to electron and hole behavior in atomic lattice and due to the lattice periodicity both photonic crystal and solid state provide band gap and these are the range of frequencies that particle or photonic crystal will not allow inside that structure. So from the theoretical point of view, determination of the eigenfunctions in a photonic crystal is very similar to the calculation of particle wave functions in the solid state. So this similarity is used to obtain the photonic band structure.

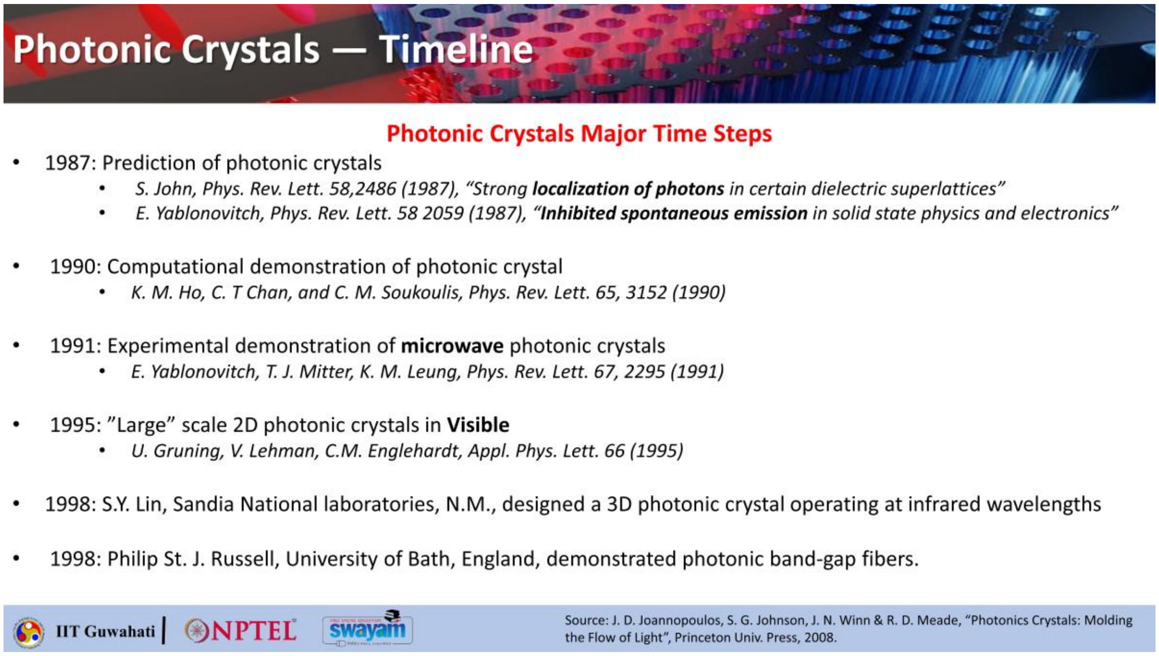
Photonic Crystals & Solid-State Physics

- There exist some essential differences between PhCs and solid-state physics.
- One of the main differences is the particle energy distribution.
  - **Electrons obey the Fermi–Dirac distribution while photons obey Bose–Einstein distribution.**
- Besides, electrons are affected by intra crystalline field which leads to the necessity of taking it into account while Photons are not affected by intra crystalline field.
- The most important property which determines practical significance of the PhC is the presence of the photonic band gap.
- The **photonic band gap (PBG)** refers to the energy or frequency range where the light propagation is prohibited inside the PhC.
- When the radiation with frequency inside the PBG incidents the structure, it appears to be completely reflected.

Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So the method that you follow to calculate electronic band structure in semiconductors you can actually use the same thing same concept here to obtain the photonic band structure for photonic crystals but there are some essential differences as well between photonic crystals and solid state physics so one main difference is the particle energy distribution So electrons in semiconductors or solid state physics, they obey the Fermi Dirac distribution while photons obey Bose-Einstein distribution. Now Bose-Einstein statistics basically apply only to particles that do not follow the Pauli's exclusion principle. So particles that follow Bose-Einstein statistics are called bosons. which have integer number of integer value of spin okay. But in contrast particles that follow Fermi Dirac statistics are called fermions and they have half integer spins right.

Besides electrons in solid state physics are affected by the intracrystalline field which leads to the necessity of taking into account right while photons are not affected by that intracrystalline field. most important property which determines the practical significance of the photonic crystals is basically the presence of the photonic bandgap. So, photonic bandgap engineering is really an art that optical engineers master by changing the constituent unit cells and the periodicity to match their requirement. Now why that is particularly important? The photonic band gap refers to the energy or the frequency range where the light propagation is prohibited inside the photonic crystal. So when the radiation with frequency inside the photonic band gap incident on the structure, it appears to be completely reflected.



So let us have a brief look at the photonic crystal timeline. We have briefly seen this in the introductory lecture, but let us have a quick overview again. So, in 1987, the predictions of photonic crystals were made, okay, and this were done by Sajeev John and Ali Avalonovich. They have written these papers on you know strong localization of photons in certain dielectric super lattices and also inhibited spontaneous emission in solid state physics and electronics. So, after that in 1990 the computational demonstration of photonic crystal was done by K.

M. Ho. 1991, Yablonovitch demonstrated experimentally the first microwave photonic crystal. If you remember, we have studied the scaling properties of electromagnetism and that is why it was possible to demonstrate the properties for you know microwave photonic crystals in instead of making you know photonic crystals for light so in 1995 large scale 2D photonic crystals were made invisible so this is where the technology for you know uh photonic crystals which handle light was developed and then in 1998 a 3D photonic crystal was designed to operate at infrared wavelength. In 1998, Philip Russell at the University of Bath, England demonstrated photonic bandgap fibers. So this is the historic timeline of photonic crystal research and after that slowly the photonic crystal research has gained momentum. So let us now go into the details of 1D photonic crystals.

One Dimensional (1D) Photonic Crystals

- Let us analyze photonic crystals by considering the simplest possible case, a one-dimensional system, and applying the principles of electromagnetism and symmetry that we developed in the previous lectures.
- Even in this simple system, we can discern some of the most important features of photonic crystals in general, such as photonic band gaps and modes that are localized around defects.
- The optical properties of a one-dimensional layered system may be familiar, but by expressing the results in the language of band structures and band gaps, new phenomena such as omnidirectional reflectivity can be discovered, as well as prepare for the more complicated two- and three-dimensional systems that lie ahead.

Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So let us analyze the photonic crystals by considering the simplest case, which is the one-dimensional periodicity. And we apply the principles of electromagnetism and symmetry that we have developed in the previous lectures. So even in this simple system, we can discern some of the important features of photonic crystal in general, something like the photonic band gap and the modes that are localized around defects. So the optical properties of a one-dimensional layered system may be familiar by, but by expressing the results in the language of band structures and band gaps, okay, a new phenomena such as omnidirectional reflectivity can be discovered. ok and this is where you know the concept of 1D photonic crystal is important.

1D Photonic Crystal—The Multilayer Film

- The simplest possible photonic crystal, shown in Figure, consists of alternating layers of material with different dielectric constants: a **multilayer film**.
- The term "***one-dimensional***" is used because the dielectric function $\varepsilon(z)$ varies along one direction (z) only.
- The system consists of alternating layers of materials (**blue** and **green**) with different dielectric constants, with a spatial period a.
- Consider that each layer is uniform and extends to infinity along the *x* and *y* directions, and that the periodicity in the *z* direction also extends to infinity.

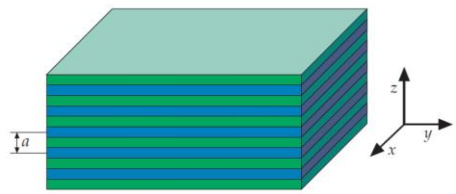


Figure: The multilayer film, a one-dimensional photonic crystal.

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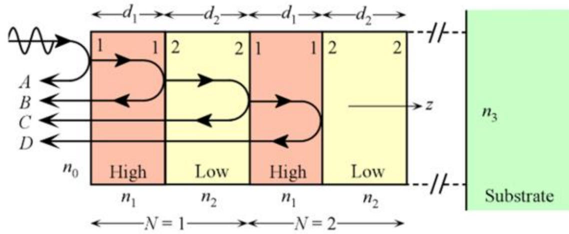
Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, what you are getting is omnidirectional reflectivity not only reflectivity at certain angle. So, although you know you have this concept of you know layered systems ok you are actually focusing on the photonic band gap and band structure to obtain this omnidirectional feature. And 1D photonic crystal is the easiest one to study as you go on more complicated systems like 2D or 3D photonic crystals, same concept will be applied but things will get more complicated. So, the simplest possible photonic crystal can be thought of as an alternating layer of materials of different dielectric constant something like a multi-layer film as you see in this particular figure. So, here if you think the permittivity along x and y a same remains constant, but then permittivity along z is basically changing along z right.

So, $\varepsilon(z)$ is changing periodically So the system consists of alternating layers of materials blue and green which have different dielectric constant and the spatial period is marked as a. So if you can consider two layers as the unit cell so this is the periodicity a which is basically the total thickness of these two layers. Now we consider each layer is uniform and it extends to infinity along the lateral dimensions that is along x and y and the periodicity in the z direction also extends to infinity. So these are needed for the theoretical calculation but if you think in practice if you want to make a device you will have finite dimensions along x and y and also along z.

1D Photonic Crystal—The Multilayer Film

- This arrangement is not a new idea. Lord Rayleigh published one of the first analyses of the optical properties of multilayer films.
- As we will see, this type of photonic crystal can act as a *mirror* (a **Bragg mirror**) for light with a frequency within a specified range, and it can localize light modes if there are any defects in its structure.
- These concepts are commonly used in dielectric mirrors and optical filters.



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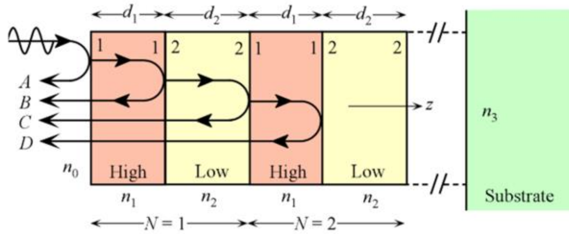
Now this arrangement is not a new idea. Lord Rayleigh published one of the first analyses of the optical properties of multilayer film, right? So that was pretty, you know, in 1800s. So as we will see, this type of photonic crystal can act as a mirror. also known as Bragg mirror for light with a frequency within a specific range and it can localize light modes if there are any defects in the structure. So, this concepts are commonly used in dielectric mirrors and optical filters. So, you can see here what is happening you have a high and low dielectric material of thickness d_1 and d_2 respectively and then this pattern is repeated over here.

So, you say n equals 1 here the first period this is the second period. The refractive index is marked as n_1, n_2. Outside it is air and then you have this infinitely long periodic structure and then finally you have a substrate supporting the structure. So when there is an incident light you see some part is getting reflected you can call that as A. some part getting transmitted but again some part of that transmitted light when it hits this particular interface between this high and low medium.

a portion of it will get reflected and the remaining gets transmitted and then you will have this repeated reflection transmission and then and finally, you get one reflection. So, this continues. So, what happens you are basically getting reflected light from all those different interfaces. Now, if all these reflected light are in phase and then they do a constructive interference you basically see a reflection right so that behaves like a dielectric mirror but now if you change the width of the of each layer or the periodicity in such a way that all this reflected light destructively interfere with each other and they cancel out it means you will not see any reflection or you may see reflection at a particular frequency and remaining light is getting transmitted.

1D Photonic Crystal—The Multilayer Film

- The traditional way to analyze this system, pioneered by Lord Rayleigh, is to imagine that a plane wave propagates through the material and to consider the sum of the multiple reflections and refractions that occur at each interface.
- Let's use a different approach—the analysis of band structures—that is easily generalized to the more complex two- and three dimensional photonic crystals.



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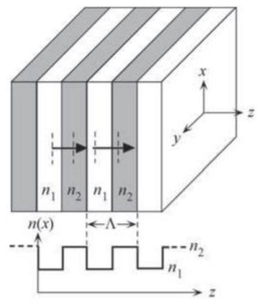
So, that behaves like a filter. So, you will have a notch kind of a filter where only a particular frequency is reflected remaining is getting transmitted. So, that is how you can use the periodic structures as dielectric mirrors and optical filters. So, the traditional way to analyze the system as I mentioned. you have to consider you know the sum of multiple reflections and refraction that occur at each of these interfaces ok. But when you try to analyze the bench structure for you know more complicated photonic crystals like 2D and 3D you have to use a slightly different approach.

Photonic Crystals —Overview of Bloch waves

- The periodicity of a photonic crystal implies that any property at a location z will be the same at $z \pm \Lambda$, $z \pm 2\Lambda$ and so on; that is, there is translational symmetry along z (in 1D).
- The EM waves that are allowed to propagate along z through the periodic structure are called the **modes of the photonic crystal**.



$$\varepsilon(z+\Lambda) = \varepsilon(z)$$

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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

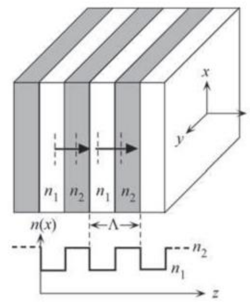
So we will try to generalize that approach and show it using Bloch waves in 1D photonic crystal first. So the periodicity of a photonic crystal implies that any property at a location z will be same as that $z \pm \Lambda$ location or even $z \pm 2\Lambda$ or $z \pm 3\Lambda$and so on so it means there is a translational symmetry along the z direction that is along 1D Now the EM waves that are allowed to propagate along z through this photonic structure are called the modes of the photonic crystal. So the ones which are allowed to propagate are basically the solution. of electromagnetic waves in this particular system. So, that is why they are also called the modes of the photonic crystal.

Photonic Crystals —Overview of Bloch waves

- They have a special waveform that must bear the periodicity of the structure, and are called **Bloch waves**.
- Such a wave for the field E_x, for example, has the form

  $$E_x(z,t) = A(z)\exp(-jkz),$$

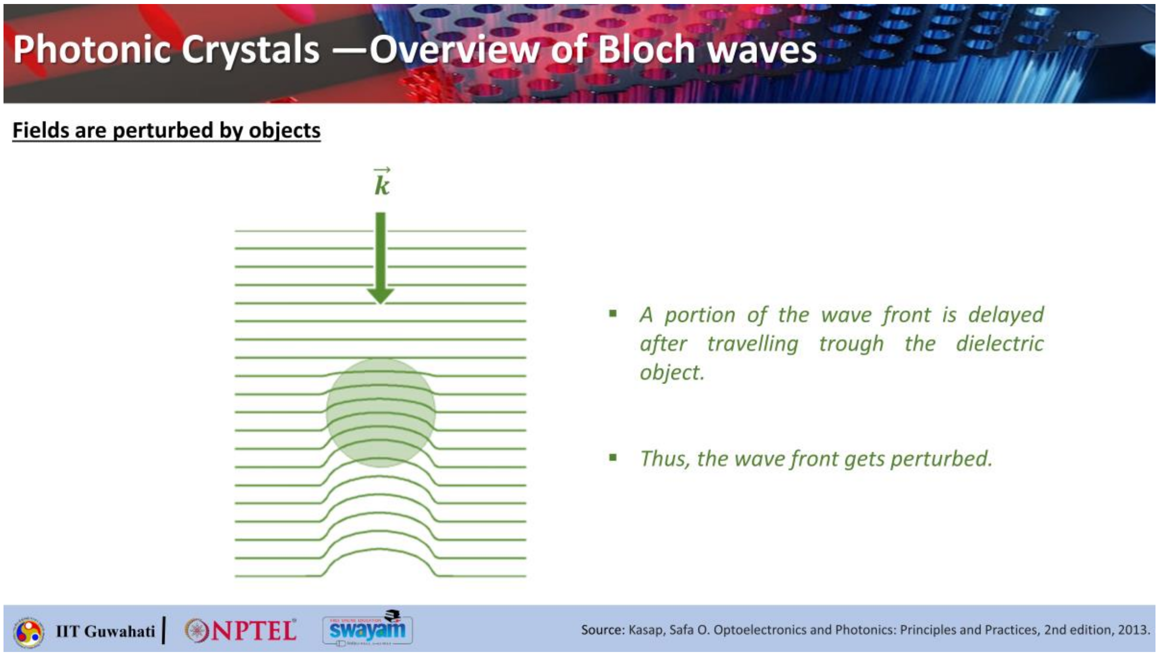
  which represents a traveling wave along z and $A(z)$ is an amplitude function that has the periodicity of the structure, that is, it is periodic along z with a period Λ.
- $A(z)$ varies with the periodic refractive index function $n(z)$.
- $A(z+\Lambda) = \mathrm{A(z)}$



Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition, 2013.

So, you can write that the permittivity is varying as this that $\varepsilon(z+\Lambda)$ is basically $\varepsilon(z)$. It means the permittivity is repeating periodically while Λ is the period. So they have a special waveform. They must bear the periodicity of the structure and are called bloch waves.

okay. So, such a wave for electric field E_x has the form of E_x(z,t) okay. So, E along x and its property along z and it is also a function of t has got A(z) which is basically an amplitude function that contains the information of the periodicity of the structure okay. That is A(z) will also be periodic, the amplitude will have the periodicity of period Λ and it propagates along z. So, A(z) varies with the periodic refractive index which is n_z.

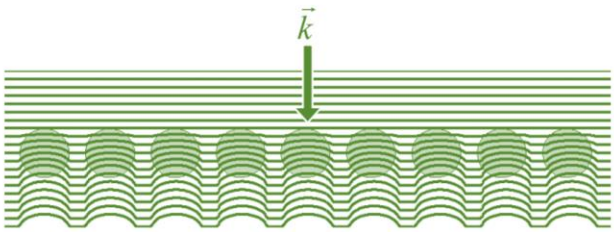


So, this refractive index is obtained from the dielectric constant which you have seen in the previous slide and if you remember the relationship that the refractive index is nothing, but square root of the dielectric constant or epsilon.

So, you can also write that $A(z+\Lambda)$ is equal to $A(z)$. So, this is how you know if you consider a field ok getting perturbed by the object. So, wherever the wave front is entering the material so any material will have refractive index more than one that means light basically slows down inside the material as compared to the light that is traveling in the air so your wave front will get actually distorted like this so this is how you can actually get the impression of that dielectric material which is present in the path of the light okay

Photonic Crystals —Overview of Bloch waves

Fields in periodic structures



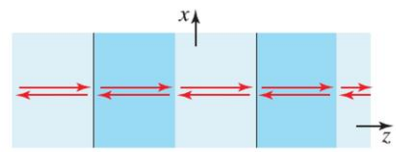
Waves in periodic structures take on the same symmetry and periodicity as their host.

Source: Kasap, Safa O. Optoelectronics and Photonics: Principles and Practices, 2nd edition, 2013.

Now, if you consider field in a periodic structure like this. So, this is how the dielectric material are arranged periodically and when a you know light with you know face front going like this you can actually see that the amplitude also acquires that periodicity right. So, that was the concept that we discussed earlier that $A(z+\Lambda)$ will be equal to $A(z)$. So, the waves in periodic structures take on the symmetry and periodicity as their host material so if you think of you know periodic crystal there if you incident light beam so the light will also you know take on the same kind of symmetry and periodicity in the amplitude as it is there in the host photonic crystal.

One-dimensional (1D) Photonic Crystals

- ***One-dimensional (1D) photonic crystals*** are dielectric structures whose optical properties vary periodically in one direction, called the axis of periodicity, and are constant in the orthogonal directions.
- Consider first a homogeneous medium, which is invariant to an arbitrary translation of the coordinate system.
- For this medium, an optical mode is a wave that is unaltered by such a translation; it changes only by a multiplicative constant of unity magnitude (a phase factor).



- The plane wave $\exp(-jkz)$ is such a mode since, upon translation by a distance d, it becomes:

$$\exp[-jk(z + d)] = \exp(-jkd)\exp(-jkz).$$

- The phase factor $\exp(-jkd)$ is the eigenvalue of the translation operation.

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

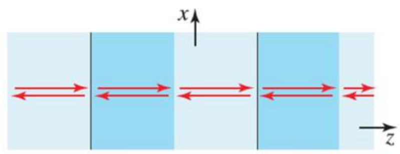
So 1D photonic crystals are basically structures as you have seen that the properties are constant in orthogonal dimension and only it is changing in one direction. So we will consider first a homogeneous system which is invariant to the arbitrary translation of the coordinate system. So, for this medium an optical mode is basically a wave that is unaltered by such a translation. So, it only changes by a multiplicative constant of unity magnitude which is the phase factor.

So, if you consider plane wave which is $\exp(-jkz)$. is such a mode since upon translation by distance of d it becomes $\exp(-jk(z+d))$. So, when you split this exponential into the two terms you get $\exp(-jkd)$ $\exp(-jkz)$. So, you are basically getting you know this phase factor $\exp(-jkd)$ as eigenvalue of the translation operation. So, we can consider the on axis block mode okay where in this photonic 1D photonic crystal or 1D periodic medium you could see that this block mode is invariant to the translation by distance Λ along the axis of the periodicity. So its optical modes are basically waves that could maintain their form upon the translation and it is changing only by a phase factor which is given as $\exp(-jkd)$ okay.

1D Photonic Crystals —Bloch mode

- This form satisfies the condition that a translation Λ alters the wave by only a phase factor $\exp(-jK\Lambda)$ since the periodic function is unaltered by such translation.

- This optical wave is known as a **Bloch mode**, and the parameter K(k in a periodic medium), which specifies the mode and its associated periodic function $p_K(z)$, is called the **Bloch wavenumber**.



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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So you can write that this modes have this particular form. So, U_z can be written as $pK(z)$ exp($-j$Kz). So, what is U? U is basically any field component it can be E_x , E_y , H_x , or H_y. So, it is a very generic term it is showing any field electric or magnetic field any component along x or y in this case. What is capital K? This is a propagation constant and pKz is basically a periodic function of period Λ. So this form satisfies the condition that a translation Λ alters the wave by a phase factor which is given by exp($-j$KΛ) since the periodic function is unaltered by this translation.

1D Photonic Crystals —Bloch mode

- The Bloch mode is thus a plane wave $\exp(-jKz)$ with propagation constant K, modulated by a periodic function $p_K(z)$, which has the character of a standing wave, as illustrated by its real part displayed in Fig. (a).
- Since a periodic function of period Λ can be expanded in a Fourier series as a superposition of harmonic functions of the form $\exp(-jmgz)$, $m = 0, \pm1, \pm2,...$, with

$$g = 2\pi/\Lambda$$

- It follows that the Bloch wave is a superposition of plane waves of multiple spatial frequencies $K + mg$.

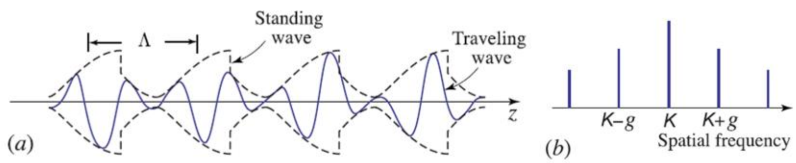

Figure: (a) The Bloch mode. (b) Spatial spectrum of the Bloch mode

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So this kind of optical wave is known as bloch mode. The parameter K, capital K which is denoted as the propagation constant or basically the wave factor or wave number in a periodic medium, okay. And it is specified, it specifies the mode and the associated periodic function which is $p_K z$. So, this parameter K, capital K is called the block wave number. So once again what is the difference between this small k? Small k is the wave vector of the incident light and capital K is basically the wave number or wave vector you can say of the wave in a periodic medium. So the block mode is thus nothing but a plane wave which is exponential minus jkz.

with a propagation constant capital K modulated by a periodic function pkz, so which has the character of a standing wave. So, if this is the way you know the travelling wave is, so the standing wave pattern is the dotted line and you can actually find that this is the period. So, since a periodic function of period Λ can be expanded into a Fourier series as a superposition of harmonic functions of the form of $\exp(-j\,mgz)$ where M is nothing but you know 0, ±1, ±2,…. and you can take small g as the spatial frequency.

So, g can be written as $2\pi/\Lambda$. So, g is the spatial frequency and it is a measure how often the sinusoidal components as determined by the Fourier transform So, the sinusoidal constants or components of the structure repeat per unit distance. So, that is g. So, it follows that the bloch wave is basically superposition of plane waves of multiple spatial frequencies like K + mg. So, here you can see this is a bloch mode.

here you can see the spatial spectrum of the bloch mode. So it has got one frequency K okay and you have K+g and K-g, K+2g, K-2g and so on okay. So the fundamental spatial frequency g of the periodic structure and its harmonics mg added to the bloch wave number K, they constitute the spatial spectrum of the bloch wave which is shown here. What is m? m is 0, ±1, ±2,…, and so on. So, the spatial frequency shift which is introduced by the periodic medium. So the shift is here right, so this is analogous to the temporal shift or temporal frequency shift like Doppler okay that is introduced by reflection from a moving object.

1D Photonic Crystals —Bloch mode

- The fundamental spatial frequency g of the periodic structure and its harmonics mg, added to the **Bloch wavenumber** K, constitute the spatial spectrum of the Bloch wave, as shown in Fig. (b).

- The spatial frequency shift introduced by the periodic medium is analogous to the temporal frequency (Doppler) shift introduced by reflection from a moving object.

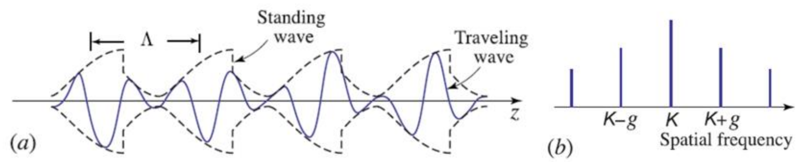

Figure: (a) The Bloch mode.
(b) Spatial spectrum of the Bloch mode

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

if you take two modes with block numbers K and K' where K' is given as $K + g$ they are basically you know equivalent since they correspond to the same phase factor. So, if you find out the phase factor associated with this. bloch wave number $\exp(-jK'\Lambda)$ this can be written as K' can be written as $K + g$.

1D Photonic Crystals —Bloch mode

- Two modes with Bloch wavenumbers K and $K' = K + g$ are equivalent since they correspond to the same phase factor, $\exp(-jK'\Lambda) = \exp(-jK\Lambda)\exp(-j2\pi) = \exp(-jK\Lambda)$.
- This is also evident since the factor $\exp(-jgz)$ is itself periodic & can be lumped with the periodic function $p_K(z)$.
- Therefore, for a complete specification of all modes, we need only consider values of K in a spatial-frequency interval of width $g = 2\pi/\Lambda$.
- The interval $[-g/2, g/2] = [-\pi/\Lambda, \pi/\Lambda]$, known as the first **Brillouin zone**, is a commonly used construct.

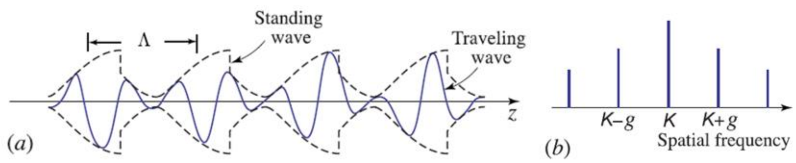

Figure: (a) The Bloch mode. (b) Spatial spectrum of the Bloch mode

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So, you can write it like this. So, this essentially gives you 1. So, you finally get $\exp(-jK\Lambda)$. okay, g if you remember g is coming from here $2\pi/\Lambda$, so that is how this particular term gets into picture right. So, instead of g you are you are writing 2 pi okay, so g lambda instead of g lambda you are writing 2 pi. So, this is evident that since the factor exponential minus jgz is itself periodic and it can be lumped with the periodic function pkz, right. So, therefore, for a complete specification of all the modes, we only need to consider the values of capital K in a special frequency interval of width g equals $2\pi/\Lambda$.

So ideally the interval is starting from minus g by 2 to g by 2. So that is from minus pi by lambda to pi by lambda. So if you only consider this interval that can give you all information because after that it is repeating periodically. So this region is also known as the first Brillouin zone. right and is a commonly used construct when you study the dispersion relation. So, now that we have established the mathematical form of the modes as imposed by the translational symmetry of the periodic medium.

The next step is basically to solve for eigenvalue problems which are described by the generalized Helmholtz equation right. So, it is basically the wave equation right. So, we have to find solutions for wave equation in this particular photonic crystal. So, those are the modes and those modes have their frequencies and only those frequencies are allowed to propagate inside the crystal.

Where there is no solution it means that frequency falls within the band gap ok. So for a mode with a block wave number K the eigenvalues provide a discrete set of frequencies ω and these values are used to construct the ω−K dispersion relation and this is a dispersion relation as you can see the y axis is basically ω. and what you have in the x axis is K so it is $\omega-K$ dispersion diagram you can also see that you have plotted it for you know g equals $2\pi/\Lambda$ so you are considering g/2 to -g/2 to g/2 okay and then you are you know the structure is repeating for integral multiples of g/2. The Eigen functions, so this figure here shows the dispersion relation as a multivalued periodic function with period g equals $2\pi/\Lambda$. and discontinuities at different K values, okay. So, you are seeing discontinuities here, you are seeing discontinuities here which are basically integral multiple of g/2.

So, here it is in g/2 into 1, here it is 2 into g/2, so that is g. So, you are actually getting discontinuities at all these points. So the eigen functions help us determine the block periodic function which is pKz for each values of omega associated with each K. So this diagram is particularly very very interesting and you are only bothered about the Brillouin zone basically because that contains all the information okay and you can have Brillouin zone. ranging from -g/2 to g/2 as we have described earlier. So, here also you can see that this region is marked at Brillouin zone and what is happening next is basically an extension of it okay.

1D Photonic Crystals —Dispersion Relation

- Now that we have established the mathematical form of the modes, as imposed by the translational symmetry of the periodic medium, the next step is to solve the eigenvalue problem described by the generalized Helmholtz equation.

- For a mode with a Bloch wavenumber K, the eigenvalues provide a discrete set of frequencies ω. These values are used to construct the $\omega - K$ **dispersion relation**.

- The eigenfunctions help us determine the Bloch periodic functions $p_K(z)$ for each of the values of ω associated with each K.

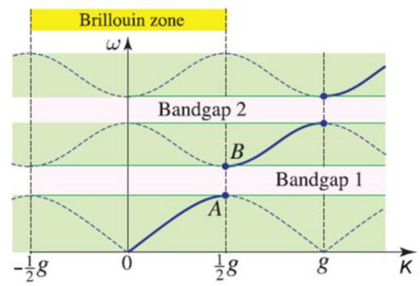

Figure: The dispersion relation is a multivalued periodic function with period $g = 2\pi/\Lambda$ and discontinuities at k equals integer multiples of $g/2$.

IIT Guwahati | NPTEL swayam — Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So, when visualized as a monotonically increasing function of K like this okay you see that there is some discrete jump okay at the values where K is basically integral multiple of g/2 okay. So, these jumps or discontinuities basically correspond to the bandgap. So, this is bandgap 1, this is bandgap 2 okay. It means spectral bands are not crossed by the dispersion lines okay in this case. So, you do not have any propagation mode existing at those particular frequencies which lie within the band gap.

So, the origin of discontinuities in the dispersion relation basically lies in the special symmetry that emerges when k = $g/2$. That is when the period of the medium equals exactly half the period of the travelling wave. So, consider two modes with k equals $\pm g/2$ and bloch periodic function $p_K(z)$then can be written as $p_{(\pm g/2)}(z)$. So, you are replacing k which is the wave number of the propagating mode with g/2. So, this since this modes travel with the same wave number, but they are traveling in the opposite direction.

So, you can actually see the inverted version of the medium. So, what you are basically seeing is $p_{(-g/2)}(z)$ will give you $p_{(g/2)}(-z)$ right. So they are basically inverted version of each other okay but these two modes are basically in fact one mode and they are same because they have the same block numbers which are different by g. So, for every g you get this same modes because g is your spatial frequency. So, it therefore follows that at the edge of the brilliant zone which is here and here there are two block periodic functions that are inverted version of one another.

1D Photonic Crystals —Dispersion Relation

- The $\omega - K$ relation is a periodic multivalued function of K with period g, the fundamental spatial frequency of the periodic structure; it is often plotted over the Brillouin zone $[-g/2 < k \leq g/2]$, as illustrated schematically in Fig.
- When visualized as a monotonically increasing function of k, it appears as a continuous function with discrete jumps at values of K equal to integer multiples of $g/2$.
- These discontinuities correspond to the photonic bandgaps, which are spectral bands not crossed by the dispersion lines, so that no propagating modes exist.

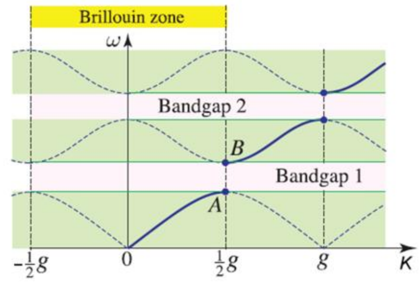

Figure: The dispersion relation is a multivalued periodic function with period $g = 2\pi/\Lambda$ and discontinuities at k equals integer multiples of $g/2$.

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So, you can see at the edges. okay since the medium is inhomogeneous or you can say piecewise homogeneous within a unit cell these two distinct functions they interact with the medium differently and therefore they have two different or distinct eigenvalues. That means they will have different values of ω. So one is having, say, this value of ω. The other one is having this value of ω.

So they are not having the same frequency, although they have the same K value. And this actually explains why you have got this discontinuity in the ω−K line across the boundary of the Brillouin zone. A similar argument could also explain the discontinuities that occur when K equals the other integer multiples of g/2. So, you can also use the same argument here and so on. So, if you simply compare the dispersion relation of photon in vacuum which is given by you know this is the ω−k diagram.

1D Photonic Crystals —Dispersion Relation

- The origin of the discontinuities in the dispersion relation lies in the special symmetry that emerges when $k = g/2$, *i.e.*, when the period of the medium equals exactly half the period of the traveling wave.

- Consider the two modes with $k = \pm g/2$ and Bloch periodic functions $p_K(z) = p_{\pm g/2}(z)$.

- Since these modes travel with the same wavenumber, but in opposite directions, *i.e.* (see inverted versions of the medium), $p_{-g/2}(z) = p_{g/2}(-z)$. But these two modes are in fact one and the same, because their Bloch wavenumbers differ by g.

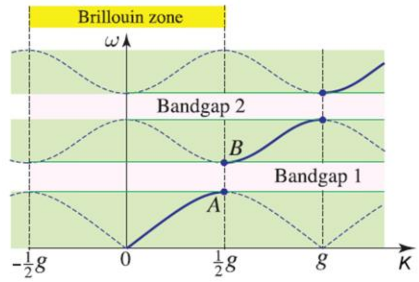

Figure: The dispersion relation is a multivalued periodic function with period $g = 2\pi/\Lambda$ and discontinuities at k equals integer multiples of $g/2$.

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So, dispersion relation is very simple is given by ω equals ck. in periodic dielectric or photonic crystal you can see that the ω–k diagram is basically follow a straight line but there are discontinuities in between and these are those photonic band gaps so who is giving rise to these band gaps there is this repeating periodic structure they are giving rise to this forbidden zone so this particular bands of frequencies are not allowed to propagate inside the crystal.

1D Photonic Crystals —Dispersion Relation

- It therefore follows that at the edge of a **Brillouin zone**, there are two Bloch periodic functions that are inverted versions of one another.
- Since the medium is inhomogeneous or piecewise homogeneous within a unit cell, these two distinct functions interact with the medium differently, and therefore have two distinct eigenvalues, *i.e.*, distinct values of ω.
- This explains the discontinuity that emerges as the continuous $\omega - K$ line crosses the boundary of the Brillouin zone.
- A similar argument explains the discontinuities that occur when K equals other integer multiples of $g/2$.

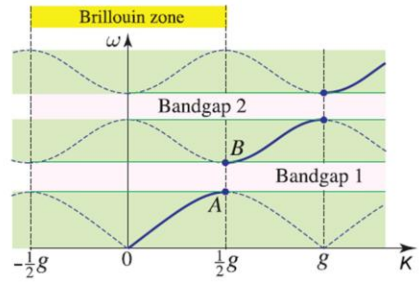

Figure: The dispersion relation is a multivalued periodic function with period $g = 2\pi/\Lambda$ and discontinuities at k equals integer multiples of $g/2$.

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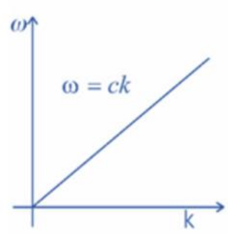
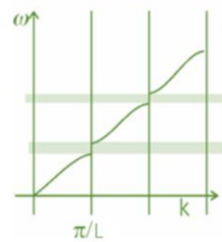
Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

The band gap size dependent primarily on the difference in dielectric constant, frequency dependent primarily on the cell size. So, the difference in the tube material that you are using to construct the periodic crystal that actually has a role in deciding the bandgap size and also the frequency dependent primarily on the cell size. you can look for high contrast alternative dielectric material that can give you wider bandgap okay so here you can see you know you are basically getting a complete bandgap it means the gap covers all the phases or all the values of k So that also tells you one interesting thing that if a light of this frequency falls from any direction that is not going to enter the crystal rather it is going to get reflected. So it is an omnidirectional reflection that is possible if the frequency lies within this particular band gap.

1D Photonic Crystals —Dispersion Relation

Photon: **In Vacuum** **In Periodic Dielectric**





- Repeating periodic structure give rise to forbidden zones
- Band gap size dependent primarily on a difference in dielectric constant, frequency dependent primarily on cell size.
- Alternating dielectrics: high contrast
- Example here id 1D (layers: 1D always has a complete band gap (gap covers all phase, k)

IIT Guwahati | NPTEL swayam Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So, one last time I will show you a pictorial representation of how waves behave inside a 1D photonic band gap crystal ok. So, PBG is photonic band gap ok. So, here you consider a wave incident on a 1D band gap material ok and you see that there is partial reflection coming from you know all the different interfaces. When the reflected waves are in phase and they reinforce one another, they basically combine with the incident wave and produce a standing wave. And if you remember from your school days, the standing wave does not propagate and it does not travel to the material, right? And this is how the frequency within the band gap does not propagate inside the material.

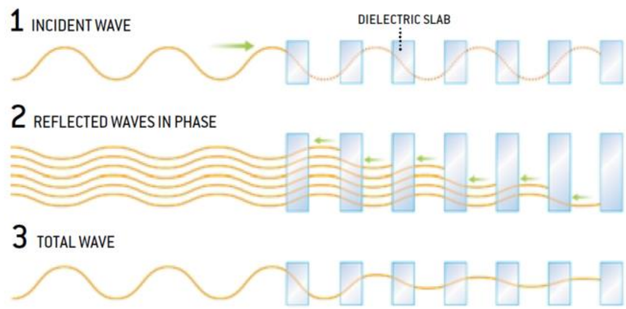
1D Photonic Crystals

- **Wavelength in a 1D PBG**

(1) A wave incident on a 1D band gap material partially reflects off each layer of the structure.

(2) The reflected waves are in phase and reinforce one another.

(3) They combine with the incident wave to produce a standing wave that does not travel through the material.



Source: Yablonovitch, Eli. "Photonic crystals: semiconductors of light." Scientific American 285, no. 6, 2001.

Whereas if the wavelength is not in the 1D photonic band gap, what happens? The reflected waves that you get from all the different interfaces are basically out of phase and they cancel out each other.

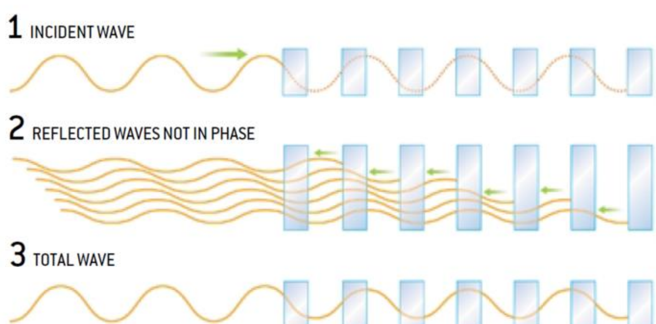
1D Photonic Crystals

- **Wavelength not in a 1D PBG**

(1) A wavelength outside the band gap enters the 1D material.

(2) The reflected waves are out of phase and cancel out one another.

(3) The light propagates through the material only slightly attenuated.



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Source: Yablonovitch, Eli. "Photonic crystals: semiconductors of light." Scientific American 285, no. 6, 2001.

So, the incident light can freely propagate through this material, only it is slightly attenuated because some part of it is reflect loss in the reflection . which essentially cancel out each other, but then the transmitted intensity will have slightly less intensity than the incident light because of the attenuation So with that we conclude this lecture. So we will start about the discussion of analysis and engineering of 1D photonic crystal van structures in the next lecture. If you have got any queries or doubt regarding this lecture you can write an email to me at this particular email address mentioning MOOC and photonic crystal on the subject line. You