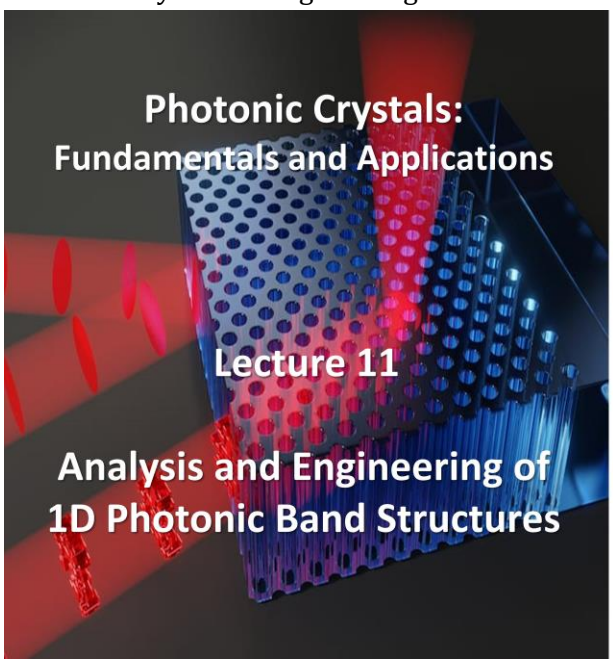
Lec 11: Analysis and Engineering of 1D Photonic Band Structure



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Hello students, welcome to lecture 11 of the online course on Photonic Crystals Fundamentals and Applications.

- The Physical Origin of Photonic Band Gaps
- The Size of the Band Gap
- Evanescent Modes in Photonic Band Gaps

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

Today's lecture will be on analysis and engineering of 1D photonic band structures. So, here is the lecture outline. So, we will be briefly looking into the physical origin of photonic band gaps. We'll discuss about the size of the bandgap, how you can actually make bandgap narrow or wide depending on your application requirement. We'll also discuss about the evanescent modes in photonic bandgaps which are not able to propagate into the photonic crystal.

The Physical Origin of Photonic Band Gaps

- Consider a one-dimensional photonic crystal with the dielectric function $\varepsilon(z)$ varies along one direction (z) only.
- The system consists of alternating layers of materials (blue and green) with different dielectric constants, with a spatial period a.
- Each layer is uniform and extends to infinity along the x and y directions, and the periodicity in the z direction also extends to infinity.
- By applying symmetry arguments, the electromagnetic modes sustainable by the crystal can be described.

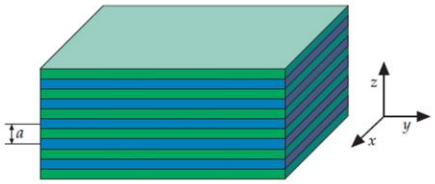

Figure: The multilayer film.

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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So let's first start with the physical origin of the photonic bandgaps. So we will begin our discussion with this multilayer thin film that we have seen in the previous lecture. So we considered this one-dimensional photonic crystal. with dielectric function of $\varepsilon(z)$ okay that varies only along the z direction right.

So along x and y it remains same. So if you remember our discussion from the previous lecture this green and blue alternating regions are basically showing you the different permittivity regions or different materials which are periodically repeated and the period here is a. So, each layer is uniform and extends to infinity along both x and y directions and the periodicity along z direction also extends to infinity. So, this is what is a 1D photonic crystal. Now, by applying the symmetry arguments, the electromagnetic modes sustainable to this crystal can be described.

The Physical Origin of Photonic Band Gaps

- The material is periodic in the z direction, and homogeneous in the xy plane.
- This allows us to classify the modes using $\mathbf{k}_{\parallel}$, k_z, and n: the wave vector in the plane, the wave vector in the z direction, and the band number.
- The wave vectors specify how the mode transforms under translation operators, and the band number increases with frequency.

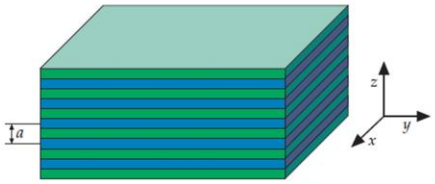

Figure: The multilayer film.

- We can write the modes in the Bloch form:

$$\mathbf{H}_{n,k_z,\mathbf{k}_{\parallel}}(r) = e^{i\mathbf{k}_{\parallel}\cdot\boldsymbol{\rho}} e^{ik_z z} \mathbf{u}_{n,k_z,\mathbf{k}_{\parallel}}(z)$$

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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So once again remember that the material is periodic along z direction and it is homogeneous in xy plane. So this basically allows us to classify modes using $\mathbf{k}_{\parallel}$, k_z and n. So these are the three parameters. So $\mathbf{k}_{\parallel}$ is basically the wave vector in the plane. k_z is the wave vector in the z direction and n represents the band number.

So, the wave vectors they specify how the mode transforms under translation operators and the band number increases with frequency. So, we can write the modes in the block form. So, you can write $\mathbf{H}_{n,kz,\mathbf{k}\parallel}(r)$ to be $e^{i\mathbf{k}\parallel\cdot\boldsymbol{\rho}} e^{ikzz} \mathbf{u}_{n,kz,\mathbf{k}\parallel}(z)$.

The Physical Origin of Photonic Band Gaps

$$\mathbf{H}_{n,k_z,\mathbf{k}_\parallel}(r) = e^{i\mathbf{k}_\parallel \cdot \boldsymbol{\rho}} e^{ik_z z} \mathbf{u}_{n,k_z,\mathbf{k}_\parallel}(z)$$

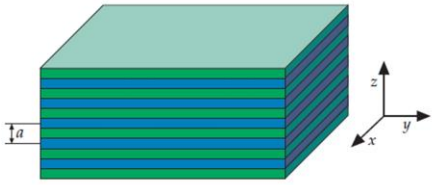

Figure: The multilayer film.

- The function $\mathbf{u}(z)$ is periodic, with the property $\mathbf{u}(z) = \mathbf{u}(z + R)$ whenever R is an integral multiple of the spatial period a.
- Because the crystal has continuous translational symmetry in the xy plane, the wave vector $\mathbf{k}_\parallel$ can assume any value.

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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

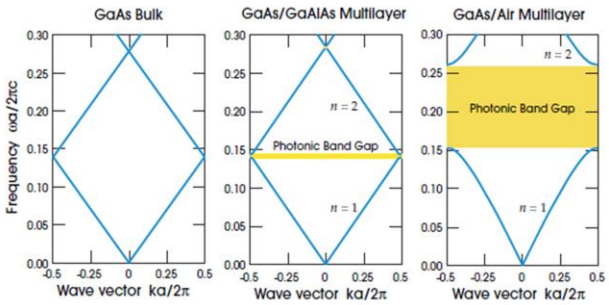
So, this is basically a term that explains if there is any wave factor along if there is any wave propagation along the plane.

okay. But usually here we consider all the wave factors along only the z direction, okay normal propagation. So, in that case this term gives you a 0 and this entire term $e^{(i\mathbf{k}\parallel.\boldsymbol{\rho})}$ will become 1. So, we will be basically dealing with this particular term. Now what is $\mathbf{u}(_{n,kz,\mathbf{k}\parallel})$? So, u_n is basically called the envelope function that is basically also periodic.

And that we have seen that when a block wave hits, when a plane wave hits a periodic medium, the amplitude gets modulated with the same periodicity of that particular periodic medium or periodic crystal. So, you can see that $\mathbf{u}(z)$ can also be written as $\mathbf{u}(z + R)$ where R is the integral multiple of the special period which is a. Now because the crystal has continuous translational symmetry in the xy plane okay, the $\mathbf{k}_\parallel$ vector this particular wave vector can assume any value okay.

The Physical Origin of Photonic Band Gaps

- However, the wave vector k_z can be restricted to a finite interval, the one-dimensional Brillouin zone, because the crystal has discrete translational symmetry in the z direction.
- Using the prescriptions of the previous chapter, if the primitive lattice vector is $a\hat{\mathbf{z}}$ then the primitive reciprocal lattice vector is $(2\pi/a)\hat{\mathbf{z}}$ and the Brillouin zone is $-\pi/a < k_z \leq \pi/a$.



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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

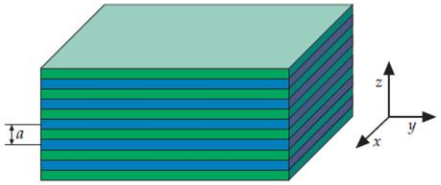
However, the wave vector k_z that is the vector along the z direction can be restricted to a finite interval okay and that interval actually tells you about the one-dimensional Brillouin zone because you know the crystal has discrete translational symmetry in the z direction right. So, it is the crystal is basically repeating after every period a.

So, you can actually restrict that you know wave vector k_z within the first Brillouin zone. So using this prescriptions of the previous chapter, if the lattice vector is $a\hat{z}$, then the primitive reciprocal lattice vector can be written as $(2\pi/a)\,\hat{z}$, right? And the Brillouin zone is defined as, you know, ranging between $-\pi/a$ to π/a. So this is a plot where you see you have done it for gallium arsenide bulk okay. So we are plotting it for k and $k_a/2\pi$. So, if you normalize it by this okay, so $k_a/2\pi$ will be ranging from minus half to half okay, that is why the scales are shown as minus half to half okay.

Similarly, the frequency is also a normalized frequency, so this is $\omega a/2\pi c$ okay. So, you can see that if you have gallium arsenide bulk okay, you do not have any band gap. But if you have you know a multilayer of gallium arsenide and gallium aluminum arsenide which are repeating okay periodically you find a very thin band gap or very narrow band gap. And if you have a multilayer formed by you know alternating layers of gallium arsenide in air you actually can have a very wide photonic band gap. So, we will come to the origin of the physical origin of the band gap to discuss how and what are those factors which contribute to the band gap and why some band gaps are narrow and some are really wide.

The Physical Origin of Photonic Band Gaps

- For now, consider waves that propagate entirely in the z direction, crossing the sheets of dielectric at normal incidence.
- In this case, $\mathbf{k}_{\parallel} = 0$ and only the wave vector component k_z is important.
- Without possibility of confusion, we can abbreviate k_z by k.



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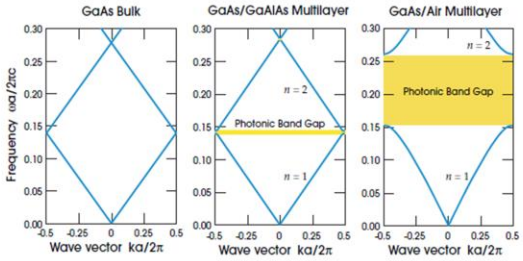
Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

Now we will consider that you know the waves propagate entirely in the z direction which I have already stated that this is an assumption when you take that you know the waves cross the sheets of this dielectric at normal incidence. So, in that case, it becomes pretty simple. You can consider $\mathbf{k}_{\parallel}$ that is the wave vector in the plane to be 0. And only the wave vector along the z direction that is k_z becomes important. So, removing all kind of possibilities of confusion, you can simply write k_z as 0.

The Physical Origin of Photonic Band Gaps

- Lets plot $\omega_n(k)$ for three different multilayer films.
- The left-hand plot is for a system in which all of the layers have the same dielectric constant; the medium is actually uniform in all three directions.
- The center plot is for a structure with alternating dielectric constants of 13 and 12, and the right-hand plot is for a structure with a much higher dielectric contrast of 13 to 1.2.



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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

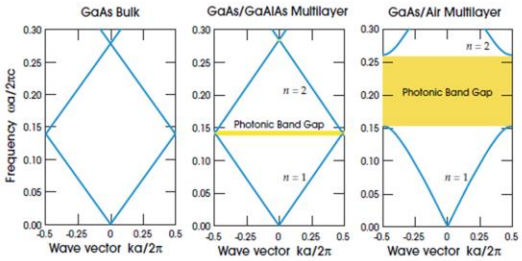
So, then we plot these three different multilayer films that I have already discussed okay. So, this is a system where you just have gallium arsenide bulk. So, you do not have a you know periodic variation of refractive index. So, it is like a you know homogeneous medium where the medium is uniform in all three dimensions and you can see that there is no band gap. So, here gallium arsenide and gallium aluminum arsenide the permittivity for gallium arsenide is 13 okay and this is 12.

So, they do not have a large you know dielectric constant contrast. However, this one has much higher contrast between the dielectric constants of the two materials involved gallium arsenide and air. This gallium arsenide has 13 air is 1. So, you can actually see that larger is the contrast between the two materials that you are using for making your 1D photonic crystal wider will be the photonic band gap. So, n equals 1 tells you the first band, n equals 2 tells you the second band and so on, fine.

The Physical Origin of Photonic Band Gaps

- The left-hand plot is for a homogeneous dielectric medium for which we have arbitrarily assigned a periodicity of a.
- But we already know that in a homogeneous medium, the speed of light is reduced by the index of refraction.
- The modes lie along the *light line* as given by:

$$\omega(k) = \frac{ck}{\sqrt{\varepsilon}}$$



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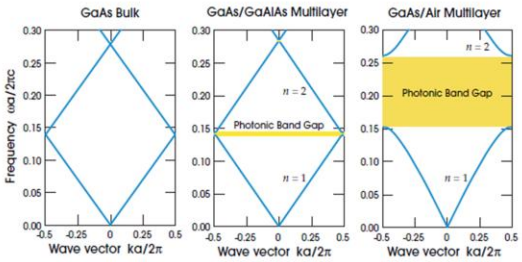
Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, you may ask that if gallium arsenide is bulk in this particular figure, okay, you have considered it as a homogeneous medium, okay, then what is the role of a here? You can actually consider any value a, but that actually does not make any difference because it is an homogeneous medium. And you already know that you know in a homogeneous medium the speed of light is basically reduced by the refractive index of that particular medium. So you can simply draw these light lines okay which are written as $\omega(k)=ck/n$ or you can write instead of n you can write $\sqrt{\varepsilon}$ right.

The Physical Origin of Photonic Band Gaps

- ***As k repeat itself outside the Brillouin zone, the light line folds back into the zone when it reaches an edge.***
- One can regard this as simply an eccentric way of relabeling of the solutions, in which $k + 2\pi/a$ is replaced by k.
- The center plot, which is for a *nearly*-homogeneous medium, looks like the homogeneous case with one important difference:
  - ***there is a gap in frequency between the upper and lower branches of the lines***



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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

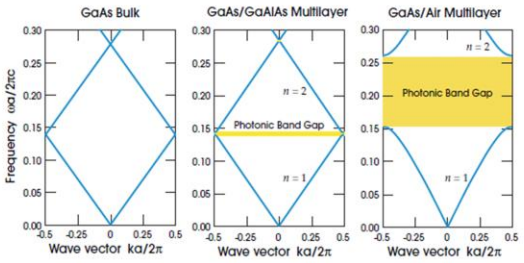
Now, wave vector k will repeat itself outside this Brillouin zone. So, the normalized wave vector is taken from minus half to half okay and after that it repeats.

So, the light line actually folds back into the region when it reaches the edge. So, that is how you actually get this particular curves. So, one can regard this as simply and eccentric way of re-leveling the solutions in which you know $k + 2\pi/a$ is simply replaced by k. So, you are not showing the solutions for the entire crystal you just showing it for one brilliant zone because you know the solutions repeat for you know $k + 2\pi/a$. Here in this case the center plot is also a nearly homogeneous medium, but there is a slight difference between the permittivity.

So, that builds in that gap of frequency between the upper and the lower branches that is n equals 1 and n equals 2 right.

The Physical Origin of Photonic Band Gaps

- There is no allowed mode in the crystal that has a frequency within this gap, regardless of *k*.
- We call such a gap a ***photonic band gap***.
- The right-hand plot shows that the gap widens considerably as the dielectric contrast is increased.

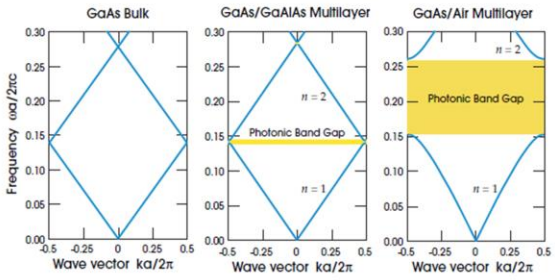


Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

And what happens in this gap? There is no frequency allowed in this particular gap. So, you know, regardless of this value of k, no frequency is supported. So, you can call this gap as a photonic bandgap. And when you go to the next figure, you can see that when the contrast increases, the bandgap significantly increases.

The Physical Origin of Photonic Band Gaps

- ***So, why does the photonic band gap appear?***
- Understand the gap's physical origin by considering the electric field mode profiles for the states immediately *above* and *below* the gap.
- The gap between bands $n = 1$ and $n = 2$ occurs at the edge of the Brillouin zone, at $k = \pi/a$.



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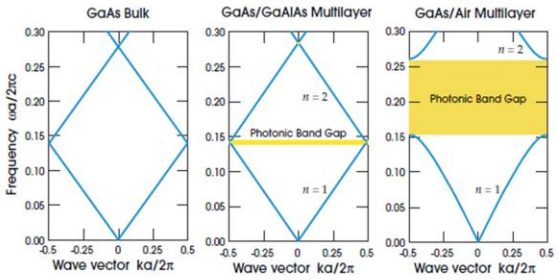
Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, the first question is why does photonic band gap appear at all? So, you can understand this by you know analyzing the electric field mode profile for the states immediately above and below the photonic band gap. So here we are considering the gap between bands n equal 1 and n equal 2 and that occurs at the edge of the Brillouin zone that is where k is you know π/a.

The Physical Origin of Photonic Band Gaps

- For now, focus on the band structure in the center panel, corresponding to the configuration that is a small perturbation of the homogeneous system.

- For $k = \pi/a$, the modes have a wavelength of $2a$, twice the crystal's spatial period (or *lattice constant*).



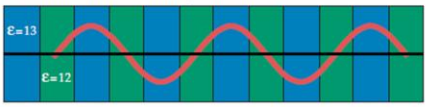
Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So if you consider this one that is a very nice system because it is a small perturbed system from the you know homogeneous system because here the contrast is very little this is 13 this is 12 right. So, here you can see that for k equals π/a that is at this particular edge at this point the modes have wavelength of 2a which is basically you know twice that of the crystal spatial period or you can say the lattice constant.

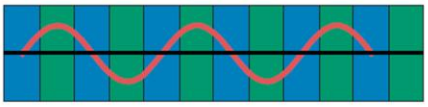
The Physical Origin of Photonic Band Gaps

- There are two ways to center a mode of this type.
- We can position the nodes in each low-ε layer, as in figure (a), or in each high-ε layer, as in figure (b).
- Any other position would violate the symmetry of the unit cell about its center.

(a) *E*-field for mode at top of band 1



(b) *E*-field for mode at bottom of band 2



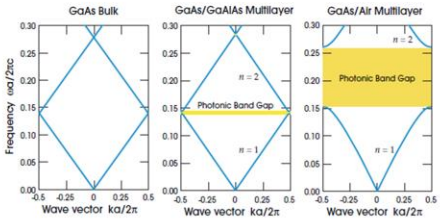


Figure: The modes associated with the lowest band gap of the band structure plotted in the center panel of figure (left), at $k = \pi/a$. (a) Electric field of band 1; (b) electric field of band 2.

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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, there are two ways to center a mode of this type.

So, you can either position ok. the electric field in low permittivity region as you can see here. So, what happens you have positioned the nodes that is where the amplitude of the field is 0 at the low permittivity region. So, here blue is the high permittivity region and green is the low permittivity region and you can see all the nodes are basically lying in the low permittivity region ok. And, the other one is also possible like where you have the nodes lying in the high permittivity region.

So, these two possibilities are there right. So, any other position if you consider that would violate the symmetry of the unit cell about its centre. So, only these two you know variations you can think of. So, if you look into the electric field energy density you can say that the low frequency mode that is the mode of band 1 they concentrate their energy mostly in the high permittivity region right. for higher frequency mode, that is for band 2, you can see that the electric field density is mostly concentrated in the low permittivity region.

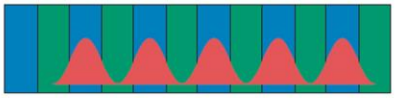
The Physical Origin of Photonic Band Gaps

- The low frequency modes concentrate their energy in the high−ε regions, and the high frequency modes have a larger fraction of their energy (although not necessarily a majority) in the low−ε regions.

- With this in mind, it is understandable why there is a frequency difference between the two cases.

- The mode just *under* the gap has more of its energy concentrated in the $\varepsilon = 13$ regions as shown in figure (a), giving it a lower frequency than the next band, most of whose energy is in the $\varepsilon = 12$ regions as shown in figure (b).

(a) Local energy density in *E*-field, top of band 1



(b) Local energy density in *E*-field, bottom of band 2

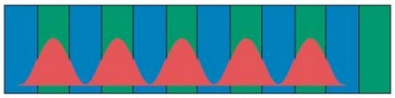


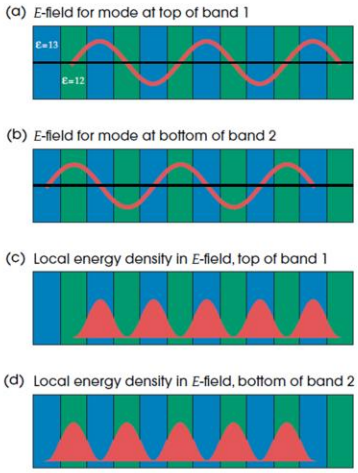
Figure: (a) electric-field energy density $\varepsilon|\mathbf{E}|^2/8\pi$ of band 1; (b) electric-field energy density of band 2. In the depiction of the multilayer film, blue indicates the region of *higher* dielectric constant ($\varepsilon = 13$).

Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, with these things in mind, it is understandable that why there is frequency difference between the two cases. So, the mode which is just under the photonic band gap will have its energy concentrated in the high permittivity region that is epsilon equals 13 region and the next band which is slightly above the band gap. okay that will have its energy concentrated you know in the low permittivity region. So, this we are talking about the top of the band 1 and this is for the bottom of the band 2. So, this is just below the photonic band gap and this is just above the photonic band gap.

The Physical Origin of Photonic Band Gaps

- The bands above and below the gap can be distinguished by where the energy of their modes is concentrated: in the high-ε regions, or in the low-ε regions.
- Often, especially in the two- and three-dimensional crystals, the low-ε regions are air regions.
- For this reason, it is convenient to refer to the band *above* a photonic band gap as the **air band**, and the band *below* a gap as the **dielectric band**.
- The situation is analogous to the electronic band structure of semiconductors, in which the ***conduction band*** and the ***valence band*** bracket the fundamental gap.



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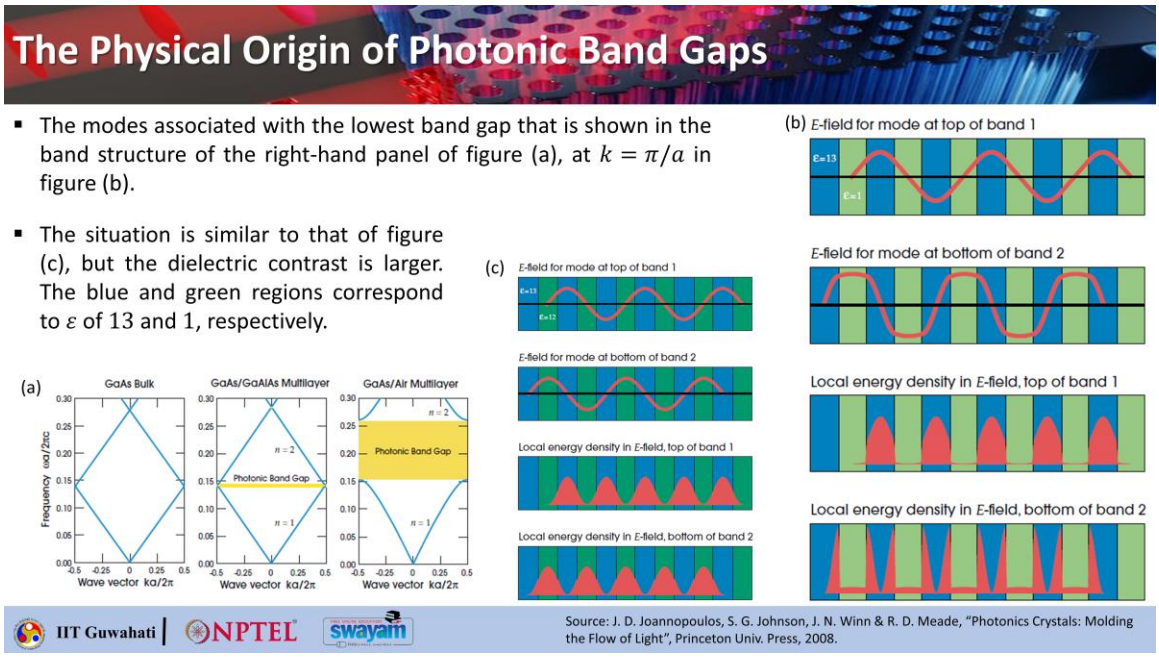
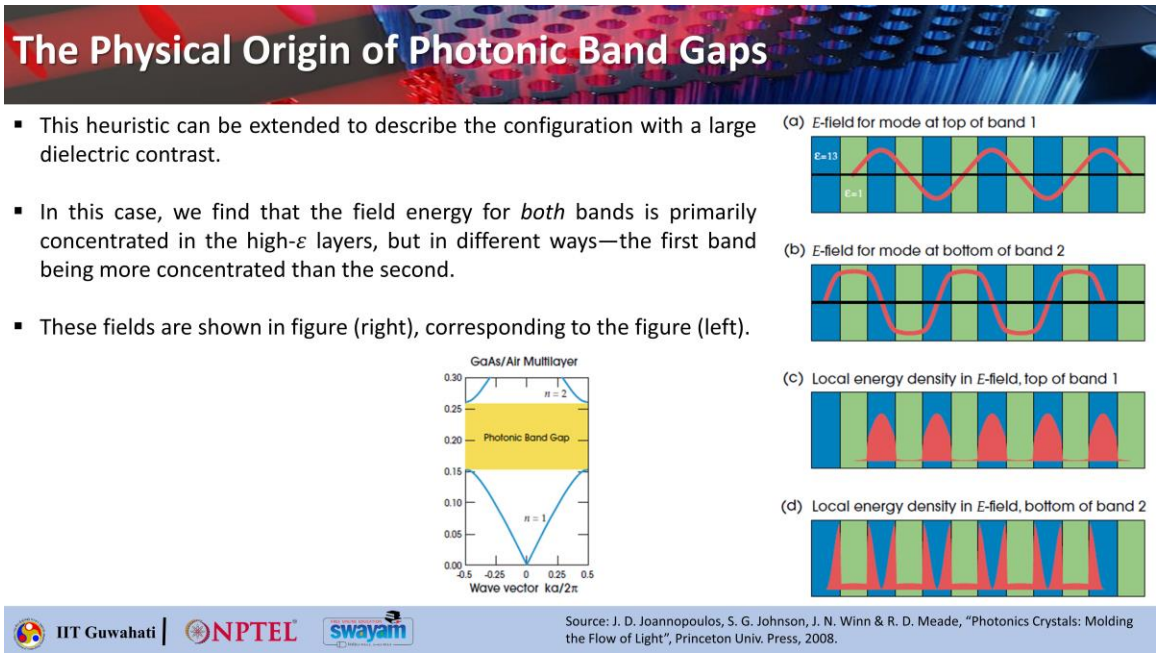
Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, the bands above and below the gap can be distinguished by whether the energy of their modes is concentrated right. So, whether it is in high permittivity region or it is in low permittivity region. So, especially in 2 and 3 dimensional crystals the low permittivity region is air and for this reason it is convenient to refer The band which is above the photonic band gap is referred to as air band and the bands which lie below the band gap is referred to as dielectric band. And this situation is very analogous or you can say this situation is analogous to electronic band structure of the semiconductors where you have conduction band and valence band above and below the band gap.

right. So, this is one more quick look all the electric field profile and their local energy density are plotted together. So, here you can see the electric field for mode at top of band 1 as we told the nodes are in low permittivity region. So, that means the energy is mostly concentrated in the high permittivity region as you can see here. And for the mode at bottom of band 2 which will have slightly different frequency than this one okay. There the node is at high permittivity region it means the majority of the energy is focused in the low permittivity region fine.



So, the modes associated with the lowest band gap Okay, is shown in the band structure as you can see here. Okay, so in this case, these are called the air bands and this is the dielectric band and this is the photonic band gap. and what happens in this case we are considering gallium arsenide and air multilayer structure. So, you again can think of you know n equals 1 where the nodes are in air region okay and n equals 2 that is you know for band 2 the energy is mostly the nodes are at the high permittivity region. So, if you look into the energy density of the electric field for the top of band 1 that is here you can see that the most energy is concentrated in the high permittivity dielectric and for the bottom of band 2 you can see there cannot be anything in the air region.



So, they are basically mostly along the side of the high permittivity material ok. So this is in contrast to what we saw in case there is a dielectric material present okay. This is typically the case for 2D and 3D photonic crystals where you know air is used as the low permittivity region to create the largest contrast of the dielectric constants. So why we need to do this as I mentioned it can give you a large dielectric contrast and as a result of that you can get a very wide photonic band gap. So this can be extended to describe the configuration with a large dielectric constant.

So you can see here that for 2D and 3D photonic crystals where the low permittivity region is typically considered to be air, there the electric field is mostly concentrated. towards the edge of the high permittivity region and this is what is the difference when you have you know high and low permittivity regions in photonic crystals with lesser contrast and when you use air as the low permittivity region as shown here. So, we understood that the gap basically arises from the difference in the field energy localization And we can still refer the upper band as the air band and the lower band as the dielectric band. So this is the first photonic band gap that you can see between n equals 1 and n equals 2, that is between band 1 and band 2. You can again have a gap between this and this.

The Physical Origin of Photonic Band Gaps

- The gap arises from this difference in field energy location.
- Consequently, we will still refer to the upper band as the air band and the lower as the dielectric band.
- Conclude this with the observation that in one dimension, a gap usually occurs between *every* set of bands, at either the Brillouin zone's edge or its center.
- This is illustrated for the band structure of a multilayer film in figure.

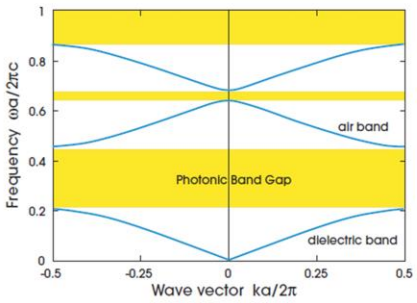


Figure: The photonic band structure of a multilayer film with lattice constant a and alternating layers of different widths. The width of the $\varepsilon = 13$ layer is $0.2a$, and the width of the $\varepsilon = 1$ layer is $0.8a$.

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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So that will be called another photonic band gap and so on. So what you see here is that this is a numerically calculated photonic band structure of a multi-layer film where the lattice constant is taken as a and alternating layers have different widths. So in this particular case, they have considered gallium arsenide layer and air as the you know unit cell. So, epsilon equals 13 that is a gallium arsenide layer has got a width of 0.2 a and the air portion okay that is epsilon equals 1 layer has got the remaining you know width that is 0.8 a and when you consider that this is what you get as the band diagram. So we can conclude this with the observation that in one dimension the gap occurs between every set of bands at either the brillouin zone's edge or at its center. So here you can actually have the band that is the gap that is created at the center of the Brillouin zone, whereas here the gap is created mainly at the edge of the Brillouin zone.

Lecture Outline

- The Physical Origin of Photonic Band Gaps
- **The Size of the Band Gap**
- Evanescent Modes in Photonic Band Gaps

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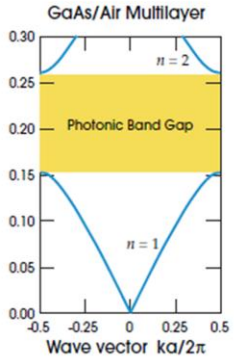
Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

The Size of the Band Gap

- The extent of a photonic band gap can be characterized by its frequency width $\Delta\omega$, but this is not a really useful measure.
- All of our results are scalable as Scaling Properties of the Maxwell Equations hold.
- If the crystal were expanded by a factor s, the corresponding band gap would have a width $\Delta\omega/s$.
- A more useful characterization, which is independent of the scale of the crystal, is the **gap–midgap ratio**.



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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

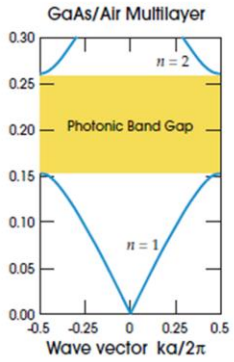
So now we'll move on to the next subtopic, which is the size of the band gap. So the extent of a photonic band gap can be characterized with its frequency width, which is $\Delta\omega$.

but it is usually not a very useful measure. So, all of our results are scalable as we have studied the scaling properties of Maxwell's equation that is valid. So, all these results are scalable to different frequencies and good thing is that all this you know band diagram that are discussed here or you can see they are all normalized ok normalized frequency versus normalized wave vector. So, if you expand a crystal by a factor of s, you can understand that the band gap of that crystal will now have a width of $\Delta\omega/s$. So, that is how the scaling will

work. So, a more useful characterization which is basically independent of the scale of the crystal is the gap-midgap ratio.

The Size of the Band Gap

- Letting ω_m be the frequency at the middle of the gap, define the gap–midgap ratio as $\frac{\Delta\omega}{\omega_m}$.
- Generally expressed as a percentage (e.g., *a "10% gap" refers to a gap–midgap ratio of* 0.1).
- If the system is scaled up or down, all of the frequencies scale accordingly, but ***the gap–midgap ratio remains the same***.
- *Thus, when we refer to the "size" of a gap, we are generally referring to the gap–midgap ratio.*



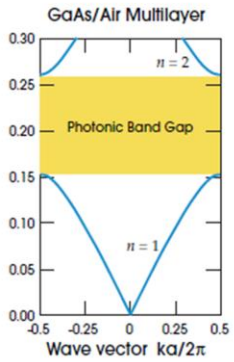
Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, what is that gap-midgap ratio? As the name suggests, if you consider ω_m as the frequency at the middle of the gap, you can define this gap mid-gap ratio as $\Delta\omega/\omega_m$. So ω_m is nothing but the middle frequency or mid-gap frequency, okay. So it is generally expressed as percentage. So you can say, you know, if you can say 10% gap, it means gap to mid-gap ratio is basically 0.1. Now, if the system is scaled down or scaled up, all of the frequencies will scale accordingly, but this gap mid-gap ratio will remain same. That is the beauty of talking in terms of this gap mid-gap ratio.

Thus when we refer to the size of the gap we are talking about not $\Delta\omega$ we are talking about $\Delta\omega$ by ω_m that is this particular gap mid gap ratio.

The Size of the Band Gap

- For the same reason, in the band diagrams, the frequency and wave vector are plotted in *dimensionless* units $\omega a/2\pi c$ and $ka/2\pi$.
- The dimensionless frequency is equivalent to a/λ, where λ is the vacuum wavelength (given by $\lambda = 2\pi c/\omega$).
- In a multilayer film with *weak* periodicity, we can derive a simple formula for the size of the band gap from the perturbation theory.



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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So for the same reason we can also see that you know the frequency and the wave vector that are used for any band diagram are basically dimensionless. So you represent frequency as $\omega a/2\pi c$ and wave vector is $ka/2\pi$ okay.

So, the dimensionless frequency is basically. So, this one what is this? This is basically you know if you consider λ to be equal to you know $2\pi c/\omega$. So, $2\pi c/\omega$. So, this one is nothing, but a/λ. So, a is what? a is the periodicity.

So, it is length scale. λ is also length scale. So, it is a dimensionless quantity. So, that way you can find out the ratio how it helps. So, if you want to have a the gap band gap at a particular wavelength say 1550 accordingly you can find out what will be a. Because you will be obtaining the ratio say a/λ comes out to be 0.2. So, a/λ is 0.2 if lambda is 1.55 micrometer or 1550 nanometer you can find out what should be the period accordingly. right. Now, in a multilayer film that we have seen with weak periodicity, we can derive a simple formula for the size of the band gap from the perturbation theory.

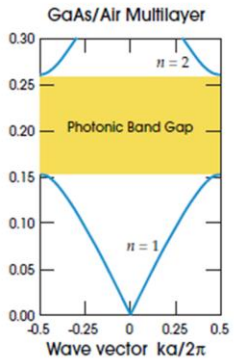
The Size of the Band Gap

- Suppose that the two materials in a multilayer film have dielectric constants ε and $\varepsilon + \Delta\varepsilon$, and thicknesses $a - d$ and d.

- If *either* the dielectric contrast is weak ($\Delta\varepsilon/\varepsilon \ll 1$) *or* the thickness d/a is small, then the gap–midgap ratio between the first two bands is approximately:

$$\frac{\Delta\omega}{\omega_m} \approx \frac{\Delta\varepsilon}{\varepsilon}.\frac{\sin(\pi d/a)}{\pi} \qquad \text{(L11.1)}$$

- *This quantifies that even an arbitrarily weak periodicity gives rise to a band gap in a one-dimensional crystal.*



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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

Now, what is this weak periodicity? So, we have seen that it is possible to relabel k plus 2π/a as k from the phenomena of quasi-phase matching, right? So, it is periodic and the wave vector repeats after you know 2π/a. So, it tells you that states at the same frequency can couple to one another if their k values are different or the k values differ by multiples of 2π/a.

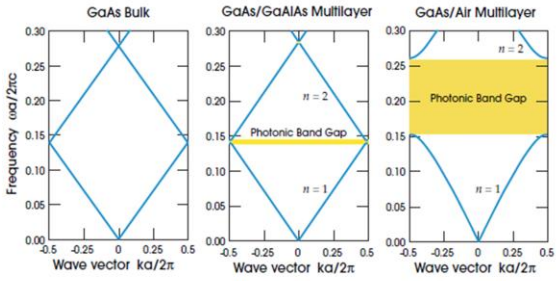
And in such case a weak periodicity A is introduced into the medium. So this is the case we have been dealing with. So in this case you can actually obtain a simple formula for the size of the band gap from the perturbation theory. So we are emphasizing general principles of periodic systems that will apply equally well to the more complicated two and three-dimensional structures which we will be discussing later on. And it is worthwhile, however, to point a few exceedingly useful analytical results that are only possible for the special case of one-dimensional problems.

Now, suppose that the two materials that we have considered in a multilayer film have dielectric constant ε and $\varepsilon + \Delta\varepsilon$. And their thickness, the thickness for the material with permittivity ε is $a - d$ and the other material which has got permittivity $\varepsilon + \Delta\varepsilon$ has got thickness of d. So, $a - d$ and d you add them together you get the period a fine. Now, either the if either the dielectric contrast is weak that means if $\Delta\varepsilon/\varepsilon$ is very small or as compared to 1 or the thickness of that you know higher permittivity layer d by epsilon is small then you can write the gap mid gap ratio between the first two bands as this.

So, this is $\Delta\omega/\omega_m \approx \Delta\varepsilon/\varepsilon.(\sin(\pi d/a))/\pi$. So, this quantifies that even at an arbitrarily weak periodicity can give rise to a band gap in one dimensional crystal.

The Size of the Band Gap

- For one of the structures shown below, with $\Delta\varepsilon/\varepsilon = 1/12$ and $d = 0.5a$ (see the center panel of figure 2), the perturbative formula (L11.1) predicts a 2.65% gap, which is in good agreement with the results of a more accurate numerical calculation (2.55%).



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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, for one of the structures that shown below, if you consider that $\Delta\varepsilon$ by ε is 1 by 12. So, which is that structure? This is that structure, right? So, because this one has got 13, this has got 12. So, we consider the lower one to be ε and this is basically 12 plus 1. So, that is your $\Delta\varepsilon$ is 1 right and if you consider that d equals 0.5a that means both the materials are of equal thickness okay. Then the formula that we saw here it predicts a band gap of 2.65 percent that means you know the gap mid gap ratio is 0.265 okay. Now 0.0265 ok, but when you do you know more accurate numerical calculation you also obtain 2.55 percent. So, we will be talking in terms of percentage here that the gap is 2.65 percent. So, you can see that the numerical calculation is pretty accurate to our analytical formula that we obtained for perturbation theory.

The Size of the Band Gap

$$\frac{\Delta\omega}{\omega_m} \approx \frac{\Delta\varepsilon}{\varepsilon}.\frac{\sin(\pi d/a)}{\pi} \qquad \text{(L11.1)}$$

- Equation would predict that the gap–midgap ratio is maximized for $d = 0.5a$
  but this is valid only for small $\Delta\varepsilon/\varepsilon$.
- For two materials with refractive indices $(\sqrt{\varepsilon})$ n_1 and n_2 and thicknesses d_1 and $d_2 = a - d_1$, respectively, the normal-incidence gap is maximized when $d_1 n_1 = d_2 n_2$, or, equivalently, $d_1 = an_2/(n_1+n_2)$.
- In this specific case, it can be shown that the midgap frequency ω_m is:

$$\omega_m = \frac{n_1 + n_2}{4n_1 n_2}.\frac{2\pi c}{a} \qquad \text{(L11.2)}$$

Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So this is that you know magical formula that gives us result which is pretty close to you know numerical solutions. So remember that the equation could predict the gap mid-gap ratio okay and it will be maximized for $d = 0.5a$ that means when the both thicknesses of the two types of material are equal. But this is valid only for small $\boldsymbol{\Delta\varepsilon/\varepsilon}$. So, for low contrast multilayer film this one works well. Now, for the two materials with refractive indices which are refractive index is basically square root of epsilon. So, if you consider n_1 and n_2 as the refractive indices and the thicknesses are d_1 and d_2.

So, d_2 can be written as $a - d_1$. So, when you have normal incidence gap that is maximized when d_1 n_1 will be equal to d_2 n_2 or you can say that n_1 sorry you can say that d_1 is a n_2 over $n_1 + n_2$. So, if you satisfy this condition you can actually maximize the gap. So, in this particular case you can also find out that the mid gap frequency ω_m is nothing but $(n_1+n_2)/(4n_1 n_2).2\pi c/a$, right.

The Size of the Band Gap

$$\omega_m = \frac{n_1 + n_2}{4n_1 n_2} \cdot \frac{2\pi c}{a} \qquad \text{(L11.2)}$$

- The corresponding vacuum wavelength $\lambda_m = 2\pi c/\omega_m$ satisfies the relations $\lambda_m/n_1 = 4d_1$ and $\lambda_m/n_2 = 4d_2$, which means that the individual layers are exactly a ***quarter-wavelength*** in thickness.
- For this reason, this type of multilayer film is called a ***quarter-wave stack***.
- The reason why the gap is maximized for a quarterwave stack is related to the property that the reflected waves from each layer are all exactly in phase at the midgap frequency.
- For the gap between the first two bands of a quarter-wave stack, the gap–midgap ratio is:

$$\frac{\Delta\omega}{\omega_m} = \frac{4}{\pi}\sin^{-1}\left(\frac{|n_1 - n_2|}{n_1 + n_2}\right) \qquad \text{(L11.3)}$$

Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

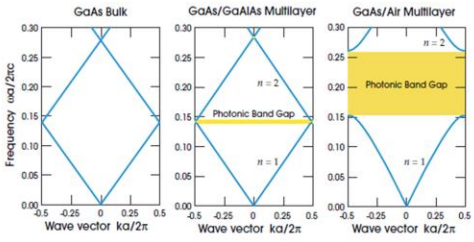
So the corresponding vacuum wavelength, so from ω_m, you can also find out what is a vacuum wavelength, which is λ_m. So you find that to be $2\pi c/\omega_m$. And that satisfy the relation that λ_m/n_1 is basically $4d_1$ or you can say that d_1 the thickness of the first layer is $\lambda_m/4n_1$. That means the thickness of the first layer is basically quarter wavelength, wavelength of light in that medium. So, λ_m/n_1 is basically the lambda in that particular medium n_1. Similarly, you can also see that d_2 is basically 4m by, sorry, d_2 equals $\lambda_m/4n_2$. It means if the individual layer thicknesses are exactly equal to their quarter wavelength, okay, in that case, you can maximize the band gap.

Okay, so for this region, you know, this type of multi-layer film is also called quarter wave stack. The reason why the gap is maximized for quarter wave stack is related to the property that the reflected waves from each layer are all exactly in phase at the mid-gap frequency. Okay, So, you can think of it your quarter wavelength. So, full path you are getting half the wavelength and that way you can think of ok. So, what happens the reflected waves from each layer ok are exactly in phase when you are at the mid gap frequency.

So for the gap between the first two bands for a quarter wave stack the mid gap ratio gives you something like this. So delta M gap mid gap ratio becomes this. So $\Delta\omega/\omega_m = 4/\pi$ sine inverse of modulus of the difference between the two refractive indices divided by the sum of the two refractive indices.

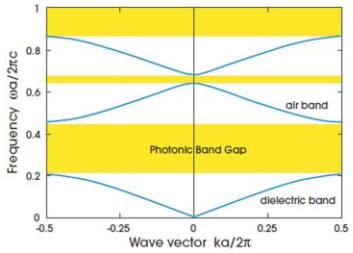
The Size of the Band Gap

- **Figure (left):** the case that is shown in the right-hand panel is a multilayer film with a dielectric contrast of 13: 1 and $d_1 = d_2 = 0.5a$, which is ***not a quarterwave stack***.
- Numerically, we find that this structure produces a 51.9% gap. If instead we had chosen $d_1 \approx 0.217$, the structure would be a quarter-wave stack with a 76.6% gap, as computed from equation (L11.3).
- **Figure (right):** shows the results for $d_1 = 0.2a$, which is nearly a quarter-wave stack, and has a computed band gap of 76.3%.



L11. 3

$$\frac{\Delta\omega}{\omega_m} = \frac{4}{\pi}\sin^{-1}\left(\frac{|n_1 - n_2|}{n_1 + n_2}\right)$$



Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, here you can see that we have considered this is a pretty wide band gap and it has got a very large dielectric contrast 13 is to 1, but here d_1 and d_2 are equal to half of a.

So, this is not a quarter wave stack right. So, numerically we find that this structure produces 51.9% gap. So, that is the gap, fine. So, we usually accept this as a pretty wide band gap.

But is that the maximum? The answer is no. Instead, if you would have chosen d_1 equals 0.217, okay, and d_2 would be, you know, a minus that, that will make the structure a quarter wave stack and you would have got a band gap which is 76.6 percent okay and as you can compute from you know this particular equation.

if you simply round it off and consider d_1 equals 0.2a it is nearly a quarter wave stack it is not exactly a quarter wave stack but you can say it is like nearly quarter wave stack and then if you do a computer simulation okay you can find out that the band gap comes out to be pretty close to what we predicted 76.3 percent okay so don't look at this scale and this scale here this scale is from 0 to 1 whether whereas this one is from 0 to 0.3 so this is a much larger band gap than this one okay So here you can see this band gap is more typically from 0.2 to 0.4 yeah 0.45 or something like that. Here you can see it is from 0.15 to 0.26 or something like that. So this typically gives you much wider band gap. So what we understood is that larger contrast is good but then as you move as you make quarter wave stack you can really get wide band gap.

Lecture Outline

- The Physical Origin of Photonic Band Gaps
- The Size of the Band Gap
- **Evanescent Modes in Photonic Band Gaps**

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

Evanescent Modes in Photonic Band Gaps

- The key observation of the previous discussions were that the periodicity of the crystal induced a gap in its band structure.
- *No electromagnetic modes are allowed to have frequencies in the gap.*
- **But if this is indeed the case, what happens when we send a light wave (with a frequency in the photonic band gap) onto the face of the crystal from outside?**
- No purely real wave vector exists for any mode at that frequency. Instead, the wave vector is complex. The wave amplitude decays exponentially into the crystal.
- When we say that there are no states in the photonic band gap, we mean that there are no *extended* states like the mode given by equation:

$$\mathbf{H}_{n,k_z,\mathbf{k}_\parallel}(r) = e^{i\mathbf{k}_\parallel\cdot\boldsymbol{\rho}} e^{ik_z z} \mathbf{u}_{n,k_z,\mathbf{k}_\parallel}(z)$$

- Instead, the modes are **evanescent**, decaying exponentially.

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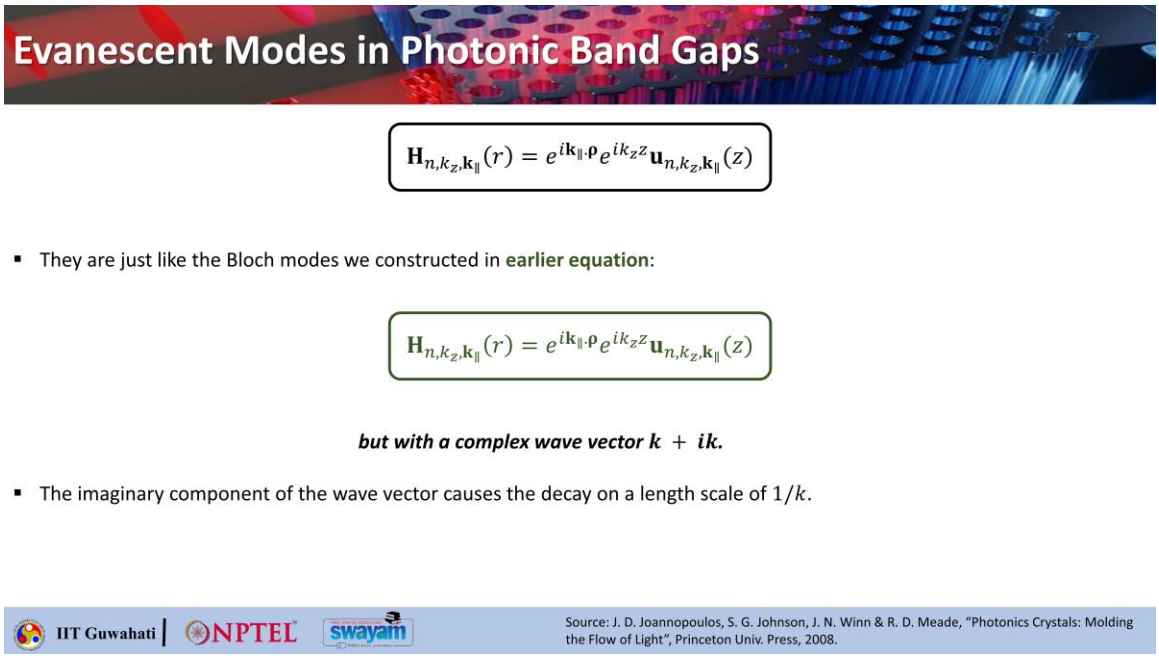
Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

Now we move on to the understanding of evanescent modes in photonic bandgaps. So, the key observation of the previous discussion were that the periodicity of the crystal that induced a gap in the band structure.

No electromagnetic modes are allowed to have frequencies that lie within the photonic bandgap. But if this is indeed the case, what happens when we send light which are having frequency within the photonic band gap, right? Something should happen to the light, right?

So no purely real wave vector exists for any mode at that frequency. Instead the wave vector is complex and hence the amplitude decays exponentially into the crystal okay. So the wave is not able to propagate inside the crystal. So when you say that there are no states in the photonic bandgap we mean that there are no extended states like the mode which were given by this kind of equation earlier okay.

So here also we will consider the case that we are talking about only you know z direction waves. So you can simply take $\mathbf{k}_\parallel$ to be 0. and you can also replace that k_z by k. So, when we are writing it about evanescent modes you can simply write $H_{n\,kz\,\mathbf{k}\parallel}$ to be equal to e to the power ikz okay, but then the amplitude decays.



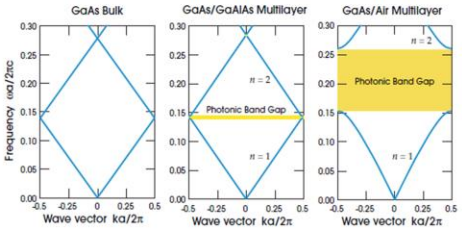
So, e to the power minus kappa z. So, this is where you know the decay factor exponential decay factor comes from. So what we are doing here, we have taken this earlier equation of the block mode.

Their $\mathbf{k}_\parallel$ is 0. So this term gives you 1. That is good. k_z can be replaced as k. That is this one. So we are having e to the power ikz. That is fine. And then this k is also having because it is complex wave factor, you will have two components k plus then the attenuation factor i κ.

So, that is what is giving you e to the power -κz. So, the wave actually exponentially decay inside the crystal. the imaginary component of the wave vector this one causes the decay on a length scale of $1/\kappa$.

Evanescent Modes in Photonic Band Gaps

- Let's understand how these evanescent modes originate, and what determines k.
- This can be accomplished by examining the bands in the immediate vicinity of the gap.
- **Return to the right-hand plot of the figure:**



- Suppose we try to approximate the second band near the gap by expanding $\omega_2(k)$ in powers of k about the zone edge $k = \pi/a$.

Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

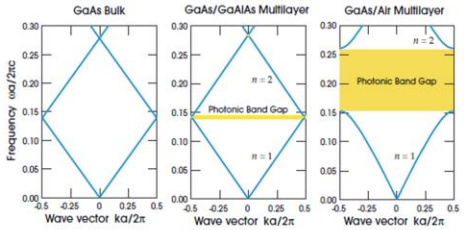
So, let us understand how this evanescent modes originate and what determines this κ. So, this can be accomplished by examining the bands in the immediate vicinity of the gap. So, if you focus on the right hand plot okay like this one okay you can try to approximate the second band near the gap okay by expanding $\omega(k)$ in powers of k about the zone H that is k equals π/a.

Evanescent Modes in Photonic Band Gaps

- Because of time-reversal symmetry, the expansion cannot contain odd powers of k, so to lowest order:

$$\Delta\omega = \omega_2(k) - \omega_2\left(\frac{\pi}{a}\right) \approx \alpha\left(k - \frac{\pi}{a}\right)^2 = \alpha(\Delta k)^2 \qquad \text{(L11.4)}$$

where α is a constant depending on the curvature of the band (i.e., the second derivative).



Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, let us do that. So, what you will see because of time reversal symmetry, the expansion cannot contain any odd powers of k. So, to the lowest order you can write $\Delta\omega$ okay is nothing but $\omega_2(k) - \omega_2(\pi/a)$ that is at this particular point okay and you can write this as $\alpha(k-\pi/a)^2$ and you get $\alpha(\Delta k)^2$. Okay, so what is α? α basically is a constant depending on the curvature of the band. So it's basically tells you about the second derivative of the band.

Evanescent Modes in Photonic Band Gaps

- Now lets see where the complex wave vector originates.
- For frequencies slightly higher than the top of the gap, $\Delta\omega > 0$.
- In this case, Δk is purely real, and we are within band 2.
- However, for $\Delta\omega < 0$, when we are within the gap, Δk is purely imaginary.
- The states decay exponentially since $\Delta k = ik$.
- As we traverse the gap, the decay constant k grows as the frequency reaches the gap's center, then disappears again at the lower gap edge.

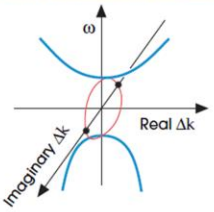
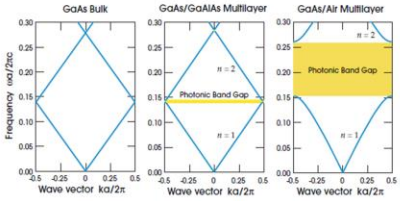


Figure : Schematic illustration of the complex band structure of the multilayer film.



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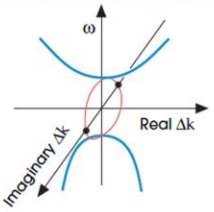
Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

And let us now see why this complex wave vector originates. So, this is a schematic illustration of the complex band structure of the multilayer film. The upper and lower blue lines correspond to you know bottom of band 2 and top of band 1 respectively. And the evanescent states occur on the red line that you see here, okay, which extends along the imaginary k-axis. So you can consider this as your real k-axis and a and a orthogonal line that is coming out of this plane which is shown like this that marks the imaginary Δk and the maximum decay occurs roughly at the center of the gap.

So, for frequencies which are slightly higher than the top of the gap that is $\Delta\omega$ is positive for them. Okay you can consider that you know Δk is purely real and you are basically within the band 2. However, when $\Delta\omega$ is negative okay that means you are basically within the gap okay and that makes your Δk to be purely imaginary. And this states decay exponentially since you can consider $\Delta k = i\kappa$. So, as we traverse the gap. the decay constant κ grows as the frequency reaches the gap centers and then disappears again at the lower gap edge right.

Evanescent Modes in Photonic Band Gaps

- By the same token, larger gaps usually result in a larger k at midgap.
- Thus, less penetration of light into the crystal.
- For a multilayer film, minimal penetration is therefore achieved in the quarter-wave stack as described earlier.
- Although evanescent modes are genuine solutions of the eigenvalue problem, they diverge as z goes to $\pm\infty$ depending on the sign of k).



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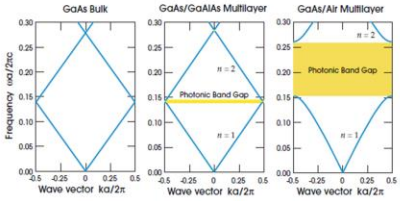

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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, by the same token larger gaps usually result in a larger κ at the mid gap okay. So, there will be less penetration into the crystal that has got a larger band gap. So, for a multilayer film minimum penetration is therefore achieved by the quarter wave stack as described earlier because quarter wave stack gives you the largest band gap. So, they will allow minimum penetration of light into that crystal as we understood. All the evanescent modes are you know. genuine solutions of the eigen problem they diverge as z goes to plus minus infinity depending on the sign of κ.

Evanescent Modes in Photonic Band Gaps

- Consequently, there is no physical way to excite them within an idealized crystal of infinite extent.
- However, a defect or an edge in an otherwise perfect crystal can terminate this exponential growth and thereby sustain an evanescent mode.
- If one or more evanescent modes is compatible with the structure and symmetry of a given crystal defect, we can then excite a *localized* mode within the photonic band gap.
- And, as a general rule of thumb, we can localize states near the middle of the gap much more tightly than states near the gap's edge.

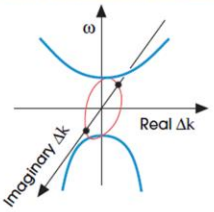
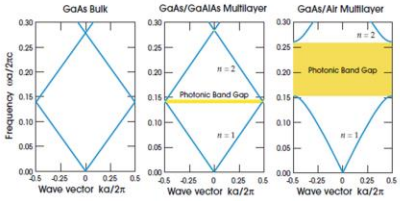


Figure : Schematic illustration of the complex band structure of the multilayer film.



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So, consequently there are no physical ways to excite them within an idealized crystal of infinite extent. So, even as in modes will not be able to you know exist. a defect or an edge in otherwise perfect crystal can terminate this exponential growth and therefore sustain an evanescent mode. So, that is where you can actually introduce a defect in your crystal and trap some light which still have frequency within that photonic band gap. If one or more evanescent modes is compatible with the structure and symmetry, of a given crystal defect, we can then excite a localized mode within the photonic band gap. So that is how you can do band gap engineering by introducing defects and you can make cavities, you can make a lot of you know other like waveguides and other applications from photonic crystal.

So a general rule of thumb is that we can localize states near the middle of the gap much more tightly than you know any states which are near the gaps edge okay.

So, anything that you do around the center of the gap you will be able to handle more localization of those states.

So, with that we will stop here and we would like to discuss the applications of 1D photonic crystal in the next lecture. If you have got any queries regarding this lecture, you can drop an email to my email address shown on the slide okay and mention MOOC and photonic crystal on the subject line. Thank you.